Prospect Theory: An Analysis of Decision Under Risk

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Outline

Introduction

This paper presents a **critique of expected utility theory** as a descriptive model of decision making under risk, and develops an alternative model, called **prospect theory**

Published by *Econometrica* in 1979, it is one of the most cited papers in social sciences

In 2002, Daniel Kahneman received the Nobel Prize *for having integrated insights from psychological research into economic science, especially concerning human judgment and decision-making under uncertainty*. Amos Tversky died in 1996 and hence he could not be awarded

Expected Utility Theories

- In 1657, Christiaan Huygens states that the fair price to pay to participate in a game involving risk is equal to the **expected monetary value** of the game
- In the early 1710s, Nicholas Bernoulli presents the so-called **St. Petersburg paradox**, a lottery with an infinite expected value that seems to be worth only a very small amount to any sensible man
- In 1738, his cousin Daniel Bernoulli resolves the paradox proposing a new assumption: individuals evaluate risks taking into account the **expected utility** of the prospect
- He noticed that most people are risk averse and if they can choose between a gamble and an amount equal to its expected value they will pick the sure thing

Expected Utility Theories

In their seminal work *Theory of games* (1944) von Neumann and Morgenstern give an axiomatic foundation to Bernoulli's hypothesis

Preferences over lotteries are binary relations with the following features

- completeness
- transitivity
- continuity or archimedean property
- independence

Expected Utility Theories

Von Neumann and Morgenstern also proved the following theorem:

Theorem

A preference relation $\succ$ over lotteries satisfies the four axioms if and only if there exists a function u over the lottery outcomes such that:
Lottery A $\succ$ Lottery B if and only if

$$\sum_A u(x_i)p_i > \sum_B u(x_i)p_i$$

Expected Utility Theories

Four years later, in a fundamental article, Milton Friedman and Leonard Jimmie Savage introduced

- the well-known **graphical representation** of the utility function
- its interpretation in terms of **attitudes towards risk**
- its **as-if interpretation**

Nowadays expected utility theory is the foundation of rational-agent model and is one of the most important theories in social sciences

Critique

Expected Utility Theory

- **normative**: prescribes how decisions should be made
- **descriptive**: describes how Econs (rational and selfish individuals with stable tastes) make choice

Prospect Theory

- **purely descriptive**: describes how Humans make choice
- the paper presents several classes of decision problems in which preferences systematically violate the axioms of expected utility theory and an alternative model of decision making under risk

Framework

- Decision making under risk can be viewed as a choice between prospects or gambles
- A **prospect** $(x_1; p_1; ...; x_n, p_n)$ is a contract that yields outcome x_i with probability p_i, where $p_1 + p_2 + ... + p_n = 1$. Kahneman and Tversky's prospects correspond to von Neumann and Morgenstern's lotteries
- The analysis is restricted to prospects with **objective** probabilities
- The demonstrations are based on the responses of students and university faculty to hypothetical choice problems

Certainty, Probability and Possibility

The following series of choice problems induces preferences that **violate** a main tenet of expected utility theory

Consider the following pair of choice problems:

PROBLEM 1: Choose between

$(2500, 0.33; 2400, 0.66; 0, 0.01)$	(2400)
[18]	[82]*

PROBLEM 2: Choose between

$(2500, 0.33)$	$(2400, 0.34)$
[83]*	[17]

Certainty, Probability and Possibility

Here we have a simpler demonstration of the same phenomenon

PROBLEM 3: Choose between

$(4000, 0.80)$	(3000)
$[20]$	$[80]^*$

PROBLEM 4: Choose between

$(4000, 0.20)$	$(3000, 0.25)$
$[65]^*$	$[35]$

Certainty, Probability and Possibility

It is easy to verify that this pattern of preferences does not obey the independence axiom

The French economist Maurice Allais was the first one to propose this paradox in 1953

Certainty Effect

Overweighting of outcomes that are considered certain, relative to outcomes which are merely probable

This effect undermines the validity of the **independence axiom** for choice of risks bordering on certainty

Certainty, Probability and Possibility

The following problems present a situation in which the independence axiom fails, but not because of the certainty effect:

PROBLEM 7: Choose between

$(6000, 0.45)$	$(3000, 0.90)$
[14]	[86]*

PROBLEM 8: Choose between

$(6000, 0.001)$	$(3000, 0.002)$
[73]*	[27]

Certainty, Probability and Possibility

The results suggest the following empirical generalization concerning the manner in which the independence axiom is violated:

- if (y, pq) is equivalent to (x, p), then (y, pqr) is preferred to (x, pr), $0 < p, q, r < 1$

This property is incorporated in the new theory proposed by the authors in the second part of the paper

The Reflection Effect

What happens when we move from the domain of gains to the domain of losses? Look at the following table:

TABLE I

PREFERENCES BETWEEN POSITIVE AND NEGATIVE PROSPECTS

Positive prospects				Negative prospects			
Problem 3:	(4,000, .80)	<	(3,000).	Problem 3′:	(−4,000, .80)	>	(−3,000).
$N = 95$	[20]		[80]*	$N = 95$	[92]*		[8]
Problem 4:	(4,000, .20)	>	(3,000, .25).	Problem 4′:	(−4,000, .20)	<	(−3,000, .25).
$N = 95$	[65]*		[35]	$N = 95$	[42]		[58]
Problem 7:	(3,000, .90)	>	(6,000, .45).	Problem 7′:	(−3,000, .90)	<	(−6,000, .45).
$N = 66$	[86]*		[14]	$N = 66$	[8]		[92]*
Problem 8:	(3,000, .002)	<	(6,000, .001).	Problem 8′:	(−3,000, .002)	>	(−6,000, .001).
$N = 66$	[27]		[73]*	$N = 66$	[70]*		[30]

The Reflection Effect

We may notice a peculiar pattern of preferences: in each case, the preference between negative prospects is the **mirror image** of the preference between positive prospects

Reflection Effect

Reversal of the preference order caused by the reflection of prospects around 0

This phenomenon had been noted early by Markowitz (1952) and Williams (1966)

The Reflection Effect

Implications of the reflection effect:

- also the preferences between negative prospects are inconsistent with expected utility theory
- risk aversion in the positive domain is accompanied by **risk seeking** in the negative domain
- **aversion for uncertainty** or variability, proposed by Allais, is eliminated as an explanation of the certainty effect

The Isolation Effect

PROBLEM 10: Consider the following two-stage game. In the first stage, there is a probability of 0.75 to end the game without winning anything, and a probability of 0.25 to move into the second stage. If you reach the second stage you have a choice between:

$$(4000, 0.80) \qquad\qquad (3000)$$

Your choice must be made before the game starts

Now recall Problem 4:

Choose between

$$(4000, 0.20) \qquad\qquad (3000, 0.25)$$

The Isolation Effect

- You may notice that in terms of final outcomes and probabilities, this is equivalent to PROBLEM 4
- However, the dominant preferences are opposite in the two problems: 78 per cent of subjects chose the latter prospects
- It seems that people ignored the first stage of the game, whose outcomes are shared by both prospects. In this case, PROBLEM 10 resembles PROBLEM 3, as confirmed by preferences

The Isolation Effect

We can see two different representations of the problem, that may induce a change of preferences:

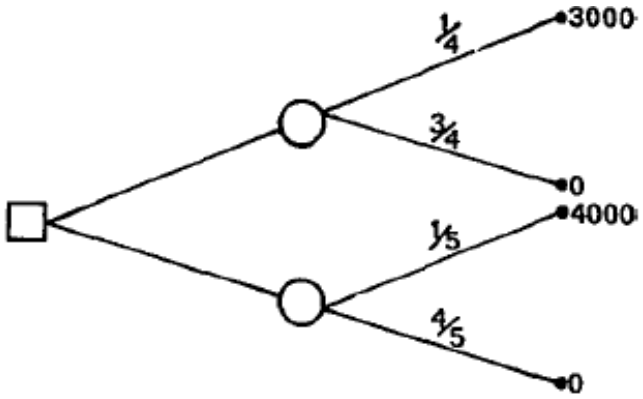

FIGURE 1.—The representation of Problem 4 as a decision tree (standard formulation).

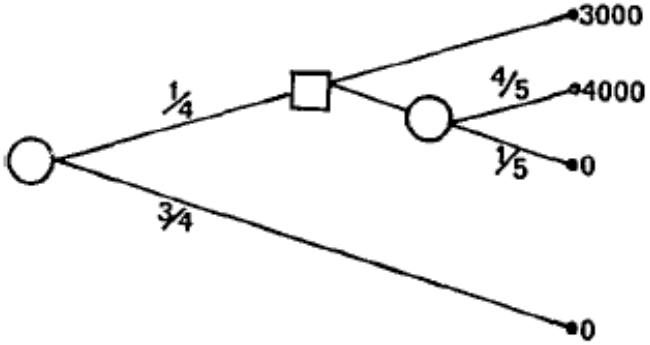

FIGURE 2.—The representation of Problem 10 as a decision tree (sequential formulation).

The Isolation Effect

In the previous case, preferences are altered by **different representations of probabilities**. We now show how choices may be altered by **varying the representation of outcomes**

Consider the following problems:

PROBLEM 11: In addition to whatever you own, you have been given 1,000. Choose between

$(1000, 0.50)$	(500)
$[16]$	$[84]^*$

PROBLEM 12: In addition to whatever you own, you have been given 2,000.Choose between

$(-1000, 0.50)$	(-500)
$[69]^*$	$[31]$

The Isolation Effect

You may note that when viewed in terms of final states, the two choice problems are identical.They ask to choose between:

$$(2000, 0.50; 1000, 0.50) \qquad\qquad (1500)$$

Evidently, the initial bonus does not enter into the comparison of prospects beacause it was common to both options in each problem

This represents another violation of the theory, according to which the domain of utility function is **final states**. This demonstration implies that the carriers of value are in fact **changes in wealth**

Isolation Effect

Disregard of components shared by two different alternatives and focus on the components that distinguish them. Since a pair of prospects can be decomposed in several ways, this effect may bring about inconsistent preferences

The Editing Phase

Two phases of the choice process:

- *Editing phase*: reorganizing outcomes and probabilities
- *Evaluation phase*: choice of the prospect with the highest value

The *editing phase* can be decomposed in four major operations:

1. **Coding**: outcomes are perceived as gains and losses
2. **Combination**: prospects are simplified by combining probabilities associated with identical outcomes
3. **Segregation**: the sure component of a prospect is segregated from the risky component
4. **Cancellation**: common components of different prospects are discarded (entire phases or outcome-probability pairs)

The Evaluation Phase

In the *evaluation phase*, the DM evaluates prospects of the form $(x, p; y, q)$ and chooses the one with highest value

The overall value V of an edited prospect depends on two scales:

- π, that associates to each probability p a **decision weight** $\pi(p)$. π is not a probability measure, since $\pi(p) + \pi(1-p) < 1$
- v, that associates to each outcome x the **subjective value** of the outcome $v(x)$

The Evaluation Phase

If $(x, p; y, q)$ is a **regular prospect**, i.e. either $p + q < 1$ or $x \geq 0 \geq y$ or $x \leq 0 \leq y$, the basic equation that describes the evaluation of a prospect is

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y) \tag{1}$$

where $v(0) = 0$, $\pi(0) = 0$ and $\pi(1) = 1$

If $(x, p; y, q)$ is a **strictly positive** or **negative** prospect, i.e. $p + q = 1$ and either $x > y > 0$ or $x < y < 0$, the equation becomes

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)] \tag{2}$$

Equation (2) reduces to equation (1) if $\pi(p) + \pi(1 - p) = 1$, but as we said this is not always the case

The Evaluation Phase

To sum up, **prospect theory** retains the bilinear form that underlies expected utility maximization, but

- values are assigned to *changes* rather than to final states
- *decision weights* do not coincide with stated probabilities

In this way prospect theory is able to predict **departures** from expected utility maximization that lead to **normatively unacceptable consequences** (inconsistencies, intransitivities, violations of dominance...)

The Value Function

- The assumption that the **value function** is defined over changes in wealth rather than final states is consistent with general principles of perception and judgement
- This does not imply that changes should be evaluated *independently* of the reference point
- However, when it comes to relatively small or moderate variations in asset positions, the preference order is not significantly altered by the reference point

The Value Function

Consider the following prospects

PROBLEM 13:

$$(6000, 0.25) \qquad\qquad (4000, 0.25; 2000, 0.25)$$

$$[18] \qquad\qquad [82]$$

and

PROBLEM 13':

$$(-6000, 0.25) \qquad\qquad (-4000, 0.25; -2000, 0.25)$$

$$[70] \qquad\qquad [30]$$

As one would expect, the value function is **concave in gains** and **convex in losses**, i.e. the marginal value of gains and losses decreases with their magnitude

The Value Function

To see this, note that the above mentioned prospects are regular, so applying equation (1) yields

$$\pi(0.25)v(6000) < \pi(0.25)[v(4000) + v(2000)]$$

$$\pi(0.25)v(-6000) > \pi(0.25)[v(-4000) + v(-2000)]$$

hence,

$$v(6000) < v(4000) + v(2000)$$

and

$$v(-6000) > v(-4000) + v(-2000)$$

The Value Function

Another property of the value function can be derived by the following facts

- people consider symmetric bets, like $(x, 0.50; -x, 0.50)$, unattractive
- the aversiveness to symmetric bets increases with the size of the stake, i.e. if $x > y \geq 0$ then

$$(y, 0.50; -y, 0.50) \succsim (x, 0.50; -x, 0.50)$$

So again by equation (1) and letting y approach to x it can be derived that $v'(x) < v'(-x)$, which means that the value function is **steeper for losses** than for gains

The Value Function

In summary, the value function

1. is defined on deviation from the reference point
2. is concave in gains and convex in losses
3. is steeper for losses than for gains

Fishburn and Kochenberger (1979) provided evidence that a standard von Neumann-Morgenstern utility function shows the same properties for changes of wealth

The Value Function

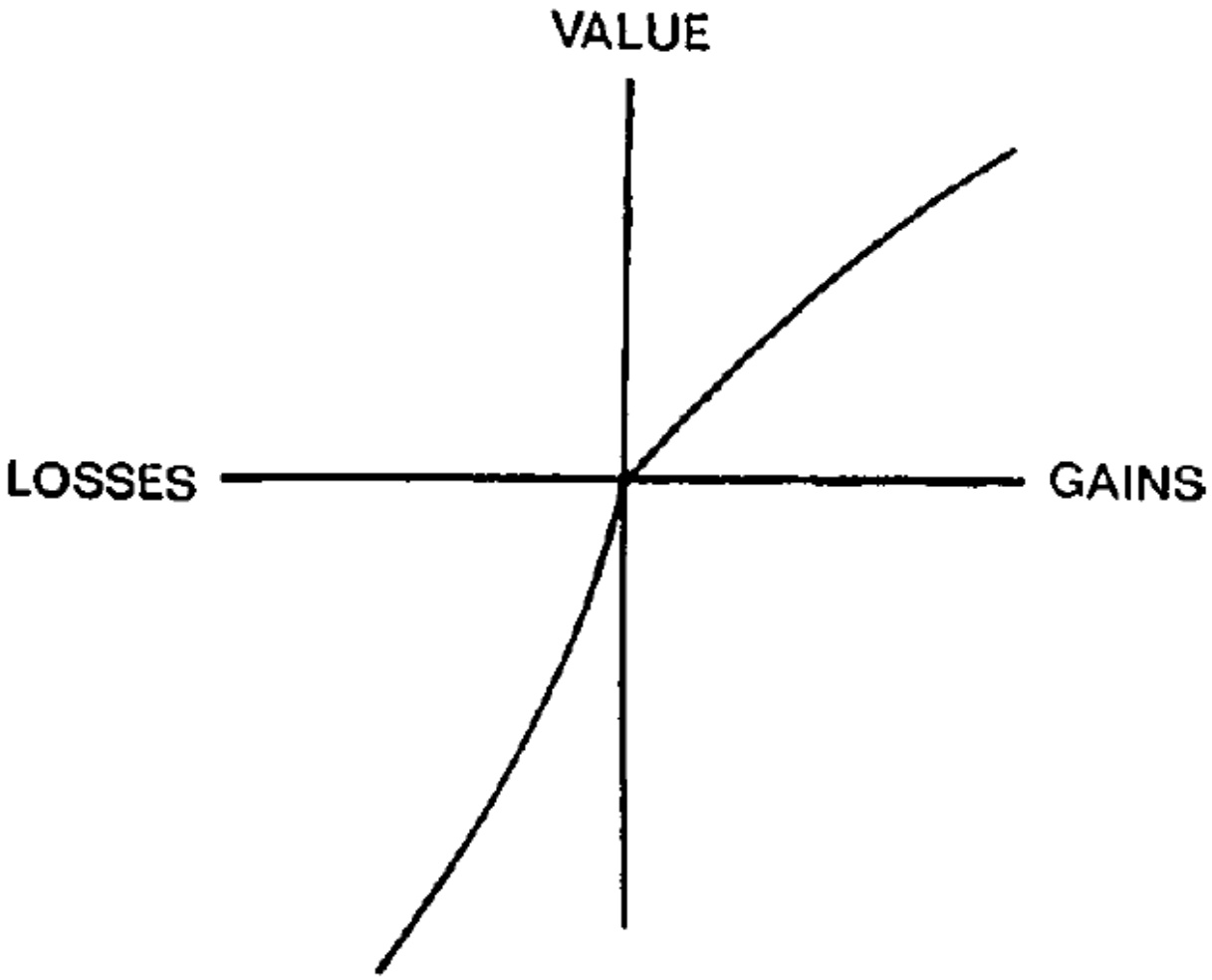

FIGURE 3.—A hypothetical value function

The Weighting Function

- In prospect theory, the value of each outcome is multiplied by a **decision weight**
- Decision weights measure the impact of events on the desirability of a prospect, not merely the perceived likelihood of these events
- They are inferred from choices of prospects as subjective probabilities are inferred from preferences in the Ramsey-Savage approach
- In the present theory, the weights are function of stated probabilities

The Weighting Function

We discuss the properties of the weighting function

1. π is an increasing function of p, with $\pi(0) = 0$ and $\pi(1) = 1$, but is not well behaved at the endpoints
2. $\pi(p) > p$ for small p, so small probabilities are generally **overweighted**. A simple example is a lottery tickets that can be represented by the following prospect

   PROBLEM 14:

$$(5000, 0.001) \qquad (5)$$

$$[72] \qquad [28]$$

   so that $\pi(0.001) > v(5)/v(5000) > 0.001$ by concavity of v

The Weighting Function

3. $\forall p \in (0,1)$, $\pi(p) + \pi(1-p) < 1$. This property is labelled as **subcertainty** and can be derived from the first two prospects we saw, yielding $\pi(0.66) + \pi(0.34) < 1$
4. if (x, p) is equivalent to (y, pq), then (x, pr) is not preferred to (y, pqr), which turns out to be equivalent to

$$\frac{\pi(pq)}{\pi(p)} \leq \frac{\pi(pqr)}{\pi(pr)}$$

   This property is called **subproportionality**: for a fixed ratio of probabilities, the corresponding ratio of decision weights is closer to unity when the the probabilities are low than when they are high
5. subproportionality and overweighting of small probabilities give **subadditivity** over all the range, i.e. $\pi(rp) > r\pi(p)$

The Weighting Function

In summary, the main properties of the weighting funcition are:

1. not well-behaved at endpoints
2. overweighting of small probabilities
3. subcertainty
4. subproportionality
5. subadditivity

The Weighting Function

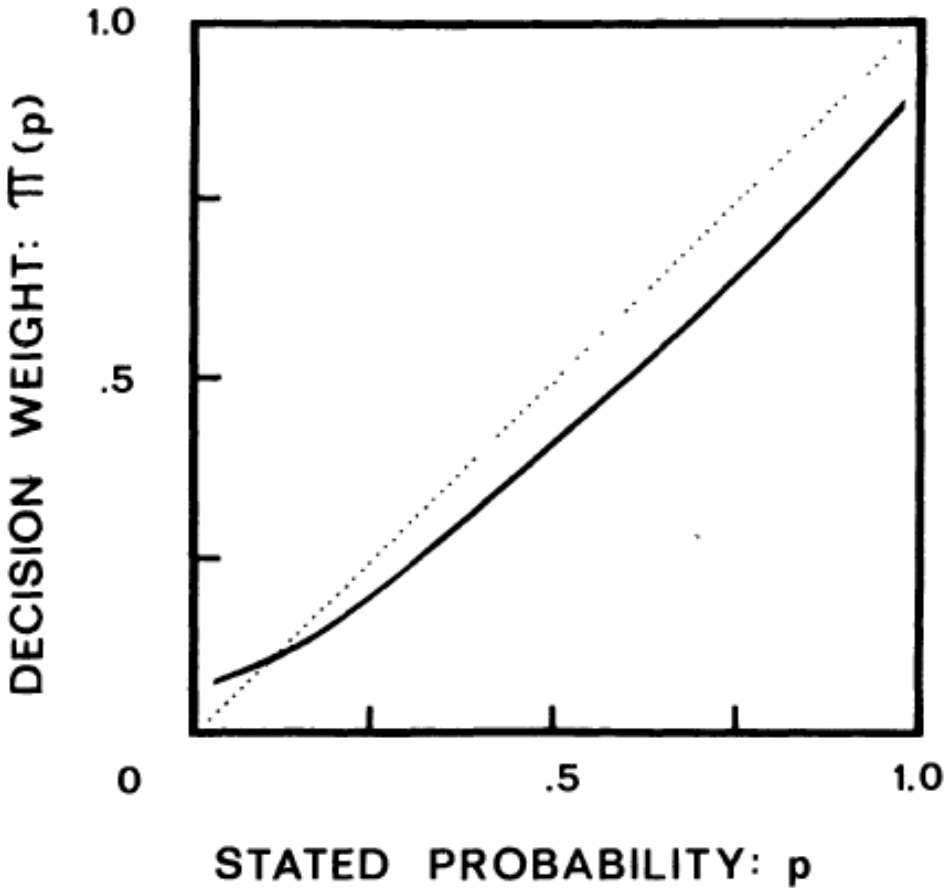

FIGURE 4.—A hypothetical weighting function.

Violation of Independence

Problem 1 and 2 provided a first example of violation of independence:

$$\frac{\pi(0.33)}{\pi(0.34)} > \frac{v(2400)}{v(2500)} > \frac{\pi(0.33)}{1-\pi(0.66)}$$

This paradox is explained by prospect theory as a result of **subcertainty** of π

Problem 7 and 8 are another example:

$$\frac{\pi(0.001)}{\pi(0.002)} > \frac{v(3000)}{v(6000)} > \frac{\pi(0.45)}{\pi(0.90)}$$

This paradox is explained by prospect theory as a result of **subproportionality** of π

Shifts of Reference

- The reference point is usually assumed to be the current asset position
- Sometimes gains and losses are coded relative to an expectation
- More importantly, changes in the reference point alters the **preference order** for prospects
- As a result, incomplete adaptation to recent losses increases **risk seeking** in some situation

Shifts of Reference

Consider a risky prospect $(x, p; -y, 1-p)$ that is just acceptable:

$$V(x, p; -y, 1-p) = 0 \quad \Longleftrightarrow \quad \pi(p)v(x) = -\pi(1-p)v(-y)$$

A negative translation of the propspect is $(x-z, p; -y-z, 1-p)$, which turns out to be preferred over $(-z)$:

$$\begin{aligned}
&V(x-z, p; -y-z, 1-p) = \\
&\quad = \pi(p)v(x-z) + \pi(1-p)v(-y-z) \\
&\quad > \pi(p)[v(x) - v(z)] + \pi(1-p)[v(-y) + v(-z)] \\
&\quad = -\pi(1-p)v(-y) - \pi(p)v(z) + \pi(1-p)[v(-y) + v(-z)] \\
&\quad = -\pi(p)v(z) + \pi(1-p)v(-z) \\
&\quad > v(-z)[\pi(p) + \pi(1-p)] \\
&\quad > v(-z)
\end{aligned}$$

Risk Attitudes

Prospect theory allows to predict these risk attitudes:

	Gains	Losses
High Prob	.95 chance of winning 1000	.95 chance of losing 1000
Low Prob	.05 chance of winning 1000	.05 chance of losing 1000

where, given the prospect (x, p) and its expected value (px), risk aversion is given by $\pi(p)v(x) < v(px)$

Risk Attitudes

Prospect theory allows to predict these risk attitudes:

	Gains	Losses
High Prob	.95 chance of winning 1000	.95 chance of losing 1000
Low Prob	RISK SEEKING .05 chance of winning 1000	RISK AVERSE .05 chance of losing 1000

where, given the prospect (x, p) and its expected value (px), risk aversion is given by $\pi(p)v(x) < v(px)$

Risk Attitudes

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Risk Attitudes

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High Prob	RISK AVERSE	RISK SEEKING
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Low Prob	RISK SEEKING	RISK AVERSE
	.05 chance of winning 1000	.05 chance of losing 1000

where, given the prospect (x, p) and its expected value (px), risk aversion is given by $\pi(p)v(x) < v(px)$

Main Problems

Main problems of prospect theory:

- Individuals are real but choices are **merely hypothetical**
- It has **no clear axiomatic foundation**. That is, Kahneman and Tversky do not describe basic characteristics of preferences that drive the behavior
- This original version gives rise to violations of first-order stochastic dominance. **Cumulative prospect theory**, proposed in 1992 by the authors, overcomes this problem
- It provides no ex-ante prediction since the realizations of V depend on a non-predictable combination of factors. Ex post it can rationalize almost any observed decision pattern. Therefore it **cannot be falsified**
- It fails to allow for emotions like regret and disappointment, even if decision makers anticipate them when making their choices

Conclusions

- *How do we explain the longevity of expected utility theory?*

  Theory induced blindness: once you have accepted a theory, it is extremely difficult notice its flaws

- *Why is EUT still taught at the undergraduate level?*
  - The standard models are relatively easier to understand
  - They allow a better understanding of the discipline
  - Failure of rationality is often irrelevant

- *Why is PT the main alternative?*

  It yields **low cost** in terms of complexity and **high benefits** in terms of explanatory power