Social Interactions and Long-Term Fertility Dynamics. 
A Simulation Experiment in the Context of the French Fertility Decline

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ABSTRACT

We build an agent-based simulation model that incorporates both historical data on population characteristics and spatial information on the geography of France to experimentally study the role of social interactions in fertility decisions. We assess how different behavioural and interdependence assumptions cause variations in macro dynamics and diffusion patterns. The analyses show that incorporating social interactions into the model contribute to mimic empirically observed behaviour. Our findings suggest individual-level mechanisms through which the observed demographic transition was materialised.

Keywords fertility decline, demographic transition, diffusion, France, simulation experiments, agent-based models, decision-making, social norms, social interactions.

JEL classification N33, J13, C15.

Introduction

Fertility transitions generate both great interest and considerable controversy among social scientists. The systematic fall in birth rates represented a major demographic break in many regions around the world, and was arguably a crucial intermediate step for those regions to enter modern economic growth (Galor 2005). Economic, social, and cultural factors have all been brought forward as potential

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drivers of the fall in birth rates (e.g. Cleland and Wilson 1987; Kirk 1996; van de Kaa 1996; Friedlander et al. 1999; or Guinnane –forthcoming–), but no single explanation dominates, generating a persistent disagreement on the actual forces behind the transition (Mason 1997). This paper suggests a way to integrate these various components into a single exploratory framework. We present an agent-based simulation that combines a behavioural model of fertility choice with the fragmentary historical information we have on nineteenth's century France. We use this model to shed light into the relative success of different decision rules to reproduce the observed aggregate patterns of this demographic transition.

Agent-based simulation has been identified as a promising yet underexplored methodology in demography (e.g. Burch 1996; Billari and Prskawetz 2003; Hobcraft 2006). Here we use it to build a setting for experimentation that allows us to play with counterfactuals and measure their effects on population dynamics. This approach provides a novel way to study demographic dynamics and interpret sparse empirical data in terms of a formal theoretical structure. Since historical data provide incomplete evidence on how the fertility decline took place, we use the simulation model to reproduce what we know about the demographic landscape of the time and explore how sensitive aggregated patterns are to different behavioural assumptions. This strategy allows us to incorporate two components normally neglected in the more quantitative literature on historical fertility decline: the role of social interactions in shaping fertility decisions (e.g. Kohler 2000a, 2000b), and the connection between major social transformations and individual behaviour.

We chose to focus on the French decline because it offers a paradigmatic case study to analyse the effects of social interaction in demographic behaviour. First, the decline followed a very characteristic pattern of spatial diffusion that seems to hide a process of social influence. And second, the decline coincided with the advent of the Revolution, which shook old normative foundations about fertility control and might have triggered a learning process. Our paper then connects with a recent line of research that sees the French Revolution as a natural experiment (Acemoglu et al. 2009a, 2009b). A growing body of research points towards a regular connection between social upheavals and fertility decline (Lesthaeghe and Wilson 1986; Binion 2001; Caldwell 2004; Bailey 2009), of which the French transition is a particular case. In this context, we assess the specific hypothesis that the dismantlement of the Catholic Church that followed the summer of 1789 contributed to bring down fertility rates (Sutherland 2003, p. 345). Our simulation models these mechanisms and evaluates whether their effects in the dynamics of the system are consistent with empirical data. This exercise is exploratory rather than explanatory in nature. None of the hypotheses we assess can be directly tested, as historical records lack the empirical resolution required to identify decision-making mechanisms at individual level. Our experiment, however, allows us to explore the link between individual motivations and aggregated patterns of demographic behaviour in a systematic way, taking the use of counterfactuals to a more sophisticated level of analysis.
Our study then responds to several of the points raised by John Hobcraft in his plea to revise the research on demographic behaviour (Hobcraft 2006, pp. 155-173). Firstly, we put together concepts from several disciplines such as the standard family decision making process typical of economics, and the role of social interaction analysed by sociologists. Secondly, we focus on how these factors feed on dynamic processes, with a particular interest in the role of heuristics and social interaction. And thirdly, we acknowledge the importance of context and model it explicitly; in doing so, we attempt to connect with the literature that focuses on how, rather than why, the fertility decline took place (e.g. Bocquet-Appel and Jakobi 1998).

Our simulated environment resembles nineteenth century France by combining historical demographic data (e.g. mortality rates, or proportion of women married) with spatial information (e.g. geography and location of major cities). Thousands of agents are born, live, and die in this artificial society following different behavioural rules. Those rules—which are at the core of this study—consider the effects of different factors on fertility decisions, with a particular focus on child mortality and social influence. We evaluate how changes in the parametric specification of these rules cause variations in long-term demographic trends.

The experiments show that the assumptions we make about social interaction are not neutral. We show, for instance, that the standard assumption that social interaction plays no role in fertility dynamics is only a special case among the possible scenarios capable of replicating trends at the macro level. Since there is limited evidence to discriminate between alternative decision-making mechanisms, the simulation experiments are not capable of providing causal explanations; but, as an empirically calibrated thought experiment, they help us identify theoretical dimensions of this fertility transition that require more attention. Our findings, for example, provide support to the hypotheses that during the pre-transitional period couples were most likely not maximising family size (i.e. they could have had more children than they did) and they were aiming at surviving children (and not fertility itself), which highlights the influence that child mortality had on fertility decisions. Finally, we also show that a simple interpretation of how the Revolution affected religious practice and beliefs (and these, in turn, behaviour) we can replicate some stylised facts about the transition; this supports the argument that the dismantlement of the Catholic Church during the revolutionary period played a role in triggering the fertility decline in France.

**Fertility choices and social interaction**

The role of social interaction on reproductive behaviour has lately gained considerable attention in demography (e.g. Casterline 2001; Kohler 2001). Hypotheses linking social interaction with fertility, however, can be traced back at least to the late nineteenth century, when some authors attributed the fall in French birth rates to changes in the nature of social dynamics (Dumont 1890, p. 130) or in the moral order of the society (Leroy-Beaulieu 1896, p. 614). These ideas gained special support in the 1970s with the publication of the first results of the European Fertility Pro-
ject (Coale and Watkins 1986), which run counter to the predictions of the then dominant demographic transition theory. Child mortality, urbanisation and industrialisation helped to explain some local differences in the decline, but not substantially; countries that were different in terms of development had almost simultaneous transitions; and fertility patterns were strongly correlated with the distribution of various cultural traits (e.g. language). This evidence suggested that the diffusion of reproductive behaviour was driven by social interactions (Knodel and van de Walle 1979, p. 239), and that it was the spread of new ideas – and not the change in material conditions – that accounted for the decline (Cleland and Wilson 1987, p. 27).

The importance of social interactions to explain fertility has been, however, very controversial. Demographers and economists have been reluctant to consider peer effects, diffusion, or other forms of social interactions not mediated by the market. Economists in particular look at diffusion stories of the fertility decline with scepticism because they appear to be at odds with the idea of rational agents. The interpretation is that, given the availability of contraceptive technology, high fertility in the pre-transition period would reflect a high demand of children, not the unwillingness to control fertility on moral grounds (e.g. Brown and Guinnane 2002, p. 40). Of course, social constructs like moral norms are not necessarily outside the calculations of a rational agent (see e.g. Iannaccone 1992, 1998) as fertility choice models developed in recent years clearly exemplify (Durlauf and Walker 2001; Kohler 2001).

Empirical implementation of these models, however, still faces a series of challenges. Models with social effects, in particular, need a tight integration of theory and application (Durlauf and Walker 2001, pp. 131) that is not always easy to attain. In this paper we suggest that using agent-based modelling to look at counterfactual scenarios provides a favourable environment to achieve that integration: while anchored in empirical data, they can be based on formal theory, incorporating elements of both micro-economic foundations and of social influence, transmitted vertically (across generations) but also horizontally (across neighbours). Before introducing the details of the simulation model, the next section discusses the theoretical context that motivates the study.

An encompassing theoretical context

Following the increasing interest on how different aspects of social interaction impact on fertility (Rosero-Bixby and Casterline 1993; Montgomery and Casterline 1993, 1996; Mason 1997; Montgomery et al. 2001), recent research has proposed micro-founded models where social interaction affects rational, utility maximising couples facing the possibility of adopting a strategy of low fertility (e.g. Durlauf and Walker 2001; Kohler 2001). According to these models, fertility choices are seen as coordination problems: the benefits of choosing low or high fertility are dependent

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1 And contraceptive technology was apparently available, as most family planning techniques used during the nineteenth century (basically coitus interruptus and abortion) were already extensively known (McLaren 1978, 1990; Van de Walle and Muhsam 1995).
on the fertility choices of others. Agents face a value function that has the following general shape:

\[
V(n(f_i), Z_i, F_i^*, \varepsilon_i(f_i)) = u(n(f_i), Z_i; \alpha) - \frac{J}{2}(f_i - F_i^*)^2 + \varepsilon_i(f_i)
\]

Each agent \(i\) is characterised by a vector \(Z_i\) of personal attributes (including tastes, values, environmental factors, etc.) and chooses a fertility strategy \(f_i\) (typically \(f_c\) –contraception– or \(f_{nc}\) –no contraception–) that takes into consideration her expectations \(F_i^*\) of what the rest of the population are doing in terms of fertility. The terms \(\alpha\) and \(J\) are parameters that define the general shape of the utility function and the importance given to other people’s fertility behaviour. The right hand side of this equation is divided in three parts: the personal utility that agents obtain from choosing a particular strategy that produces \(n(f_i)\) children, the cost faced if they deviate from the average behaviour of the other agents, and an external personal shock \(\varepsilon_i(f_i)\), also dependent of the fertility strategy chosen.

Much of what it is now the standard economist’s approach to fertility (van de Kaa 1996, pp. 409-414) is conveyed by the first term: changing environmental conditions –such as increases in urbanisation rates or female wages– directly affect the personal utility leading couples to adjust their fertility strategy. Yet, however natural this idea seems when applied to modern societies, the literature is somewhat divided on whether the assumption that couples cared for completed family size is valid for pre-transitional societies (e.g. Carlsson 1966, Cleland and Wilson 1987); this is a hypothesis to which we are going to pay attention in our simulation exercise.

Related to this, an equally contested issue in the literature is the role that death rates played in decision making. There are a number of reasons why infant or child mortality could affect fertility (van de Kaa 1996, pp. 405-409), the most obvious being that families –if they care for a completed size at all– in the end do not care about fertility \textit{per se}, but about having \(n(f_i)\) surviving children. Empirical evidence on the connection between mortality and fertility is mixed: the findings of the Princeton project provided little support for this idea (e.g. van de Walle 1986), yet recent research suggests that before the transition parents were in fact taking child mortality into account (e.g. Reher 1999, Reher and Sanz-Gimeno 2007). This is another aspect that our simulations will allow us to explore.

Although not normally used for this purpose, the first component of equation (1) can be used to illustrate the influence of one type of non-market effect in fertility choices: religion. The role of religious influence on fertility choice is a recurrent theme in the literature (e.g. Derosas and van Poppel 2006), yet it is rarely treated in a formal way. Religion affects the utility of individuals directly, through a sub-component of \(Z_i\), assigning a positive or negative impact to a particular fertility
strategy choice. A similar interpretation has been applied by Botticini and Eckstein in their research on Jews and education (Botticini and Eckstein 2007, pp. 893-894; also Iannacone 1992): if belonging to a religious group imposes norms of behaviour (here in terms of fertility) that contrast with the strategy individuals want to pursue, they will face a cost. If we denote as $x(f_i)$ the reward an individual receives from her religious institution for choosing a particular strategy, we can modify the value function above as:

\[
V(f_i, Z_i, x(f_i), F^e_i, \varepsilon_i(f_i)) = u(n(f_i), Z_i) + x(f_i) - \frac{J}{2}(f_i - F^e_i)^2 + \varepsilon_i(f_i)
\]

For a religious person $\partial u / \partial x > 0$ and for a non-religious person $\partial u / \partial x = 0$. If the religious institution condemns contraception in any way, we should have that $x(f_i) \leq 0 < x(f_{nc})$. When religious ideals are enforced, controlling fertility results in a disutility.

Then, we have the term $(f_i - F^e_i)^2$, which deals with other aspects of social interaction. There are various reasons that motivate its introduction, the most straightforward being simple social pressure. Another is the uncertainty associated with infrequent events. The decision to reproduce is an important, yet relatively infrequent choice in a lifetime; this makes people rely on the experience and judgment of others to make their own assessment, which introduces a particular form of social interaction effects by means of learning. In addition, from the point of view of the agent a deviation from the norm can have negative consequences. Kohler (2000a) has shown that the presence of this sort of component in the value function of potential parents can lead to very particular birth rate dynamics, which explain a series of empirical puzzles associated with the presence of multiple equilibria, high fertility ‘traps’ or the timing of some transitions (e.g. Kohler 2000b, 2001).

Our modified version of the standard social influence model including the effect of religion summarises good part of the discussion on the impact of non-economic factors on fertility choices. If we take equation (2) to be a reasonable approximation to the way an agent chooses her family size, it is easy to see why economic modernisation does not necessarily result (at least immediately) in a fall of birth rates. Any improvement in utility stemming from the reaction to different economic conditions –conveyed by $Z_i$– must first offset both the religious and other social costs. Once this threshold is surpassed, an endogenous mechanism is triggered: the expectations on the behaviour of other agents $(F^e_i)$ begin to change and this leads to self-reinforcing dynamics towards a new generalised fertility strategy. At the same time, non-economic modernisation taking the form of a relaxation in

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2 This expression is of course a special case of the more general formulation where $x(f)$ is simply another argument the personal utility. Making the assumption of separability and writing it in this way makes the expression clearer without changing its main implications.
religious norms (a decrease in $x(f_i)$) or a weakening of social ties (a fall in $J$) can make the value function more sensitive to (even small) changes in the fundamentals.

As we pointed above, the complex nature of this type of models generates a series of challenges for empirical analysis, especially regarding their econometric implementation (Durlauf and Walker 2001, pp. 131-133). Some studies have successfully addressed these problems (e.g. Kohler 2001), but in this paper we pursue an alternative strategy: we make the agents in our simulation model follow different versions of a behavioural rule inspired by equation (2), and we assess how changes in these rules affect the aggregated demographic outcomes. In order to facilitate empirical calibration, the model reproduces the geography and demographic history of France in the eighteenth and nineteenth centuries, and attempts to replicate the observed patterns of fertility decline. The following section gives more details of why the French case offers an interesting example of these dynamics, and of how we relate that demographic transition to the simulation experiment.

Modelling French demographic history

As Figure 1 shows, the decline in fertility rates arrived earlier in France than in other European country, and the fall does not seem to be triggered by any major economic change. The timing does coincide with the onset of the Revolution, but the actual mechanisms by which these two events are connected are far from clear (Weir 1983, Wrigley 1985a, 1985b). Within France, fertility rates also followed interesting patterns. Systematic information covering different geographical areas for the whole country is available only at the département level since the early nineteenth century (van de Walle 1974; Coale and Watkins 1986; Bonneuil 1997). Figure 2 plots, for some selected dates, the Princeton $I_g$ index of marital fertility, which relates the number of births to the maximum biologically attainable given the age structure of married women.

[Figure 1 and 2 about here]

All throughout the period there are two distinct zones of low fertility: the valley of the Seine (the Bassin Parisien) and the region of Aquitaine (the Bassin Aquitaine, in the south-west); over time, these two areas spread to the detriment of two ‘islands’ of high fertility: the region of Bretagne in the north-west and the Massif Central in the centre-south-east. As early as 1831, for example, one can find départements with indices below 0.40 (evidencing clear fertility limitation), such as Gironde, Lot-et-Garonne or Eure, whereas as late as 1911 places like Finistère or Côtes-du-Nord were resisting change and still had indices above 0.70 (showing little or no limitation at all). The maps suggest a slow process of diffusion from the Parisian and Aquitaine basins towards these islands of high fertility, making France stand again in contrast with other European regions where such a process was either too fast, or not obvious at all.
Here the comparison with England, the new industrial economy across the channel, seems inevitable (although it should be taken cautiously, as the size of the region is only half of the French one in terms of population). Regional comparable data is available only after 1851 but then again, as seen in Figure 1, England arrived quite late to the fertility transition. Figure 3 shows a clear contrast with the French case. Throughout the five decades displayed, it is quite difficult to say whether a particular region behaved as a leader or follower in the decline. Changes in fertility seem to be fairly homogeneous across the country and it is difficult to point to heterogeneity among counties at any given time. If there was a process of diffusion taking place in England, it happened at much faster pace or in a non-spatial dimension —e.g. educational level or socio-economic status— (see, e.g. Szreter 1996, Bocquet-Appel and Jakobi 1998, Garrett et al. 2001).

[Figure 3 about here]

Both the presence of clustering and the spatial evolution of rates depicted by Figure 2 points towards diffusion as an appealing way of describing what happened in France (Bocquet-Appel and Jakobi 1998, p. 190), but it is certainly not the only plausible way to understand the evidence. One problem is that data limitations do not allow assessing the process when it actually started. By 1831 there is some degree of heterogeneity within France, but we can only speculate on whether that heterogeneity was (at least partly) already present there in the eighteenth century or not. Henry and Houdaille found in their analysis of the INED sample that there were some regional differences then, though age of marriage still largely appeared to explain fertility levels (Henry 1972, 1978; Henry and Houdaille 1973; Houdaille 1976).

One potential explanation for the spatial evolution of fertility rates is that it results from a process of (downward) homogenisation motivated by a change affecting the whole country, as the one suggested by Le Play (1874) in relation to the introduction of the Napoleonic Code. If this hypothesis of homogenisation towards lower fertility levels were supported by the data, we should see a declining mean fertility and a declining variance among départements. Under the hypothesis of diffusion, however, the data should show an increase in population heterogeneity and, after a peak, a decrease. Figure 4 plots a time series of the mean and the coefficient of variation across départements for the period covered by our data.

[Figure 4 about here]

The mean level of fertility falls as expected, until it stabilises around 0.32, a value that is maintained at least until the mid-twentieth century. The coefficient of variation shows a clear upward trend throughout the nineteenth century, falling sharply around the turn of the century, and reaching values of 0.13 for 1961. Heterogeneity across départements in marital fertility was not the greatest in the early nineteenth century, but towards the end of the century. It is certainly possible that differences in fertility levels existed beforehand and that these differences were
rooted in socio-economic differences across the regions; but Figure 4 suggests that even in that case, something else correlated with fertility was also diffusing throughout the nineteenth century.

The Simulation Experiment

The aim of the simulation is to compare the effects of different behavioural assumptions on the aggregated patterns of fertility rates over space and time. The simulation treats the evolution of family size as the dependent variable and the demographic and geographical constraints, calibrated empirically, as controls; the two main experimental factors are the rules that determine how agents interact with each other, captured in equation (2) by the term \((f_i - F^*)^2\); and the exogenous impact of the Revolution, which prompted a change in the normative setting that we interpret in terms of the component \(x(f_i)\). The model allows us to evaluate the effects of different counterfactuals in the pre-transitional period and in the context of the French Revolution by switching on or off alternative assumptions of how individuals made their fertility choices.

A detailed description of the model appears in the appendix, but the main features can be summarised as follows. The simulation covers the historical period between 1740 (when we first have information covering the whole of France) and 1900 (when the transition was well underway), and it is connected to empirical data in at least two levels: in the initial demographic set-up, by defining how many agents of each demographic group populate each \(département\); and in its dynamics, by defining how likely it is for an agent to have children and die at different stages of its life. Agents interact in a geography that reproduces the demographic reality of France during this period, as assessed by available data. Fertility decisions are affected by this reality (e.g. child mortality rates) but also by the local knowledge agents have of their neighbours’ choices. As stated above, the theoretical debate has so far tended to emphasise the relevance of one of those two factors: material conditions vs. social interactions. Our simulation model plays them simultaneously to evaluate their relative effects on the overall dynamics.

The rule that guides agents’ behaviour considers their own willingness to reproduce, weighted by local conditions, including the ratio of child mortality, but also the desired offspring of their neighbours. When agent \(i\) reaches reproductive age at time \(t\), they establish their desired number of offspring \((y_{i,t})\) by considering their own inclination to reproduce \((z_i)\), how likely it is that the child will survive (adjusting by the level of child mortality \(d\)), and the average desired number of offspring that other fertile agents around them were inclined to have in \(t - 1\):

\[
y_{i,t} = \alpha z_i (1 + d) + (1 - \alpha) \frac{1}{m} \sum_{j=1}^{m} y_{j,t-1}
\]
It is clear that with the introduction of \( d \) in this initial behavioural rule we are explicitly assuming agents care about completed family size, a thing that—the point we made above—is somewhat contested issue in the literature. We are going to deal with this issue later, in a set of experiments where we compare the outcomes of this rule with one that tells families to aim at fertility instead of surviving children.

The degree of social interaction is captured by \( \alpha \). This parameter determines, in a similar way as \( J \) did in equation (2), the relative weight agents give to their own preferences with respect to the behaviour of those around them. The larger the value of \( \alpha \), the more the agent cares about its inclination, and the less about that of their neighbours. Since individuals most likely look at the generation closest to them, the behavioural rule makes agents take as reference the behaviour of all other agents in the vicinity that were fertile in the previous period. In our model ‘vicinity’ takes a very specific meaning. Geographical proximity is defined in terms of the grid that underlies the simulated map of France, where each cell accounts for about 100 km\(^2\), and the ‘neighbourhood’ is defined by the cell where the agent lives and the eight cells immediately surrounding it (roughly the area within 3 to 4 hours of walking distance), as illustrated in Figure 5.

\[ \text{[Figure 5 about here]} \]

Our strategy to model the transition brings together the theoretical approaches summarised in the previous section. We derive two main assumptions from them. First, the majority of agents in the economy are relatively close to the threshold that separates the old and new fertility order, but not yet there; that is, fundamentals are such that non-religious agents in isolation (i.e., with \( x\left(f_i\right) = \frac{J}{2}\left(f_i - F^*_i\right)^3 = 0 \) ) would adjust their fertility to a new level. In the context of a modernising society with many factors encouraging individuals to have smaller families (e.g. Galor and Weil 1999), this is probably not a costly simplification. Second, agents have two alternative strategies: to follow the fertility behaviour conventional in the ancien régime (labelled with the superscript \( \text{ar} \)), or to modernise (labelled with the superscript \( \text{mo} \)), which they exercise by picking a fertility level from two alternative random distributions:

\begin{align*}
Z^{\text{ar}} & \sim \log N\left(\mu^{\text{ar}}, \sigma\right) \\
Z^{\text{mo}} & \sim \log N\left(\mu^{\text{mo}}, \sigma\right)
\end{align*}

where \( \mu^{\text{ar}} > \mu^{\text{mo}} \). These distributions reflect the heterogeneity inherent to individuals that are affected by different vectors \( Z_i \) or some occasional shock \( \epsilon_i(f_i) \). These choices take place along changing material conditions, such as child mortality, that appears in the behavioural rule.

At the beginning of the simulations all agents draw their inclination to reproduce \( z_i \) from \( Z^{\text{ar}} \), which captures the pre-transitional equilibrium. Once the exogenous impact of the Revolution takes place, a number of agents switch to draw their desired \( z_i \) from \( Z^{\text{mo}} \). This aims to capture a shift of values and normative expectations, which in terms of equation (2) can be interpreted as a reduction in the
reward $x(f_i)$ of adopting the no contraception strategy. The shock takes place only once, randomly affecting agents of all ages. Those that are young will take the new distribution into consideration when choosing their family size. The fertility of mature agents will not be affected, because they already made their choice, but they will pass this trait to their offspring with probability 1. In terms of the current debate on cultural transmission (e.g. Bisin and Verdier 2001) this feature means that direct vertical socialisation—that is, the one coming from the family (Cavalli-Sforza and Feldman 1981, pp. 78-84)—is perfect: daughters behave exactly like their mothers.

Non-family influence is partly accounted for by parameter $\alpha$, but the behavioural rule (3) does not allow agents to decide whether they want to change their type (e.g. go from $Z^ur$ to $Z^mo$). For that, we introduced an additional parameter $\gamma$, which allows agents to undergo the change. Agents not affected by the initial shock of the Revolution look at their ‘neighbourhood’ and decide to change the distribution from which they draw their desired $z_i$ (that is, they become ‘modern’) if a proportion equal to or larger than a threshold $\gamma$ of their neighbours are already doing so (we do not consider the possibility of turning traditional if you are modern). This parameter $\gamma$ opens an additional experimental space that allows us to test the effects of a second, post-Revolution channel of horizontal social influence.

There are several reasons to believe that such an additional channel of cultural transmission (normally labelled oblique or horizontal socialisation, see Cavalli-Sforza and Feldman 1981, pp. 130-133) is important after a shock like the one we are describing. The presence of modern parents in a neighbourhood, for instance, reduces the uncertainty of deviating from the traditional behaviour by showing that smaller families are economically viable and perhaps desirable, and increasing the expected utility of a low fertility strategy. As suggested by the economics of religion literature (Iannaccone 1998, pp. 1482-1484), if a considerable amount of agents in a community diverge from a religious norm, the value of following that norm (here the reward $x(f_i)$) decreases for all agents in the congregation, even those not originally affected by the shock. In either case, one can expect a feedback mechanism to enter into play and amplify the effects of an original shock in a local community.

In the following two sections we use the simulation model to explore the hypotheses discussed above. First, we will ask whether different assumptions on social interaction generate different population dynamics, and whether the introduction of child mortality in the behavioural rule contributes to reproduce more accurately the empirical profile, to shed some light on whether couples might have been aiming—before the transitional—to completed family size. Then, we will assess

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3 We experimented relaxing this assumption that agents cannot change their choice when they are mature. Since the five-year periods we use are rather coarse, the main dynamics of the simulations were not affected and—to avoid additional computational costs—we decided to keep this simplifying assumption.
whether the presence of social influence, in both its horizontal and vertical form, affect the long term trends during the transition.

**Fertility decisions in pre-transitional France**

Most of the evidence on pre-transitional Europe suggests fertility levels were more or less stable over time (e.g. Flinn 1981), and France does not seem distinct in that respect (Henry 1972, 1978; Henry and Houdaille 1973; Houdaille 1976). For the French case it is also well established that signs of a downturn became evident only after 1790 (Weir 1983, Wrigley 1985a). These two stylised facts make it plausible to assume that before the transition all individuals were drawing their fertility levels from a single, stable distribution ($Z^*$), and only after the transition started there was a higher heterogeneity of choices (here captured by the second distribution, $Z^{**}$). We present in this section a series of simulation experiments related to the pre-transitional period that aim to uncover the implicit fertility decisions in the ancien régime under alternative behavioural assumptions. We identify the parametric configurations that allow us to reproduce empirical patterns of population growth and fertility levels. This essentially entails finding values of $\mu^*$ that are consistent with aggregated trends of population dynamics under different values of $\alpha$.

Setting $\sigma$ equal to 0.45, which is the average value for empirical populations as estimated from age-specific fertility tables (Flinn 1981), we began by generating sets of simulations starting in 1740 and up to 1790 for different values of $\mu^*$: from 1.0 (equivalent to 2 children per family in actual data) to 3.0 (equivalent to 6 children), with increments of 0.05; and for different values of $\alpha$: from 0.2 (virtually complete social influence) to 1 (no social influence at all). We assessed how these different parametric combinations affected the evolution of population levels by plotting the average of 100 simulations against the empirical data. Figure 6 shows our results for a representative selection of parametric combinations.

*Figure 6 about here*

The figure shows that several parametric combinations ($\mu^*, \alpha$) lead to good fits of the population trend, with alternative degrees of social influence being consistent with different levels of $\mu^*$. Simulations where $\alpha$ was smaller required lower means to sustain the same population levels. This is probably a consequence of having less agents aiming at lower values of the distribution and, since there is an upper limit to the amount of children an agent can have in her lifetime, this generates a tendency to have on average larger families. In general, for every $\alpha_i$ there is a $\mu_i^*$ that allows us to track population growth well, and the match we ob-
tain from an optimal pair \((\mu_i^{\pi_i}, \alpha_i)\) is comparable to that of any other pair as assessed by alternative goodness of fit measures.

The second result of interest is that, for every degree of social influence, there is a level of \(\mu^{\pi_i}\) for which population can grow too much. This outcome is not trivial: it suggests that a population with these demographic characteristics could have grown more. Since by construction the model does not allow agents to have more children than what is plausible given basic biological limitations (like fecundity and mortality rates, and age structure of the population) and social considerations (effective marriage rates), the fact that there are values of \(\mu^{\pi_i}\) for which population growth is considerably higher than observed implies that families were not maximising the number of offspring either because families were actively controlling births or certain social practices (e.g. post-partum abstinence or extensive breast-feeding) reduced fertility. In effect, the graphs suggest that a mean of around 3 surviving children per family (the empirical equivalent of \(\mu^{\pi_i} = 1.5\)) is enough to replicate the population growth of France during the period.

The model produces slightly lower fertility rates than those observed, yet with the same stable trend. For the country as a whole, our estimates of crude birth rates in the simulation are in the order of 30 to 31 per 1000, whereas the empirical average for the second part of the eighteenth century is about 39 (INED 1977, pp. 332-333). The margin is not so large if we take other (less coarse) measures, such as the Princeton indices. For the period 1740-1790, for example, studies suggest for the whole of France average \(I_f\) values of around 0.41 (Weir 1994, pp. 330-331), whereas those we get are about 12 per cent smaller (in the order of 0.36), a difference that is smaller than discrepancies within alternative empirical estimates of the Princeton indexes for the early nineteenth century (e.g. Weir 1994 versus Bonneuil 1997).

Empirical data before 1800 are partial and scattered, so there is no direct way to assess whether simulated fertility rates for the different départements across France do in fact reflect actual rates in the pre-transition period. One alternative is to compare them with the estimates for the INED sample (Weir 1983, pp. 189, 194). Since these values correspond to specific villages, one cannot really take the \(I_f\) values to be representative of the départements they were in, but they could provide a first (yet noisy) indicator of the simulations’ performance. We find, for various \((\mu_i^{\pi_i}, \alpha_i)\) combinations, a positive correlation of around 0.37 between Weir’s estimates for the villages in the 1690-1769 period and our values for the respective départements in the 1740-1790 simulated interval, which is reassuring. We can also compare the results of the simulation with the earliest fertility figures available for all French départements, the \(I_f\) values calculated by Bonneuil (1997) for the period 1806-1811. By this time the decline had already started in some places, yet regional differences were probably dominated by the pre-transitional dynamics. With respect to these figures, we also find a positive, and stronger association to
our results. Correlations were between .50 and .59, where pairs with high levels of \( \alpha \) performed marginally better.

Finally, the third interesting result of this set of experiments is that when the influence of child mortality is removed from the decision rule, the simulated fertility trends do not resemble the observed values as closely. When building the behavioural rule (3) we made the assumption that parents take local mortality rates into account at the moment of deciding their family size. However, as stated above, this is a contested issue in the literature. We assessed this issue running a set of simulation experiments that apply a behavioural rule that does not account for child mortality of the following form:

\[
y_{i,t} = \alpha z_i + (1-\alpha) \frac{1}{m} \sum_{j=1}^{m} y_{j,t-1}
\]

The aim of these experiments is to determine if there are significant changes in the empirical performance of the model. When running simulations using (3'), the population trends are comparable to those coming from the original behavioural rule (3), but the results on fertility are remarkably worse. As expected, the values of \( \mu^* \) providing a good fit are higher: agents must aim to a mean close to 2 (equivalent of 4 children in the real world) to maintain the empirical population growth rate. Besides somewhat higher volatility in the series, the results at macro level are not substantially different for this alternative behavioural rule. Regarding fertility, however, the results are markedly poorer, particularly at regional level. There is no relationship between our estimates at the local level and those of the INED sample or the early nineteenth century départements figures: correlations are significantly lower in every case, and close to zero in most of them. The results of these simulations suggest parents were indeed looking at surviving children when deciding the size of their families, and some of the pre-transition differences in fertility rates can be explained simply by differences in child mortality.

**Revolution, religion and social influence in the transition**

Simulations that maintain pre-1790 parameters overestimate, in all cases, population growth for the nineteenth century. This implies that at least some agents must have switched to draw their fertility decision from another distribution, as we suggested in our theoretical discussion. In this section we consider a series of experiments that explore the possible causes behind the decline, paying special attention to the effects of vertical and horizontal social influence. We assess how well the model performs when a proportion of agents in each département aim at a (common) lower fertility, and this proportion is correlated with support for the Revolution. Modelling a proportion of agents aiming at a common level (as opposed to making all agents aim at different lower levels) addresses the empirical observation made by Weir that the fertility decline in France was the consequence of the effort of an efficient group and not a collective choice in the population (Weir 1983,
This also connects with the theoretical suggestion made by Kohler (2000a) that fertility choice can be partly understood as a coordination problem with multiple equilibria. Exogenous shocks can cause agents to update their expectations and coordinate in a new equilibrium and we explore how the French Revolution could have acted as such a shock.

*Revolution and the fertility decline*

Recent studies suggest social upheavals can have profound effects on the evolution of birth rates (e.g. Caldwell 2004; Bailey 2009). The specific connection between the Revolution of 1789 and the French fertility decline has been apparent for some time (Spengler 1938, pp. 163-174; Flandrin 1979, p. 238). Timing itself makes the Revolution a good candidate to explain the decline, as the first signs of reductions in birth rates appear after 1790 (Weir 1983, p. 39). At least two types of theoretical arguments further support the hypothesis that the Revolution prompted the decline: one is linked to the ideological shift associated with the rise of a more egalitarian and democratic society (Dumont 1890; Leroy-Beaulieu 1913), with individuals realising they could actively decide aspects of their lives that were historically taken as given (Binion 2001); the other stresses the institutional aspects of the new order, like modifications in inheritance laws (Le Play 1874) or the revolutionaries’ promotion of agricultural capitalism (Weir 1983, p. 280). In either case, there are reasons to believe the Catholic Church, and in particular its disintegration, played a key role in those changes.4

There is extensive evidence suggesting a connection between religion and fertility behaviour (e.g. Derosas and van Poppel 2006). Up to the early nineteenth century Catholicism, which held a particular code with respect to family behaviour, remained the main norm-setter in France and had a strong attitude against contraception (Flandrin 1979, pp. 194-196; Gibson 1989, pp. 185-186). Regarding ‘ideational shift’ stories, the Revolution shook the Church to its very foundations allowing “at least some French men and women to break free from old constraints” (Gibson 1989, pp. 244-245), and enabling them to reach a new ideal normative equilibrium in terms of fertility behaviour.

Yet the National Assembly also interfered in the regular functioning of the Church in a more literal way by suddenly curtailing its liberties, along with its resources, and shaking its whole apparatus with the purge of its members. Towards the end of 1790, for example, the revolutionaries imposed a clerical oath of allegiance to the new Constitution that split the clergy into jurors (*constitutionnel*) or non-jurors (*réfractaire*), fuelling confrontations within the clergy and at different levels of society. The nature and consequences of the oath are rather complex (see 4 Despite the many parallelisms between the American and the French fertility decline (Binion 2001), discontinuity in religion does not seem to have played a crucial role in the former. Although both regions underwent ‘democratic’ revolutions, and these seem to be associated somehow with the fertility decline, in the French case several aspects of the subsequent evolution of the Catholic Church appear to be better proximate determinants of fertility than republican (versus monarchical) characteristics (see, e.g. Murphy 2010).
Tackett 1986), but some authors have ventured the idea that the relaxation of clerical discipline in ‘constitutional’ regions can partly explain the rapid spread of birth control in those areas where the Church was debilitating. Most notably, Sutherland pointed out that the oath contributed to put an end to a quasi-universal religious practice in France and, in particular, limited the ways in which local priests could influence birth control practices; this facilitated the rise of ‘anomalies’ in sexual behaviour such as contraceptive practices, illegitimacy, and bridal pregnancies (Sutherland 2003, p. 345). Arguments not primarily religious are consistent with this story. Given the extent of the influence of the Church, it is not a stretch to think that weakly religious areas could have been more sensitive to the institutional changes brought by the Revolution and that these changes could have had an impact on fertility.

We tested this story in a second set of experiments aimed to mimic population growth after the Revolution. One way of interpreting Sutherland’s hypothesis in terms of our model is that in oath-taking areas the Revolution reduced the costs of not following the prevalent norms (i.e. those mandated by the church) via a drop in $x(f_i)$ for some of the agents. If those agents were close to the threshold to change their fertility strategy, which for early modern France is a reasonable assumption, they would have decided to become modern. Since the proportion of priests taking the oath varied substantially throughout the country, we use this variation to model spatial differences in this attitudinal shift. We do so using a simple direct proportionality; if, for example, 25 per cent of the priests took the oath in a département, we assume a quarter of our simulated agents in that same département will now draw their personal fertility inclination ($z_i$) from a distribution that has a mean of $\mu^{mo}$ instead of $\mu^{ar}$. This new fertility inclination can then be transmitted vertically (from parents to offspring) or horizontally, via the parameter $\gamma$, which determines how easily this behaviour spreads in time to other agents.

_**Simulating the transition**_

As we discussed in the previous section, more than one combination of parameters $(\mu^{ar}, \alpha_i)$ was consistent with pre-transition demographic trends. We tried several of them as alternative starting points for this set of experiments, obtaining similar results with all of them. Figure 7 illustrates those corresponding to the initial combination $(\mu^{ar}, \alpha_i) = (1.5, 0.7)$.

*[Figure 7 about here]*

The first thing to notice is that we need relative small drops from $\mu^{ar}$ to $\mu^{mo}$ to replicate the evolution of population. Depending on the value of $\gamma$, with higher values making more restrictive horizontal diffusion, a decline of less than 20 per cent in the average desired family size is enough to achieve the empirical popula-
tion growth rates. Another feature of interest is that the dynamics at the aggregated level are quite sensitive to $\gamma$, with high values requiring a larger fall in the mean of the distribution. As in the previous section, we can find a series of parametric combinations $\left(\mu_0^a, \mu_0^{mo}, \alpha_i, \gamma_i\right)$ that maximise the goodness of fit of simulated population patterns. Figure 8 shows how different parametric values affect fertility trends: the upper pane (a) holds $\gamma$ constant and varies $\alpha$; the lower panel (b) holds $\alpha$ constant and varies, instead, $\gamma$.

[Figure 8 about here]

The matching of simulated and observed data is far from perfect, but the simulated trends change at the right time. The best results in both panels come from parametric combinations where social interaction effects were present but did not dominate the dynamics. In panel (a), for example, a steep and persistent decline is achieved when social interaction is moderate ($\alpha = 0.7$), and not so much at high levels ($\alpha = 0.4$) or when it is absent ($\alpha = 1.0$). In panel (b) the least appropriate combination is that for which $\gamma = 0$.

Although timing and pace of the fertility decline are matched relatively well in the first few decades of the transition, the model does not perform as well in the later part of the nineteenth century. The most likely explanation for that is the inherent simplicity of the intervention we impose: since we only allow for a once-and-for-all decline in $\mu$, we are ruling out the possibility of further declines in $\mu^{mo}$, which are otherwise quite plausible (e.g. motivated by the secular increase in wages or the expansion of schooling in the later part of the century). Other parametric rigidities might as well be important. We assume throughout that both $\alpha$ and $\gamma$ are constant across time, and these could well be changing, helping to reinforce some of the dynamics that drive down fertility in the model. The type of information we use to describe the environment can also explain part of this performance. Since data limitations force us to use child mortality as one of our main empirical anchors, we are probably missing aspects of the long-term evolution of non-infant child mortality (i.e. 4q1) that some authors claim is crucial to fully understand the dynamics of fertility decline (see e.g. Reher 1999, p. 15). And as the coarseness of child mortality might affect the results of the simulation, the roughness of $I_7$ as a measure of fertility could be playing a role too in our reading of its outputs. As Figure 9 shows, since $I_7$ incorporates the fact that people are marrying earlier in the later period, its tracking of the fertility decline is much better.$^5$

[Figure 9 about here]

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$^5$ All these issues highlight crucial aspects of the discussion on fertility decline that simulation models of this sort help to identify and that further research could address.
For these sets of experiments it is interesting to look closer at the performance of the simulation vis-à-vis the actual data at départements level. Figures 10 and 11, for example, look at some areas that were leaders and laggards in the decline. The model perceives well the absence of downward trend in the sluggish areas of France (many of them largely oath-rejecters), yet tracks better the Massif Central than Brittany. A number of factors might account for that difference. Brittany, for example, was relatively richer and appears to have been largely under-taxed (Jones 1988, p. 36); this means that parents had more disposable income to spend on children. If children were normal goods, families in Brittany would have been motivated ex-ante to draw fertility from a distribution with higher mean. The North-West also experienced higher child mortality. This led most départements in the area to start from higher levels of fertility—which is remarkably well reflected in the results of our model—yet, if the point we made earlier about infant versus non-infant child mortality is indeed relevant, this higher level of fertility might have well be underestimated.

[Figure 10 and 11 about here]

As Figure 11 shows, the simulation somewhat overstates the levels of fertility for the leaders, although it replicates well the general downward trend. A few characteristics of the model could account for some of these discrepancies. The model assumes homogeneity across all individuals in terms of how vulnerable they are to social interaction effects (that is, $\alpha$ and $\gamma$ are and remain constant for all agents). It is certainly plausible that the propensity to follow or learn from others could vary across regions; in particular, it is likely that areas leading the decline were more ‘individualistic’. It is also possible that the relationship between oath-taking and the change in desired fertility is not linear. One could speculate that while in conservative or moderate areas the correlation might be good, political reasons could have motivated church leaders to pressure priests in very liberal areas to take a stand against the Revolution as a way to make an example. If this was the case, the impact of the Revolution could be underestimated in the leading areas. In fact, it is interesting to note that—in contrast to the sluggish areas that were all among the top oath-rejecters—none of the leading départements in the fertility decline were among the top oath-takers. These effects might of course be reinforced by other sources of heterogeneity that the model is simply not incorporating and are ‘hidden’ in the normal distribution that agents use to draw their desired family size, such as differences in income, or education.

[Figure 12 about here]

Figure 12 illustrates the performance of the model in a representative sample of the remaining départements and are tracked more closely. Often starting from slightly different pre-transitional levels, which—as we showed in the previous section—probably reflects heterogeneities in child mortality, most départements show a decline that is consistent with the available empirical information. Early
adopters and latecomers in the simulated results are generally early adopters and latecomers in the empirical data, and the evolution of heterogeneity across départements as measured by the coefficient of variation increases and then decreases, as empirical data showed in Figure 4. Although the tracking is not perfect, the model is able to replicate many of the stylised facts of the decline, including this regional diversity, underscoring the importance of social interactions and influence.

Conclusion

Recent literature has argued that agent-based simulation provides a fruitful avenue to explore the individual-level mechanisms that underlie demographic trends (e.g. Billari and Prskawetz 2005; Hobcraft 2006, p. 176). By being anchored in various types of empirical information and making explicit assumptions about the theory behind agents’ behaviour, experiments using simulation models are particularly useful to test theoretical arguments that cannot be resolved using available data. This paper provides a concrete example, showing how different behavioural assumptions impact on traceable patterns, and testing the relative weight of social interactions and influence in those changes.

This simulation exercise is fundamentally exploratory in nature. It creates a framework to assess controlled thought experiments and counterfactuals in a context where scarce data does not allow us to apply other empirical strategies. With that spirit, our results provide evidence that emphasise the relevance of social links to reproduce the observed patterns of fertility decline. Different parametric combinations are able to replicate the observed demographic trends, yet simulations that incorporated social interaction effects (through parameters $\alpha$ and $\gamma$) appear to track these trends better than those where these effects were ignored. These findings highlight that interpersonal interactions and do matter to explain fertility transitions – an issue only marginally discussed in the literature but that, in light of these findings, deserves more consideration.

Our experiments allowed us to test some hypotheses of interest for the debate on early modern demographic dynamics. For the pre-transitional period, for example, the simulations suggested that families were not maximising offspring (as many Malthusian arguments imply), and parents probably considered the risk of their children dying when deciding how many they wanted to have (which is an issue still contested in the literature). Regarding the transition, results at micro level insinuate that part of the different regional trends could be traced back to the heterogeneous impact of the Revolution, at least as Sutherland (2003) envisaged it and we interpreted it in our model, via the weakening of the Church that in turn affected the high fertility norm. If this reading is indeed correct, an interesting political economy corollary stems from this argument. Since the revolutionary government had the typical pro-natalistic interest of modern states (that need people to pay taxes and fight wars), its success in taking to pieces (at least partly) the Church’s structure might have been Pyrrhic, as it dismantled the institution that was helping to sustain high levels of fertility.
Although having certain clear limitations, the simulation strategy we propose in this paper is able to tackle many issues that other empirical methods struggle to address. As such, this approach is not a substitute, but a suitable complement in contexts where data is limited or theoretical modelling is too complex to generate treatable closed-form solutions. Simulation like the one we present here can contribute to operationalise empirical research, validate certain claims, and draw attention to aspects of theoretical debates that other approaches fail to perceive.
References


Appendix

Agents in the model are born to reproduce. From the moment they are created they have an inclination to have a certain amount of children, but they can actually have them only when they reach a mature age, and they do so at a rate of one child per period. Agents can be interpreted as the female part of the population: for the sake of simplicity we have abstained from gender distinctions and marriage dynamics, or the actual dynamic of the couples jointly determining the fertility they are aiming at (Miller et al. 2004). Nevertheless, we allowed them to live for fifteen periods to facilitate comparison with demographic data, which usually comes in five-year ranges. Agents are classified into different groups of ‘age’: newborns, young1 to young3, mature1 to mature5, and old1 to old6 (that is, the maximum of fifteen time periods they are allowed to live). They have two attributes associated with their age: the probability of death, a rate that is determined empirically; and fertility, which results from rules of the model. Only agents classified as mature are able to create new agents and therefore reproduce the population, following the behavioural rules discussed in the text.

To make the model resemble reality, we incorporated some elements from the geography and demographic history of France in the set-up of the environment where the agents interact. The space that agents occupy is a grid that reproduces the map of France, each cell representing more or less 100 square kilometres (i.e. a 10x10 km area), with a total of 5308 cells. The simulation starts with roughly 100,000 agents that are placed on the grid following empirical estimates of population composition and density. Due to the lack of estimates about the amount of people in the different age groups for each département around 1740 (let alone for every other 100 square kilometres), we had to make some assumptions. Henry and Blayo have estimated age pyramids for early modern France and we have taken as reference the one corresponding to 1740 (Henry and Blayo 1975, pp. 92-93). As can be seen in panel (a) in Figure A1, the correspondence between model and actual data is nearly perfect; the most substantial differences affect the oldest population because, for simplicity, we only allow agents to live until they are 75. We assume that age pyramids were similar throughout France (this was probably not the case but is not a major drawback for the purposes of the model). Population densities provide the second anchoring point between the set-up of the model and empirical data. The earliest year for which we have some information about population density is 1801 (Service de la Statistique Général de France 1878), and agents are distributed in the grid according to these data. We considered the population of each département and that of their major cities and produced a rough estimate of the proportion of the total population living in a particular geographical area. We applied this proportion to the initial 100,000 agents to determine the size of the population in each cell of the grid, in line with the age structure described before. The map in Panel (b) of Figure A1 illustrates this set-up.

[Figure A1 about here]
According to this initialisation, not all agents will eventually have children. Europe was characterised by a particular marriage pattern, where women married late and some did not marry at all (Hajnal 1965; Voigtländer and Voth 2010). We follow here the estimates of Henry and Houdaille (1979, p. 421) for the mean age of marriage for ten different regions within France at five different times in the period 1740-1900. To translate mean marriage ages into proportions of married agents by age-group we had to make the rough assumption all possible marriages take place before agents become Mature 3 (i.e. age 30-34), that the proportion of Mature 2 agents married is smaller but constant over time, and that differences in age of marriage translate into different proportions of Mature 1 agents married. In Figure A2 we show how the averages of our model at the departmental level relate to the empirical estimates available for the whole of France.

[Figure A2 about here]

It is also important to take into account the role of decreasing fecundity with age, or lack of fecundity. For simplicity, in the model we assume that only mature agents can have children, so we are implicitly considering that all agents are sterile (or unmarried) until then. We can thus distinguish primary sterility –women that can never be fecund— and secondary sterility, which kicks off at some stage after being fertile for a period (Boongarts 1975, p. 293). There are different biological factors affecting both types of sterility, so estimates could vary between populations considerably. Sterility is often difficult to disentangle from actual contraception using historical data, especially for younger ages. Hence, we take the conservative approach of assuming no primary sterility at all, and secondary sterility affecting only the last two groups of matures. For this, we take Henry’s estimates for a series of European populations in the modern period (Henry 1961, p. 85) as upper-bounds and impede procreation of 15 per cent of mature4 and 30 per cent of mature5 (that might be married or not). With these data we obtain a series of expected proportions of agents in the risk of having children. Following this rule, mature agents can generate new agents until they reach the maximum determined by their behavioural rule or until they enter the old age category.

The simulation runs for a total of 36 periods, each representing five years, starting from 1720 and stopping in 1900. At every time step, agents move upwards in the age scale. Once an agent is born, it will live for up to 15 periods, although random agents in all categories can disappear at any time in proportion to the mortality rate attached to their age. Mortality rates were estimated by Bonnieul (1997) for all age ranges every five years throughout the nineteenth century and, though far from perfect (see e.g. Guinnane 1999, pp. 171-172), remain probably the best proxies we have for every department. For pre-1800 simulations we assumed the earliest rates available. Post-1800 we adjusted child mortality (4q0) every ten years according to the empirical estimates of Bonnieul to account for the sharp decline over the century, but kept constant those for other ages which are more or less stable till the twentieth century.
The simulation keeps track of the number of agents in each age group; it also records the number of offspring that agents want to have and calculates the average for each cell in the map. This creates a census of the simulated population as it evolves over time. The simulation then applies the mortality rates in accordance to the age of the agents and the département in which they are located; it next shifts the remaining agents one level up: agents with age > 70 all die and are replaced by the agents in the previous age group; and the agents entering the mature category are given a desired number of offspring as determined by the behavioural equation (4). New agents classified as newborns are then created: if a mature agent has not yet reached the maximum number of offspring she wants to have, is married and not sterile, she will create a new agent. This loop, depicted in Figure A3, is repeated 36 times, at which point the simulation stops.

[Figure A3 about here]
Figure 1. Crude birth rates (births per 1000 population) for selected European countries, 1770-1900

Sources: For France, INED (1977, pp. 332-333); Wrigley and Schofield (1981, pp. 531-535) for England and Wales; for Sweden, Denmark, and Finland, Gillie (1949, p. 83) and Chesnais (1992, pp. 518-541). Values are 5-year averages, centred in the year.
**Figure 2.** Marital fertility index (Ig) in France for each département, 1831-1911

Sources: Maps are ours, constructed using data from Coale and Watkins (1986, pp. 94-107).
Figure 3. Marital fertility index ($I_g$) in England for each county, 1871-1911

Sources: Maps are ours, constructed using data from Coale and Watkins (1986, pp. 88-93).
**Figure 4.** Mean and coefficient of variation of marital fertility (Ig) within departments, 1831-1921


**Figure 5.** Agent's neighbours in the grid
**Figure 6.** Actual and simulated levels of population for different pairs of \((\mu^\omega, \alpha)\)

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**Notes:** Dotted lines indicate actual values and smooth lines correspond to average of 100 simulations. Actual and simulated populations are set equal to 100 in 1740. Actual population is from INED (1977, pp. 332-333) and INSEE (1961, p. 36).
Figure 7. Actual and simulated levels of population for different pairs of \((\mu^m, \gamma)\), when \((\mu^m, \alpha) = (1.5, 0.7)\)

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Notes: Dotted lines indicate actual values, smooth lines correspond to average of 100 simulations, and dashed lines 95 per cent confidence intervals. Actual and simulated populations are set equal to 100 in 1740. Actual population is from INED (1977, pp. 332-333) and INSEE (1961, p. 36).
Figure 8. Actual and simulated overall fertility ($I$) at macro level when $\mu^\alpha = 1.50$, 1740-1900

(a) Changing $\alpha$ (with $\gamma = 0.3$)

Sources: Dotted lines indicate actual values and smooth lines correspond to average of 100 simulations. Overall fertility 1740-1900 as estimated by Weir (1994, pp. 330-331). Simulations hold constant $\mu^\alpha$ and one parameter (either $\gamma$ or $\alpha$), while changing the other (either $\alpha$ or $\gamma$) for the value of $\mu^\alpha$ that maximised the goodness of fit for the evolution of population.
Figure 9. Actual and simulated fertility at macro level when \((\mu^r, \mu^m, \alpha, \gamma) = (1.50, 1.3, 0.7, 0.3), 1740-1900\)

(a) Overall fertility \((I_0)\)

(b) Marital fertility \((I_0)\)

Sources: Dotted lines indicate actual values and smooth lines correspond to average of 100 simulations (dashed lines indicate 95 per cent confidence interval). Marital and overall fertility 1740-1900 (indicated with dots) as estimated by Weir (1994, pp. 330-331), and shorter series of overall fertility 1806-1901 (indicated with triangles) as estimated by Bonneuil (1997, pp. 197-205).
**Figure 10.** Actual and simulated marital fertility levels when \((\mu^r, \mu^w, \alpha, \gamma) = (1.50, 1.3, 0.7, 0.3)\), lagging départements, 1740-1900

**Notes:** Dotted lines indicate actual values starting in 1831 (van de Walle, 1974), whereas smooth lines correspond to simulation starting in 1741. Both finish in 1896.
**Figure 11.** Actual and simulated marital fertility levels when \((\mu^\mu, \mu^m, \alpha, \gamma) = (1.50, 1.3, 0.7, 0.3)\), leading départements, 1740-1900

Notes: Dotted lines indicate actual values starting in 1831 (van de Walle, 1974), whereas smooth lines correspond to simulation starting in 1741. Both finish in 1896.
Figure 12. Actual and simulated marital fertility levels when \((\mu', \mu'', \alpha, \gamma) = (1.50, 1.3, 0.7, 0.3)\), other départements, 1740-1900

**Notes:** Dotted lines indicate actual values starting in 1831 (van de Walle, 1974), whereas smooth lines correspond to simulation starting in 1741. Both finish in 1896.
Figure 12 (cont.) Actual and simulated marital fertility levels when \((\mu^*, \mu^**, \alpha, \gamma) = (1.50, 1.3, 0.7, 0.3)\), other départements, 1740-1900

Notes: Dotted lines indicate actual values starting in 1831 (van de Walle, 1974), whereas smooth lines correspond to simulation starting in 1741. Both finish in 1896.
Figure 12 (cont.)  Actual and simulated marital fertility levels when \((\mu, \mu', \alpha, \gamma) = (1.50, 1.3, 0.7, 0.3)\), other départements, 1740-1900

Loir-et-Cher

Yonne

Vienne

Dordogne

Loire

Notes: Dotted lines indicate actual values starting in 1831 (van de Walle, 1974), whereas smooth lines correspond to simulation starting in 1741. Both finish in 1896.
Figure A1. Demographic features of the simulation

(a) Age structure

(b) Population density in simulation

Notes: Panel (a): The axis in the bottom indicates the proportion of each age-group with respect to the total population. Actual data for 1740 France comes from Henry and Blayo (1975, pp. 92-93). Panel (b): Population density as simulated in the model; darker patches are more populated.

Figure A2. Proportions ‘married’ in the simulation model and real data

Notes: Dotted lines indicate actual values (Weir, 1994, p. 329) and smooth lines simulation.

Figure A3. Simulation dynamics

Notes: Census → Agents die → Shift generation → Give birth

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