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# Occasionally binding emission caps and real business cycles

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## Abstract

Recent applications to the modeling of emission permit markets by means of stochastic dynamic general equilibrium models look into the relative merits of different policy mechanisms under uncertainty. The approach taken in these studies is to assume the existence of an emission constraints that is always binding (i.e. the emission cap is always smaller than what actual emissions would be in the absence of climate policy). Although this might seem a reasonable assumption in the longer term, as policies will be increasingly stringent, in the short run there might be instances where this assumption is in sharp contrast with reality. A notable example would be the current status of the European Emission Trading Scheme. This paper explores the implications of adopting a technique that allows occasionally, rather than strictly, binding constraints. With this new setup the paper sets out to investigate the relative merits of different climate policy instruments under different macro-economic shocks.

*Keywords:* Dynamic Stochastic General Equilibrium model, emission trading, carbon tax, occasionally binding constraints.

*JEL codes:* Q58, Q54, E2.

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# 1 Introduction

The European Union Emissions Trading Scheme (EU-ETS) is the first, multi-region large scale implementation of a pollution permit market. What is traded in this market are carbon emission permits and the initial allocation of such permits is a combination of long term nation-wide caps and within nations allocation. As in any market, the carbon price obviously reflects the equilibrium between emissions demand and supply of emissions. Current prices are around €4 a tonne of carbon dioxide and have been lower than this earlier this year, but the shock in prices have started much earlier, around 2008. The reason for such low prices is a combination of overallocation during the early phase of the scheme, the overlapping of policies favoring renewable technologies throughout Europe and the economics crisis. Since 2008 the EU-ETS has experienced a surplus of allowances and international credits compared to emissions, which have accumulated and rolled over in time. Figure 1 reports, on the left, data on total EU-ETS allowances and verified emission for the 2005-2012 period (spanning the first two “trading periods” of the scheme) published by the European Environment Agency (EEA), and, on the right, the *deficit* of allowances (for the sake of exposition), i.e. the difference between verified emission and allowances, and the industrial production gap for EU28 countries: there is an evident positive correlation between the allowance deficit and the business cycle, at least considering industrial production.<sup>1</sup>

The EEA “*Trends and projections in Europe 2013*” report<sup>2</sup> announces that: “Aggregated projections from Member States indicate that total EU 28 emissions will further decrease between 2012 and 2020. With the current set of national domestic measures in place, EU emissions are expected to reach a level in 2020 which is 21 % below 1990 levels (including emissions from international aviation). Implementing the additional measures at planning stage in Member States is expected to achieve a reduction of 24 % below 1990 levels in 2020.”

This is a clear and dramatic example of how, in reality, the assumption of binding caps can be far from what really turns out to be. It is thus important to model the possibility of non binding caps as otherwise the assessment of a cap system would tend to be biased unfavorably both in economic and in environmental terms.

The debate concerning the optimal choice of a market based instruments to internalize an externality under uncertainty has been pioneered by Weitzman (1974). The basic finding of this seminal theoretical paper, is that it is the slope of marginal damages relative to that of marginal abatement costs that determines whether price versus quantity market

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<sup>1</sup>The industrial production gap in percentage terms is obtained by log-linearly detrending the 1975:III-2013:III quarterly index of industrial production for the EU28 countries published by the OECD. The annual figures are computed as averages of the quarterly ones.

<sup>2</sup>See <http://www.eea.europa.eu/publications/trends-and-projections-2013> for further details.

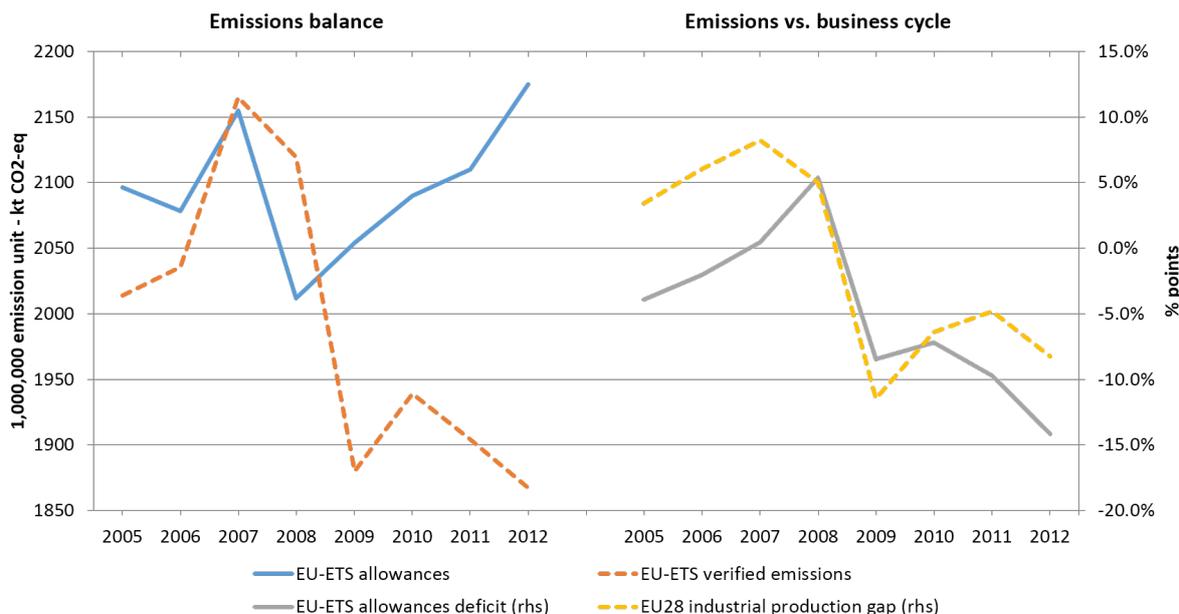


Figure 1: Emission gap vs. business cycles.

based instruments should be preferred. Recently, specific attention has been devoted to the study of this very same question (analysis of the relative merits of carbon taxes versus quantity instruments) in more realistic settings by means of real business cycle models. Alternative mechanisms as well as allocation rules are typically investigated by means of single sector (Fischer and Springborn, 2011, and Heutel, 2012) or a multi-sector frameworks Dissou and Karnizova (2012). Heutel (2012) finds that optimal emissions are pro-cyclical and that the optimal emission policy should respond accordingly to economic fluctuations and cycles. Angelopoulos et al. (2010) find that the cap policy is always the worst in terms of lifetime utility. Different policy tools are evaluated in Fischer and Springborn (2011), where the authors find that a cap system would achieve a given emission reduction with a slightly higher welfare cost than the tax, but it would ensure that the cut is achieved without lag, resulting in higher welfare if these additional reductions are valued; the cap system also features a lower level of labor variance than all other policies considered.

In the present paper we start by highlighting some key differences in the face of business cycle uncertainty of price versus quantity based instruments by means of a very simplified model, in Section 2. Section 3 then removes most of these simplifying assumptions and presents a dynamic stochastic general equilibrium model that we use to extend our analysis. Section 4 describes the calibration process and data while Section 5 reports results. In Section 6 we conclude.

## 2 A toy model

Let us first introduce a very simplified model that we employ to present on of the main mechanisms behind the results of the more complex model presented in Section 3. There exists a representative household who directly produces a single homogenous final good, via a constant-returns-to-scale production function, using an exogenous endowment of capital,  $k$ , and energy,  $e$ . Energy is imported from abroad: its supply is perfectly elastic, its international price  $p$  is given, and trade is balanced by assumption. The production function is of the ‘‘Cobb-Douglas’’ form:  $q = Ak^{1-\gamma}e^\gamma$ , where  $\gamma \in (0, 1)$  is the share of energy in output and  $A$  denotes Total Factor Productivity (*TFP*). For the sake of simplicity, assume that  $k = 1$  and model *TFP* as a discrete random variable, such that  $A \in \{a_1, a_2\}$ , where  $a_2 > a_1 > 0$  and the probability of each of the two states to occur is given by  $P(a = a_i) = \omega_i$ , which realizes after  $k$  has been installed but before  $e$  is chosen. Assume that emissions are proportional to the use of energy, and choose units of emissions such that the quantity of emissions is equal to  $e$ .

The representative household observes  $k$ , and optimally chooses its state-contingent plan for consumption and energy use before the uncertainty about  $A$  resolves, maximizing the following expected utility function:<sup>3</sup>

$$U = \mathbb{E} [\ln (c_i)] = \sum_{i=1}^2 \omega_i \ln (c_i),$$

where  $c_i$  denotes state-contingent consumption, under the budget constraint  $c_i \leq \pi_i + g_i$ , where  $\pi_i$  denotes profits and  $g_i$  per-capita government lump-sum transfers.

### 2.1 No-policy case

Absent any climate policy, household’s profits  $\pi_i = q_i - pe_i$ , and the government budget constraint implies  $g_i = 0$ . It can easily be shown that the optimal state-contingent demand functions for energy boil down to  $e_i^{NP} = (\gamma a_i / p)^{\frac{1}{1-\gamma}}$ . Hence, the *ex-ante* expected level of emissions, denoted  $\bar{e}$ , can be expressed as:

$$\mathbb{E}(e) = \bar{e} \equiv \left(\frac{\gamma}{p}\right)^{\frac{1}{1-\gamma}} \sum_{i=1}^2 \omega_i a_i^{\frac{1}{1-\gamma}}.$$

In equilibrium, state-contingent consumption will depend on state-contingent energy use:  $c_i = q_i - pe_i = \frac{1-\gamma}{\gamma} pe_i$ . The household’s expected utility level in the no-policy case

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<sup>3</sup>Emissions do not explicitly affect the household welfare level in our formulation: given the purpose of our analysis, this turns out to be irrelevant.

is therefore given by:

$$U_{NP} = \frac{\sum_{j=1}^2 \omega_j \ln(a_j)}{1 - \gamma} + \ln(1 - \gamma) - \frac{\gamma}{1 - \gamma} \ln\left(\frac{p}{\gamma}\right).$$

In order to reduce emissions, the government faces two possible alternatives: setting up cap-and-trade scheme or imposing a carbon tax.

## 2.2 Carbon tax

If the government imposes a carbon tax,  $\tau_E \geq 0$ , then  $\pi_i = q_i - (1 + \tau_E)pe_i$ , and  $g_i = \tau_E pe_i$ . In this case, the *ex-ante* expected level of emissions, denoted  $\hat{e}_{TAX}$ , for a given value of  $\tau_E$  can be expressed as:

$$\mathbb{E}(e | \tau_E) = \hat{e}_{TAX} \equiv \left(\frac{1}{1 + \tau_E}\right)^{\frac{1}{\gamma-1}} \bar{e}.$$

In equilibrium, state-contingent consumption equals  $c_i = \left(\frac{1-\gamma+\tau_E}{\gamma}\right)pe_i$ , and the household's expected utility for a given value of  $\tau_E$  drops to:

$$U_{TAX} = U_{NP} + \ln\left(1 + \frac{\tau_E}{1 - \gamma}\right) - \frac{1}{1 - \gamma} \ln(1 + \tau_E). \quad (1)$$

It is straightforward to show that, not surprisingly,  $\partial U_{TAX} / \partial \tau_E < 0$ . Indeed, in our toy model we are not considering any distortionary pre-existing tax, that might be reduced with revenues from carbon taxation thus leading to a positive welfare effect of the policy (as in Bovenberg and Goulder, 1996, and Bovenberg and van der Ploeg, 1994); nor we consider environmental welfare implication of the climate policy. Furthermore, note that, if  $\tau_E < 1 - \gamma$ , then<sup>4</sup>  $U_{TAX} - U_{NP} \approx -\frac{\gamma\tau_E^2}{2(1-\gamma)^2} < 0$ .

## 2.3 Cap and trade

Alternatively, the government imposes an exogenous cap on emissions, denoted  $m$ , so that  $e_i \leq m$  in both states of the world. In this case,  $\pi_i = q_i - pe_i$ , and  $g_i = 0$ .<sup>5</sup> Assume that  $m$  is such that the cap is binding only in the state of the world characterized by the highest productivity: hence,  $e_1 = (\alpha a_1 / p)^{\frac{1}{1-\gamma}}$ ,  $e_2 = m$ ,  $\mu_1 = 0$  and  $\mu_2 = \gamma a_2 m^{\gamma-1} - p$ , where  $\mu_i$  denotes the multipliers associated with the emissions constraint. Furthermore,

<sup>4</sup>Recall that, if  $x \in (-1, 1)$ , then  $\ln(1 + x) \approx x - \frac{1}{2}x^2$ .

<sup>5</sup>An equivalent setting would be the following: the government sells emission permits at the unit price  $z > 0$  before uncertainty resolves, and the representative households purchases the optimal number of permits, denoted  $m$ , knowing that, once uncertainty is resolved, the emission constraint  $e_i \leq m$  will be potentially binding. Government revenues,  $g_i = zE$ , are as before paid back via lump-sum transfers. In equilibrium, the price of emission permits, for a given cap  $m$ , is equal to  $z = \mu_2 \left(\frac{\omega_2}{c_2} / \sum_{i=1}^2 \frac{\omega_i}{c_i}\right)$ .

we impose that the expected level of emissions obtained under the emission cap is equal to the expected level obtained under the carbon tax; this requires that:

$$(1 + \tau_E)^{-\frac{1}{1-\gamma}} = \phi + (1 - \phi) \left(1 + \frac{\mu_2}{p}\right)^{-\frac{1}{1-\gamma}}, \quad (2)$$

where  $\phi \equiv \omega_i a_i^{\frac{1}{1-\gamma}} / \left(\sum_{j=1}^2 \omega_j a_j^{\frac{1}{1-\gamma}}\right) \in (0, 1)$ . The previous condition can be approximated as follows:  $\mu_2/p \approx \tau_E / (1 - \phi)$ .<sup>6</sup>

In equilibrium, state-contingent consumption equals  $c_i = \frac{(1-\alpha)p+\mu_i}{\alpha} e_i$ , and the household's expected utility for a given value of  $m$  drops to:

$$U_{CAP} = U_{NP} + \omega_2 \left[ \ln \left(1 + \frac{\mu_2}{p}\right) - \frac{1}{1-\gamma} \ln \left(1 + \frac{\mu_2}{p}\right) \right]. \quad (3)$$

Note that, if  $\frac{\mu_2}{p} < 1 - \gamma$ , then  $U_{CAP} - U_{NP} \approx -\frac{\omega_2 \gamma (\mu_2/p)^2}{2(1-\gamma)^2} < 0$ .

## 2.4 Comparison

Figure 2 provides a graphical representation of the two scenarios, in case of linear energy demand functions and two equiprobable states of the world. Absent any climate policy, the equilibrium would be characterized by two state-contingent optimal emission levels,  $e_i$ , and an expected emission level equal to  $\bar{e} = (e_1 + e_2) / 2$ .

The upper panel of Figure 2 compares the no-policy case with the outcome of imposing a carbon tax  $\tau_E > 0$ : the tax lowers emissions in both states of the world, and reduces the expected level of emissions to  $\hat{e}$ . Note furthermore that the volatility of emissions (and therefore output) is essentially unaffected by the carbon tax. The lower panel, instead, compares the no-policy case to the outcome of a cap-and-trade scheme that imposes an exogenous upper bound, denoted  $m$ , on the level of emissions, such that their expected level is equal to  $\hat{e}$ , the one obtained under carbon taxation. In order to lower the expected level of emissions, we just need the cap to be binding in one of the two states of the world, characterized by the highest TFP level. Hence, the reduction in expected emissions is accompanied by a drop in the volatility of emissions themselves, and of output and consumption as a consequence. Note furthermore that, under the cap-and-trade scheme, the equilibrium allocation has to deviate from the Pareto-efficient one, i.e. the no policy case, only in one of the two possible states of the world, while the carbon tax implies a deviation in both states of the world.

Combining (1) and (3), we conclude that  $|U_{CAP} - U_{NP}| \lesssim |U_{TAX} - U_{NP}|$  if  $\mu_2/p < \tau_E/\omega_2^2$ . Condition (2) implies that  $\mu_2/p \approx \tau_E / (1 - \phi)$ . Hence, if  $\phi$  is sufficiently smaller than  $1 - \omega_2^2$ , then the *CAP* scheme reaches the same decrease in expected emissions as

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<sup>6</sup>Note that  $(1 + x)^{-n} \approx 1 - nx$  if  $n > 0$  and  $x \in (-1, 1)$ .

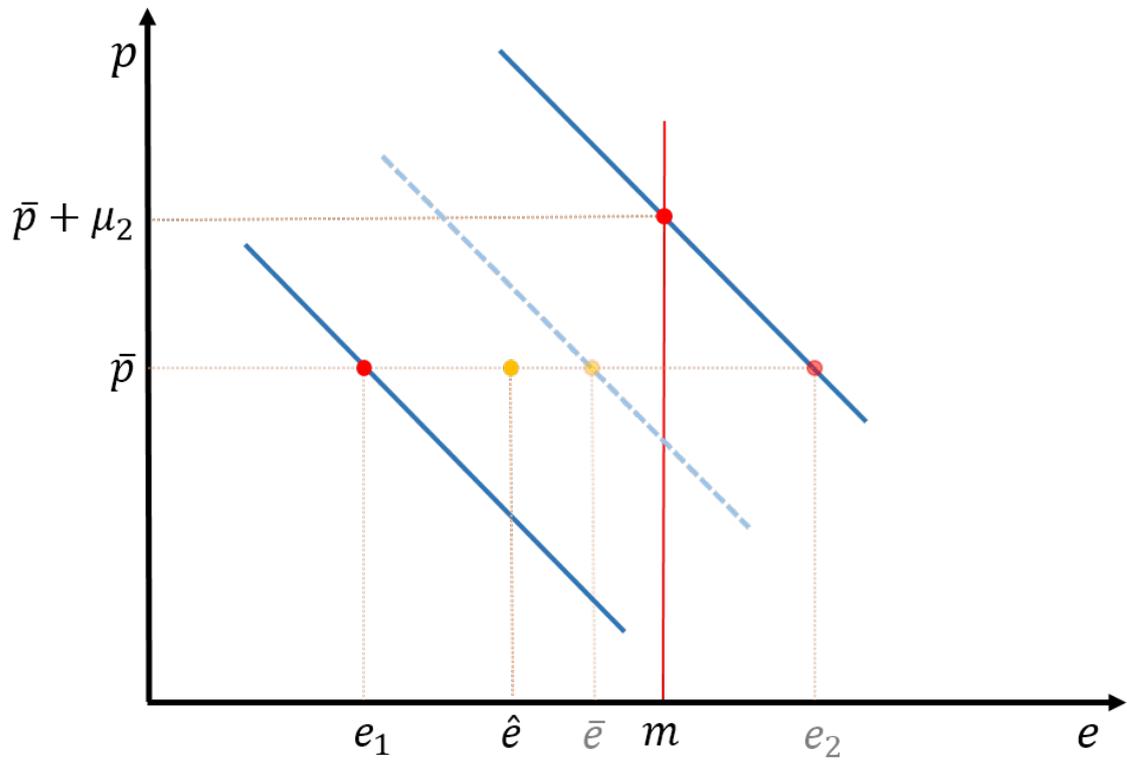
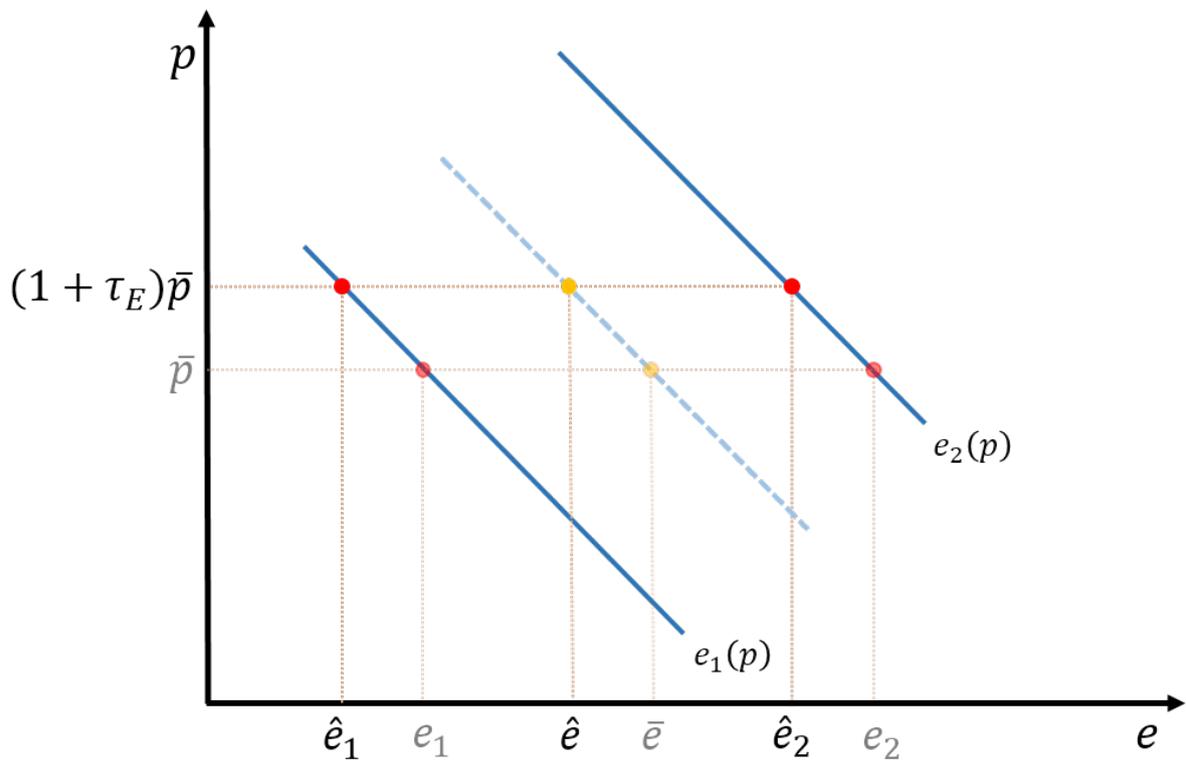


Figure 2: Example of equilibrium effects of environmental policies.

the *TAX* scheme, but with a smaller reduction in expected utility.<sup>7</sup>

The intuition of this result can be easily read in the light of Weitzman (1974). Rather than an analysis of optimal taxation, we are investigating the optimal instrument under uncertainty when the environmental target, in terms of emission reduction, is given. Uncertainty affects marginal abatement costs, as it shocks economic growth and the price of energy. Marginal damage is represented by a vertical line, i.e. most likely steeper than the slope of marginal abatement costs. Thus quantity based instruments, will tend to be more efficient, the more the likelihood of instances of shocks to the economy of the price of energy that make that induce emissions below the given target.

### 3 The model

Time is discrete, indexed by  $t \in \{0, 1, \dots, \infty\}$ . There exists a continuum of *ex-ante* identical and infinitely lived households, with total mass equal to one. Households own both factors of production, capital and labor. Firms, directly owned by the households, produce a single homogenous final good competitively, via a constant-returns-to-scale production function, using capital, labor and energy. Energy is bought on the international market and firms are price takers. Depending on the scenario, the government can rise the price of energy through an environmental market based policy. The final good can be used for consumption and investment. Asset markets are complete. The next Sections will describe the model components more in detail.<sup>8</sup>

#### 3.1 Households

Each household owns a single private firm. Firms employ labor and purchase intermediate goods in competitive markets but use the capital stock accumulated by the respective owner. The capital income of a generic household, excluding the non-depreciated capital stock installed in the firm, is given by the firm's earnings net of factor costs:

$$\pi_t = q_t - (1 + \tau_N) w_t n_t - (1 + \tau_E) p_t e_t, \quad (4)$$

where:

$$q_t \equiv \phi_t \left( k_t^\alpha n_t^{1-\alpha} \right)^{1-\gamma} e_t^\gamma, \quad (5)$$

denotes the firm's output,  $k_t$  the stock of capital in place at the beginning of period  $t$ ,  $n_t$  the amount of labor hired,  $w_t$  the wage rate,  $p_t$  the price of energy,  $\tau_E$  the carbon tax, when we assume a price mechanism is adopted,  $\tau_N$  is the payroll tax,  $\phi_t$  the level of

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<sup>7</sup>Note that, if  $\omega_i = \frac{1}{2}$ , then  $1 - \omega_2^2 = \frac{3}{4}$ , and the previous condition is most likely satisfied, because  $\omega_1 = \omega_2$  and  $a_2 > a_1$  imply  $\phi < \frac{1}{2}$ .

<sup>8</sup>The model is a simplified representative-agent version of the framework developed in Bosetti and Maffezzoli (2013), with a few twists.

aggregate productivity (common to all households),  $\gamma$  the share of energy in gross output, while  $\alpha$  and  $1 - \alpha$  are respectively the share of capital and labor in value added. The aggregate productivity level and the international price of energy are subject to aggregate shocks: in particular, we assume that the logs of  $\phi_t$  and the log of  $p_t$  follow stationary discrete Markov processes, characterized by transition matrices  $\pi_\phi$  and  $\pi_p$ , and we assume  $\mathbb{E}(\phi) = 1$  for normalization purposes. The aggregate shocks are realized at the beginning of period  $t$ , after capital is installed but before labor  $n_t$  and  $e_t$  are chosen.

We assume that emissions at the firm level are proportional to the use of energy,  $e_t$ , and units of emissions are chosen such that the quantity of emissions is equal to  $e_t$ . Firms face an emission constraint which requires that:

$$e_t \leq m_t, \quad (6)$$

where  $m_t$  is the stock of emissions permits purchased (or set aside when banking is allowed for) in the previous period and available for use at the beginning of period  $t$ .

### 3.1.1 The optimization problem

Household's preferences over stochastic consumption streams are given by:

$$u_t \equiv \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{\left( c_s - \kappa \frac{l_s^{1+\eta}}{1+\eta} \right)^{1-\sigma} - 1}{1-\sigma} \right] \right\}, \quad (7)$$

where  $c_t$  is the consumption level,  $l_t$  the share of time devoted to labor,  $\beta \in (0, 1)$  the intertemporal discount factor,  $\sigma > 0$  the reciprocal of the elasticity of intertemporal substitution, and  $\eta > 0$  a parameter linked to the Frisch elasticity of labor supply.

The stock of physical capital evolves over time according to the following accumulation equation:

$$k_{t+1} = (1 - \delta_K) k_t + d_t - z_t [x_t - \chi(k_t)] - c_t, \quad (8)$$

where  $d_t \equiv (1 - \tau_Y)(\pi_t + w_t l_t) + G_t$  denotes disposable income; note that  $\tau_Y$  represents a proportional tax rate on income,  $G_t \geq 0$  the per-capita government lump-sum transfers,  $\delta_K \in [0, 1]$  a physical depreciation rate,  $z_t$  the price of emissions permits,  $x_t$  the total number of emission permits withheld in period  $t$ , and  $\chi(k_t)$  the number of permits obtained through grandfathering, that possibly depend on the size of the firm (hence,  $x_t - \chi_t$  represents the number of permits purchased, or sold, in the period). The stock of emission permits evolves according to:

$$m_{t+1} = (1 - \delta_M)(m_t - e_t) + x_t. \quad (9)$$

The parameter  $\delta_M$  determines whether banking of permits is allowed: if  $\delta_M = 1$ , then

no banking is allowed, and the emission constraint reduces to  $e_{t+1} \leq x_t$ ; if  $\delta_M = 0$ , then banking is allowed, and permits can be set aside forever; finally, if  $\delta_M \in (0, 1)$ , then banking is still allowed, but permits have a finite half-life.

We can now put all the elements together; for given sequences of factor prices, the dynamic optimization problem of a generic household is as follows:

$$\begin{aligned} \max_{\{c_s, l_s, n_s, e_s, k_{s+1}, m_{s+1}\}_{s=t}^{\infty}} \quad & \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{\left( c_s - \kappa \frac{l_s^{1+\eta}}{1+\eta} \right)^{1-\sigma} - 1}{1-\sigma} \right] \right\}, \\ \text{s.t.} \quad & k_{t+1} = (1 - \delta_K) k_t + d_t - z_t (x_t - \chi_t) - c_t, \\ & m_{t+1} = (1 - \delta_M) (m_t - e_t) + x_t, \\ & e_t \leq m_t. \end{aligned} \tag{10}$$

## 3.2 Aggregate variables

### 3.2.1 The Government

The government might choose a price or a quantity instrument to regulate carbon emissions. In the first case, the government sets a carbon tax which acts effectively as a sales tax on energy imports. In the second case, the government sets the initial number of emission permits,  $M_0$ , and allocate them to the households for free according to a preferred rule (permits can obviously be auctioned and we do analyze the implication of alternative ways of allocating permits in Bosetti and Maffezzoli, 2013). At the beginning of each period, the government issues  $X_t$  new emission permits and sells  $(X_t - \chi_t) \geq 0$  of them on the secondary market at the price  $z_t$ . Apart from this, the government plays a minimalist role, collecting tax revenues, selling permits, and paying everything back to the households via lump-sum sum transfers<sup>9</sup> (capital letters denote aggregate variables):

$$G_t = \tau_Y Y_t + \tau_Y w_t (L_t - N_t) + (1 - \tau_Y) (\tau_N w_t N_t + \tau_{EP_t} E_t) + z_t (X_t - \chi_t), \tag{11}$$

where  $Y_t = Q_t - p_t E_t$  denotes GDP.

### 3.2.2 Energy

Energy is imported from abroad, at a given international price  $p_t$ , and its supply is perfectly elastic. In other words, our economy can be characterized as a small open economy in the international market for energy; however, recall that households do not have access to international financial markets, and can only invest in physical capital. This implies that trade is balanced by assumption: energy imports are financed via final

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<sup>9</sup>More complex ways of redistributing the revenues from the carbon policy and their implications are discussed in Bosetti and Maffezzoli (2013).

good exports.

### 3.3 Equilibrium

In equilibrium, the demand and supply of labor coincide:  $N_t = L_t$ . In the Appendix we describe the equilibrium conditions in detail. It seems useful to highlight here that, thanks to the ‘‘Cobb-Douglas’’ form of the production function, we can easily solve for energy and hours as functions of the capital stock. We start by expressing hours as a function of capital and energy:

$$N_t = \left( \frac{1 - \tau_Y}{1 + \tau_N} \frac{s_N}{\kappa} \phi_t K_t^{s_K} E_t^\gamma \right)^{\frac{1}{\eta + 1 - s_N}}. \quad (12)$$

Then, we can express energy as a function of the capital stock alone:

$$E_t = \min \left\{ \left[ \frac{\gamma (\phi_t K_t^{s_K})^\xi \left( \frac{1 + \tau_N}{1 - \tau_Y} \frac{\kappa}{s_N} \right)^{1 - \xi}}{(1 + \tau_E) p_t + \frac{z_t(1 - \delta_M)}{1 - \tau_Y}} \right]^{\frac{1}{1 - \gamma \xi}}, M_t \right\}, \quad (13)$$

where  $\xi \equiv (\eta + 1) / (\eta + 1 - s_N)$ .

## 4 Calibration

The parameters that characterize household’s preferences are selected in the following way: the intertemporal discount factor and the reciprocal of the elasticity of intertemporal substitution are set to standard values in the literature,  $\beta = 0.985$  and  $\mu = 2$ . There is still no general consensus on how to parametrize the elasticity of labor supply in macro models, due to the somehow conflicting empirical evidence at the macro (where large estimates are typically obtained) and micro level (where the estimates tend to be much lower). However, Reichling and Whalen (2012) report that the Congressional Budget Office incorporates into its analysis an estimate of the Frisch elasticity of labor supply that ranges from 0.27 to 0.53. Bargain et al. (2013) perform a large-scale international comparison of labor supply elasticities for 17 European countries and the US, and report own-wage elasticities that range from 0 to 0.65 for women and single men, and from 0 to 0.2 for married men. Jäntti et al. (2013) obtain broadly comparable results with a different methodology and sample, and show that macro estimates on the same data are not far from the micro ones. The previously cited studies suggest that elasticities higher than unity are unlikely, in particular for many European countries. Therefore, we set  $\eta = 1.9$  in order to make the model reproduce a Frisch elasticity equal to 0.53, the upper limit in the CBO estimated range. OECD (2009) considers 18 OECD countries and reports the 24-hour breakdown of time spent in main activities for individuals aged 15 and over: on

Parameter	Value	Parameter	Value	Parameter	Value
$\beta$	0.985	$\alpha$	0.33	$\rho_\phi$	0.95
$\sigma$	2	$\gamma$	8.26%	$\sigma_\phi$	0.070
$\eta$	1.9	$\tau_N$	19%	$\rho_p$	0.73
$\xi$	5.293	$\tau_Y$	27%	$\sigma_p$	0.124
$\delta$	0.025	$\bar{p}$	0.73		

Table 1: Summary of the benchmark parametrization.

average, those individuals devote 67% of their time to leisure and personal care. Hence, we calibrate  $\xi$  so that the average number of hours worked, in absence of climate policies, correspond to 33% of the time endowment.

The depreciation rate is set to  $\delta = 0.025$ , while the share of capital in value added,  $\alpha$ , is assumed to be 0.33: both values are standard in the literature. The share of energy in gross output,  $\gamma$ , is calibrated in order to make the model reproduce, in absence of climate policies, a share of total energy expenditure in GDP equal to 9%, a figure in line with the empirical evidence for EU countries.

Using data for 27 EU countries in 2012 provided in Rogers and Philippe (2012), we compute the cross-country average Social Security payroll tax rate and the average income tax rate, equal respectively to  $\tau_N = 19\%$  and  $\tau_Y = 27\%$ .

As far as the supply of emission permits is concerned, we assume that:

$$X_t = \bar{X} + v_1 K_t, \quad (14)$$

$$\chi_t = v_2 X_t. \quad (15)$$

In our benchmark parametrization, we assume that permits are fully auctioned, and set  $v_1 = 0$ ,  $v_2 = 0$ , and  $\bar{X}$  equal to the desired level of aggregate emissions.

The log of the aggregate productivity level is assumed to follow an AR(1) process of the form:

$$\begin{aligned} \ln \phi_{t+1} &= \rho_\phi \ln \phi_t + \epsilon_{\phi,t+1}, \\ \epsilon_{\phi,t} &\sim N(0, \sigma_\phi^2). \end{aligned} \quad (16)$$

Following Cooley and Prescott (1995), we set we set  $\rho_\phi = 0.95$  and  $\sigma_\phi = 0.07$ . Similarly, the log of the international price of energy follows:

$$\begin{aligned} \ln p_{t+1} &= \ln(\bar{p}) + \rho_p \ln p_t + \epsilon_{p,t+1}, \\ \epsilon_{p,t} &\sim N(0, \sigma_p^2). \end{aligned} \quad (17)$$

We take the quarterly average imported crude oil price for the 1974:I-2013:II period, as published by the U.S. Energy Information Administration (EIA) in its *Short-Term Energy*

*Outlook*, as a proxy for the overall price of energy  $p_t$ : we apply the Hodrick-Prescott filter (with the smoothing parameter equal to 1600) to the time series and estimate (17) on the cyclical component. The estimated parameters values are  $\rho_p = 0.73$  and  $\sigma_p = 0.124$ . Both stochastic processes are then approximated with a 5-state discrete Markov chain computed using Rouwenhorst’s method, as suggested in Kopecky and Suen (2010).

Finally, the average price  $\bar{p}$  is calibrated to make the model reproduce in steady state, again in absence of climate policies, the average energy intensity of GDP at constant purchasing power parities (expressed in koe/\$2005p) for EU countries over the 2002-12 period, equal to 0.128, computed using data from the *Global Energy Statistical Yearbook 2013* published by *Enerdata*. The parameter constellation is summarized in Table 1.

The model is solved using fully non-linear methods: the policy functions are computed using the Euler equation approach discussed in Rendhal (2013), while the ergodic distribution of the endogenous state variables is obtained with the binning approach discussed in Young (2010) and extended to the bivariate case in Maffezzoli (2011).<sup>10</sup>

## 5 Results

In order to evaluate the implications of alternative carbon policies we discuss six scenarios. All of them, but the no policy case, are calibrated in order to have the same environmental effect, on average, that is a 10% reduction in emissions with respect to the no policy case. The European Commission claimed that the EU-ETS managed to reduce overall emission by 8.3% over the 2005-10 period.<sup>11</sup> This is a rather modest target when compared to the more challenging targets that were advocated for at the Copenhagen Climate Change Conference of Parties in 2009; Still it is more aggressive mitigation action than most nations of the world are currently doing or planning to do. A 10% reduction *vis a vis* the no policy case in 2011 translates into a 23% (27%) cut with respect to 1990 emissions if we account for the fact that greenhouse gasses emissions have been decreasing by 15% (18%) in the EU15 (EU27) in 2011 with respect to 1990. This is actually more than the 2020 goal for the European Union that is to reduce emissions by 20% with respect to 1990 emission levels.

When looking at the welfare implications of each scenario, the reader should bear in mind an important caveat, that is we are not including in the analysis environmental welfare implications of the policy. This would be problematic if we were to compare the no policy case with the climate policy scenarios or if our aim were that of computing the optimal climate policy. However, we are working under the assumption that the level of

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<sup>10</sup>We use a grid of 1000 nodes for capital and 200 nodes for the stock of permits: further increasing the density of the grid has no significant impact on the results. The policy functions are approximated via multivariate linear interpolation.

<sup>11</sup>See [http://ec.europa.eu/clima/publications/docs/factsheet\\_ets\\_emissions\\_en.pdf](http://ec.europa.eu/clima/publications/docs/factsheet_ets_emissions_en.pdf) for further details.

commitment, in terms of emission reduction, will most likely be the result of some national/international political process and it is given. Our goal is that of evaluating welfare implications of imposing such policy by means of different instruments and alternative property rights allocations.

The six scenarios discussed in the next sections are the following:

1. **NoPolicy:** this is the benchmark model where no climate policies are in place, i.e.  $\tau_E = 0$  and  $M_t = \infty$ .
2. **Tax:** the government uses a price instrument to limit emissions, thus  $M_t = \infty$  and no market for emission permits is in place. The carbon tax is calibrated in order to achieve a 10% decrease in emissions in steady state (the resulting tax is  $\tau_E = 0.0915$ ). Revenues are rebated through a lump-sum transfer to the household.
3. **Quantity - Cap (*fully auctioned permits*):** the government chooses a cap to limit emissions in line with previous scenarios, in average, but now the constraint on emissions,  $E_t \geq M_t$ , is *not necessarily* binding in equilibrium. Banking of unused permits is not allowed, i.e.  $\delta_M = 0$ . Again, the calibrated level of  $M_t$  is constant over time and equal to 0.0653. Permits are fully auctioned.
4. **Quantity - Banking (*fully auctioned permits*):** the government adopts a quantity instrument to limit emissions in line with previous scenarios, in average; this times banking of unused permits is allowed, with (arbitrarily)  $\delta_M = 0.25$ . As in the previous scenarios, the calibrated level of  $M_t$  is constant and equal to 0.0660. Permits are fully auctioned.
5. **Quantity - Cap (*output-based allocated permits*):** the government chooses a cap to limit emissions in line with previous scenarios, in average, but now the constraint on emissions,  $E_t \geq M_t$ , is *not necessarily* binding in equilibrium. Banking of unused permits is not allowed, i.e.  $\delta_M = 0$ . Currently, in the EU-ETS only 5% of permits are auctioned, hence we set  $v_2 = 0.95$ ; the remaining permits are allocated following an output-based rule, i.e. in our case proportionally to the installed capital stock. We calibrate the proportionality parameter  $v_1$  in order to make the model replicate the desired long-run level of emissions: the calibrated value is 0.01999.
6. **Quantity - Banking (*output-based allocated permits*):** the government adopts a quantity instrument to limit emissions in line with previous scenarios, in average; this times banking of unused permits is allowed, with (arbitrarily)  $\delta_M = 0.25$ . As in the previous scenarios, we set  $v_2 = 0.95$ ; the proportionality parameter  $v_1$  is set to 0.02023.

<i>Unconditional mean</i>						
		FULLY AUCTIONED			OUT. BASED.	
	<i>NoPol.</i>	<i>Tax</i>	<i>Cap</i>	<i>Bank.</i>	<i>Cap</i>	<i>Bank.</i>
<b><i>Output (Q)</i></b>	<b>0.572</b>	0.562	0.563	0.564	0.570	0.570
<i>%Δ from NoPolicy</i>		-1.78%	-1.56%	-1.54%	-0.36%	-0.37%
<b><i>GDP (Y)</i></b>	<b>0.525</b>	0.520	0.520	0.520	0.527	0.527
<i>%Δ from NoPolicy</i>		-1.04%	-0.95%	-0.92%	0.34%	0.35%
<b><i>Cons. (C)</i></b>	<b>0.446</b>	0.442	0.442	0.443	0.447	0.447
<i>%Δ from NoPolicy</i>		-0.91%	-0.85%	-0.80%	0.12%	0.15%
<b><i>Investment</i></b>	<b>0.079</b>	0.077	0.078	0.078	0.080	0.080
<i>%Δ from NoPolicy</i>		-1.76%	-1.53%	-1.57%	1.60%	1.50%
<b><i>Gov. rev. (G)</i></b>	<b>0.183</b>	0.183	0.183	0.183	0.183	0.183
<i>%Δ from NoPolicy</i>		0.36%	0.16%	0.18%	0.26%	0.26%
<b><i>Capital (K)</i></b>	<b>3.154</b>	3.098	3.106	3.108	3.204	3.204
<i>%Δ from NoPolicy</i>		-1.77%	-1.53%	-1.48%	1.59%	1.58%
<b><i>Hours (N)</i></b>	<b>0.330</b>	0.328	0.328	0.328	0.330	0.330
<i>%Δ from NoPolicy</i>		-0.62%	-0.54%	-0.53%	-0.12%	-0.12%
<b><i>Energy (E)</i></b>	<b>0.067</b>	0.061	0.061	0.061	0.061	0.061
<i>%Δ from NoPolicy</i>		-10%	-10%	-10%	-10%	-10%
<b><i>Price of Per. (z)</i></b>			0.035	0.034	0.042	0.040
<b><i>Banked Per. (M)</i></b>				0.082		0.077
<b><i>CEV</i></b>		-0.61%	-0.58%	-0.53%	0.19%	0.22%
<b><i>Prob. of the cap being binding</i></b>			<b>50.3%</b>	<b>25.7%</b>	<b>67.0%</b>	<b>26.9%</b>

Table 2: Stochastic properties of the main variables I: unconditional means.

## 5.1 Long-run properties

A summary of results is presented in Tables 2 and 3, where each policy scenario corresponds to a separate column and each row represents an aggregate macro economic variable (we report both absolute values and percentage deviations from the NoPolicy case). In particular, we report the unconditional means of the variables (i.e. their “steady-state” values) in Table 2 and their volatilities, as measured by the standard deviation, in Table 3.<sup>12</sup> Furthermore we report in Table 2 the *Consumption Equivalent Variation*<sup>13</sup> (CEV) with respect to the NoPolicy case, and the probability of the emission constraint being binding, computed from the ergodic distribution of the model.

All policy simulations, but for those based on output based allocation, imply a decrease

<sup>12</sup>The statistics are computed directly from the model’s ergodic distribution: hence, the small-sample bias problem that affects alternative solution procedures is absent here.

<sup>13</sup>The CEV is computed in the following way: denote as  $C_0$ ,  $N_0$ , and  $\lambda_0$  the policy functions and the ergodic distribution in the NoPolicy case, and as  $V_1$  and  $\lambda_1$  the value function and ergodic distribution in one of the alternative scenarios. Define, for a given scalar  $\theta$ , the value function  $V_0(\theta) = E_0 \{ \sum_{t=0}^{\infty} \beta^t U[(1+\theta)C_{0,t}, N_{0,t}] \}$ . Then, solve for a  $\theta$  such that  $\int V_0(\theta) d\lambda_0 = \int V_1 d\lambda_1$ . Note that we do not condition on any particular current state, and therefore we place ourselves “behind the veil of ignorance.”

<b>Volatility (Std. Dev.)</b>						
		FULLY AUCTIONED			OUT. BASED	
	<i>NoPol.</i>	<i>Tax</i>	<i>Cap</i>	<i>Bank.</i>	<i>Cap</i>	<i>Bank.</i>
<b>Output (Q)</b>	<b>0.035</b>	0.034	0.030	0.030	0.032	0.032
<i>%Δ from NoPolicy</i>		-1.58%	-13.15%	-13.01%	-7.40%	-7.08%
<b>GDP (Y)</b>	<b>0.032</b>	0.032	0.030	0.030	0.031	0.031
<i>%Δ from NoPolicy</i>		-0.85%	-6.53%	-6.65%	-4.08%	-3.61%
<b>Cons. (C)</b>	<b>0.022</b>	0.022	0.021	0.021	0.021	0.021
<i>%Δ from NoPolicy</i>		-0.59%	-4.94%	-5.80%	-2.56%	-2.38%
<b>Investment</b>	<b>0.012</b>	0.012	0.011	0.011	0.011	0.012
<i>%Δ from NoPolicy</i>		-1.23%	-9.13%	-8.31%	-6.09%	-4.93%
<b>Gov. rev. (G)</b>	<b>0.011</b>	0.011	0.012	0.012	0.011	0.011
<i>%Δ from NoPolicy</i>		0.54%	6.13%	5.68%	-4.69%	-4.24%
<b>Capital (K)</b>	<b>0.234</b>	0.230	0.221	0.217	0.225	0.226
<i>%Δ from NoPolicy</i>		-1.37%	-5.46%	-7.19%	-3.85%	-3.08%
<b>Hours (N)</b>	<b>0.007</b>	0.007	0.006	0.006	0.006	0.006
<i>%Δ from NoPolicy</i>		-0.43%	-11.98%	-11.83%	-7.07%	-6.78%
<b>Energy (E)</b>	<b>0.014</b>	0.013	0.007	0.007	0.007	0.008
<i>%Δ from NoPolicy</i>		-10.01%	-51.18%	-47.98%	-47.91%	-46.11%
<b>Price of Per. (z)</b>			0.037	0.036	0.037	0.036
<b>Banked Per. (M)</b>				0.017		0.015

Table 3: Stochastic properties of the main variables II: standard deviations.

in welfare.<sup>14</sup> The carbon tax and the fully auctioned allowable permit systems imply welfare costs in the order of 0.5% – 0.6% of lifetime consumption, within the bounds of the EMF22 assessment for the EU 20/20/2020 policy costs done with a suite of CGE models (a welfare loss of 0.5–2.0% by 2020 as in Böhringer et al., 2009) and of estimates reported in Fischer and Springborn (2011). Both cap and trade systems outperform the tax in welfare terms. Here it is important to notice that, were we to report results assuming that the cap has to be always binding as in Fischer and Springborn (2011), Heutel (2012) and Angelopoulos et al. (2010), then welfare implications of the cap and trade systems would always be worse than that of the tax. Indeed, the assumption of a binding cap tantamount forcing emissions to be sub-optimal by means of a subsidy.

As reported in the last row of Table 2, Given assumptions in our analysis, the cap is not binding approximately half of the time, as economic performance is such that emissions are lower than the cap. When banking is allowed for the constraint on emissions,  $E_t \leq M_t$  becomes binding in only a quarter of simulations (again last row of Table 2), as permits can be rolled over to subsequent periods and used up when more needed, partially relaxing the constraint. Henceforth properly modeling the emission constraint can have important

<sup>14</sup>Interestingly, Bosetti and Maffezzoli (2013) do find a double dividend effect for a comparable price of carbon and most policy mechanisms when policies are evaluated using an heterogeneous agents model set up as opposed to the representative agent model.

effects on the relative merits of alternative policy mechanisms.

Although banking is clearly improving efficiency, given the implications for intertemporal flexibility, the game changer in terms of welfare implications of policies is the choice of permits allocation. This is in line with recent literature that shows how different allocations can have huge implications for overall policy costs, see for example Goulder et al. (2010). The last two columns of Table 2 report results for the output based allocation simulations, with and without banking. Under this allocation scheme, the environmental policy works as subsidy to investments, thus increasing capital accumulation and in turn overall output and welfare.

If we look into hours of work, as an important indicator of policy performance, we see that, although all policies imply a deteriorating effect, this is almost negligible under the output based allocation scheme.

Table 3 reports result on volatility of macroeconomic indicators. In general internalizing the external cost of energy works in the direction of stabilizing the economy and reducing volatility of most macroeconomic indicators as well as energy demand. This is more pronounced under the quantity based than the price based instruments.

## 6 Conclusions

The literature that has so far performed welfare analyses of alternative market based instruments to reduce carbon emissions under the real business cycles (Fischer and Springborn, 2011, Heutel, 2012 and Angelopoulos et al., 2010) has typically considered a model framework where the emission constraint has to be binding. Although this should be the case for very stringent climate policies, we argue this might not be true under more realistic short term assumptions, also considering what has so far happened in the EU emission trading scheme. Thus, by allowing for the constraint on emissions to be occasionally binding, we can assess more appropriately the relative merits of different mechanisms. Emission volatility, thus being notably lower than in the TAX case (where it fluctuates with productivity shocks), is far from being null under a cap and trade system as well, as when economic conditions are bad, emissions can be lower than the cap.

In general, pricing carbon functions as a stabilizer, reducing volatility in all major macro economic indicators. This effect is more pronounced under an emission cap than under a carbon tax.

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## A Appendix

Being in equilibrium  $L_t = N_t$ , the equilibrium conditions can be combined and summarized as:

$$V_t = \left( C_t - \kappa \frac{N_t^{1+\eta}}{1+\eta} \right)^{-\sigma} \quad (18)$$

$$\kappa N_t^{\eta+1} = \frac{1 - \tau_Y}{1 + \tau_N} s_N Q_t, \quad (19)$$

$$\gamma \frac{Q_t}{E_t} = (1 + \tau_E) p_t + \frac{z_t (1 - \delta_M) + \tilde{\mu}_t}{1 - \tau_Y}, \quad (20)$$

$$V_t = \beta \mathbb{E}_t \left\{ V_{t+1} \left[ 1 - \delta_K + (1 - \tau_Y) s_K \frac{Q_{t+1}}{K_{t+1}} + z_{t+1} \chi'_{t+1} \right] \right\}, \quad (21)$$

$$z_t = \beta \mathbb{E}_t \left\{ \frac{V_{t+1}}{V_t} [z_{t+1} (1 - \delta_M) + \tilde{\mu}_{t+1}] \right\}, \quad (22)$$

$$K_{t+1} = (1 - \delta) K_t + Y_t - C_t, \quad (23)$$

$$M_{t+1} = (1 - \delta_M) (M_t - E_t) + X_t, \quad (24)$$

$$\tilde{\mu}_t (E_t - M_t) = 0 \quad (25)$$

$$E_t \leq M_t \quad (26)$$

$$\tilde{\mu} \geq 0 \quad (27)$$

where  $s_N \equiv (1 - \alpha)(1 - \gamma)$ ,  $s_K \equiv \alpha(1 - \gamma)$  and  $\tilde{\mu}_t \equiv \mu_t/v_t$ .  
 Note that (19) can be solved for  $N_t$ :

$$N_t = \left( \frac{1 - \tau_Y}{1 + \tau_N} \frac{s_N}{\kappa} \phi_t K_t^{s_K} E_t^\gamma \right)^{\frac{1}{\eta+1-s_N}}. \quad (28)$$

Imposing  $\tilde{\mu}_t = 0$ , we can combine (19) and (20) in order to get:

$$\tilde{E}_t = \left[ \frac{\gamma (\phi_t K_t^{s_K})^\xi \left( \frac{1 + \tau_N}{1 - \tau_Y} \frac{\kappa}{s_N} \right)^{1-\xi}}{(1 + \tau_E) p_t + \frac{z_t(1-\delta_M)}{1-\tau_Y}} \right]^{\frac{1}{1-\gamma\xi}}, \quad (29)$$

where  $\xi \equiv (\eta + 1) / (\eta + 1 - s_N)$ . If  $\tilde{E}_t < M_t$ , then  $E_t = \tilde{E}_t$  and  $\tilde{\mu}_t = 0$ ; otherwise,  $E_t = M_t$  and:

$$\tilde{\mu}_t = (1 - \tau_Y) \left[ \gamma \frac{Q_t}{E_t} - (1 + \tau_E) p_t \right] - z_t (1 - \delta_M). \quad (30)$$