Structural Change, Growth and Volatility*

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Abstract

I construct a two-sector general equilibrium model to study the implications of the sectoral composition of GDP on cross-country differences in GDP growth and volatility. Typically, high income economies grow slower, are less volatile and have a larger share of services in GDP with respect to middle income economies. I show that even when total factor productivity (TFP) growth and volatility are the same in manufacturing and services at the gross output level, the larger intensity of intermediate goods in gross output production in manufacturing implies a larger growth and volatility of TFP at the value added level in manufacturing than in services. In the model, this implies that structural change towards services induces a decline in both aggregate TFP growth and volatility, which in turn reduces GDP growth and volatility. When the model is calibrated to the U.S. manufacturing and services sectors, structural change alone is able to account for measured differences in per-capita GDP growth and volatility between high and middle income economies during the 1970-2006 period.

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1 Introduction

This paper puts forth the idea that the composition of GDP represents an important channel in shaping GDP growth and volatility. Cross-country evidence suggests that typically i) per-capita GDP of high income economies grows slower than that of middle income economies; ii) richer economies display lower per-capita GDP volatility than poorer ones; and iii) richer economies display a larger share of services in GDP. Motivated by this evidence, I use a two-sector general equilibrium model to analyze how the relative size of the broadly defined manufacturing and services sectors affects GDP growth and volatility of an economy.

The production side of the model is composed of two sectors producing, respectively, manufacturing and services. In each sector, output is produced by means of a gross output production function in labor services and intermediate goods purchased from the sector itself and from the other sector. This production structure implies a well defined production possibility frontier, that determines all the feasible combinations of manufacturing and services that can be consumed given the aggregate amount of labor used in production in the economy. The size of the services sector is then determined by Stone-Geary type non-homothetic preferences, that imply an income elasticity of services consumption larger than one. When GDP in the model economy increases, the model endogenously generates a rise in the share of services in GDP.

In the model, aggregate TFP growth and volatility depend on value added TFP growth and volatility in the two sectors and on the shares of the two sectors in GDP.¹ This implies that when one sector grows relative to the other, value added TFP growth and volatility of that sector have a larger impact on aggregate TFP growth and volatility. Thus, for invariant value added TFP processes in the two sectors, structural change affects the performance of aggregate TFP. In turn, value added TFP depends on two factors: one is TFP at the gross output level while the other is the share of intermediate goods in gross output in the sector considered.² This implies that, for a given growth and volatility of gross output TFP, the intensity of intermediate goods in gross output production provides a multiplier on value added TFP growth and volatility. Thus, even if two sectors display the same TFP growth and volatility at the gross output level, value added TFP volatility is larger in the sector

¹In this paper I abstract from explicitly modelling the agricultural sector. The aim here is to compare per-capita GDP growth and volatility of economies that already moved away from being mainly agricultural.
²This relationship is showed in Hulten (1978).
with the largest share of intermediate goods in gross output. I use data from the KLEMS dataset, 2008, to show that the multiplier associated to the share of intermediate goods is larger in manufacturing than in services in 25 developing and developed countries over the 1970-2005 period. This implies that the same growth rate and volatility of TFP at the gross output level in manufacturing and services delivers, when averaging across countries, a TFP growth and volatility at the value added level 71% larger in manufacturing.

Thus, the main prediction of the model is that, given a common process for sectoral (gross output) TFP in the two sectors, when the services sector grows the economy endogenously generates a decline in aggregate TFP growth and volatility. This, in turn, reduces GDP growth and volatility. The theory proposed is then qualitatively consistent with the cross-country evidence on growth and volatility: High Income economies grow slower, are less volatile and display a larger share of services in GDP with respect to Middle Income economies. Following these results the model can be calibrated to quantify the effect of the size of the services sector on cross-country differences in per-capita GDP growth and volatility.

To perform the quantitative analysis, I use data of the two groups of countries defined Middle and High Income economies by the World Bank. The share of services in Middle Income economies increases from 0.41 to 0.53 during the 1970-2006 period while that of High Income economies increases from 0.55 to 0.73 during the same period. Also, Middle Income economies display an average growth rate and a volatility of per-capita GDP respectively 22% and 19% larger than High Income economies during the 1970-2006 period. To compare the model with the data, I study a single model economy calibrated such that, in 74 periods, it generates a transition path in which the share of services increases from the level displayed by Middle Income economies in 1970 to that displayed by High Income economies in 2006. Sectoral TFP in the two sectors follows a common stochastic process that implies that, in the first 37 periods of the transition path, the calibrated economy matches per-capita GDP growth and volatility of Middle Income economies during the 1970-2006 period. Along the transition path, per-capita GDP growth is 21% larger and 18% more volatile in the first 37 periods with respect to the last 37. Thus, when a common TFP process is assumed in the two sectors, structural change alone is able to account for 95% of the difference in both per-capita GDP growth and per-capita GDP volatility between Middle Income and High Income economies during the 1970-2006 period.
Along the transition path, the model also generates a marked decline in the volatility of real value added of the services sector. This decline is due to the non-homotheticity of preferences and is consistent with the observed difference between Middle and High Income economies in the 1970-2006 period. Stone-Geary preferences imply that the income elasticity of services consumption is larger than one. When GDP increases in the model, this elasticity declines towards one so that services consumption responds less to changes in income, and services real value added becomes less volatile, even when TFP processes in the two sectors have not changed. This result suggests that Stone-Geary preferences that can account for long run facts of the structural transformation, can at the same time account for changes in single sectors business cycle properties.

Finally, I calibrate TFP processes in the two sectors to the U.S. manufacturing and services sectors. The U.S. experienced a large process of structural transformation between manufacturing and services during the second part of the last century, so they provide a representative source to calibrate sectoral TFP processes in a model of structural change. In this case, the difference between the first and the second part of the transition is 20% in terms of growth and 25% in terms of volatility. This implies that also when TFP processes are calibrated to the US, the model accounts for 95% of the difference in growth rates and for the entire difference in volatility between middle and high income economies.

The remaining of the paper is as follows: section 2 describes the related literature; section 3 describes the relationship between gross output and value added measures of TFP growth and volatility; section 4 presents the model and the quantitative results; finally, section 5 concludes.

2 Related Literature

In the literature on economic development and economic growth, several papers point out the importance of the sectoral composition for GDP growth. Most of these papers focus on differences in value added total factor productivity (TFP) or in labor productivity across sectors. Baumol (1967) is a pioneering work in the structural change literature. His conclusion is that in an economy with two sectors, one with productivity growth and one with constant productivity, the production cost of the stagnant productivity sector grows unbounded and that sector attracts the entire labor force as time passes. In the limit, the
The economy converges to the zero growth rate of the stagnant sector. Echevarria (1997) provides evidence that middle income countries display the largest per-capita GDP growth rates while low income countries the smallest. High income countries display growth rates that lie in between. She also shows that, across countries, TFP growth at the value added level is larger in manufacturing than in services which, in turn, exhibit higher TFP growth than agriculture. Based on non-homotheticity of preferences, she constructs a model in which, as the economy becomes richer, the manufacturing sector expands with respect to agriculture and services. Eventually, when income in the economy is sufficiently high, the services sector expands with respect to agriculture and manufacturing. During this transition, when manufacturing is the largest sector in the economy, GDP growth is larger than when services is the largest sector. A similar mechanism is used in this paper to generate a transition from manufacturing to services. With respect to Echevarria (1997), I show that the larger growth rate of manufacturing TFP is mainly due to the different intensity of intermediate goods in gross output production with respect to services. This allows me to show that even when manufacturing and services display the same growth rate of TFP at the gross output level, the TFP measure at the value added level is around 70% larger in manufacturing than services. It follows that structural change towards services implies a reduction in aggregate TFP growth even when sectoral (gross output) TFP growth is the same in both sectors. In addition, this paper studies the effect of the structural transformation on both GDP growth and volatility, while Echevarria (1997) focuses on GDP growth. More recently, Duarte and Restuccia (2010) find that sectoral differences in labor productivity play a key role in the process of structural transformation and aggregate productivity differences across countries. Other recent papers relating the sectoral composition of an economy to economic growth are Kongsamut, Rebelo and Xie (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008).

In the literature on economic development and output volatility, Lucas (1988) notes that growth rates of advanced countries tend to be more stable than growth rates of poorer countries. This is true not only when comparing rich countries with poor ones, but also when comparing high and middle income economies. Acemoglu and Zilibotti (1997) provide ev-

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4 Ramey and Ramey (1995) find a negative relationship between growth and volatility. Martin and Rogers (2000), instead, find that this relationship is robust only for developed countries.
idence of a negative correlation between GDP levels and GDP volatility. They propose a theory in which volatility is high at early stages of development because of a lack of diversification in investment projects. Koren and Tenreyro (2008) report that GDP volatility declines with development, both in the cross section of countries and for each country over time. Furthermore, Koren and Tenreyro (2007) find that, as countries develop, they switch production from more volatile to less volatile sectors. This fact implies a reduction in aggregate GDP volatility. They estimate that the sectoral composition can account for up to 60% of the difference in aggregate volatility between poor and rich countries. They also find that a large part of this difference is due to a reduction in volatility within sectors. In this paper, I show that other conditions equal, structural change can induce a change in the volatility within sectors. I find that, for common TFP processes in manufacturing and services, structural change towards services implies a marked decline in the volatility of services real value added. This is due to the non-homotheticity of preferences, which display an income elasticity of services consumption larger than one, which declines towards one as GDP increases.

Another strand of the literature on economic development focuses on the role of intermediate goods for TFP levels. Jorgenson et al. (1987) argue that gross output based measures of TFP should be preferred to value added measures. This is because gross output TFP is the “pure” part of output growth that cannot be explained by changes in capital, labor and intermediate goods in the sector considered. Hulten (1978), Ciccone (2002) and Jones (2007) show that intermediate goods utilization in the production process can provide a multiplier effect on the aggregate TFP level. This multiplier effect is absent in standard models in capital and labor only and, according to Jones (2007) is able to explain up to 32-fold differences in TFP levels across countries. Ngai and Samaniego (2009) show that the multiplier associated with intermediate goods raises the contribution of investment specific technical change to post-war U.S. growth from around 60% to 96%. In Moro (2009), I show that in the U.S. the multiplier effect due to intermediate goods is larger in manufacturing than in services. This implies that the same TFP volatility at the gross output level in manufacturing and services implies a 55% larger value added TFP volatility in manufacturing with respect to

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5Koren and Tenreyro (2008) propose a theory based on inputs diversification to explain the decline in volatility within sectors during the development process. See also Jaimovich (forthcoming) for a theory of development with similar predictions.
services. Using this mechanism it is possible to show that the increase in the services sector in the U.S. can account for 32% of the decline in GDP volatility over time.

This paper presents a theory, based on structural change, consistent with the evidence from the growth, the volatility and the development literature described above. In particular, I show that, other conditions equal, an economy with a larger share of services in GDP (high income economies) grows less and is less volatile than an economy with a smaller share (middle income economies). This implies that per-capita GDP growth and volatility decline along a transition path in which the share of services in GDP increases at the expenses of the share of manufacturing. As the transition from manufacturing to services appears to be a common feature across countries as income grows, the theory is also consistent with the reduction in GDP growth and volatility observed for a single country over time.

3 From Gross Output to Value Added TFP in Manufacturing and Services

Most macroeconomic models build on production functions in which capital and labor are the only inputs. This is the case both when the production function is an aggregate one and when it is sectoral. In general, at the sectoral level, intermediate goods contribute, together with capital and labor, to produce gross output of that sector. Value added is then measured as the contribution of capital, labor and technical change to gross output growth. This procedure implies that what is measured as total factor productivity (TFP) growth at the value added level depends on two factors: one is TFP growth at the gross output level, defined as the part of gross output growth that cannot be explained by changes in capital, labor and intermediate goods; the other is the intensity of intermediate goods in gross output

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6 See also Carvalho (2009), for the theoretical relationship between the share of intermediate goods and output volatility.

7 The decline in output volatility has been extensively studied for the U.S. See, for instance McConnell and Perez-Quiros (2000), Blanchard and Simon (2001) and Stock and Watson (2002). However, these papers disregard the sectoral composition explanation for the GDP volatility decline. In Alcalá and Sancho (2003), Moro (2009) and Carvalho and Gabaix (2010) it is showed that the composition of the economy represents an important factor in the determination of GDP volatility in the U.S.

8 The transition from agriculture to manufacturing is not considered in this paper. Echevarria (1997) shows that this transition implies an increase in the GDP growth rate, while Da-Rocha and Restuccia (2006) show that a decline in the share of agriculture in GDP implies a reduction in GDP volatility.

9 See Sato (1976). Equivalently, value added can be defined as the part of gross output growth that cannot be explained by growth in intermediate goods.
production.

To see this, consider a firm that produces gross output using capital, labor and intermediate goods. Consider now an increase in productivity embodied in intermediate goods. Holding fixed the amount of all inputs, the firm can now produce more gross output. This also implies an increase in capital and labor productivity in gross output. Furthermore, the more intensive the production process in intermediate goods, the larger the increase in gross output. Thus, the increase in capital and labor productivity due to productivity embodied in intermediates increases with the intensity of intermediates in gross output. Value added TFP is defined as the part of real value added growth not explained by the growth in capital and labor inputs while real value added itself is defined as the contribution of capital, labor and productivity to gross output growth.\textsuperscript{10} It follows that any change in productivity embodied in intermediate goods impacts value added TFP through an increasing function of the intensity of intermediate goods in gross output. The bottom line is that value added TFP reflects both changes in productivity embodied in capital and labor and changes in productivity embodied in intermediate goods.

More formally, consider a generic sector in which the representative firm produces gross output using a Cobb-Douglas production function in intermediate goods $M$ and a function of capital and labor $f(K,N)$.\textsuperscript{11} With competitive markets the firm takes the price of labor $w$, of capital $r$, of gross output $p_g$ and of intermediate goods $p_m$ as given. The profit maximization problem of the firm is

$$\max_{K,N,M} \left\{ p_g B f(K, N)^\omega M^{1-\omega} - rK - wN - p_m M \right\}, \quad (1)$$

where $B f(K, N)^\omega M^{1-\omega}$ is the gross output production function, $B$ is gross output TFP and $0 < \omega < 1$.

By taking the first order condition of (1) with respect to intermediate goods and using it to substitute for intermediate goods again in (1), the following reduced form problem is obtained

$$\max_{K,N} \left\{ p_g B^\frac{\omega}{1-\omega} f(K, N) - rK - wN \right\}. \quad (2)$$

\textsuperscript{10}See Sato (1976). More precisely, he writes, pag. 441: “Real value added is the contribution of primary inputs, economies of scale, and technical change in the production process”. Technical change here is represented by changes of productivity embodied in inputs. In the formal example later in the text, the technology is constant returns so economies of scale do not have any effect on gross output growth. See again Sato (1976), pag. 440.

\textsuperscript{11}Assume $f(K, N)$ to be homogeneous of degree one in capital and labor.
In (2), $B^\frac{1}{2} f(K, N)$ is the value added production function of the sector considered and $p_v = \omega(1 - \omega)\frac{1}{\omega} p_g \frac{1}{\omega} p_m$ represents its price. It follows that TFP at the value added level, defined as the part of value added growth not explained by the growth in capital and labor inputs, is given by $B^\frac{1}{2}$.\(^{12}\)

The growth rate of the variable $B_t$ at time $t$ is given by $b_t = \log(B_t) - \log(B_{t-1})$ while that of the variable $B_t^{\frac{1}{2}}$ is given by $\bar{b}_t = (1/\omega)[\log(B_t) - \log(B_{t-1})] = (1/\omega)b_t$. As a result, the value of $\omega$ affects value added TFP growth through its effect on the sector’s gross output TFP, $B$. In the Cobb-Douglas case, $1 - \omega$ is equal to the share of intermediate goods in gross output in equilibrium. Thus, the larger the share of intermediate goods in one sector, the larger TFP growth at the value added level for a given TFP growth at the gross output level.

In the real business cycle literature, before computing volatility statistics, each variable is logged and detrended using the Hodrick-Prescott (HP) filter. For the variable $\log(B_t)$ and its HP filter $\log(\hat{B}_t)$, the deviation $\hat{b}_t$ at time $t$ is given by $\hat{b}_t = \log(B_t) - \log(\hat{B}_t)$. Instead, for the variable $\log(B_t^{\frac{1}{2}})$ the deviation $\bar{b}_t$ at $t$ is given by $\bar{b}_t = (1/\omega)[\log(B_t) - \log(\hat{B}_t)]$. As a result, the value of $\omega$ affects value added TFP volatility through its effect on sectoral TFP $B$. The result also extends to the case in which volatility is computed as the standard deviation of the variable’s growth rate, as the above formula for $\bar{b}_t$ makes clear. It follows that a larger share of intermediate goods in gross output implies both a larger growth and a larger volatility of TFP at the value added level. Aggregate GDP growth can be computed as a weighted average of value added growth of the various sectors in the economy. Thus, what matters for aggregate TFP determination is value added TFP in each sector of the economy and the size of each sector in GDP. It follows that for a common growth and volatility of $B$ across the sectors in the economy, the larger the share of sectors with a smaller $\omega$, the larger aggregate TFP growth and volatility.

Figure 1 reports the average share of intermediate goods in gross output in the manufacturing and in the services sectors across countries for the 1970-2005 period. The average share in the services sector across countries is 0.40 while that of manufacturing is 0.65. Figure 1 highlights the different production technology used in the two sectors, which implies

\(^{12}\)An interpretation of the term $B$ in the production function $B^\frac{1}{2} f(K, N)^{\omega} M^{1-\omega}$ is that productivity is embodied in the three inputs, $[f(BK, BN)]^{\omega} [BM]^{1-\omega}$. Thus, as described in the text, value added TFP depends linearly on capital and labor productivity (through $B$) and non-linearly on intermediate goods productivity (through $B^{\frac{1}{2-\omega}}$), such that the two effects jointly deliver $B B^{\frac{1}{2-\omega}} = B^{\frac{1}{2}}$. 

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that in manufacturing the incidence of intermediate goods in gross output is on average 63% larger than in services. When $B$ displays the same growth and volatility level in the two sectors, that is the same $b_t$ and the same $\hat{b}_t$, the difference in $\omega$ between the two sectors implies a value added TFP growth and volatility in manufacturing 71% larger than in services. This result suggests that, even if manufacturing and services had the same growth and volatility of $B$ in each country and across countries, those economies with a larger share of services would display a smaller growth rate and a smaller volatility of aggregate TFP and, consequently, of GDP. Note also that in Moro (2009) I document that for the U.S. the share of intermediate goods in gross output has been around 0.6 in manufacturing and around 0.38 in services during the entire period 1960-2005.

A larger share of services in GDP implies a smaller aggregate TFP growth and volatility as long as manufacturing and services gross output TFP is similar. If $B$ grows at a sufficiently larger pace in services than in manufacturing, it is possible that an increase in the services sector provide an increase in aggregate TFP instead of a decline. The same reasoning holds for volatility. Figure 2 reports $B$ and $B^\perp$ for manufacturing and services during the 1960-2005 period in the U.S. As showed in the left panel, the growth rate of $B$ is larger in manufacturing than in services, with a 28% larger average growth rate in manufacturing. When measuring
value added TFP growth instead, $B^2$, the difference between the two sectors enlarges, with an average TFP growth rate 93% larger in manufacturing. The same argument holds for volatility.\(^{13}\) It follows that an increase in the services sector is expected to provide a decline in aggregate TFP growth and volatility.

The partial equilibrium analysis in this section shows that the multiplier effect that affects value added TFP through the share of intermediate goods is consistent with the growth/development facts observed: i) economies that are more intensive in manufacturing than in services (typically middle income economies) are expected to grow faster and display higher volatility of GDP, as the manufacturing sector displays a larger share of intermediate goods in gross output; ii) economies that are more intensive in services (typically high income economies) should instead display a smaller growth rate and volatility level of GDP, as the services sector displays a smaller share of intermediate goods in gross output.

In the next section I construct a two-sector model that allows me to show that the main insights of this section also hold in general equilibrium. In particular, the model allows me to show that along a transition path in which the share of services endogenously increases, aggregate TFP growth and volatility decline, which implies a decline in GDP growth and volatility. Once calibrated, the model can be used to quantify how much of the observed difference in growth rates and volatility across countries can be explained by the different

\(^{13}\)See Moro (2009) for a detailed quantitative analysis of U.S. TFP volatility.
composition of GDP alone.

4 The Model

The model presented in this section is based on Moro (2009). For tractability, I abstract from capital accumulation in this version.

4.1 Firms

There are two sectors in the economy, manufacturing and services. The representative firm in each sector produces gross output using a Cobb-Douglas production function in labor, manufactured intermediate goods and intermediate services. The gross output production function of the representative firm in the manufacturing sector is

\[ G_m = B_m N_m^{\nu_m} \left( M_m^{\varepsilon_m} S_m^{1-\varepsilon_m} \right)^{1-\nu_m}, \]  

(3)

and that of the representative firm in the services sector is

\[ G_s = B_s N_s^{\nu_s} \left( M_s^{1-\varepsilon_s} S_s^{\varepsilon_s} \right)^{1-\nu_s}, \]

(4)

where \( 0 < \nu_j < 1, 0 < \varepsilon_j < 1, N_j \) is labor, \( M_j \) is the manufactured intermediate good, \( S_j \) is intermediate services and \( B_j \) is gross output TFP, with \( j = m, s \). Gross output TFP in each sector follows a stochastic process, unspecified for the time being.

The profit maximization problem of the representative firm in manufacturing is

\[ \max_{N_m, M_m, S_m} \left[ p_m G_m - w N_m - p_m M_m - p_s S_m \right] \]

subject to (3),

where \( w \) is the wage rate, \( p_m \) the price of the manufacturing good and \( p_s \) the price of services. The problem of the representative firm in services is

\[ \max_{N_s, M_s, S_s} \left[ p_s G_s - w N_s - p_m M_s - p_s S_s \right] \]

subject to (4).

Given the structure of the supply side of the economy, competitive markets imply that the relative price of services with respect to manufacturing, \( p_s/p_m \), is independent of the
quantities produced of the two goods. This is given by

\[
p_s \frac{p_s}{p_m} = \Omega(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) \left( \frac{B_m^{\nu_m}}{B_s^{\nu_s}} \right)^{\nu_m (1-\varepsilon_s) (1-\nu_s) + \nu_s (1-\varepsilon_m) (1-\nu_m)} - \frac{1}{\nu_s} - \frac{1}{\nu_m}.
\]  

(7)

Details of the derivation are given in appendix A. In (7), \( \Omega \) is a function of the parameters \( \nu_m, \nu_s, \varepsilon_m \) and \( \varepsilon_s \). The relative price of the two goods is technologically determined as it depends on the parameters of the production functions and on sectoral TFP, \( B_m \) and \( B_s \). This result follows from the non-substitution theorem (Samuelson, 1951).

The supply side of this economy allows the construction of an aggregate production function similar to the standard neoclassical aggregate production function. As there are two goods in the economy, the aggregate production function can be expressed either in manufacturing or in services units. I define an aggregate production function for the economy under study as a mapping from the total amount of labor available in the economy at a point in time into the maximum amount of manufacturing or services that can be produced for consumption purposes. To find the aggregate production function in manufacturing units at time \( t \) it is sufficient to solve the following problem

\[
\max_{N_m, M_m, S_m, M_s} \left[ B_m N_m^{\nu_m} \left( M_m^{\varepsilon_m} S_m^{1-\varepsilon_m} \right)^{1-\nu_m} - M_m - M_s \right]
\]

subject to

\[
B_s (N - N_m)^{\nu_s} \left( M_s^{1-\varepsilon_s} S_s^{\varepsilon_s} \right)^{1-\nu_s} = S_m + S_s,
\]

where \( B_m N_m^{\nu_m} \left( M_m^{\varepsilon_m} S_m^{1-\varepsilon_m} \right)^{1-\nu_m} \) and \( B_s (N - N_m)^{\nu_s} \left( M_s^{1-\varepsilon_s} S_s^{\varepsilon_s} \right)^{1-\nu_s} \) are the gross output production functions defined in (3) and (4) and \( N \) is the total amount of labor used in production in the economy in the period considered. The solution to problem (8) determines the maximum amount of manufacturing that can be consumed in the economy given the amount of labor services \( N \) used in production at the economy level. That is, the solution to (8) determines the point in which the production possibility frontier of this economy crosses the manufacturing axis. Note that the constraint in (8) implies that the services sector becomes an intermediate goods sector as it produces only intermediate services used in the production of manufacturing and of services themselves.\(^{14}\)

\(^{14}\)Note that the entire production possibility frontier can be found by substituting the constraint in (8) with \( B_s (N - N_m)^{\nu_s} \left( M_s^{1-\varepsilon_s} S_s^{\varepsilon_s} \right)^{1-\nu_s} = S_m + S_s + c_s \). In this case, the solution to (8) would provide the amount of manufacturing that can be consumed - or invested - as a function of the amount of services consumption \( \varepsilon_s \).
The solution to problem (8) at time \( t \) is

\[
V_{m,t} = \Theta_m(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) B_{m,t}^{f_1(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s)} B_{s,t}^{f_2(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s)} N_t,
\]

where \( \Theta_m, f_1 \) and \( f_2 \) are functions of \( \nu_m, \nu_s, \varepsilon_m \) and \( \varepsilon_s \). By dividing (9) by (7) it is possible to derive the maximum amount of services that can be consumed when the manufacturing sector produces only intermediate goods

\[
V_{s,t} = \Theta_s(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) B_{m,t}^{f_3(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s)} B_{s,t}^{f_4(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s)} N_t,
\]

where \( \Theta_s, f_3 \) and \( f_4 \) are also functions of \( \nu_m, \nu_s, \varepsilon_m \) and \( \varepsilon_s \). Details of the derivation and the explicit functional form of \( \Theta_m, \Theta_s, f_1, f_2, f_3 \) and \( f_4 \) are given in appendix A. As a standard neoclassical aggregate production function, (9) and (10) represent the economy’s resources in two extreme cases, one in which only manufacturing is consumed and services is only an intermediate sector, and another in which the opposite situation holds. Note also that (7) implies that the production possibility frontier of this economy is linear. Thus, \( p_s/p_m \) gives the feasible amount of manufacturing that can be produced in the economy by reducing the production of services by one unit. The difference between (9) and (10) lies in the TFP term that multiplies aggregate labor services. For given processes of \( B_{m,t} \) and \( B_{s,t} \), the pattern of the TFP term depends on the value of the functions \( f_1 \) and \( f_2 \) when the economy produces only manufacturing, and on the value of \( f_3 \) and \( f_4 \) when the economy produces only services.

Consider the case in which TFP in manufacturing and services follow the same process over time, \( B_{m,t} = B_{s,t} = B_t \ \forall \ t \). Aggregate TFP growth and volatility are larger in (9) than in (10) if and only if \( f_1 + f_2 > f_3 + f_4 \). By using the explicit functional forms of \( f_1, f_2, f_3 \) and \( f_4 \) it can be shown that this is true when

\[
\nu_m < \nu_s,
\]

that is, when the share of intermediate goods in gross output is larger in manufacturing than in services. The proof is provided in Appendix A. Condition (11) is empirically true across countries, as figure 1 and the discussion in section 3 make clear. Thus, given the same sectoral TFP process in the two sectors, aggregate TFP growth and volatility decline along the production possibility frontier of the economy when reducing manufacturing and increasing services.
4.2 Households

The model economy is inhabited by a measure one of identical households, indexed in the interval $i \in [0, 1]$. Households in this economy have preferences over manufacturing and services consumption and are endowed with one unit of labor services each period.

The utility function of the consumer at date $t$ is given by

$$u = \log \left[ bc_m^\rho + (1 - b) (c_s + \bar{s})^{\rho_1} \right]^{\frac{1}{\rho_1}} + \varphi \log(1 - n), \quad (12)$$

where $c_m$ and $c_s$ are the per-capita consumption levels of manufacturing and services and $n$ are per-capita labor services.\(^{15}\) In (12), $\rho \leq 1$, $b \in [0, 1]$, $\bar{s} > 0$ and $\varphi > 0$. As in Kongsamut, Rebelo and Xie (2001), $\bar{s}$ is interpreted as home production of services.\(^{16}\)

Once the consumption index is defined as $c = \left[ bc_m^\rho + (1 - b) (c_s + \bar{s})^{\rho_1} \right]^{\frac{1}{\rho_1}}$, the utility function in (12) coincides with the one often used in growth theory and in the real business cycle literature. In this paper, $n$ is defined as labor services and not as hours worked. Average hours worked in the U.S. slightly fell from 1950 to 2000.\(^{17}\) Instead, per-capita labor services increased by a factor of 1.21 between 1960 and 2005 as measured in Jorgenson Dataset, 2007. Growth in labor services is computed as a weighted average of growth in hours worked of several types of labor, where the weights are given by the share of each type of labor in total labor compensation.\(^{18}\)

Each period, the household decides how much labor services to supply, earns a wage $wn$, and spends it in manufacturing and services consumption. The problem of each household at time $t$ is

$$\max_{c_s, c_m, n} \left\{ \log \left[ bc_m^\rho + (1 - b) (c_s + \bar{s})^{\rho_1} \right]^{\frac{1}{\rho_1}} + \varphi \log(1 - n) \right\} \quad (13)$$

subject to

$$p_s c_s + p_m c_m = wn.$$  

\(^{15}\) As households are identical I avoid the use of the index $i$ for the time being.

\(^{16}\) Herrendorf, Rogerson and Valentinyi (2009) show that Stone-Geary preferences in agriculture, manufacturing and services provide a good fit of post-war expenditure shares in these three types of goods in the U.S.

\(^{17}\) See Duarte and Restuccia (2007), among others.

\(^{18}\) See Jorgenson, Gollop and Fraumeni (1987) and O’Mahony and Timmer (2009) for a detailed description of the methodology to construct series of labor services. I use labor services instead of hours worked because the data on manufacturing and services used in the calibration are from Jorgenson Dataset, 2007. This dataset provides labor services data instead of hours worked. As TFP in manufacturing and services is computed using this dataset, the appropriate measure of labor in the model is labor services.
4.3 The Competitive Equilibrium

A competitive equilibrium for this economy is a set of prices \( \{p_s, p_m, w\} \), allocations for the households \( \{c_s, c_m, n\} \), for the manufacturing firm \( \{N_m, M_m, S_m\} \) and for the services firm \( \{N_s, M_s, S_s\} \) such that, given prices: a) \( \{c_s, c_m, n\} \) solve the household problem; b) \( \{N_m, M_m, S_m\} \) solve the manufacturing firm problem; c) \( \{N_s, M_s, S_s\} \) solve the services firm problem; and d) markets clear:

\[
G_m = \int_0^1 c_m di + M_m + M_s = c_m + M_m + M_s,
\]

\[
G_s = \int_0^1 c_s di + S_m + S_s = c_s + S_m + S_s,
\]

\[
\int_0^1 n di = n = N_m + N_s.
\]

The numeraire of the economy is the price of manufacturing, \( p_m = 1 \). Absent capital accumulation, the dynamic equilibrium of the economy is a sequence of static equilibria. Once the equilibrium \( c_{m,t} \) and \( c_{s,t} \) are found at each \( t \), real value added in the two sectors, \( y_{m,t} \) and \( y_{s,t} \), is given by

\[
y_{m,t} = \frac{c_{m,t}}{p_{ym,t}} \quad \text{and} \quad y_{s,t} = \frac{p_{s,t} c_{s,t}}{p_{ys,t}}
\]

where

\[
p_{ym,t} = \nu_m (1 - \nu_m)^{\frac{1-\nu_m}{\epsilon_m}} \left( \frac{p_{s,t}^{1-\epsilon_m}}{\epsilon_m (1 - \epsilon_m)^{1-\epsilon_m}} \right)^{-\frac{1-\nu_m}{\epsilon_m}}, \quad (14)
\]

and

\[
p_{ys,t} = \nu_s (1 - \nu_s)^{\frac{1-\nu_s}{\epsilon_s}} \left( \frac{p_{s,t}^{1-\epsilon_s}}{\epsilon_s (1 - \epsilon_s)^{1-\epsilon_s}} \right)^{-\frac{1-\nu_s}{\epsilon_s}}.
\]

Here \( p_{ym,t} \) and \( p_{ys,t} \) are the value added deflators in manufacturing and services.\(^{19}\) Aggregate value added at \( t \), which is the model’s counterpart of real GDP in the data, is computed as a chain-weighted Fisher index of sectoral value added. This is the same concept suggested by NIPA and used to construct the U.S. real GDP series.

4.4 Strategy and Simulations

In this section I calibrate the model to quantify the importance of the size of the services sector on observed differences in per-capita GDP growth and volatility between high and

\(^{19}\)See Appendix B for the derivation of \( p_{ym,t} \) and \( p_{ys,t} \).
middle income countries. As noted in Lucas (1988), the cross country variability in growth rates is high for middle income economies while it is low for high income economies. Thus, the comparison of per-capita GDP growth and volatility between a high income and a middle income economy crucially depends on the middle income country chosen, even when controlling for the share of services in GDP. To deal with this issue, I compare the two groups of countries defined High Income and Middle Income by the Word Bank in 2008. As these two groups include a large number of countries, idiosyncratic conditions of single countries are netted out. Figure 3 reports the share of services in GDP in the two groups of countries. The share of services of Middle Income increases from 0.41 to 0.53 from 1970 to 2006 while that of High Income increases from 0.55 to 0.73. During the 1970-2006 period, the Middle Income GDP per-capita grows at 2.69% per-year while the High Income one at 2.20%. Volatility, measured as the standard deviation of per-capita GDP growth rates over the period, is 1.70% for Middle Income and 1.42% for High Income. Thus, the group of Middle Income economies experiences an average growth rate 22% larger and a volatility 19% larger than the High Income group during the 1970-2006 period.

In the model, given sectoral TFP processes in manufacturing and services, the share of services in GDP is the unique determinant of GDP growth and volatility. It follows that to isolate the effect of structural change on GDP growth and volatility it is useful to study the case in which manufacturing and services sectoral TFP is driven by a common process. Thus, the model can be used to answer the following: if sectoral TFP processes in manufacturing and services were to be generated by the same stochastic process, what would be the effect of structural change on GDP growth and volatility? And further, how much of the difference in growth and volatility observed in the data between Middle and High Income economies can be accounted for by structural change in the model? To address these questions I proceed as follows. As one model period is one year in the data, I calibrate the model such that, given a common TFP process in the two sectors, the share of services in GDP increases

---

20 This approach implies that all countries that belong to the High (Middle) Income group in 2008 are used to construct the time series of the share of services and GDP growth of the High (Middle) Income group from 1970 to 2006. Thus, although it is possible that a country defined High (Middle) Income in 2008 was not so in 1970, this methodology provides data for two homogenous groups of countries over time. Economies are defined according to 2008 GNI per capita, calculated using the World Bank Atlas method. High Income countries are those for which per capita GNI in 2008 was $11,906 or more. Middle Income countries are those for which per capita GNI in 2008 was between $976 and $11,905.

21 GDP is measured in 2000 U.S. dollars.
from 0.41 to 0.73 in 74 periods and in the first 37 periods GDP growth and volatility match those measured in the data for Middle Income economies during the 1970-2006 period. In this way, the calibrated model generates a transition path that matches GDP growth and volatility of Middle Income economies in the first half of the transition. Next, by measuring GDP growth and volatility generated by the model in the last 37 periods, and comparing them with the corresponding figures for High Income economies it is possible to quantify the effect of structural change on growth and volatility.

More in detail, the calibration strategy is as follows. To simulate the model it is necessary to calibrate ten parameters and the gross output TFP processes in the two sectors. Calibration of the technology parameters $\nu_m$, $\nu_s$, $\varepsilon_m$ and $\varepsilon_s$ requires data on the value of gross output, labor and intermediate inputs for the manufacturing and the services sector. These data are not available for the two groups of High and Middle Income countries. However, equation (11) states that the condition for the decline in aggregate TFP growth and volatility when the share of services in GDP increases is $\nu_m < \nu_s$, which is true for all countries in figure 1 and for the U.S. In addition, the absolute values of $\nu_m$ and $\nu_s$ are roughly constant across countries. Thus, I calibrate $\nu_m$, $\nu_s$, $\varepsilon_m$ and $\varepsilon_s$ using U.S. data from Jorgenson dataset, 2007. The Cobb-Douglas assumption implies that the elasticities of output with respect to inputs are equal to the inputs shares of gross output in equilibrium. Parameters are then
set equal to the average shares during the 1960-2005 period. I obtain $\nu_m = 0.32$, $\nu_s = 0.51$, $\varepsilon_m = 0.71$ and $\varepsilon_s = 0.72$. As in Duarte and Restuccia (2010), I normalize to one sectoral TFP levels in the first period in both sectors, $B_{m,1} = B_{s,1} = 1$. On the demand side, I set $\rho$ according to Rogerson (2008) and Duarte and Restuccia (2010), equal to $-1.5$.

The remaining three preference parameters, $\bar{s}$, $b$ and $\varphi$, and the common sectoral TFP growth rate in the two sectors, $\psi$, are set to match four targets in the data. Two targets are the share of services in GDP in the first period (0.41), which corresponds to that of Middle Income economies in 1970, and that in the last period (0.73), which is the share in High Income economies in 2006. That is, given the growth rate and the initial level of $B_{m,t}$ and $B_{s,t}$, the model matches a share of services in GDP of 0.41 in period 1 and of 0.73 in period 74. The third target is the increase in per-capita labor services over time. This target is not available for the two groups of High and Middle Income economies. I measure the total growth in per-capital labor services in the U.S. during the 1960-2005 period using Jorgenson dataset and find an average yearly growth rate of 0.43%. Using this measure, the third target of the calibration is an increase in per-capita labor services of a factor $(1+0.0043)^{73} = 1.3678$ between periods 1 and 74. The fourth target is an average growth rate of GDP equal to 2.69% during the first 37 periods, which is that of Middle Income economies in the 1970-2006 period. The calibrated parameters are $\bar{s} = 0.0664$, $b = 0.0227$, $\varphi = 0.6121$, $\psi = 0.0083$. To interpret the calibrated value of $\bar{s}$, note that this implies a share of market consumption in total services consumption, $c_s/(c_s + \bar{s})$, of 23% in period one and of 73% in period 74.

Finally, I need to calibrate the stochastic process for sectoral TFP shocks. Sectoral TFP is defined as

$$B_{j,t} = B_{j,t-1}(1 + \psi)e^{z_{j,t}},$$

(15)

with $j = m, s$, $z_{j,t} = \rho_z z_{j,t-1} + \varepsilon_{j,t}$, $\varepsilon_{j,t} \sim N(0, \sigma^2)$ and i.i.d. over time. Equation (15) states that in each sector sectoral TFP grows at the constant rate $\psi$ and receives a shock $z_{j,t}$ at each $t$. The shock $z_{j,t}$ follows the same stochastic process in both sectors. If shocks are set to zero, $z_{j,t} = 0$ $\forall j, t$, sectoral TFP grows at the deterministic rate $\psi = 0.0083$. To calibrate $\rho_z$ I use data from the U.S. manufacturing and services sectors in Jorgenson dataset, 2007. I first compute series for $B_{m,t}$ and $B_{s,t}$ using the production functions (3) and (4) and data of sectoral gross output, labor and intermediate goods in manufacturing and services. Then, I log and detrend the series $B_{m,t}$ and $B_{s,t}$ with an Hodrick-Prescott filter, and estimate
an AR(1) process of the percentage deviations. The estimated coefficient is 0.63 for both sectors and statistically significant. Finally, I set $\sigma = 0.0062$, so that the model matches a standard deviation of GDP growth rates equal to 1.70% in the first 37 periods, which is that of Middle Income economies in the 1970-2006 period. Parameter values are reported in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Calibrated Parameters for the Benchmark Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$\nu_m$</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
</tr>
<tr>
<td>$\nu_s$</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
</tr>
<tr>
<td>$\rho$</td>
</tr>
<tr>
<td>$b$</td>
</tr>
<tr>
<td>$\bar{s}$</td>
</tr>
<tr>
<td>$\varphi$</td>
</tr>
<tr>
<td>$B_{m,1}$</td>
</tr>
<tr>
<td>$B_{s,1}$</td>
</tr>
<tr>
<td>$\psi$</td>
</tr>
<tr>
<td>$\rho_z$</td>
</tr>
<tr>
<td>$\sigma$</td>
</tr>
</tbody>
</table>

Average results of 100,000 simulations are reported in table 3. The first row reports the average per-capita GDP growth rate and the standard deviation of per-capita GDP growth rates for the first 37 model periods and two calibration targets, the per-capita GDP growth rate and the standard deviation of per-capita GDP growth for Middle Income economies during the 1970-2006 period. The second row reports the same statistics for the last 37 model periods and for High Income economies during the 1970-2006 period. Finally, the third row reports the ratio between the two cases. The model delivers a decline in growth and volatility very close to that observed in the data. The average growth rate of per-capita GDP is 21% larger in the first 37 periods with respect to the last 37, compared to a difference in the data of 22%. Thus, the structural transformation alone is able to explain around 95% of the difference in growth rates between Middle and High income economies. The ratio of volatility is 18% in the model compared to 19% in the data. Thus, the different size

\[ \frac{22}{22} \] There are other ways to calibrate the autoregressive parameter $\rho_z$. The procedure adopted here has the advantage to provide the same estimate for manufacturing and services, which is used to calibrate the common TFP process in the two sectors. Note, however, that the quantitative results are not sensitive to the value of the autoregressive parameter $\rho_z$.\[ \frac{20}{20} \]
of the services sector also explains 95% of the difference in volatility between Middle and High income economies.\textsuperscript{23} By assuming an identical TFP process at the sectoral level, the structural transformation between manufacturing and services can explain almost the entire difference in growth and volatility between Middle and High income countries.

An interesting feature of the model is the change in volatility within sectors along the transition path. Koren and Tenreyro (2007 and 2008) document that a large part of the volatility decline observed along the development process is due to a reduction of volatility within sectors. The fifth and sixth columns of table 3 report the volatility of manufacturing and services real value added in the model in the two subperiods. While the volatility of manufacturing remains almost unchanged, the volatility of services is 44% larger in the first subperiod with respect to the second. In the data, the ratio of volatility between Middle Income and High Income economies is 0.79 in manufacturing and 1.76 in services.\textsuperscript{24} In the model, the large reduction of services volatility is due to the non-homotheticity of preferences, which implies an income elasticity of services larger than one. As TFP grows, the income elasticity of services declines over time. Instead, the income elasticity of manufacturing is always equal to one along the transition path. Thus, the model suggests that a changing income elasticity can have an important role in shaping sectors’ volatility, even when TFP processes do not change over time. This represents a new channel to investigate in the study.

\textsuperscript{23}The results in table 3 are robust to cross-countries changes in the technological parameters $\nu_m$, $\nu_s$, $\varepsilon_m$, and $\varepsilon_s$. I perform the same calibration exercise as in table 2 using KLEMS data for Poland, which is a Middle Income economy in 2008, instead of the U.S. In this case, the quantitative results deliver a difference between the first and the second part of the transition of 24% in terms of growth and 25% in terms of volatility.

\textsuperscript{24}Value added in manufacturing and services is computed at constant 2000 dollars. The series of manufacturing value added for high income economies is available only from 1998. Thus, manufacturing value added volatility for High Income economies is the standard deviation of growth rates over the period 1999-2006.
of the time varying properties of a country business cycle. It suggests that non-homothetic preferences that can account for long-run changes in the composition of GDP, can also play a role in determining sectors volatility and, in turn, GDP volatility.

The results in table 3 provide a quantitative assessment of the importance of the relative size of manufacturing and services for per-capita GDP growth and volatility under the assumption that $B_{m,t}$ and $B_{s,t}$ follow a common process. However, this assumption is not necessarily true in the data. As discussed in section 3, if gross output TFP grows faster and is more volatile in services than manufacturing, it is possible that a larger share of services in GDP imply a faster and more volatile per-capita GDP growth. I now turn to perform simulations of the model where the gross output TFP processes in manufacturing and services are calibrated using U.S. data for the 1960-2005 period from Jorgenson dataset, 2007. Calibrating TFP processes to the U.S. manufacturing and services sectors is relevant to study the quantitative properties of the model because the U.S. experienced a large process of structural transformation between these two sectors.

As for the calibration in table 2, I compute series for $B_{m,t}$ and $B_{s,t}$ for the U.S. using the model’s production functions (3) and (4) and data for gross output, labor services and intermediate goods. Average gross output TFP growth is 0.0079 in manufacturing and 0.0082 in services. Note that the larger gross output TFP growth in services than manufacturing is in contrast with figure 2, where gross output TFP in manufacturing displays a larger growth than services. This is due to the fact that series in figure 2 are computed using production functions in capital, labor and intermediate goods while the model’s production functions (3) and (4) include only labor and intermediates. Thus, in this case the structural transformation towards services implies an increase in the size of the sector with the largest growth rate of sectoral TFP.
Table 4: Calibrated Parameters for the U.S. Economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_m$</td>
<td>Share of $N_m$ in $G_m$</td>
<td>0.32</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>Share of $M$ in manufac. interm.</td>
<td>0.71</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>Share of $N_s$ in $G_s$</td>
<td>0.51</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Share of $S$ in services intern.</td>
<td>0.72</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elasticity parameter in preferences</td>
<td>-1.5</td>
</tr>
<tr>
<td>$b$</td>
<td>Weight of manufacturing in preferences</td>
<td>0.0202</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>Home production of services</td>
<td>0.0728</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Weight of labor services in preferences</td>
<td>0.5617</td>
</tr>
<tr>
<td>$B_{m,1}$</td>
<td>Manufacturing TFP level in the first period</td>
<td>1</td>
</tr>
<tr>
<td>$B_{s,1}$</td>
<td>Services TFP level in the first period</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{\psi}_m$</td>
<td>Average growth rate of manufacturing TFP</td>
<td>0.0079</td>
</tr>
<tr>
<td>$\bar{\psi}_s$</td>
<td>Average growth rate of services TFP</td>
<td>0.0082</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Autoregressive Parameter in manufacturing</td>
<td>0.20</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Autoregressive Parameter in services</td>
<td>0.32</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Standard deviation of shocks in manufact.</td>
<td>0.0104</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Standard deviation of shocks in services</td>
<td>0.0078</td>
</tr>
</tbody>
</table>

Gross output TFP in manufacturing follows

$$B_{m,t} = B_{m,t-1}(1 + \bar{\psi}_m)e^{z_{m,t}},$$  \hspace{0.5cm} (16)

with $z_{m,t} = \rho_m z_{m,t-1} + \epsilon_{m,t}$, $\epsilon_{m,t} \sim N(0, \sigma_m^2)$ and i.i.d. over time, and gross output TFP in services is

$$B_{s,t} = B_{s,t-1}(1 + \bar{\psi}_s)e^{z_{s,t}},$$

with $z_{s,t} = \rho_s z_{s,t-1} + \epsilon_{s,t}$, $\epsilon_{s,t} \sim N(0, \sigma_s^2)$ and i.i.d. over time.

Equation (16) implies that the shock $z_{m,t}$ is the difference of two components

$$z_{m,t} = \log(B_{m,t}) - \log(B_{m,t-1}) - \frac{\log(1 + \bar{\psi}_m)}{\text{Trend growth rate}},$$  \hspace{0.5cm} (17)

the realized growth rate of $B_m$ at $t$ and the trend growth rate. I construct series of $z_{m,t}$ using (17), the series $B_{m,t}$ and the average growth rate $\bar{\psi}_m$ measured in the data. I then use the series of $z_{m,t}$ so constructed to estimate

$$z_{m,t} = \rho_m z_{m,t-1} + \epsilon_{m,t}.$$  

The estimated $\rho_m$ is 0.20 and the standard deviation of residuals is $\sigma_m = 0.0104$. The same procedure for services delivers $\rho_s = 0.32$ and $\sigma_s = 0.0078$. As in the calibration in table 2,
the initial period TFP is normalized to one in both sectors $B_{m,1} = 1$, $B_{s,1} = 1$. With the new processes for $B_m$ and $B_s$, the parameters $\bar{s}$, $b$ and $\varphi$ have to be calibrated to match a share of services in GDP of 0.41 in the initial period, of 0.73 in the final period and an average growth of per-capita labor services of 0.43%. The calibrated parameters are $\bar{s} = 0.0728$, $b = 0.0202$ and $\varphi = 0.5617$. Parameter values are reported in table 4.

Table 5 reports average statistics of 100,000 simulations. As in table 3, the first row reports results for Middle Income and the second row results for High Income economies, while the third row reports the ratios between the two economies. In terms of growth rates, the difference between the two economies, 21%, is equal to table 3 even if services sectoral TFP grows faster than manufacturing. Thus, the sectoral composition accounts again for 95% of the difference between the two types of economies. Instead, the ratio of the standard deviations of GDP growth rates in the two economies is now 25% in the model while it is 19% in the data. In this case, the different size of the services sector can account for more than the observed difference in volatility between Middle and High income economies. This is due to the larger TFP volatility in manufacturing with respect to services. Thus, also when the model is calibrated on a country that experienced a large process of structural transformation like the U.S., it is able to account for cross-country differences in GDP growth and volatility.

Finally, note that some care must be taken when interpreting the absolute value of the standard deviations generated by the model in table 4. These values are sensibly larger than the ones in the data. This is because the model is calibrated on the U.S. economy, which has a smaller size than the two groups of countries considered in the data. To see this, suppose that each aggregate of countries is composed by $n$ economies of the size of the U.S. Thus, the GDP growth rate $\gamma$ of the aggregate economy can be written as

$$\gamma = \sum_{i=1}^{n} \frac{\gamma_i}{n}$$
where $\gamma_i$ is the growth rate of GDP of the $i$ economy. Consider now two extreme cases, the one in which the $\gamma_i$’s are perfectly correlated and that in which they are uncorrelated. If the standard deviation of each $\gamma_i$ is $\sigma$, and the growth rates are perfectly correlated across $i$’s, the standard deviation of $\gamma$ is $\sigma$ itself. Instead, when the growth rates are perfectly uncorrelated across $i$’s, the standard deviation of $\gamma$ is $\sigma/\sqrt{n}$. In this case, the absolute value of the volatility generated by the model is similar to that of an economy that receives U.S. shocks but also that is similar in size to the U.S. The purchasing power parity GDP in international dollars in 2008 of the aggregate of middle income countries in 2008 is 2.04 times that of the U.S. while that of high income economies is 2.79 times that of the U.S. in the same year. Thus, under the assumption of perfectly uncorrelated $\gamma_i$’s, the standard deviations generated by the model should be divided by $\sqrt{2.04} = 1.43$ for Middle Income economies and by $\sqrt{2.79} = 1.67$ for High Income economies. The last column of table 4 reports the adjusted values. Volatility in the model is now lower than in the data and the ratio of standard deviations becomes 47%.

The numerical results of this section point to a crucial role of the sectoral composition for the growth rate and the volatility of GDP. The analysis suggests that, other conditions equal, economies that differ in the size of the services sector relative to manufacturing display different patterns of GDP growth and volatility. That is, the transmission of sectoral TFP to aggregate TFP changes as an economy develops. Thus, the growth performance of aggregate TFP should be evaluated by taking into account the size of manufacturing and services in the economy. The same argument holds for aggregate TFP volatility.

5 Conclusion

In an influential contribution, Jorgenson et al. (1987) argue that gross output measures of TFP should be preferred to value added measures. This is because gross output measures take explicitly into account intermediate goods contribution to output growth. Instead, value added TFP is an increasing function of the share of intermediate goods in gross output. In this paper, I show that the share of intermediate goods in gross output is larger in manufacturing than in services for 26 developing and developed countries. This fact implies that, for the same growth and volatility of TFP at the gross output level in the two sectors, manufacturing displays a higher valued added TFP growth and volatility. Under these
conditions, an economy that is more intensive in services displays a lower GDP volatility and a lower GDP growth with respect to an economy more intensive in manufacturing.

In this paper, I exploit this intuition to construct a model of the structural transformation which is qualitatively consistent with the main difference in growth an volatility observed between Middle Income and High Income economies. I also use the model to quantify the importance of the sectoral composition for these observed differences. I find that the structure of the economy has a quantitatively important effect both on GDP growth and volatility. Thus, this paper represents an attempt to reconcile cross-country and single country times series evidence on GDP growth and volatility in a unique environment.
Data Appendix

**KLEMS dataset, 2008.** This dataset provides harmonized data for 30 countries. These are the countries reported in figure 1 plus Malta, Luxemburg and the U.S. The first two are excluded for their size. For the U.S. I use data from Jorgenson Dataset, 2007, that reports a longer time period. The share of intermediate goods in gross output in manufacturing is computed as the total value of intermediate goods used in manufacturing divided the total value of gross output produced in manufacturing. The share of intermediate goods in gross output in services is accordingly constructed. The sectors used in manufacturing computations are 1) Agriculture, hunting and forestry, 2) Fishing, mining and quarrying, 3) Total Manufacturing, 4) Electricity, gas and water supply, 5) Construction. The sectors used in services computations are 6) Wholesale and retail trade, 7) Hotels and restaurants, 8) Transport, storage and communication, 9) Financial intermediation, 10) Real estate, renting and business activities, 11) Public administration and defense, 12) Education, 13) Health and social work, 14) Other community, social and personal services, 15) Private households with employed persons, 16), Extra-territorial organizations and bodies.

**Jorgenson Dataset, 2007.** This dataset provides data for 35 sectors that cover the U.S. economy for the period 1960-2005. For each sector and for each year the dataset provides quantity and price indices for gross output, capital, labor and intermediate goods coming from the same 35 sectors. The share of intermediate goods in gross output in manufacturing is computed as the total value of intermediate goods used in manufacturing divided the total value of gross output produced in manufacturing. The share of intermediate goods in gross output in services is accordingly constructed. The manufacturing sector includes 1) Agriculture, forestry and fisheries, 2) Metal mining, 3) Coal mining, 4) Crude oil and gas extraction, 5) Non-metallic mineral mining, 6) Construction, 7) Food and kindred products, 8) Tobacco manufactures, 9) Textile mill products, 10) Apparel and other textile products, 11) Lumber and wood products, 12) Furniture and fixtures, 13) Paper and allied products, 14) Printing and publishing, 15) Chemicals and allied products, 16) Petroleum refining, 17) Rubber and plastic products, 18) Leather and leather products, 19) Stone, clay and glass products, 20) Primary metals, 21) Fabricated metal products, 22) Non-electrical machinery, 23) Electrical machinery, 24) Motor vehicles, 25) Other transportation equipment.

\(^{25}\text{See Moro (2009) for further details.}\)

**World Bank Data (World Development Indicators).** The World Bank reports time series of per capita-GDP levels for the group of countries defined High Income economies and the group defined Middle Income economies. High-income economies are those in which 2008 GNI per capita was $11,906 or more. Middle-income economies are those in which 2008 GNI per capita was between $976 and $11,905. The per-capita GDP is measured in constant 2000 U.S. dollars. The World Bank also reports the share of services in GDP for these two groups of countries. The share of services in GDP for High Income countries is available only from 1970 to 2006, so I use this time period to compare the model with the data. I use the per-capita GDP series to compute the per-capita GDP growth rate and the standard deviation of per-capita GDP growth reported in tables 3 and in table 5. The World Bank also reports series of real value added in manufacturing and services for High Income and Middle Income economies. These are measured in constant 2000 U.S. dollars. The series of manufacturing real value added in High Income economies starts in 1998. Finally, I use the purchasing power parity GDP in international dollars in 2008 to perform the adjustment of GDP volatility in the model in table 5.

**Appendix A: Relative Price and Aggregate Production Functions**

The maximization problem of the representative firm in manufacturing is

\[
\max_{N_m, M_m, S_m} \left[ p_m G_m - w N_m - p_m M_m - p_s S_m \right] \tag{18}
\]

subject to \( G_m = B_m N_m^{\nu_m} \left( M_m^{\varepsilon_m} S_m^{1-\varepsilon_m} \right)^{1-\nu_m} \),

where \( w \) is the wage rate, \( p_m \) the price of the manufacturing good and \( p_s \) the price of services. The problem of the firm in services is

\[
\max_{N_s, M_s, S_s} \left[ p_s G_s - w N_s - p_m M_s - p_s S_s \right] \tag{19}
\]

subject to \( G_s = B_s N_s^{\nu_s} \left( M_s^{1-\varepsilon_s} S_s^{\varepsilon_s} \right)^{1-\nu_s} \).
As the gross output production function in the two sectors is Cobb-Douglas, it is possible to derive a net production function, defined as the amount of gross output produced in one sector minus the amount of inputs produced and used in the same sector. The net production function in the manufacturing sector is obtained by solving

\[ Y_m = \max_{M_m} \left\{ B_m N_m^{\nu_m} (M_m^{1-\varepsilon_m} S_m^{1-\varepsilon_m})^{1-\nu_m} - M_m \right\}, \tag{20} \]

and it is equal to

\[ Y_m = \Phi_{m1} B_m^{1-\varepsilon_m(1-\nu_m)} N_m^{-\varepsilon_m(1-\nu_m)} S_m^{-\varepsilon_m(1-\nu_m)}, \]

where \(\Phi_{m1} = [1 - \varepsilon_m (1 - \nu_m)] [\varepsilon_m (1 - \nu_m)]^{1-\varepsilon_m(1-\nu_m)}. \) Equation (20) can be re-written as

\[ Y_m = A_m N_m^{\theta} S^{1-\theta}, \tag{21} \]

where

\[ A_m = \Phi_{m1} B_m^{1-\varepsilon_m(1-\nu_m)}, \tag{22} \]

\(0 < \theta < 1\) is equal to \(\frac{\nu_m}{1-\varepsilon_m(1-\nu_m)}\) and \(S_m = S\). The problem of the firm in the manufacturing sectors becomes

\[ \max_{N_m,S} [p_m Y_m - w N_m - p_s S] \tag{23} \]

subject to (21) and (22).

By using the same derivation, the net production function in the services sector is given by

\[ Y_s = A_s N_s^{\gamma} M^{1-\gamma}, \tag{24} \]

where \(0 < \gamma < 1\) is equal to \(\frac{\nu_s}{1-\varepsilon_s(1-\nu_s)}\), \(N_s\) is the amount of labor and \(M\) is the amount of manufacturing used as intermediate good in the services sector, with

\[ A_s = \Phi_{s1} B_s^{1-\varepsilon_s(1-\nu_s)}, \tag{25} \]

and \(\Phi_{s1} = [1 - \varepsilon_s (1 - \nu_s)] [\varepsilon_s (1 - \nu_s)]^{1-\varepsilon_s(1-\nu_s)}. \)

The problem of the representative firm in services becomes

\[ \max_{N_s,M} [p_s Y_s - w N_s - p_m M] \tag{26} \]

subject to (24) and (25).
The production functions (21) and (24) and competitive markets imply that the representative firms in the two sectors set prices as

\[ p_m = \frac{w^\theta p_s^{1-\theta}}{\Phi_{m2} A_m}, \]  

and

\[ p_s = \frac{w^{\gamma} p_m^{1-\gamma}}{\Phi_{s2} A_s}, \]

where \( \Phi_{m2} = \theta^\theta(1-\theta)^{1-\theta} \) and \( \Phi_{s2} = \gamma^\gamma(1-\gamma)^{1-\gamma} \). The relative price of the two goods is

\[ \frac{p_s}{p_m} = \frac{(\Phi_{m2} A_m)^{\gamma/(\gamma+\theta-\gamma)\gamma}}{(\Phi_{s2} A_s)^{\theta/(\gamma+\theta-\gamma)\theta}}. \]  

By substituting (22) and (25), (29) can be rewritten as

\[ \frac{p_s}{p_m} = \frac{\left(\Phi_{m1}\Phi_{m2} B_m^{1-\nu_m(1-\nu_m)}\right)^{\gamma/(\gamma+\theta-\gamma)\gamma}}{\left(\Phi_{s1}\Phi_{s2} B_s^{1-\nu_s(1-\nu_s)}\right)^{\theta/(\gamma+\theta-\gamma)\theta}}. \]

Finally, using \( \theta = \frac{\nu_m}{1-\nu_m(1-\nu_m)} \) and \( \gamma = \frac{\nu_s}{1-\nu_s(1-\nu_s)} \),

\[ \frac{p_s}{p_m} = \Omega(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) \left( \frac{B_m^{\nu_s}}{B_m^{\nu_m}} \right)^{\frac{\nu_m[1-\varepsilon_m(1-\nu_m)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_s\nu_m}{\nu_m[1-\varepsilon_m(1-\nu_m)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_s\nu_m}}, \]  

where

\[ \Omega(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) = \frac{\Phi_{m1}\Phi_{m2}}{\Phi_{s1}\Phi_{s2}} \left( \frac{B_m^{\nu_s}}{B_m^{\nu_m}} \right)^{\frac{\nu_m[1-\varepsilon_m(1-\nu_m)]}{\nu_m[1-\varepsilon_m(1-\nu_m)]+\nu_s[1-\varepsilon_m(1-\nu_m)]-\nu_s\nu_m}}. \]

To find the aggregate production function in manufacturing units at time \( t \) it useful to rely again on the net production functions, and solve

\[ \max_{N_m, M, S} \left[ A_m N_m^{\theta} S^{1-\theta} - M \right] \]

subject to

\[ A_s (N - N_m)^{\gamma} M^{1-\gamma} = S, \]

where \( N \) is the total amount of labor available in the economy. Note that (31) corresponds to a reduced form of problem (8) in the main text, in which the first order conditions with respect to \( M_m \) and \( S_s \) already hold. The solution to problem (31) determines the maximum amount of manufacturing that can be produced for consumption purposes when the services
sector produces only the intermediate goods needed in the manufacturing sector, $S_t$, that is, when services is only an intermediate goods sector. This is given by

$$V_m = \Phi_{m3} A_m^{\frac{1}{\gamma + \theta - \gamma \theta}} A_s^{\frac{1-\theta}{\theta + \gamma - \gamma \theta}} N,$$

(32)

with $\Phi_{m3} = [1 - (1 - \theta)(1 - \gamma)][(1 - \theta)(1 - \gamma)]^{\frac{1-\theta}{\theta + \gamma - \gamma \theta}} \left(\frac{\theta}{\gamma + \theta - \gamma \theta}\right)^{\frac{\gamma(1-\theta)}{\frac{1-\theta}{\theta + \gamma - \gamma \theta}} \left(\frac{\gamma(1-\theta)}{\gamma + \theta - \gamma \theta}\right)}$. By substituting the definitions of $A_m$ and $A_s$, (32) becomes

$$V_m = \Phi_{m3} \Phi_{m1}^{\frac{1}{\gamma + \theta - \gamma \theta}} \Phi_{s1}^{\frac{1-\theta}{\theta + \gamma - \gamma \theta}} B_m^{\frac{1}{\nu_m(1-\nu_m)}} B_s^{\frac{1}{\nu_s(1-\nu_s)}} N,$$

and by defining $\Theta_m = \Phi_{m1}^{\frac{1}{\gamma + \theta - \gamma \theta}} \Phi_{m3} \Phi_{s1}^{\frac{1-\theta}{\theta + \gamma - \gamma \theta}}$ and using the definitions of $\theta$ and $\gamma$ it is possible to write (32) as

$$V_m = \Theta_m (\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) B_m f_1(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) B_s f_2(\nu_m, \nu_s, \varepsilon_m, \varepsilon_s) N,$$

(33)

which is equation (9) in the main text. In (33)

$$f_1 = \frac{1 - \varepsilon_s (1 - \nu_s)}{\nu_m [1 - \varepsilon_s(1 - \nu_s)] + \nu_s [1 - \varepsilon_m(1 - \nu_m)] - \nu_m \nu_s},$$

and

$$f_2 = \frac{(1 - \varepsilon_m)(1 - \nu_m)}{\nu_m [1 - \varepsilon_s(1 - \nu_s)] + \nu_s [1 - \varepsilon_m(1 - \nu_m)] - \nu_m \nu_s}.$$  

By using the numerators $f_1 + f_2 > f_3 + f_4$ if and only if $\nu_m < \nu_s$, note that the denominator in $f_1$, $f_2$, $f_3$ and $f_4$ is always positive. This is evident by re-writing it as $\nu_m (1 - \varepsilon_s) + \nu_s (1 - \varepsilon_m) + \nu_m \nu_s \varepsilon_s + \nu_m \nu_s \varepsilon_m$. Thus, by using the numerators $f_1 + f_2 > f_3 + f_4$ reads

$$1 - \varepsilon_s (1 - \nu_s) + (1 - \varepsilon_m)(1 - \nu_m) > (1 - \varepsilon_s)(1 - \nu_s) + 1 - \varepsilon_m (1 - \nu_m)$$

which can be simplified to obtain $\nu_m < \nu_s$. 

31
Appendix B: Real Value Added Deflators

Consider again the maximization problem of the manufacturing firm

$$\max_{N_m, M_m, S_m} \left[ p_m G_m - w N_m - p_m M_m - p_s S_m \right]$$  \hspace{1cm} (35)

subject to $G_m = B_m N_m^\nu_m \left( M_m S_m^{1-\varepsilon_m} \right)^{1-\nu_m}$.

Define

$$R_m = M_m^{\varepsilon_m} S_m^{1-\varepsilon_m},$$  \hspace{1cm} (36)

as the intermediate goods index in the manufacturing sector. Given the Cobb-Douglas form of this index, with competitive markets the price of $R_m$ is

$$p_r = \frac{p_m^{\varepsilon_m} p_s^{1-\varepsilon_m}}{\varepsilon_m (1-\varepsilon_m)^{1-\varepsilon_m}}.$$  \hspace{1cm} (37)

Thus, problem (35) can be written as

$$\max_{N_m, R_m} \left[ p_m G_m - w N_m - p_r R_m \right]$$  \hspace{1cm} (38)

subject to $G_m = B_m N_m^\nu_m R_m^{1-\nu_m}$.

The first order condition of (38) with respect to $R_m$ delivers the following condition

$$R_m = (1 - \nu_m)\frac{1}{\varepsilon_m} \left( \frac{p_m}{p_r} \right)^{\frac{1}{\nu_m}} \frac{1}{B_m^{\nu_m} N_m}.$$  \hspace{1cm} (39)

By plugging (39) into (38) it is possible to obtain the reduced form problem

$$\max_{K_m, N_m} \left[ p_{vm} VA_m - w N_m \right]$$  \hspace{1cm} (32)

subject to $p_{vm} VA_m = \nu_m (1 - \nu_m)\frac{1-\varepsilon_m}{\nu_m} \left( \frac{p_m}{p_r^{1-\nu_m}} \right)^{\frac{1}{\varepsilon_m}} \frac{1}{B_m^{\nu_m} N_m}$.

Here $p_{vm} VA_m$ represents nominal value added. Real value added $VA_m$ is defined, as in Sato (1976), as the contribution to gross output growth of primary inputs (here only labor) and technical change. It follows that the real value added function is given by $VA_m = B_m^{\nu_m} N_m$ and its price is $p_{vm} = \nu_m (1 - \nu_m)\frac{1-\varepsilon_m}{\nu_m} \left( \frac{p_m}{p_r^{1-\nu_m}} \right)^{\frac{1}{\varepsilon_m}}$. By using $p_r = \frac{p_m^{\varepsilon_m} p_s^{1-\varepsilon_m}}{\varepsilon_m (1-\varepsilon_m)^{1-\varepsilon_m}}$ in the last expression, the price of manufacturing value added in (14) is obtained. The value added price for services is accordingly constructed.
References


