Nominal Frictions, Monetary Policy, and Long-Run Risk

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Abstract

I show that long-run risk — highly persistent variation in expected consumption growth — arises endogenously in a production economy with nominal frictions. The ‘long-run’ part comes from price stickiness. Nominal frictions in the model generate a consumption growth process that shows low persistence unconditionally, but has a highly persistent conditional mean. The ‘risk’ part comes from Epstein-Zin preferences, which result in a large risk premium being associated with variation in the conditional mean. The model provides new testable implications for long-run-risk models, and restricts the joint distribution of consumption and nominal equity and bond risk premia. A calibrated version of the model generates consumption, a risk-free interest rate, and equity risk premium behavior that are consistent with U.S. data.

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1 Introduction

Bansal and Yaron (2004) show that long-run consumption risk — highly persistent variation in the conditional mean of consumption growth — can have quantitatively important implications for asset prices if investors have recursive preferences. This finding has given rise to an active empirical debate. At its heart lies a dilemma. Observed consumption growth looks i.i.d., implying that any predictable variation must be small and hard to detect. Bansal and Yaron’s insight was that a small component is enough. Recursive preferences do not need the predictable variation to be large. The dilemma, however, is that the basic story is difficult to verify. There exist many alternative models of the equity risk premium. How are we to discriminate between them and the long-run risk model if a tenet of the latter is “it’s there but you can’t see it?”

One approach for making progress is to use a more fully-articulated general equilibrium model to enlarge the set of testable restrictions. In such a model, consumption is an endogenous variable. So are output, capital and labor, wages, the return on capital, and so on. If consumption contains a magical, yet difficult-to-detect, ingredient then it is likely that these other variables are affected by the same ingredient. Examining them all in unison, through the lens of a model, might shed light on the plausibility of long-run consumption risk. This is the approach of Kaltenbrunner and Lochstoer (2010). They endogenize long-run risk in a real business cycle model. Their basic mechanism is the consumption smoothing motive that arises in a model with capital and investment. They show that it gives rise to high persistence in expected consumption growth. More recently, alternative mechanisms have been proposed. Kung and Schmid (2011) show that long-run risk arises in a production economy in which technological progress is determined endogenously by firms’ R&D activity. Kuehn, Petrosky-Nadeau, and Zhang (2010) embed search frictions in the labor market of a real business cycle model and show that they generate high persistence in output and a large equity premium. My paper follows a similar path. I also endogenize long-run risk, but I propose a substantially different mechanism.

The mechanism emphasized here is nominal frictions. This paper asks if nominal frictions — and therefore monetary policy — imply consumption outcomes that display long-run risk behavior. There is, of course, no shortage of motivation for examining this mechanism. Economic theory — ranging from the classical economists, to both old and New Keynesian models, to the modern search-theoretic models of money — gives us many coherent reasons
to believe in monetary non-neutrality. There is also much empirical evidence. Christiano, Eichenbaum, and Evans (1999) and Christiano, Eichenbaum, and Evans (2005) estimate the effects of monetary policy shocks using a structural vector autoregressive model and find that output, consumption, investments, real profits, real wages, and labor productivity fall in response to an exogenous monetary policy tightening. Sims (1992), Gál (1992), Bernanke and Mihov (1998), and Uhlig (2005) reach similar conclusions. Romer and Romer (1989) use an alternative identification approach and find that monetary policy shocks have large and persistent effects on production and unemployment. Closer to my paper is Rudebusch and Swanson (2008). They show that a New Keynesian model of nominal frictions can explain a fairly wide set of nominal term structure facts. My approach shares much in common. My model, while placing primary emphasis on the equity premium, also has sharp predictions for how nominal frictions affect real and nominal interest rates. But where my approach is distinctly different is in its emphasis on the distribution of consumption. I develop a general equilibrium model that formally ties all of this nominal-friction motivation to the consumption behavior emphasized by Bansal and Yaron (2004) and its descendants.

Examining how nominal frictions are connected to long-run risk, then, is my primary objective. But something that comes along with this is of equal importance. My model generates an endogenous process for inflation. It allows me to address questions like: What are the effects of nominal frictions on the inflation risk premium, the equilibrium consumption process, the term premium, and the equity premium? What are the effects of monetary policy? Much of the existing long-run risk literature is mute concerning these questions. It either focuses on real asset prices — thereby making it necessary to estimate real expected returns from nominal data — or it treats inflation as an exogenous process, which is appended to the model once real allocations and real asset prices are already determined (e.g., Bansal and Shaliastovich (2009)).

My model works as follows. It is a production economy in which the representative agent derives utility from the consumption of a basket of goods and disutility from supplying labor. Preferences are recursive as in Epstein and Zin (1989). Firms produce differentiated goods in a monopolistically competitive environment using a labor-only production function. The nominal frictions come from price stickiness, as in Calvo (1983). The only shock in the economy is a permanent technology shock. Finally, to close the model, the monetary authority follows an interest rate rule. Two specifications that have received particular attention in the literature are considered. In the first one, the nominal interest rate is set as a function of current inflation.
and the current output gap, as in Taylor (1993). In the second one, the monetary authority smooths interest rates over time, as in Judd and Rudebusch (1998) and Clarida, Galí, and Gertler (2000).

The model generates long-run consumption risk in the following sense. First, if there is no price stickiness, then output follows the dynamics of technology. I refer to this as the model without long-run risk, because realized and expected consumption growth have the same persistence. Second, if there is price stickiness, then a wedge arises in the dynamics of realized and expected consumption growth, resulting in a consumption growth process that shows low persistence unconditionally, but has a highly persistent conditional mean. I refer to this as the model with exogenous long-run risk, because the high persistence in expected consumption growth comes from the persistence of the exogenous technology growth process. Third, if the monetary authority smooths interest rates over time, the model generates endogenous long-run risk, because the inertial behavior of monetary policy induces ‘momentum’ in the representative agent’s expectations about future consumption, resulting in a highly persistent expected consumption growth process. Note that this is a somewhat weaker result than the one obtained by Kaltenbrunner and Lochstoer (2010) who show that an i.i.d technology growth process is sufficient to generate endogenous long-run consumption risk. In a way that will be made clear soon, my model requires some of the persistence to be exogenously injected.

The price stickiness mechanism works as follows. At each point in time, only a fraction of firms can react to the technology shock affecting the economy by optimally adjusting their prices and, therefore, their production. The remaining fraction of firms, on the other hand, must keep their prices unchanged and produce a suboptimal level of output. Sluggishness in the production process induced by the nominal rigidities then reduces the persistence of realized consumption growth relative to the persistence of expected consumption growth. The larger the fraction of firms that cannot adjust prices, the lower the persistence of realized consumption growth. For levels of stickiness that are consistent with the microeconomic evidence (e.g., Bils and Klenow (2004)), this wedge is quantitatively significant, with realized consumption growth showing low persistence, as in the data, and expected consumption growth being highly persistent.

The model’s nominal frictions generate real effects of monetary policy. All else being equal, a weak reaction of the monetary authority to current inflation and the output gap magnifies the effects of price rigidities on real allocations, thus reducing the persistence of realized consumption growth. While the reaction to current inflation and the output gap has a
significant impact on the dynamics of realized consumption growth, interest rate inertia plays a crucial role in determining expectations about future consumption. Increasing the level of interest rate inertia makes firms react less aggressively in response to technology shocks, which in turn decreases the initial consumption reaction. This leads to high persistence in expected consumption growth, as production moves closer to the new steady state. High persistence in expected consumption growth then increases the amount of long-run risk in the model, resulting in a large equity premium.

An important ingredient is a permanent technology shock. This creates a positive correlation between realized and expected consumption growth. In the model of Bansal and Yaron (2004), shocks to realized consumption growth are independent of shocks to expected consumption growth. In the general equilibrium framework developed here, the correlation between realized and expected consumption growth depends on the nature of the shock. When the shock is permanent, bad times for realized consumption growth are associated with bad times for expected consumption growth. Given this, a preference for the early resolution of risk implies a large price of risk. In contrast, if the shock was transitory, bad times for realized consumption growth would be associated with good times for expected consumption growth because technology would be expected to revert to its mean. An agent with preference for the early resolution of risk would view the variation associated with expected consumption growth as a hedge for a bad realization of current consumption growth, and the overall price of risk would be small.

The model delivers new restrictions on the joint dynamics of real allocations and both real and nominal asset prices. These restrictions shed light on the ongoing debate over the predictability of consumption and dividend growth (e.g., Beeler and Campbell (2009) and Bansal, Kiku, and Yaron (2009)). The restrictions fall into two categories. First, asset prices can be used to test the predictability of not only consumption and dividends, as is standard in the long-run risk literature, but also wages and inflation. Second, nominal asset prices can be tested in their ability to predict real allocations. In the model, a high nominal interest rate and a high price-dividend ratio predict high future consumption growth, dividend growth, real wage growth, and inflation. The degree of predictability depends on the level of price stickiness and on the strength of monetary policy. Consumption, dividends, and wages are more predictable when nominal frictions are low and/or monetary policy is strong. When the price stickiness and the interest rate rule coefficients are consistent with empirical estimates for the U.S., the price-dividend ratio and the nominal interest rate positively predict future
consumption, dividends, wages, and inflation, thus supporting the findings of Bansal, Kiku, and Yaron (2009).

A calibrated version of the model matches key features of the dynamics of U.S. consumption growth and asset returns, including a large equity premium (7.02%), a volatile return on equity (20.60%), and a low and smooth real risk-free interest rate (with an average level of 1.44% and a volatility of 1.46%, both annualized). Moreover, the equilibrium dynamics of the one-period nominal interest rate and of the inflation rate closely mimic those observed in the data. The nominal interest rate is highly persistent, and has an annualized mean of 6.42% and an annualized volatility of 3.72%. These results are obtained with a risk aversion coefficient of 6.5 and a level of price stickiness consistent with a situation in which firms, on average, change their prices every 4.5 months. The risk aversion coefficient is lower than what is common in the long-run risk literature as a consequence of the permanent nature of the technology shock. The degree of price stickiness is a conservative measure of what has been documented at the microeconomic level. Among others, Bils and Klenow (2004) find that firms change prices every 4 to 6 months, while Nakamura and Steinsson (2008) suggests that, excluding price changes associated with sales, the average price duration is in the range of 8 to 11 months.

The rest of the paper is organized as follows. Section 2 develops the baseline model, Section 3 inspects the mechanism at work and comments on the main theoretical results, Section 4 shows quantitative results from a calibrated version of the model, and Section 5 investigates additional asset pricing implications. In Section 6, I extend the model to allow for interest rate inertia in the monetary policy rule. Finally, Section 7 concludes.

2 The model

I model a production economy in which the representative agent derives utility from the consumption of a basket of goods and disutility from supplying labor for production. Firms produce a differentiated good in a monopolistically competitive environment with sticky prices. Monetary policy is conducted by means of a Taylor (1993) rule. The nominal price rigidities in the model generate real effects of monetary policy. I will show that, under certain conditions, these real effects generate long-run risk in consumption growth.
2.1 Preferences

The representative agent chooses to maximize the recursive utility function given by Epstein and Zin (1989). The intertemporal utility function, $V_t$, over streams of consumption $C_t$ and labor $L_t$ is the solution to the recursive equation

$$V_t = \left\{ (1 - \beta)U(C_t, L_t)^{1-\psi} + \beta E_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1-\psi}{1-\gamma}} \right\}^{\frac{1}{1-\psi}}, \quad (1)$$

where $\beta$ characterizes impatience, $\psi^{-1}$ measures the elasticity of intertemporal substitution over the consumption-labor bundle, and $\gamma$ measures relative risk aversion towards static gambles over the bundle. The relative magnitude of $\psi$ and $\gamma$ determines whether agents prefer early resolution of risk ($\gamma > \psi$), late resolution of risk ($\gamma < \psi$), or are indifferent to the timing of resolution of risk ($\gamma = \psi$). The intratemporal utility of consumption and labor is

$$U(C_t, L_t)^{1-\psi} = \left( \frac{C_t^{1-\psi}}{1-\psi} - \chi A_t^{1-\psi} \frac{L_t^{1+\omega}}{1+\omega} \right),$$

where $\omega^{-1}$ is the non-compensated elasticity of substitution of labor supply, and $\chi$ determines the average time the representative agent spends at work. Scaling the disutility of labor by the technology trend $A_t$ is necessary for the model to be consistent with balanced growth (see Uhlig (2010)). The process for $A_t$ will be described in Section 2.2.

The consumption good $C_t$ is a Dixit-Stiglitz basket of the differentiated goods $C_t(j)$, for $j \in [0, 1]$. Specifically,

$$C_t = \left[ \int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}}, \quad (2)$$

where $\theta$ is the elasticity of substitution across goods. Likewise, aggregate labor is specified as

$$L_t = \left[ \int_0^1 L_t(j)^{1+\omega} dj \right]^{\frac{1}{1+\omega}}, \quad (3)$$

where $L_t(j)$ is the labor supplied in the production of good $j$.

Financial markets are complete. The intertemporal budget constraint faced by representa-
tive household is
\[
E_t \left[ \sum_{s=0}^{\infty} M_{t,t+s} P_{t+s} C_{t+s} \right] \leq E_t \left[ \sum_{s=0}^{\infty} M_{t,t+s} \left( \int_0^1 W_{t+s}(j) L_{t+s}(j) + P_{t+s} \Psi_{t+s} \right) \right],
\]
where \( M_{t,t+s} \) is the nominal pricing kernel, \( W_t(j) \) is nominal wage earned in the production of good \( j \), \( \Psi_t \) measures the aggregate profits from production, and
\[
P_t \equiv \left( \int_0^1 P_t(j)^{1-\theta}dj \right)^{\frac{1}{1-\theta}}
\]
is the nominal price of a unit of the basket of goods, where \( P_t(j) \) is the price of good \( j \).

The representative agent’s maximization problem involves two steps. In the first step, the agent optimally allocates her resources across each differentiated good in a purely static fashion. As is standard in the Dixit-Stiglitz environment, this static optimization implies the following demand schedule:
\[
C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} C_t.
\]
In the second step, the agent solves a dynamic programming problem, arriving at the intertemporal allocation of consumption and savings. In particular,
\[
W_t \frac{P_t}{P_t} = \chi A_t^{1-\psi} C_t^{\psi} L_t^\omega,
\]
\[
e^{-i_t} = E_t(M_{t,t+1})
\]
Details for the derivation can be found in Appendix A.1. Condition (4) describes the evolution of real wages, and condition (5) is the Euler equation for the one-period nominal interest rate \( i_t \). The intertemporal marginal rate of substitution (the real pricing kernel), \( N_{t,t+1} \equiv M_{t,t+1}(P_t/P_{t+1}) \), is given by
\[
N_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\psi} \left( \frac{V_{t+1}}{E_t(V_{t+1}^{1-\gamma})^{1-\gamma}} \right)^{\psi-\gamma}.
\]
It is a crucial ingredient of our model and will be discussed in Section 3.2.
Using (1), it is useful to express the scaled value function $v_t \equiv \log \frac{V_t}{C_t}$ as

$$e^{(1-\psi)v_t} = (1 - \beta) \left( \frac{U(C_t, L_t)}{C_t} \right)^{1-\psi} + \beta e^{(1-\psi)} \log E_t \left[ e^{(1-\gamma)(v_{t+1} + \Delta c_{t+1})} \right],$$

where $\Delta c_{t+1} \equiv \log \frac{C_{t+1}}{C_t}$ denotes (log) consumption growth. Thus, a solution to the representative agent’s optimization problem can be characterized by a solution for the process $v_t$, which, in turn, depends on the endogenous solution for consumption growth, $\Delta c_{t+1}$.

### 2.2 Firms

Each firm $j \in [0, 1]$ produces a differentiated good, but they all use an identical linear labor-only technology

$$Y_t(j) = A_t L_t(j).$$

(7)

I abstract from endogenous capital accumulation because Kaltenbrunner and Lochstoer (2010) showed that, in a real business cycle model where the representative agent has recursive preferences, endogenous investment decisions generate consumption dynamics that are typical of long-run risk models, even in the absence of nominal frictions. By abstracting from capital accumulation, I isolate the effect of nominal frictions on realized consumption growth, expected consumption growth and asset prices.

The technology shock $A_t$ evolves according to the following autoregressive process:

$$\log(A_t/A_{t-1}) \equiv \Delta a_t = (1 - \varphi_a) \theta_a + \varphi_a \Delta a_{t-1} + \sigma_a \epsilon^a_t.$$  

(8)

The assumption of a difference stationary process for technology is common in the literature. Among others, Campbell (1994) and Rouwenhorst (1995) consider it in the context of a standard real business cycle model. More recently, based on the empirical estimates of Nelson and Plosser (1982), Goodfriend and King (2009) adopt such a process in the context of a New Keynesian model.\(^2\)

Producers have market power to set the price of their differentiated goods in a Calvo (1983)\(^2\) production decision. In this context, producers choose the price of their differentiated goods such that their marginal cost equals the marginal revenue from sales. This setup allows for potential price rigidities, where firms may not adjust prices in response to changes in demand or cost, reflecting the fact that prices are determined at the beginning of each period.

\(^2\)Croce (2008), in the context of a standard real business cycle model, considers an exogenous technology growth process that contains a persistent time-varying component. He shows that such a process reproduces several features of both asset prices and macroeconomic quantities, including long-run consumption risk. In the next Section I show that, in the presence of nominal frictions, the simple autoregressive process for technology growth considered here is sufficient to deliver long-run consumption risk.
staggered price setting. That is, each firm may reset its price with probability \((1 - \alpha)\) in any given period, independently of the time elapsed since the previous adjustment. Similarly to Yun (1996), the remaining fraction \(\alpha\) must charge the previous period’s price times an exogenous inflation target \(\Pi^*\). When a producer can adjust its price optimally, the price is set to maximize the present value of expected future profits. The maximization problem of firm \(j\) can be written as

\[
\max_{P_t^*(j)} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \alpha^s M_{t,t+s} \left[ P_t^*(j)(\Pi^*) Y_{t+s|t}(j) - W_{t+s|t}(j) L_{t+s|t}(j) \right] \right\},
\]

where \(P_t^*(j)\) is the optimal price chosen by firm \(j\) at time \(t\), \(Y_{t+s|t}(j)\), \(W_{t+s|t}(j)\), and \(L_{t+s|t}(j)\) denote product, wages and labor for the firm \(j\) at time \(t + s\), when the last price adjustment was at time \(t\). The maximization problem is subject to the production function (7) and the product demand function

\[
Y_{t+s|t}(j) = \left( \frac{P_j}{P_{t+s}} \right)^{-\theta} Y_{t+s},
\]

where \(Y_t \equiv \left[ \int_0^1 Y_t(j)^{\theta-1} dj \right]^{\theta-1} \) is the aggregate output.

As each firm that chooses a new price for its good in period \(t\) faces the same decision problem, the optimal price \(P_t^*(j)\) is the same for all of them, and so in equilibrium, all prices that are chosen at time \(t\) have the common value \(P_t^*(j) = P_t^*\). The Calvo price environment described above implies

\[
P_t \equiv \left[ \int_0^1 P_t(j)^{1-\theta} \right]^{1/\theta} = [(1 - \alpha)(P_t^*)^{1-\theta} + \alpha(\Pi^* P_{t-1})^{1-\theta}]^{1/\theta},
\]

so that in order to determine the price level \(P_t\), one need only know its initial value \(P_{t-1}\) and the single new price \(P_t^*\) that is chosen each period, without any reference about the prices chosen by the firms in the past.\(^3\)

The first order condition for the firm implies that the optimal price \(P_t^*\) satisfies

\[
P_t^* = \mu \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha^s M_{t,t+s} Y_{t+s|t} MC_{t+s|t}}{\mathbb{E}_t \sum_{s=0}^{\infty} (\alpha \Pi^*)^s M_{t,t+s} Y_{t+s|t}},
\]

where \(\mu \equiv \theta/(\theta - 1)\) is the frictionless markup, and \(MC_{t+s|t} = \frac{W_{t+s|t}}{A_{t+s}}\) is the nominal marginal cost. Details for the derivation can be found in Appendix A.2.

\(^3\)This is precisely true only in the case of specific factor markets, as in Woodford (2003).
2.3 Monetary policy

The monetary authority sets the one-period nominal interest rate according to a simple Taylor rule. In particular,

\[ i_t = \tau + \tau_\pi (\pi_t - \pi^*) + \tau_x x_t \quad , \tag{12} \]

where \((\pi_t - \pi^*) \equiv \log(\Pi_t / \Pi^*)\) denotes inflation deviations from its target, with \(\Pi_t \equiv (P_t / P_{t-1})\). The output gap \(x_t\) is

\[ x_t \equiv \log \left( \frac{Y_t}{Y_t^F} \right) \quad . \]

It is a measure of the deviation of total output \(Y_t\) from the natural level of output \(Y_t^F\) that would be produced in the economy if prices were perfectly flexible. The coefficient \(\tau_\pi\) and \(\tau_x\) measure the reaction of the monetary authority to inflation and output gap, and the constant \(\tau\) is used to determine the average level of \(i_t\).

2.4 Equilibrium and Solution Method

Under the constraints that consumption equals production \((C_t(j) = Y_t(j), \text{ for all } j)\) and labor demand equals labor supply, the equilibrium allocations and prices simultaneously satisfy the first order conditions of the maximization problem of the representative agent (conditions (4), (5) and (6)), the firms’ optimality condition (11), together with equation (10) describing the evolution of the price index, and the Taylor rule (12).

I solve the model by first obtaining a log-linearized approximation of the equilibrium conditions and then exploiting the affine structure of the linearized model to find a solution to the approximated system. Details of the derivation can be found in Appendix B. After deriving the approximated equilibrium conditions of the model, I use the method of undetermined coefficients to find the equilibrium dynamics of the endogenous variables (see, among others, McCallum (1983)). In particular, I show that the output gap, inflation and the scaled value function can be written as:

\[ x_t = x_a \Delta a_t \quad , \tag{13} \]

\[ \pi_t - \pi^* = \pi_a \Delta a_t \]

\[ \psi_t = \psi_a \Delta a_t \quad , \]

where the equilibrium coefficients \(x_a, \pi_a, \psi_a, \psi, \text{ and } \psi_a\) satisfy the equilibrium conditions and
depend on the structural parameters of the economy.

3 Results

In this section, I describe the main results of the paper. I begin by studying the dynamics of realized consumption growth and how they differ from the dynamics of expected consumption growth. I then focus on the return on the consumption claim, and analyze the properties of the real pricing kernel and of the consumption risk premium.

3.1 Consumption Growth

As discussed in the introduction, three ingredients are necessary to generate endogenous consumption dynamics and asset price behavior similar to what is obtained in the typical long-run risk context of Bansal and Yaron (2004): (i) sticky prices, which deliver low correlation in realized consumption growth but high correlation in expected consumption growth; (ii) Epstein-Zin preferences; and (iii) a permanent technology shock, so that the correlation between realized and expected consumption growth is positive. Epstein-Zin preferences are necessary to price long-run risk, as only a representative agent of that kind requires a compensation for being exposed to shocks to expected consumption growth, while ingredients (i) and (iii) are necessary to obtain consumption dynamics consistent to the ones assumed in long-run risk models.

In equilibrium, realized consumption growth is equal to

$$\Delta c_t = \Delta y_t = \Delta y_t^F + \Delta x_t$$

where the second equality follows from the definition of the output gap. Understanding the dynamics of consumption therefore requires understanding the properties of the flexible output growth and the output gap. When prices are fully flexible, firms maximize profits, $P_t(j)Y_t(j) - W_t(j)L_t(j)$, subject to the technology constraint (7) and the demand function (9). It is easy to show that $\Delta y_t^F = \Delta a_t$. With flexible prices, there is no output gap, and output is always at its natural level. Therefore, if prices were flexible, equilibrium consumption growth would inherit the dynamics of the technology shock. In particular, the persistence of realized consumption growth would be equal to the persistence of expected consumption growth, that is

$$\text{Corr}(\Delta c_t^F, \Delta c_{t+1}^F) = \text{Corr}(E_t(\Delta c_{t+1}^F), E_{t+1}(\Delta c_{t+2}^F)) = \varphi_a$$
The result above makes it clear that, without nominal frictions, the model cannot recreate the requisite discrepancy in the autocorrelation dynamics of realized and expected consumption growth. In particular, the high autocorrelation in expected consumption growth required by long-run risk models necessarily implies the same high autocorrelation in realized consumption growth, which is highly at odds with the data.

When prices are sticky, monetary policy affects real allocations and moves realized output away from its flexible-price level. Combining (13) and (14), I obtain the dynamics for equilibrium consumption growth with sticky prices. Excluding constants,

$$\Delta c_{t+1} = g_a \Delta a_t + (1 + x_a)\sigma_a \epsilon_{t+1}^{a}$$

(15)

and, therefore, expected consumption growth, \(E_t \Delta c_{t+1}\), is given by

$$E_t \Delta c_{t+1} = g_a \Delta a_t$$

(16)

where the coefficient \(g_a\) depends on the structural parameters of the economy. From (15) and (16), I obtain the following Result.

**Result 1.** The first order autocorrelation of realized consumption growth is lower than the first order autocorrelation of expected consumption growth. In particular,

$$\text{Corr}(\Delta c_t, \Delta c_{t+1}) < \text{Corr}(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2}) = \varphi_a$$

Also, the first order autocorrelation of realized consumption growth is

(i) increasing in the probability \((1 - \alpha)\) of a firm being able to adjust its price optimally,

(ii) increasing in the Taylor rule reaction to inflation, \(\tau_\pi\),

(iii) increasing in the Taylor rule reaction to the output gap, \(\tau_x\).

See Appendix E for the proof. □

Important conclusions can be drawn from Result 1. First, the persistence of expected consumption growth is equal to the persistence of the output gap and of the technology shock (see equations (13) and (16)). In particular, it is not affected by the presence of nominal frictions. This result does not hold with generality and is specific to the simple model analyzed in this section, which cannot generate endogenous persistence in the dynamics of equilibrium.
allocations and prices. If the monetary policy rule allows for interest rate inertia (see Section 6), the autocorrelation of expected consumption growth depends on the structural parameters of the economy, including the level of nominal frictions and the interest rate rule coefficients.

Realized consumption growth is less persistent than expected consumption growth, and therefore technology, because of the presence of sticky prices. This is intuitive. Consider the case in which the economy is hit by a positive permanent technology shock at time $t$. The level of technology is permanently higher so that the optimal level of production goes up. As I showed before, when prices are flexible, the growth rate of production and, in equilibrium, the growth rate of consumption perfectly inherits the dynamics of the technology shock. Instead, when prices are sticky, only a fraction $(1 - \alpha)$ of the firms can adjust prices optimally and produce the newly optimal level of output. The fraction of firms that cannot revise prices will face a higher demand for their products and will therefore increase production more than they would if their prices were flexible. In the following period, a fraction of the firms will again be unable to set their prices optimally, and so on. The fact that at each point in time only a fraction of firms can optimally react to the shocks in the economy breaks the perfect tie between the autocorrelation dynamics of realized consumption growth and technology growth, making realized consumption growth less persistent than technology and, in light of Result 1, expected consumption growth. As the fraction of firms that are able to adjust their prices goes down – that is, as $\alpha$ gets larger – the persistence of realized consumption growth goes down, as more firms will be unable to react optimally to the shocks hitting the economy.

The persistence of realized consumption growth is increasing in the Taylor rule coefficients. When the Taylor rule reaction to current macroeconomic conditions is stronger, prices are smoother, and the output gap is smaller. Roughly speaking, and all else being equal, when monetary policy is strong, the component of realized consumption growth coming from changes in the output gap is less relevant (see equation (14)) and the autocorrelation of consumption growth will be closer to the autocorrelation of technology and, therefore, to the autocorrelation of expected consumption growth. In other words, for a given level of price stickiness, a weak monetary policy magnifies the effects of nominal frictions on realized consumption growth,

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4To see this, one must consider the effects of a positive technology shock on the average real marginal cost in the economy, $MC_t/P_t = (W_t/P_t)/A_t$. The technology shock affects marginal costs through two channels: a direct channel, which decreases real marginal costs; and an indirect channel, which increases real wages $W_t/P_t$, and therefore marginal costs, due to higher income. Given the permanent nature of the technology shock, the indirect channel is stronger, so that the average real marginal cost goes up. On average, firms should therefore increase their prices, but only a fraction of them can do so. Therefore, agents shift their demand towards the relatively cheap goods produced by the firms that cannot revise their prices, which eventually increase production more than they would if their prices were flexible.
thus increasing the wedge with the dynamics of expected consumption growth.

In the model, nothing would prevent the monetary authority from offsetting the welfare inefficiencies caused by the nominal frictions. If the reaction to inflation and output gap is strong enough, the effects of price stickiness disappear and the equilibrium allocation coincides with the flexible price allocation, which is Pareto efficient.\(^5\) In Appendix F, I discuss the case in which the presence of a shock to the disutility of labor introduces a trade-off between the optimal level of inflation and output. While I do not explicitly derive the optimal policy in that context, I show that the monetary authority does not have an outright incentive to completely offset the effects of price stickiness. Moreover, in Section 4, when I calibrate the baseline model, I show that when the Taylor rule coefficients are consistent with the empirical estimates of Taylor (1999), a mild level of price stickiness is sufficient to have quantitatively significant effects on the dynamics of realized and expected consumption growth.

In the original work of Bansal and Yaron (2004), realized and expected consumption growth are affected by independent shocks. This is not possible in the context of the general equilibrium model analyzed here, since shocks affect the dynamics of all endogenous variables. Consequently, the conditional correlation between realized and expected consumption growth is determined endogenously and ultimately depends on the nature of the shocks. This leads to the following Result.

**Result 2.** The conditional correlation between realized consumption consumption growth and expected consumption growth is equal to one:

\[
\text{Corr}_{t-1}(\Delta c_t, E_t \Delta c_{t+1}) = +1.
\]

See Appendix E for the proof. □

Result 2 is a consequence of the permanent nature of the technology shock. When technology is hit by a positive permanent shock, the optimal level of production is permanently higher. Realized output is high and therefore realized consumption growth is high. Since in the model technology growth is positively autocorrelated, expected consumption growth is also high. When the representative agent has preference for the early resolution of risk, shocks to expected consumption growth carry a large and positive price of risk and result in a large risk premium. Kaltenbrunner and Lochstoer (2010) make a similar point in the context of a

\(^5\)Precisely, this is true only after the introduction of an employment subsidy to offset the effects of monopolistic competition, as in Gáli (2008).
standard real business cycle model.6

Results 1 and 2 show that, for a suitable parametrization – large autocorrelation in technology growth, a sensible level of nominal frictions, and a not-too-strong reaction of the monetary authority to inflation and the output gap – the model can endogenously generate consumption dynamics consistent with the long-run risk literature. These results are independent from the preferences of the representative agent and could equivalently be derived in the standard case with power utility. However, in the latter case, a highly persistent variation in expected consumption growth would have no effect on asset prices. Only when the representative agent has recursive preferences, the high variation in expected consumption growth matters because it directly affects the agent’s marginal rate of substitution. The New Keynesian literature typically assumes power utility and therefore cannot say anything regarding the risk associated with variation in expected consumption growth.

3.2 The Real Pricing Kernel

The real pricing kernel $N_{t+1}$ in equation (6) can be expressed, up to an approximation error, as the sum of three components. Omitting constants,

$$\log N_{t,t+1} = -\psi \Delta c_{t+1} + (\psi - \gamma) \sum_j \eta^j_{vv} (E_{t+1} - E_t) \Delta c_{t+1+j} + (\psi - \gamma) \sum_j \eta^j_{vv} \eta_{vx} (E_{t+1} - E_t) n_{t+1+j},$$

where $0 < \eta_{vv} < 1$ and $\eta_{vx} > 0$ are linearization coefficients. The first component, $-\psi \Delta c_{t+1}$, is standard in the asset pricing literature and captures the short-run risk in the economy. When realized consumption growth is low, the marginal rate of substitution is high and therefore any risky asset will pay a positive risk premium whenever its return covaries positively with consumption growth.

The second and third component arise when the representative agent has Epstein-Zin preferences and capture long-run risk. These last two components say that downward revisions in future expected consumption growth and labor are bad news – that is, the marginal rate

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6In the presence of more than one shock, the correlation need not be equal to +1. In general, a non-negative correlation is needed to obtain a large risk premium when the representative agent has preference for the early resolution of risk.
of substitution is high – when agents prefer early resolution of risk \((\psi - \gamma < 0)\), and good news when agents prefer late resolution of risk \((\psi - \gamma > 0)\). In particular, when agents prefer early resolution of risk, risk premia on risky assets are large when good news about future consumption growth and labor are positively correlated with their returns. Therefore, a representative agent with preference for the early resolution of risk dislikes shocks both to realized and expected consumption growth. When low realized consumption growth is associated with low expected consumption growth, as is the case in the model in light of Result 2, technology shocks have a magnified effect on the marginal rate of substitution.\(^7\)

In order to gather more insight on the role of recursive preferences and nominal frictions on the price of risk associated with the technology shock, one can substitute the equilibrium dynamics of consumption and labor in (17) to get

\[
- \log N_{t,t+1} = \delta + \gamma_a \Delta a_t + \lambda_a \sigma_a c_{t+1}^a ,
\]

where

\[
\delta = - \log(\beta) + \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{1}{\eta_{ev}} \eta_{v} - (\psi - \gamma)\nu + \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{1 - \psi}{\eta_{ev}} \nu - \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{\eta_{ev}}{\eta_{ev}} \tilde{x} \\
+ \left[ \gamma(1 + x_a) - (\psi - \gamma)v_a \right] (1 - \phi_a) \theta_a ,
\]

\[
\gamma_a = \left[ \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{\eta_{ev}}{\eta_{evv}} + \gamma \right] x_a + \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{1 - \psi}{\eta_{ev}} v_a + \left[ \gamma(1 + x_a) - (\psi - \gamma)v_a \right] \phi_a ,
\]

\[
\lambda_a = \gamma(1 + x_a) - (\psi - \gamma)v_a .
\]

Following the affine term structure literature, I refer to \(\gamma_a\) as the real factor loading and \(\lambda_a\) as the real price of risk of technology. Equation (18) encompasses several familiar specifications for the real pricing kernel. First, when there are no nominal frictions and preferences are of

\(^7\)Uhlig (2007) notices that, in the data, high returns tend to predict upswings in economic activity – today’s return and labor changes in the future are negative correlated – and therefore the third component in (17) provides a hedge against consumption news. However, when I calibrate the model, I find that the impact on the marginal rate of substitution coming from news about future labor is small compared to the effect of news to future consumption.
the time and state separable power utility form, the real pricing kernel collapses to the familiar

$$- \log N_{t+1} = [\log \beta + \gamma (1 - \varphi_a) \theta_a] + \varphi_a \Delta a_t + \gamma \sigma_a x_{t+1}^a,$$

with the price of risk being equal to the coefficient of relative risk aversion. Allowing for
nominal frictions, while remaining in the power utility framework, monetary policy affects the
pricing kernel only through its contemporaneous effect on the output gap, that is, only through
its effect on \( \pi \) and \( x_a \). Finally, in the more general case with nominal frictions and recursive
preferences considered in this paper, monetary policy affects the pricing kernel through two
channels: first, its direct effect on the output gap; second, its indirect effect on the equilibrium
process of the scaled utility function, \( v_t \). In particular, since the sensitivity of the scaled utility
to the technology shock is positive (\( v_a > 0 \)), when the agent prefers early resolution of risk,
the price of risk \( \lambda_a \) will be larger than in the standard power utility case. A larger price of risk
is key in the determination of a large consumption risk premium.

### 3.3 The Return on the Consumption Claim

The price \( S^c_t \) of a claim to all future consumption is \( S^c_t = E_t(M_{t+1}(C_{t+1} + S^c_{t+1})) \) and its return
\( r^c_{t+1} = \log(R^c_{t+1}) \equiv \log((S^c_{t+1} + C_{t+1})/S^c_t) \) can be expressed as

$$e^{r^c_{t+1}} = \left(1 + \frac{S^c_{t+1}}{C_{t+1}}\right) \left(\frac{C_{t+1}}{C_t}\right) \left(\frac{C_t}{S^c_t}\right).$$

(20)

Following Campbell and Shiller (1988), the return on the consumption claim can be approximated as

$$r_{c,t+1} = \eta_{pc} + \eta_{pc} pc_{t+1} - pc_t + \Delta c_{t+1},$$

(21)

where \( pc_t \equiv \log(S^c_t/C_t) \) is the log price-consumption ratio and \( \eta_{pc} \) and \( \eta_{pc} \) are linearization
constants depending on the average level of the price-consumption ratio.

As is standard, I solve for the approximated return on the consumption claim by conjecturing a linear solution for the log price-consumption, that is

$$pc_t = \overline{A} + A_a \Delta a_t.$$  

(22)

Substituting this guess in (21), and then using it in (20), I obtain the solution for the endoge-
nous parameters $\overline{A}$ and $A_a$. I have

$$\overline{A} = \frac{1}{1 - \eta_{pc}} \left( -\delta + \eta_{pc} + \mu_c + \eta_{pc}A_a(1 - \varphi_a)\theta_a + (\eta_{pc}A_a + 1 + x_a - \lambda_a)^2\sigma_a^2/2 \right) ,$$

$$A_a = \frac{g_a - \gamma_a}{1 - \eta_{pc} \varphi_a} . \quad (23)$$

The sign of the $A_a$ coefficient determines the sensitivity of the price-consumption ratio to the technology shock. This coefficient bears similarities with the one derived by Bansal and Yaron (2004). First, one can show that $A_a > 0$ if and only if the intertemporal elasticity of substitution is larger than one. In response to higher expected growth, agents buy more assets, and consequently the price-consumption ratio rises. Second, $A_a$ is increasing in the persistence of expected consumption growth which, as shown in Result 1, is equal to the persistence of the technology shock. High persistence in expected consumption growth, combined with a large elasticity of intertemporal substitution, results in a large and positive sensitivity of the price-consumption ratio to the technology shock. Note that expected future consumption and future labor decisions affect the coefficient $A_a$ through their effect on $g_a$ and $\gamma_a$.

Given the lognormal structure of the model, the continuously compounded risk premium on the consumption claim is equal to

$$E_t(r_{t+1}^c - r_{f,t}) = \text{Cov}_t \left( \log N_{t+1} - E_t(\log N_{t+1}), r_{t+1}^c - E_t(r_{t+1}^c) \right) - 0.5 \text{Var}_t(\log N_{t+1}) - 0.5 \text{Var}_t(r_{t+1}^c)$$

$$\lambda_a B_a \sigma_a^2 - 0.5 \text{Var}_t(r_{t+1}^c) \quad , \quad (24)$$

where $r_{f,t} = \log R_{f,t}$ is the one-period continuously compounded real risk-free rate, and $B_a = \eta_{pc}A_a + (1 + x_a)$. The size of $B_a$ determines the size of the risk premium. In the baseline calibration of the next section, $B_a$ is positive, larger with early resolution of risk, and increasing in the persistence of expected consumption growth.

### 3.4 The Return on Dividends

Following Abel (1999), dividends $D_t$ are defined as a levered claim to aggregate consumption. Omitting constants, the logarithm of dividend growth can be expressed as

$$\log(D_{t+1}/D_t) \equiv \Delta d_{t+1} = \phi_d g_a \Delta a_t + \xi_d (1 + x_a) \sigma_a \epsilon_{t+1}^a$$
where $\phi_d$ governs leverage and $\xi_d$ affects the sensitivity of dividend volatility to consumption innovations. The solutions for the price-dividend ratio $pd_t \equiv \log(S^d_t/D_t) = \bar{A}^d + A_a^d \Delta a_t$ and the return on dividends $r^d_{t+1} = \eta_{pd} + \eta_{pd} pd_{t+1} - pd_t + \Delta d_{t+1}$ are obtained in exactly the same way as for the consumption claim.

### 3.5 Discussion

In this section, I showed that a real business cycle model with sticky prices can generate long-run risk when the representative agent has recursive preferences. In particular, I demonstrated how the presence of nominal frictions breaks the link between the autocorrelation dynamics of realized and expected consumption growth and showed that the effect of such frictions is larger when monetary policy is weak. Also, I showed how a representative agent who fears both variation in realized and expected consumption growth requires a large compensation for being exposed to such shocks.

The results were obtained in the context of a model in which the only source of uncertainty was a shock to technology growth. While the model is capable of replicating the main features of the dynamics of consumption growth and asset prices typical of the long-run risk literature, it fails in generating any endogenous persistence in the dynamics of the equilibrium variables. In Section 6, I address this issue and propose a model that can generate endogenous persistence in the dynamics of equilibrium variables by generalizing the monetary policy rule in (12) to allow for interest rate inertia.

### 4 Calibration

Following the long-run risk literature, I calibrate the model at monthly frequency. The discount factor $\beta$ is set to 0.999 as in Bansal, Kiku, and Yaron (2006) and the elasticity of substitution, $\psi^{-1}$, is equal to 1.5 as in Bansal and Yaron (2004). The Calvo parameter $\alpha$ is consistent with the results of Bils and Klenow (2004) and is set equal to $7/9$, implying an average price duration of 4.5 months. I borrow the parameters governing the elasticity of substitution across goods and the elasticity of labor from the New Keynesian literature and set them equal to $\theta = 7$ and $\omega = 1$, respectively. The parameter $\chi$ is set such that the representative agent spends one third of her time endowment working. Finally, consistently with Taylor (1999), I set $\tau_\pi = 1.1$ and $\tau_x = 0.5/12$. 
Then, I choose five parameters to match five moments in the data. The parameters are: the risk aversion coefficient $\gamma$, the Taylor rule constant $\tau$, and the three parameters governing the dynamics of technology growth ($\theta_a$, $\sigma_a$, and $\varphi_a$). The five moments I match are the mean and volatility of consumption growth, the mean and volatility of the one-month nominal interest rate, and the Sharpe Ratio. Finally, the leverage $\phi_d$ and the scaling coefficient $\xi_d$ are chosen to match the unconditional equity premium and the unconditional volatility of the return on equity. Table 1 shows the parameters of the baseline calibration and Panel A of Table 2 shows the moments that are exactly matched by the calibration exercise. Figure 1 shows the simulated path for 360 months of observations for consumption growth, the output gap, inflation, real and nominal returns, and the price-consumption ratio.

The persistence of the technology shock and the risk aversion coefficient are crucial magnitudes in the model, as they determine the persistence of expected consumption growth and the level of the Sharpe Ratio. The calibration requires a high persistence in technology growth ($\varphi_a = 0.95$) and a risk aversion coefficient of $\gamma = 6.5$. A high persistence in technology growth is needed because the baseline model cannot generate endogenous persistence. Such a level of persistence is high relative to previous studies, which suggest a first order autocorrelation of technology growth in the order of 0.2-0.4 (see Nelson and Plosser (1982) and Gomme and Rupert (2007)). However, in Section 6, where I consider a generalized interest rate rule that allows for a certain degree of policy inertia, the model generates endogenous persistence in expected consumption growth, thus allowing me to significantly reduce the autocorrelation of technology growth.

Consistently with the discussion in Section 3, the risk aversion parameter implies that the representative agent has a preference for the early resolution of risk and thus fears variation in expected consumption growth. Importantly, both the persistence of expected consumption growth and the risk aversion are slightly lower than what is usually assumed in the long-run risk literature. This is a consequence of the permanent nature of the technology shock, which generates a positive covariation between realized and expected consumption growth. Such a positive covariation magnifies the effect of technology shocks on asset prices. Therefore, compared to the long-run risk literature, the model further reduces the need for a large risk aversion coefficient to generate a large equity premium.

Panel B of Table 2 shows important statistics of the model that were not calibrated to fit. The autocorrelation of monthly realized consumption growth is 0.12, which is much lower than the autocorrelation of expected consumption growth. This is a result of the presence of sticky
prices which breaks the link between the dynamics of technology and the dynamics of realized 
consumption growth. When aggregated to annual frequency, realized consumption growth has 
a first order autocorrelation of 0.51, slightly overshooting the data. Given the absence of any 
amplification mechanism, the autocorrelation of expected consumption growth, inflation, and 
of the one-month nominal interest rate are all equal to 0.95. The one-month real interest rate 
has an annualized mean of 1.44% and an annualized volatility of 1.45%, and the annualized 
mean of the inflation rate is 4.72%, with a volatility of 0.74%.

Contrary to the data, the correlation between realized consumption growth and inflation is 
positive and equal to 0.46 (once the monthly observations are aggregate to annual frequency). 
This is a consequence of the permanent nature of the technology shock, which imposes a 
positive covariation between consumption and inflation. A positive correlation between real-
ized consumption growth and inflation implies a negative slope for the average nominal term 
structure (see Rudebusch and Swanson (2008) and the discussion in Section 5.3).

4.1 The Role of Recursive Preferences

If preferences were of the standard power utility form, the high persistence in expected con-
sumption growth implied by the model would have no effects on asset prices. To see this, I 
compare the results implied by the baseline calibration with the ones that would have been 
obtained with standard power preferences. Keeping all other parameters unaltered, I set 
\( \gamma = \psi = 6.5 \), and analyze the impact of long-run risk on asset prices. Results are shown in 
Table 3.

Standard power utility preferences are unable to capture the riskiness associated with 
high variation in expected consumption growth. The excess return on the consumption claim 
becomes negative (equal to \(-1.13\)) and, because of the low intertemporal elasticity of substi-
tution, consumption growth and interest rates become more volatile. A similar effect impacts 
the level of interest rates, with the unconditional mean of the nominal interest rate sharply 
moving from 6.42% to as high as roughly 50%.

Recursive preferences, combined with early resolution of risk, magnify the risk associated 
with variation in expected consumption growth. Define short-run risk as the amount of risk 
that would arise in the standard power utility case (\( \gamma = \psi \)), and long-run risk as the residual.
Using the expression for the price of risk of technology in (19), one can write

\[
\text{Sharpe Ratio} = \text{short-run risk} + \text{long-run risk}
\]

\[
= \gamma (1 + x_a) \sigma(\Delta c_{t+1}) + (\gamma - \psi)v_a \sigma(\Delta c_{t+1})
\]

\[
0.34 = (0.17) + (0.17)
\]

where \(\sigma(\Delta c_{t+1})\) denotes the unconditional volatility of realized consumption growth. Long-run risk has a large impact on asset prices and accounts for 50% of the total Sharpe Ratio.

Further insights can be obtained by looking at how recursive preferences affect the sensitivity \(A_a\) of the price-consumption ratio to the technology shock. Table 4 shows that, with early resolution of risk, the coefficient \(A_a\) is positive and increasing in the autocorrelation of technology. Importantly, when the intertemporal elasticity of substitution is smaller than one, as is the case in the power utility calibration, the sensitivity of the price-consumption ratio to technology becomes negative. Long-run risk has a nontrivial impact on asset prices only when expected consumption growth is highly autocorrelated. In the baseline calibration, this autocorrelation is set equal to 0.95. Figure 2 shows the risk premium on the consumption claim, the price of risk of technology \(\lambda_a\), the sensitivity \(A_a\) of the price-consumption ratio to the technology shock, and the coefficient \(B_a\), as I vary the autocorrelation of technology growth for three preference specifications: the baseline calibration, with early resolution of risk, the case of standard power utility and the case of preference for the late resolution of risk. The price of technology risk is increasing in the autocorrelation of technology growth, and larger when the representative agent has preference for the early resolution of risk. The risk premium is positive and increasing in the case of early resolution of risk, while it decreases in the case of power utility and late resolution of risk. A similar pattern applies for the \(A_a\) and \(B_a\) coefficients.

### 4.2 The Role of Nominal Frictions

Price stickiness can generate consumption dynamics that are typical of long-run risk models. Intuitively, the fact that only a fraction of firms can adjust their prices at each point in time breaks the perfect tie between realized and expected output. The more the prices are sticky, the larger the wedge between the dynamics of realized and expected consumption growth. Keeping the remaining parameters unchanged, Table 5 shows the dynamics of consumption growth for the case of flexible prices and for three different levels of price stickiness: 2, 4.5,
and 9 months of average price duration.

When prices are not very sticky, consumption is more closely tied to the dynamics of technology, the output gap is small, and therefore consumption growth closely mimics the dynamics of consumption that would arise with flexible prices. On the contrary, strong frictions generate a large output gap, moving consumption away from its flexible price counterpart. The baseline calibration, obtained with a level of price stickiness implying 4.5 months of average price duration, generates a consumption risk premium that is roughly twice the size of the one obtained with flexible prices (1.55% versus 0.87%). Interestingly, a very mild level of price stickiness is sufficient to generate a significant wedge between the dynamics of realized versus expected consumption growth. For example, when firms are allowed, on average, to reoptimize their prices every 2 months, the first order autocorrelation of realized consumption growth remains as low as 0.23. Figure 3 shows the consumption risk premium and the autocorrelation of realized and expected consumption growth as a function of the parameter $\alpha$. As expected, the risk premium is increasing in the level of price stickiness. The autocorrelation of expected consumption growth is fixed at 0.95, while the autocorrelation of realized consumption growth is sharply decreasing in the level of price stickiness.

### 4.3 The Role of Monetary Policy

When prices are sticky, monetary policy affects the dynamics of consumption growth. Thus, a natural question to ask is how the dynamics of consumption growth are affected by changes in the Taylor rule parameters. Here, I conduct the following policy exercises. The two Taylor coefficients, $\tau_\pi$ and $\tau_x$, are, in turn, increased to the level that is necessary to obtain a reduction of 1% in the unconditional mean of the one-period nominal interest rate. All remaining parameters are unchanged.

The first exercise requires an increase of $\tau_\pi$ from 1.1 to 2.17, while in the second exercise $\tau_x$ has to move from 0.5/12 up to 2.55/12. Results are shown in Table 6. Both exercises show that a stronger monetary policy reduces the effect of nominal frictions on realized consumption growth. For a given price stickiness, stronger reactions to inflation and output gap increase the autocorrelation of realized consumption growth to the point where its dynamics become highly at odds with the data. For the exercise involving an increase in $\tau_\pi$, the persistence of realized consumption growth moves from 0.12 to 0.39, while for the experiment involving an increase in $\tau_x$, the persistence moves up to 0.50. Intuitively, a stronger monetary policy smooths the
output gap and inflation, so that price stickiness matters less for realized consumption growth.

Figure 4 and 5 show the consumption risk premium and the autocorrelation of realized consumption and expected consumption growth as a function of $\tau_\pi$ and $\tau_x$, respectively. A stronger monetary policy dwarfs the effects of price stickiness on asset prices and real allocations. As $\tau_\pi$ and $\tau_x$ get larger, the model approaches the results that would be obtained with flexible prices. However, it is interesting to notice how, for Taylor rule coefficients that are common in the literature, the model still delivers a significant effect of nominal frictions on the risk premium and the autocorrelation of consumption growth. In particular, the autocorrelation of realized consumption growth remains below 0.40 for values of $\tau_\pi$ and $\tau_x$ not exceeding 2.8 and 2.0/12, respectively. Similarly, the risk premium remains roughly 35% (for $\tau_\pi < 2.8$) and 43% (for $\tau_x < 2.0/12$) larger than what would have been obtained with flexible prices.

5 Additional Asset Pricing Implications

I now focus on additional asset pricing implication of the model. The model delivers testable restrictions on the joint behavior of consumption, dividends, real wages and inflation. Some of these restrictions are novel to the framework developed in this paper. As is common in the long-run risk literature, the main source of asset price variability relative to macroeconomic aggregates is a small, predictable, and highly persistent component in consumption growth. It is therefore natural to test the model by evaluating the ability of asset prices to predict the long-run behavior of equilibrium macroeconomic variables, such as consumption, dividend, real wages, and inflation. Tables 7 to 10 show the results of model-implied predictive regressions. Some of these regressions, such as the ones of future consumption and dividend growth on the price-dividend ratio, are common to the the long-run risk literature in which the dynamics of consumption and dividends are exogenously specified. However, I can exploit the general equilibrium results of the model and test novel additional restrictions on the joint dynamics of allocations and prices.

The novelty of these regressions goes in two directions. First, I can test whether nominal asset prices and returns have predictive power for equilibrium allocations. This is not possible in standard long-run risk models, which entirely focus on the dynamics of real assets. A natural test for this exercise is the target of the monetary authority, the one-period nominal interest rate. Second, I can test the predictability power of asset prices for a larger set of macro variables, including real wages and inflation, and not limit myself to consumption and
dividend growth only.\textsuperscript{8}

5.1 The Predictive Power of the Price-Dividend Ratio

Tables 7 and 8 show regression coefficients, \( t \) statistics, and \( R^2 \) coefficients for predictive regressions of annual consumption, dividend, real wages growth, and inflation on the price-dividend ratio. In each of the tables, I first report the results I obtain from the data, and then compare them with what is implied by the model. In addition to the results from the baseline model, I report the results for two alternative calibrations. In Table 7, I consider an increase (decrease) in price stickiness, by increasing (decreasing) the probability that a firm is unable to adjust its price optimally at each point in time. In Table 8, I consider an alternative comparative static exercise, and study the impact of a stronger (weaker) monetary policy, by increasing (decreasing) the monetary authority reaction \( \tau_\pi \) to inflation.

Similar to what is obtained in the long-run risk literature, the price-dividend ratio in the model positively predicts future consumption and dividend growth. At the one- (three-) year horizon, the point estimate for dividend growth is significantly different from zero and roughly four (six) times as large as in the data. The significance of the estimates decreases with the horizon. At the five-year horizon, the price-dividend ratio does not significantly predict dividend growth. The estimates for consumption growth show a similar pattern.

In the data, univariate regressions suggest that the price-dividend ratio does not predict future consumption and dividend growth (Campbell and Shiller (1988), Fama and French (1988)). In the model, the predictability depends on the level of price stickiness and on the strength on monetary policy. The predictive power of the price-dividend ratio for consumption and dividend growth is decreasing in the level of price stickiness. When firms can, on average, adjust their prices only once every nine months (\( \alpha = 8/9 \)), the price-dividend ratio significantly predicts dividend growth only at the one-year horizon, while the point estimates for the three- and five-year horizon regressions become insignificantly different from zero. Moreover, the price-dividend ratio exhibits weaker explanatory power and, consistently with the data, only explains 8\% of the variation in the one-year-ahead dividend growth and 4\% of the variation in dividend growth for the next five years.

A similar decrease in the predictive power of the price-dividend ratio is obtained in the case

\textsuperscript{8}In the literature, long-run risk models have usually been tested in their ability to predict future excess returns. Here, risk premia are constant as a consequence of the homoscedasticity of the technology shock.
of a weaker monetary policy. When the Taylor coefficient to inflation is $\tau_\pi = 1.05$, the price-dividend ratio significantly predicts future consumption and dividends only at short horizons. Moreover, the price-dividend ratio never explains more than 8% of the variation of future consumption. These results are consistent with the discussion in Section 4, where I showed that, for a given level of price stickiness, a weaker monetary policy magnifies the effect of nominal frictions on real allocations.

In the data, the price-dividend ratio predicts neither growth in real wages nor inflation significantly. For real wages, the model delivers similar results. Although the sign of the estimated regression coefficient is opposite to the ones obtained in the data – positive in our model, while negative in the data – none of the estimates is significantly different from zero. While at the one-year horizon the price-dividend ratio explains too much variability of the growth in real wages (4% versus virtually 0% in the data), at longer horizons the $R^2$ coefficients obtained from model’s based regressions are comparable to the data. Similarly to what was obtained for the predictability of consumption and dividend growth, increasing the impact of the nominal friction on real allocations, either by directly increasing the level of price stickiness or by reducing the reaction of the monetary authority to inflation, reduces the predictive power of the price-dividend ratio.

In the model, and contrary to the data, a higher price-dividend ratio significantly predicts higher future inflation. At the one-year horizon, the price-dividend ratio explains 60% (!) of the variation of future inflation and, while the $R^2$ coefficients sharply decrease with the horizon, the point estimates remain significantly different from zero. Unlike what happens in the case of consumption, dividend and real wage growth, changing the level of nominal frictions only has a marginal impact on the predictability of future inflation.

### 5.2 The Predictive Power of the Nominal Interest Rate

Tables 9 and 10 show regression coefficients, $t$ statistics, and $R^2$ coefficients for predictive regressions of annual consumption, dividend, real wages growth, and inflation on the one-month nominal interest rate. Similarly to the case of predictive regressions for the price-dividend ratio, in each of the tables, I first report the results obtained from the data, and then compare them with what is implied by the model. Again, in addition to the results from the baseline model, I report the results for two alternative calibrations. In Table 9, I consider and increase (decrease) in price stickiness, by increasing (decreasing) the probability that a firm is
unable to adjust its price optimally at each point in time. In Table 10, I consider an alternative comparative static exercise and study the impact of a stronger (weaker) monetary policy, by increasing (decreasing) the monetary authority reaction $\tau_{\pi}$ to inflation.

In the model, the nominal interest rate predicts future consumption and dividend growth. However, the significance of the regression coefficients sharply decreases with the horizon. As was the case for the price-dividend ratio, the predictive power of the nominal interest rate is decreasing in the level of price stickiness and increasing in the strength of monetary policy. The data suggests that the nominal interest rate does not significantly predict future consumption and dividend growth, consistently with a situation of large nominal frictions and/or weak monetary policy.

In the data, the nominal interest rate positively predicts inflation and negatively predicts future wages growth. The model performs well when the predictability of inflation is considered. While the explanatory power of the nominal interest rate is too high, I can reproduce regression coefficients that, consistently with the data, are positive, statistically significant, and increasing with the horizon. As was the case for the regressions on the price-dividend ratio, varying the impact of the nominal friction only has a marginal effect on the predictability of future inflation.

In contrast, the model struggles to capture the negative relation between nominal interest rates and real wages. Both in the baseline calibration and in all of the alternative calibrations considered, the model cannot explain the statistically significant relation between the nominal interest rate and future wages observed in the data. This is a consequence of the simple structure of the labor market in the model, which is not able to account for the joint distribution of hours worked, real wages, and production.

5.3 The Term Structure of Interest Rates

Backus and Zin (1994) show that a necessary condition for the average nominal yield curve to be upward sloping is a negative autocorrelation in the nominal pricing kernel. In the model, the autocovariance of the nominal pricing kernel is given by

$$\text{Cov}(\log M_t, \log M_{t-1}) = (\gamma_a + \varphi_a \pi_a)^2 \text{Cov}(\Delta a_t, \Delta a_{t-1}) + (\gamma_a + \varphi_a \pi_a)(\lambda_a + \pi_a) \text{Cov}(\Delta a_t, \epsilon_t^a),$$

where $(\gamma_a + \varphi_a \pi_a)$ and $(\lambda_a + \pi_a)$ refer to the nominal factor loading and nominal price of risk of technology, respectively. Both the autocovariance of technology growth and the covariance
of the technology growth level with its innovation are positive (see equation (8)). Therefore, for the autocovariance of the nominal pricing kernel to be negative, the nominal factor loading and the nominal price of risk must have opposite signs. Additionally, the price of risk must be large enough relative to the factor loading to counteract the positive autocovariance term. In the model, the factor loading and the price of risk of technology have the same sign, and therefore the average nominal yield curve is downward sloping.

One can gain more insights into the reason why the average nominal yield curve is downward sloping by considering the correlation of inflation and consumption growth. Piazzesi and Schneider (2007) argue that a surprise increase in U.S. inflation, which lowers the value of nominal bonds, was typically followed by lower consumption in the future. That is, inflation is bad news about future consumption. This relationship implies that long-term nominal bonds lose value precisely when agents desire consumption the most, resulting in an upward sloping average nominal yield curve. Combining equation (13) and equation (16), inflation is related to expected future consumption according to (omitting constants):

\[ \pi_t = \frac{\pi_a}{g_a} E_t \Delta c_{t+1}. \]

The coefficient \( \pi_a \) determines the sensitivity of inflation to technology growth. As discussed in Section 3, prices rise in response to a permanently higher level of technology and therefore \( \pi_a > 0 \). The coefficients \( g_a \), which governs the sensitivity of expected consumption growth to technology, is also positive. This is because, following a permanent technology improvement, agents expect equilibrium consumption to be permanently higher. Therefore, in the model, inflation is good news about future consumption, and the average nominal yield curve is downward sloping.

Bansal and Shaliastovich (2009) develop a long-run risk model with an exogenously specified process for inflation to account for various predictability puzzles in bond and currency markets. Because of the exogeneity of the inflation process, they are free to impose a negative sensitivity of inflation to news to expected consumption growth, thus obtaining an average nominal yield curve that is upward sloping. In the general equilibrium model developed here, I do not have this freedom. When the only shock in the economy is a permanent technology shock, consumption and inflation move in the same direction, resulting in negatively sloped average nominal yield curve.

An upward sloping nominal yield curve can be obtained if the model is extended to in-
corporate additional shocks that generate a negative correlation between consumption and inflation. Examples of these shocks are a temporary technology shock, a price markup shock, a labor supply shock, and a government spending shock. When the effect of these additional shocks is strong enough to counteract the effect of the permanent technology shock, then the resulting nominal term structure is positively sloped. See Rudebusch and Swanson (2008) and Hsu (2011) for a treatment of the yield curve in a framework similar to the one developed in this paper.\(^9\)

6 A Model with Interest Rate Inertia

In this section, I generalize the interest rate rule to account for an explicit desire of the monetary authority to smooth the interest rate over time. This extension allows the model to generate endogenous persistence in expected consumption growth so that I need not rely on a very high degree of autocorrelation in the technology growth process to generate long-run risk. The interest rate rule followed by the monetary authority is

\[
i_t = \tau + \tau_\pi \left( \pi_t - \pi^* \right) + \tau_x x_t + \tau_i i_{t-1},
\]

where the \(\tau_i\) coefficient governs interest rate smoothing over time. A similar specification for the interest rate rule is used by Judd and Rudebusch (1998), Clarida, Galí, and Gertler (2000), and Orphanides (2004).

The model is solved following the same steps of Section 2.4, with the difference that the lagged interest rate, \(i_{t-1}\), is now a state variable. The equilibrium consumption growth process generalizes (15) and (16) as follows (omitting constants):

\[
\Delta c_{t+1} = g_a \Delta a_t + g_i i_{t-1} + (1 + x_a) \sigma a_{t+1}^a
\]

\[
E_t(\Delta c_{t+1}) = g_a \Delta a_t + g_i i_{t-1},
\]

where the expressions for \(g_a\) and \(g_i\) depend on the structural parameters of the economy. Details for the derivation can be found in Appendix D.

From (26) and (27), one can see that the autocorrelation of expected consumption growth

\(^9\)A version of the model in which technology has both a permanent and a transitory component can deliver both long-run consumption risk and a positively sloped average nominal term structure. Results are available upon request.
now depends not only on the persistence of the technology growth process, but also on the persistence of the nominal interest rate. The contribution to the persistence of expected consumption growth coming from technology growth, relative to the contribution of the nominal interest rate, depends on their relative variability, together with the relative magnitude of the $g_a$ and $g_i$ parameters.

In the model, interest rate inertia reduces the initial reaction of output to a technology shock. This leads to high persistence in expected consumption growth, as output is expected to move towards its new steady state. Overall, this effect manifests itself in a large $g_i$ coefficient. Intuitively, the lagged interest rate matters more, relative to the current level of technology growth, in determining today’s expectations about future consumption. When the model is calibrated to match the high autocorrelation in nominal interest rate observed in the data, the persistence of expected consumption growth is significantly higher than the persistence of technology growth. Again, as was the case in the baseline model of Section 2, the high persistence in expected consumption growth does not transfer one-to-one to the persistence of realized of consumption growth, because at each point in time the nominal friction forces some firms to act suboptimally. This wedge manifests itself in the innovation term in equation (26).

The price-consumption ratio becomes

$$pc_t = \overline{A} + A_a \Delta a_t + A_i \delta_t - 1,$$

where

$$A_a = \frac{g_a + \eta pc A_i (\tau_x x_a + \tau_\pi \pi_a) - \gamma_a}{1 - \eta pc \varphi_a}$$

(28)

$$A_i = \frac{g_i - \gamma_i}{1 - \eta pc (\tau_x x_i + \tau_\pi \pi_i + \tau_i)}.$$

Expression (28) generalizes (23) in the baseline model. When there is no interest rate inertia, the price-consumption ratio is independent of the lagged nominal interest rate, so that $A_i = 0$. When this is the case, the only way to get a large $A_a$ coefficient is to exogenously increase the autocorrelation of technology growth, $\varphi_a$. With interest rate inertia, the sign and magnitude of the $A_a$ coefficient depend on $A_i$. With strong interest rate smoothing and early resolution of risk, the coefficient $A_i$ is large and positive, resulting in a large $A_a$ coefficient, even when technology growth is only modestly autocorrelated.
The risk premium has the same functional form as (24) in the baseline model, that is

\[ E_t(r_{t+1}^c - r_{f,t}) = \lambda a B_a \sigma_a^2 - 0.5 \text{Var}_t(r_{t+1}^c) \]

with \( B_a = \eta_{pc} A_a + (1 + x_a) \). The difference is that \( B_a \) now depends on the level of interest rate inertia, through its effect on the \( A_a \) coefficient. A strong inertia delivers a large \( B_a \) coefficient, resulting in a large risk premium.

6.1 Numerical Exercise

In this section, I perform a simple numerical exercise to show how a strong interest rate smoothing coefficient can generate endogenous long-run consumption risk. I keep the calibration as close as possible to the baseline parameter values used in Section 4 and make the following changes. First, the monetary authority has a strong desire to smooth interest rates over time \( (\tau_i = 0.98) \); second, the parameter \( \tau_x \) is set to zero for parsimony. Then, I choose the autocorrelation of technology growth \( (\varphi_a) \), the conditional volatility of technology \( (\sigma_a) \), the interest rate rule constant \( (\tau) \), and the coefficient of risk aversion \( (\alpha) \) to match the volatility of consumption growth, the Sharpe Ratio, and the average level and the first order autocorrelation of the nominal interest rate. Results are shown in Table 11.

The required autocorrelation of technology growth is \( \varphi_a = 0.50 \), with a conditional volatility \( \sigma_a = 0.0045 \). This is in sharp contrast with the results of the baseline model, in which the absence of internal propagation required the calibration to rely on a highly persistent technology growth process. The model needs a coefficient of risk aversion equal to 10 to generate the Sharpe Ratio observed in the data. The increase in the coefficient of relative risk aversion from the baseline model is necessary because the introduction of a transitory state variable, \( i_{t-1} \), reduces the conditional correlation between realized and expected consumption growth, as discussed in Section 3.

Figures 7–10 show the impulse-response functions of the model following an (annualized) one-percent permanent technology shock and provide further support for the intuition behind the propagation mechanism at work. The desire of the monetary authority to smooth the interest rate over time reduces the initial reaction of the nominal interest rate to a technology shock, as the equilibrium lagged interest rate level provides an anchor to the movements of the nominal rate. Because of price stickiness, a similar mechanism affects the real interest rate, which increases following a positive technology shock, but not as much as it would have
increased in the absence of interest rate inertia. Given the lower real interest rate, firms hire less labor and, in turn, produce less output than they would have produced without the inertial behavior of the monetary authority. In equilibrium, this mechanism results in an initial smaller reaction of consumption, thus increasing the persistence of expected consumption growth, as output — and consumption — are expected to move to the new steady state. Put simply, the high persistence of nominal interest rates observed in the data, which in the model is captured by a high degree of inertia in the monetary policy rule, manifests itself in a smaller initial reaction of macro variables and asset prices to a technology shock. This smaller initial reaction then implies an increase in expected future growth rates, as the system moves to the new, higher steady state.

With this specification, the model struggles to match the volatilities of the nominal interest rate and inflation. This was expected, as a strong inertia in the monetary policy rule mechanically imposes little period-by-period changes in the interest rate. One way to increase the volatility of the nominal interest rate and inflation is to introduce an exogenous policy shock to the interest rate rule, as in Rudebusch and Swanson (2008).\footnote{Consider the following modification of the interest rate rule followed by the monetary authority:

\[ i_t = \pi + \tau_e (\pi_t - \pi^*) + \tau_x x_t + \tau_i i_{t-1} + u_t. \]

If \( u_t \) is an \textit{i.i.d.} monetary policy shock with mean zero and monthly volatility \( \sigma_u = 1.73 \times 10^{-3} \), the model provides a sensible fit for the volatilities of the nominal interest rate and of the inflation rate, leaving all the remaining calibration parameters roughly unchanged. Details of the results can be found in Table 12.}

In a way similar to the baseline model, I conduct various sensitivity exercises to inspect the mechanism at work. First, I analyze how the persistence of expected consumption growth changes with the inertial coefficient \( \tau_i \). Table 13 shows the results, together with the autocorrelation and the volatility of realized consumption growth. As was expected, an increase in \( \tau_i \) increases the weight of the lagged interest rate in the determination of the persistence of expected consumption growth. For \( \tau_i = 0.98 \), the autocorrelation of expected consumption growth is equal to 0.925, and sharply decreases as \( \tau_i \) gets smaller. When there is no policy inertia, the correlation of expected consumption growth equals the correlation of the technology shock. Table 13 also shows that the persistence of realized consumption growth is only marginally affected by the interest rate smoothing coefficient. The major driver of such a persistence is the nominal friction, which creates a wedge between the dynamics of realized and expected consumption growth.

Table 14 shows how the risk premium, the sensitivity of the price-consumption ratio to the lagged interest rate \( A_i \), the price of risk of technology \( \lambda_a \), the coefficient \( B_a \), and the persistence...
of expected consumption growth vary for various levels of price stickiness. The persistence of expected consumption growth is increasing in the level of price stickiness, and moves from 0.63 when $\alpha = 1/9$ to 0.97 when $\alpha = 8.5/9$. A large persistence in expected consumption growth manifests itself in a large price of risk, a large $B_a$ coefficient, and therefore a large risk premium. Moving from (roughly) one month to 18 months of average price duration, the consumption risk premium increases by 32.6%, from 0.72% to 0.95%. Interestingly, as the level of price stickiness goes up, so does the relative contribution to the risk premium coming from the lagged interest rate, as summarized by the coefficient $A_i$. The more the prices are sticky, the more the inertia in the interest rate rule matters for real allocations and therefore asset prices.

Long-run risk is priced only when the representative agent has recursive preferences. Following the discussion in Section 3, one would expect the risk premium to be high with early resolution of risk and low with late resolution of risk. Figure 6 confirms this intuition and shows that the consumption risk premium and the price of risk of technology are increasing in the level of the elasticity of intertemporal substitution.

Overall, the exercises performed confirm the ability of a real business cycle model with nominal frictions to generate endogenous long-run risk. Here, unlike in the case of the baseline model, high persistence in expected consumption growth is generated endogenously, as a consequence of the desire of the monetary authority to smooth interest rate over time. While expected consumption growth is highly persistent, realized consumption growth is much less so as a consequence of price stickiness.

7 Conclusion

Where does long-run consumption risk come from? I analyze a real business cycle model in which the nominal frictions generated by price stickiness endogenously deliver an equilibrium consumption growth process that shows low persistence unconditionally, but has a highly persistent conditional mean. At each point in time, only a fixed fraction of firms can optimally adjust the level of production, resulting in a realized consumption process that is not very persistent, as we observe in the data. However, since the effects of the nominal frictions on real allocations are only transitory, expected consumption growth can be highly persistent, depending on the persistence of the state variables in the economy.
The lack of an internal propagation mechanism in the baseline model requires such a high persistence to be exogenously imposed, resulting in a counterfactually high autocorrelation in technology growth. However, when the model is generalized to allow for a more general interest rate rule that allows the monetary authority to smooth interest rate over time, expected consumption growth can be extremely persistent even when technology growth is only modestly autocorrelated.

The model provides a theoretical basis for identifying a new source of long-run risk in consumption growth. The general equilibrium equations in the model link real and nominal asset prices to consumption, dividends, wages, and inflation. I test this link in time-series regressions and show that the predictability of macroeconomic variables depends on the degree of price stickiness and on the strength of monetary policy. The model struggles to reproduce the predictability of future real wages observed in the data. This is not surprising, as the simple labor market considered in this paper cannot explain the joint distribution of hours worked, real wages, and production. In the future, I plan to analyze a model with a more realistic labor market in order to account for these additional macroeconomic moments.

More generally, it would be interesting to analyze the impact of alternative nominal frictions on real allocations, and to further inspect the mechanism linking those frictions to monetary policy and asset prices. Here, I focused on the role played by price stickiness, given both its widespread use in the macroeconomic literature and its analytical tractability. However, other frictions warrant further attention. For instance, staggered wage settings, sticky information, and financial frictions affecting the behavior of financial institutes, all have a non-trivial impact on real allocations. In principle, these frictions could generate consumption dynamics that are consistent with what is usually assumed in the long-run risk literature. I leave this topic for future research.
References


Appendices

In the Appendices, I solve the model to allow for a shock to the disutility of labor $\chi_t$ and for inertia in the interest rate rule ($\tau_i \neq 0$). The model of Section 2 is obtained by setting the labor shock equal to a constant and $\tau_i = 0$.

A Equilibrium

A.1 Representative agent maximization

The representative agent problem is:

$$
\max V_t = \left\{ (1 - \beta)U(C_t, L_t)^{1-\psi} + \beta E_t[V_{t+1}^{1-\psi}] \right\}^{\frac{1}{1-\psi}}
$$

s.t.

$$
E_t \left[ \sum_{s=0}^{\infty} M_{t,t+s}P_{t+s}C_{t+s} \right] \leq E_t \left[ \sum_{s=0}^{\infty} M_{t,t+s} \left( \int_0^1 W_{t+s}(j) L_{t+s}(j) + P_{t+s} \Psi_{t+s} \right) \right],
$$

where the consumption and utility indexes are defined in Equations (2) and (3) in the main text. The intratemporal utility function $U(C_t, L_t)$ is defined as

$$
U(C_t, L_t)^{1-\psi} = \left( \frac{C_t^{1-\psi}}{1-\psi} - e^{\chi_t A_t^{1-\psi} L_t^{1+\omega}} \right),
$$

where $\Delta a_t$ evolves according to (8) in the main text and

$$
\chi_t = (1 - \varphi \chi) \theta \chi + \varphi \chi \chi_{t-1} + \sigma \chi \epsilon_t^X.
$$

The first order conditions are:

$$
\frac{\partial V_t}{\partial C_t} = 0 \quad \Rightarrow \quad \frac{1 - \beta}{1 - \psi} C_t^{1-\psi} \left[ V_t^{1-\psi} \right]^{\frac{1}{1-\psi}} - \lambda M_{t,t} P_t = 0
$$

$$
\frac{\partial V_t}{\partial L_t} = 0 \quad \Rightarrow \quad \frac{1 - \beta}{1 - \psi} e^{\chi_t A_t^{1-\psi} (-L_t)\omega} \left[ V_t^{1-\psi} \right]^{\frac{1}{1-\psi}} + \lambda M_{t,t} W_t = 0
$$

$$
\frac{\partial V_t}{\partial C_{t+1}} = 0 \quad \Rightarrow \quad \beta E_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\psi}} \left[ V_t^{1-\psi} \right]^{\frac{1}{1-\psi}} V_{t+1}^{\gamma} \frac{\partial V_{t+1}}{\partial C_{t+1}} - \lambda M_{t,t+1} P_{t+1} = 0.
$$

Rearranging, I obtain

$$
\frac{W_t}{P_t} = e^{\chi_t A_t^{1-\psi} C_t^{\psi} L_t^{\omega}},
$$

$$
e^{-i_t} = E_t(M_{t,t+1}).
$$
A.2 Firms

A.2.1 Fully flexible economy

The maximization problem of firm \( j \) under flexible prices is:

\[
\max_{P_t(j)} \quad P_t(j)Y_t(j) - W_t(j)L_t(j)
\]

subject to:

\[
Y_t(j) = A_tL_t(j)
\]

The first order condition is

\[
\frac{W_t(j)}{P_t} = \frac{1}{\mu} A_t = e^{\chi_t} A_t^{1-\psi} C_t^{\psi} L_t^{\omega} = Y_t^{\psi} \left( \frac{Y_t}{A_t} \right)^{\omega}.
\]

Therefore, the fully flexible output \( Y_t^F \) evolves according to

\[
(Y_t^F)^{\psi+\omega} = \frac{1}{\mu e^{\chi_t}} A_t^{\psi+\omega},
\]

or, in logs, \( \Delta y_{t+1}^F = \Delta a_{t+1} - \frac{1}{\psi+\omega} \Delta \chi_t \).

A.2.2 Calvo Pricing

The maximization problem of firm \( j \) is

\[
\max_{P_t(j)^*} \quad E_t \left\{ \sum_{s=0}^{\infty} \alpha^s M_{t+s} \left[ P_t(j)^*(\Pi^*)Y_{t+s}(j) - W_{t+s}(j)L_{t+s}(j) \right] \right\}
\]

subject to

\[
Y_{t+s}(j) = A_{t+s}L_{t+s}(j)
\]

The first order condition is

\[
E_t \left[ \sum_{s=0}^{\infty} \alpha^s M_{t+s}Y_{t+s}(j) \left( P_t(j)(\Pi^*)^{s} - \mu \frac{W_{t+s}(j)}{A_{t+s}} \right) \right] = 0
\]

\[
\Rightarrow E_t \left[ \sum_{s=0}^{\infty} \alpha^s M_{t+s}Y_{t+s}(j) \left( P_t(j)(\Pi^*)^{s} - \mu e^{\chi_t} A_{t+s}^{1-\psi} P_{t+s} Y_{t+s}^{\psi} \frac{Y_{t+s}(j)}{A_{t+s}} \right) \right] = 0,
\]

where the second equality uses (31). Rearranging, I get expression (11) in the main text, where I used the fact that, in equilibrium, \( P(j) = P_t^* \) for any \( j \). Replacing (32), I can write the first order condition
as

\[ \left[ P_t^\ast \left( \frac{P_t^\ast}{P_t} \right)^{-\theta} Y_t \right] H_t = \left[ \mu e^{x_t \psi} Y_t^{1+\omega+\psi} \left( \frac{P_t^\ast}{P_t} \right)^{-\theta(1+\omega)} \left( \frac{P_t}{A_t^\psi+\omega} \right) \right] F_t \]

where the expression for \( H_t \) and \( F_t \) are given by

\[ H_t = \sum_{s=0}^{\infty} \left( \alpha \Pi^\ast \right)^s M_{t,t+s} \frac{Y_{t+s}}{Y_t} \left( \frac{P_t(\Pi^\ast)^s}{P_{t+s}} \right)^{-\theta} \]

\[ F_t = \sum_{s=0}^{\infty} \alpha^s M_{t,t+s} \frac{e^{x_{t+s}}}{e^{\chi_t}} \left( \frac{Y_{t+s}}{Y_t} \right) \left( \frac{A_{t+s}}{A_t} \right)^{\psi+\omega} \left( \frac{P_t(\Pi^\ast)^s}{P_{t+s}} \right)^{-\theta(1+\omega)} \frac{P_{t+s}}{P_t} \]

The expressions for \( H_t \) and \( F_t \) can be more conveniently expressed recursively as

\[ H_t = 1 + \alpha \Pi^\ast E_t \left[ M_{t,t+1} \frac{Y_{t+1}^F}{Y_t^F} \left( \frac{X_{t+1}}{X_t} \right) \left( \frac{\Pi_{t+1}}{\Pi^\ast} \right)^{\theta} H_{t+1} \right] \]

\[ F_t = 1 + \alpha \Pi^\ast E_t \left[ M_{t,t+1} \frac{Y_{t+1}^F}{Y_t^F} \left( \frac{X_{t+1}}{X_t} \right) \left( \frac{\Pi_{t+1}}{\Pi^\ast} \right)^{1+\theta(1+\omega)} F_{t+1} \right] \]

where I used the definition \( X_t = Y_t / Y_t^F \).

### A.3 Taylor Rule

The monetary authority sets the one-period nominal interest rate according to:

\[ i_t = \tau + \tau_x (\pi_t - \pi^\ast) + \tau_s x_t + \tau_t i_{t-1} \]

In the special case of \( \tau_t = 0 \), I obtain the Taylor rule (12) in the main text.

### A.4 System

The equilibrium system of equations is:

\[ i_t = -\log E_t[M_{t,t+1}] \]  \hspace{1cm} (33)

\[ \left( \frac{P_t^\ast}{P_t} \right)^{1+\theta \omega} H_t = X_t^{\omega+\psi} F_t \]  \hspace{1cm} (34)

\[ P_t^\ast = \left[ \frac{1}{1-\alpha} \left( 1 - \alpha \left( \frac{\Pi_{t+1}}{\Pi^\ast} \right)^{1-\theta} \right) \right]^{\frac{1}{1-\theta}} \]  \hspace{1cm} (35)

\[ H_t = 1 + \alpha \Pi^\ast E_t \left[ M_{t,t+1} \frac{Y_{t+1}^F}{Y_t^F} \left( \frac{X_{t+1}}{X_t} \right) \left( \frac{\Pi_{t+1}}{\Pi^\ast} \right)^{\theta} H_{t+1} \right] \]  \hspace{1cm} (36)

\[ F_t = 1 + \alpha \Pi^\ast E_t \left[ M_{t,t+1} \frac{Y_{t+1}^F}{Y_t^F} \left( \frac{X_{t+1}}{X_t} \right) \left( \frac{\Pi_{t+1}}{\Pi^\ast} \right)^{1+\theta(1+\omega)} F_{t+1} \right] \]  \hspace{1cm} (37)
$M_{t+1} = \beta \left( \frac{Y^F_{t+1}}{Y^F_t} \right)^{\psi} \left( \frac{X_{t+1}}{X_t} \right)^{-\psi} \left( \frac{V_{t+1}}{E_t(V_{t+1})^{1-\gamma}} \right)^{\psi-\gamma} \left( \frac{P_{t+1}}{P_t} \right)^{-1}$

(38)

$i_t = \tau + \tau_\pi (\pi_t - \pi^*) + \tau_x x_t + \tau_i i_{t-1}$  

(39)

together with the law of motions for the shocks $\Delta a_t$ and $\chi_t$.

B Log-Linear Approximation

Combine equation (34) and (35) to get

$$H_t = X_t^{\psi+\psi} F_t$$

Taking logs,

$$\frac{1 + \theta \omega}{1 - \theta} \log \left[ \frac{1}{1 - \alpha} \left( 1 - \alpha e^{(1-\theta)(\pi^* - \pi_t)} \right) \right] + h_t = (\omega + \psi) x_t + f_t$$

(40)

A loglinear approximation of the above expression around $\pi_t = \bar{\pi}$ yields

$$\frac{1 + \theta \omega}{1 - \theta} \left[ D_\pi + F_\pi (\pi_t - \bar{\pi}) \right] + h_t = (\omega + \psi) x_t + f_t$$

where $\bar{\pi} = E(\pi_t)$, and

$$D_\pi = \log \left( \frac{1 - \alpha e^{-(1-\theta)(\bar{\pi} - \pi^*)}}{1 - \alpha} \right); \quad F_\pi = \frac{\alpha(1-\theta) e^{-(1-\theta)(\bar{\pi} - \pi^*)}}{1 - \alpha e^{-(1-\theta)(\bar{\pi} - \pi^*)}}$$

Next, loglinearize the value function. Let $v_t = \log \frac{V^F_t}{C_t}$ and use the homogeneity of the certainty equivalent $E_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}$ to write

$$e^{(1-\gamma)v_t} = (1-\beta) \left( \frac{C_t}{U(C_t, L_t)} \right)^{-\psi} + \beta e^{(1-\gamma)\pi_t}$$

$$= (1-\beta) \left[ \frac{1}{1-\psi} - \frac{1}{\mu(1+\omega)} e^{(\psi+\omega)x_t} \right] + \beta e^{(1-\gamma)\Xi_t}$$

where $\Xi_t = \log E_t \left[ e^{(1-\gamma)(\psi + \Delta a^F_{t+1} + \Delta x_{t+1})} \right]$. A loglinear approximation of the above expression around $x_t = \bar{x}$ and $\Psi_t = \bar{\Psi}$, yields

$$v_t = \frac{1}{1-\psi} \left\{ \bar{v} + \eta_{vx} x_t + \eta_{vv} \log E_t \left[ e^{(1-\gamma)(\psi + \Delta a^F_{t+1} + \Delta x_{t+1})} \right] \right\}$$

(41)

where $\bar{v} = E(v_t)$, $\bar{\Psi} = E(\Xi_t)$, and

$$\eta_{vx} = -\frac{(1-\beta)(\psi + \omega)}{\mu(1+\omega)D_v} e^{(\psi + \omega)\bar{m}}; \quad \eta_{vv} = \frac{(1-\psi)}{(1-\gamma)} \beta D_v e^{(1-\gamma)\bar{m}} \bar{v}$$

$$\bar{v} = \log D_v - \eta_{vx}\bar{m} + \eta_{vv}\bar{\Psi}$$
\[ D_v = (1 - \beta) \left[ \frac{1}{1 - \psi} - \frac{1}{\mu(1 + \omega)} e^{(\psi + \omega) m_x} \right] + \beta e^{(1 - \psi) m_v} \]

Next, loglinearize the expression for \( H_t \). First, combine (36) and (38) to get

\[ e^{h_t} = 1 + \alpha \beta E_t \left[ \left( \frac{Y_{t+1}^F}{Y_t^F} \right)^{1-\psi} \left( \frac{X_{t+1}}{X_t} \right)^{1-\psi} \left( \frac{\Pi^*}{\Pi_{t+1}} \right)^{1-\theta} \left( \frac{V_{t+1}}{E_t[V_{t+1}^{1-\gamma}]} \right)^{\psi-\gamma} \right] H_{t+1} \].

Note that

\[ \frac{V_{t+1}}{E_t(V_{t+1}^{1-\gamma})^{1-\gamma}} = e^{v_{t+1} + \Delta y_{t+1} + \Delta y_{t+1}} = \frac{e^{v_{t+1} + \Delta y_{t+1} + \Delta y_{t+1}}}{e^{(1-\gamma) v_{t+1} + \Delta y_{t+1} + \Delta y_{t+1}}} \]

where the second equality follows from (41). Therefore, I can write

\[ h_t = \log[1 + \alpha \beta e^{\Omega_t}] \]

where

\[ \Omega_t \equiv \log E_t \left[ e^{(1-\gamma)(\Delta y_{t+1} + \Delta y_{t+1}) + (1-\theta)(\pi^* - \pi_{t+1}) + (\psi-\gamma)v_{t+1} + h_{t+1}} \right] - \frac{\psi - \gamma}{1 - \gamma} \frac{1}{\eta_{uv}} \psi \left[ (1 - \psi) v_t - \eta_u - \eta_{vx} x_t \right] \]

A loglinear approximation of the above expression around \( \Omega_t = \overline{\Omega} \) yields

\[ h_t = \frac{\alpha \beta e^{\overline{\Omega} \psi}}{1 + \alpha \beta e^{\overline{\Omega} \psi}}, \quad \eta_{uv} = \frac{\alpha \beta e^{\overline{\Omega} \psi}}{1 - \gamma}, \quad \eta_{hx} = \frac{\eta_{hh} (\psi - \gamma) (1 - \psi)}{1 - \gamma} \]

where \( \overline{\Omega} = E(\Omega_t) \) and

\[ \eta_{hh} = \frac{\alpha \beta e^{\overline{\Omega} \psi}}{1 + \alpha \beta e^{\overline{\Omega} \psi}}, \quad \eta_{uv} = \frac{\eta_{hh}(\psi - \gamma)(1 - \psi)}{1 - \gamma}, \quad \eta_{hx} = \frac{\eta_{hh}(\psi - \gamma)\eta_{vx}}{1 - \gamma} \]

\[ \eta_h = \log(1 + \alpha e^{\overline{\Omega} \psi}) - \eta_{hh} \overline{\Omega} \eta_v + \frac{\eta_{hh}(\psi - \gamma)\eta_v}{1 - \gamma} \]

Last, I obtain a loglinearized expression for \( F_t \). First, combine (37) and (38) to get

\[ e^{f_t} = 1 + \alpha \beta E_t \left[ \left( \frac{Y_{t+1}^F}{Y_t^F} \right)^{1-\psi} \left( \frac{X_{t+1}}{X_t} \right)^{1-\psi} \left( \frac{\Pi^*}{\Pi_{t+1}} \right)^{1-\theta} \left( \frac{V_{t+1}}{E_t[V_{t+1}^{1-\gamma}]} \right)^{\psi-\gamma} \right] F_{t+1} \].

In a way similar to what was done for the expression for \( H_t \), one can write

\[ f_t = \log[1 + \alpha \beta e^{\overline{\Omega} \psi}] \],
where

\[ \Upsilon_t = \log E_t \left[ e^{(1-\gamma)\Delta y_{t+1} + (1+\omega+\psi-\gamma)\Delta x_{t+1} - \theta(1+\omega)(\pi^*-\pi_{t+1}) + (\psi-\gamma)v_{t+1} + f_{t+1}} \right] \]

\[ - \frac{\psi - \gamma}{1 - \gamma} \frac{1}{\eta_{vv}} \left[ (1 - \psi)v_t - \eta_{vv} - \eta_{vx}x_t \right] . \]

A loglinear approximation of the expression above around \( \Upsilon_t = m \) yields

\[ f_t = \eta_{ff} + \eta_{fv}v_t + \eta_{fx}x_t \]

\[ + \eta_{ff} \log E_t \left[ e^{(1-\gamma)\Delta y_{t+1} + (1+\omega+\psi-\gamma)\Delta x_{t+1} + \theta(1+\omega)(\pi_{t+1} - \pi^*) + (\psi-\gamma)v_{t+1} + g_{t+1}} \right] , \]

(43)

where \( m = E(\Upsilon_t) \) and

\[ \eta_{ff} = \frac{\alpha\beta e^{m}}{1 + \alpha\beta e^{m}}, \quad \eta_{f v} = -\frac{\eta_{ff} (\psi - \gamma)(1 - \psi)}{\eta_{vv}} \frac{1}{1 - \gamma}, \quad \eta_{f x} = \frac{\eta_{ff} (\psi - \gamma)\eta_{vx}}{\eta_{vv}} \frac{1}{1 - \gamma} \]

\[ \eta_{f f} = \log(1 + \alpha e^{m}) - \eta_{ff} m + \frac{\eta_{ff} (\psi - \gamma)}{\eta_{vv}} \frac{1}{1 - \gamma} . \]

The loglinearized system consists of equations (40)-(43), together with the Euler condition (33).

B.1 Solution to the Loglinear System

I guess that the solution to the system is affine in the state variables, that is

\[ x_t = \pi + x_a \Delta a_t + x_{\chi} \chi_t + x_{i} i_{t-1} \]

(44)

\[ \pi_t - \pi^* = \pi + \pi_a \Delta a_t + \pi_{\chi} \chi_t + \pi_{i} i_{t-1} \]

\[ h_t = \bar{h} + h_a \Delta a_t + h_{\chi} \chi_t + h_{i} i_{t-1} \]

\[ f_t = \bar{f} + g_a \Delta a_t + f_{\chi} \chi_t + f_{i} i_{t-1} \]

\[ v_t = \bar{v} + v_a \Delta a_t + v_{\chi} \chi_t + v_{i} i_{t-1} \].

Substituting the guess above in Equations (39) to (43), and using the Euler equation (33) and the expression for the nominal pricing kernel in (38), one can obtain the solution for the coefficients governing the dynamics of the endogenous variables \( x_t, \pi_t, h_t, f_t, \) and \( v_t \) by matching coefficients. The solution involves a fixed point problem, as the linearization points are endogenously determined. See Hsu and Palomino (2011) for details.

C The pricing kernel

The pricing kernel of the economy is
\[
M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\psi} \left( \frac{V_{t+1}}{E_t(V_{t+1}^{1-\gamma})} \right)^{\psi-\gamma} (\Pi_{t+1})^{-1} \\
= \beta \left( \frac{Y_{t+1}^F}{Y_t^F} \right)^{-\gamma} \left( \frac{X_{t+1}^F}{X_t^F} \right)^{-\gamma} \left( \frac{e^{(\psi-\gamma)\pi_{t+1}}}{e^{\frac{\psi-\gamma}{1-\gamma}[(1-\psi)\pi_t-\gamma x_t]} + 1} \right) e^{-\pi_{t+1}}
\]

Let \( M_{t,t+1}^* = \left\{ \begin{array}{ll}
N_{t,t+1} & \text{if } I = 0; \\
M_{t,t+1} & \text{if } I = 1.
\end{array} \right. \)

Then,

\[-\log M_{t,t+1}^* = -\log \beta + \gamma \Delta y_{t+1}^F + \gamma \Delta x_{t+1} - (\psi - \gamma)v_{t+1} + \frac{\psi - \gamma}{1 - \gamma} \frac{1}{\eta_{vv}[(1 - \psi)v_t - \eta_{xv}x_t]} + I(\pi^* + \Pi)
\]

Substituting the guesses for the endogenous variables and rearranging, I can write the expression for the pricing kernel more succinctly as

\[-\log M_{t,t+1}^* = \delta + \gamma_a \Delta a_t + \gamma_\chi \chi_t + \gamma_i i_{t+1} + \lambda_a \sigma_a \epsilon^a_{t+1} + \lambda_\chi \sigma_\chi \epsilon^\chi_{t+1},
\]

where

\[
\delta = -\log \beta - (\psi - \gamma)\overline{v} + \frac{\psi - \gamma}{1 - \gamma} \frac{1}{\eta_{vv}}[(1 - \psi)\overline{v} - \eta_{xv}\overline{x}] + I(\pi^* + \Pi)
\]
\[
+ \left[ \gamma(1 + x_a) - (\psi - \gamma)v_a + I\pi_a \right] (1 - \varphi_a)\theta_a
\]
\[
\gamma_a = -\left[ \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{\eta_{xx}}{\eta_{vv}} + \gamma \right] x_a + \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{1 - \psi}{\eta_{vv}} v_a + \left[ \gamma(1 + x_a) - (\psi - \gamma)v_a + I\pi_a \right] \varphi_a
\]
\[
+ (\tau_x x_a + \tau_\pi \pi_a)(\gamma x_a - (\psi - \gamma)v_a + I\pi_i),
\]
\[
\gamma_\chi = -\left[ \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{\eta_{xx}}{\eta_{vv}} + \gamma \right] x_\chi + \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{1 - \psi}{\eta_{vv}} v_\chi + \left[ \gamma(1 + x_a) - (\psi - \gamma)v_a + I\pi_a \right] \varphi_\chi
\]
\[
+ (\tau_x x_\chi + \tau_\pi \pi_\chi)(\gamma x_\chi - (\psi - \gamma)v_\chi + I\pi_i) - \frac{\gamma}{\psi + \omega}(\varphi_\chi - 1),
\]
\[
\gamma_i = -\left[ \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{\eta_{xx}}{\eta_{vv}} + \gamma \right] x_i + \left( \frac{\psi - \gamma}{1 - \gamma} \right) \frac{1 - \psi}{\eta_{vv}} v_i
\]
\[
+ (\tau_x x_i + \tau_\pi \pi_i + \tau_\chi \chi_i)(\gamma x_i - (\psi - \gamma)v_i + I\pi_i),
\]
\[
\lambda_a = \gamma(1 + x_a) - (\psi - \gamma)v_a + I\pi_a,
\]
\[
\lambda_\chi = \gamma \left( \frac{1}{\psi + \omega} + x_\chi \right) - (\psi - \gamma)v_\chi + I\pi_\chi.
\]
D Consumption Growth, Price-Consumption Ratio, and Risk Premium

Here I solve for the equilibrium consumption growth process, the price-consumption ratio, and the risk premium for the model with interest rate inertia in section 6. Using (14) in the main text and the guess for the output gap, realized consumption growth can be written as

\[ \Delta c_{t+1} = \Delta y_{t+1} = \mu_c + g_a \Delta \alpha_t + g_i \Delta \pi_t + (1 + x_a) \sigma_a \epsilon_{t+1}, \]

where

\[ \mu_c = (1 + x_a)(1 - \phi_a) \theta_a + x_i(\tau_x \pi + \tau_x \pi_a), \]
\[ g_a = (1 + x_a) \phi_a - x_a + x_i(\tau_x x_a + \tau_x \pi_a), \]
\[ g_i = x_i(\tau_x x_i + \tau_x \pi_i + \tau_i - 1). \]

The price-consumption ratio is

\[ pc_t = \frac{1}{1 - \eta_{pc}} \left( \delta + \eta_{pc} + \mu_c + \eta_{pc} A_a (1 - \phi_a) \theta_a + A_i (\pi + \tau_x \pi + \tau_x \pi_a) \right) \]
\[ + \left( \eta_{pc} A_a + (1 + x_a) - \lambda_a \sigma_a^2 / 2 \right), \]

\[ A_a = \frac{g_a + \eta_{pc} A_i (\tau_x x_a + \tau_x \pi_a) - \gamma_a}{1 - \eta_{pc} \phi_a}, \]
\[ A_i = \frac{g_i - \gamma_i}{1 - \eta_{pc} (\tau_x x_i + \tau_x \pi_i + \tau_i)}. \]

The risk premium on the consumption claim is

\[ E_t(r_{t+1}^c - r_{f,t}) + 0.5 Var_t(r_{t+1}^c) = -Cov(\log N_{t+1} - E_t(\log N_{t+1}), r_{t+1}^c - E_t(r_{t+1}^c)) = \lambda_a B_a \sigma_a, \]

where \( B_a = \eta_{pc} A_a + (1 + x_a). \)

E Proofs of Results

The system in (44) does not have a closed form solution (see Bekaert, Cho, and Moreno (2010)), so I rely on a numerical analysis to show that \( x_a > 0, g_a > 0, \frac{\partial x}{\partial \alpha} > 0, \frac{\partial x}{\partial \tau_x} < 0, \text{ and } \frac{\partial x}{\partial \sigma_x} < 0. \) See Figure 11 for the results.

Proof of Result 1: Using (16) in the main text, the first order autocorrelation of expected consumption
growth is

\[ \text{Corr}(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2}) = \frac{\text{Cov}(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})}{\text{Var}(E_t \Delta c_{t+1})} \]
\[ = \frac{\text{Cov}(g_a \Delta a_t, g_a \Delta a_{t+1})}{g_a^2 \text{Var}(\Delta a_t)} \]
\[ = \frac{g_a^2 \varphi_a \text{Var}(\Delta a_t)}{g_a^2 \text{Var}(\Delta a_t)} = \varphi_a . \]

Using (15) in the main text, the first order autocorrelation of realized consumption growth is

\[ \text{Corr}(\Delta c_t, \Delta c_{t+1}) = \frac{\text{Cov}(\Delta c_t, \Delta c_{t+1})}{\text{Var}(\Delta c_t)} \]
\[ = \frac{g_a^2 \varphi_a \text{Var}(\Delta a_t) + (1 + x_a) g_a \sigma_a^2}{g_a^2 \text{Var}(\Delta a_t) + (1 + x_a)^2 \sigma_a^2} \]
\[ = \varphi_a + \frac{(1 + x_a) \sigma_a^2 (g_a - \varphi_a (1 + x_a))}{g_a^2 \text{Var}(\Delta a_t) + (1 + x_a)^2 \sigma_a^2} \]
\[ = \varphi_a - \frac{(1 + x_a) \sigma_a^2 x_a}{g_a^2 \text{Var}(\Delta a_t) + (1 + x_a)^2 \sigma_a^2} < \varphi_a , \quad (45) \]

since \( x_a > 0 \). Now, define

\[ b(\alpha) = \frac{(1 + x_a) \sigma_a^2 x_a}{g_a^2 \text{Var}(\Delta a_t) + (1 + x_a)^2 \sigma_a^2} . \]

Substituting \( g_a = (1 + x_a) \varphi_a - x_a \) and \( \text{Var}(\Delta a_t) = \frac{\sigma_a^2}{1 - \varphi_a^2} \); after some algebra, I get

\[ \frac{1}{b(\alpha)} = \left( 1 + \frac{x_a}{x_a} \right) \frac{x_a}{1 + x_a} \frac{1 - \frac{\varphi_a}{1 - \varphi_a^2}}{1 - \varphi_a^2} \]
\[ = \left( 1 + \frac{x_a}{x_a} \right) \frac{x_a}{1 + x_a} \frac{1 - \frac{\varphi_a}{1 - \varphi_a^2}}{1 - \varphi_a^2} \quad (46) \]

Express \( \frac{\partial (\frac{1 + x_a}{x_a})}{\partial \alpha} \) and \( \frac{\partial (\frac{x_a}{1 + x_a})}{\partial \alpha} \) in terms of \( \frac{\partial (x_a)}{\partial \alpha} \):

\[ \frac{\partial \left( \frac{1 + x_a}{x_a} \right)}{\partial \alpha} = \frac{x_a \frac{\partial (1 + x_a)}{\partial \alpha} - (1 + x_a) \frac{\partial (x_a)}{\partial \alpha}}{x_a^2} \]
\[ = - \frac{\partial (x_a)}{x_a^2} \quad (47) \]
\[ \frac{\partial \left( \frac{x_a}{1 + x_a} \right)}{\partial \alpha} = \frac{(1 + x_a) \frac{\partial (x_a)}{\partial \alpha} - x_a \frac{\partial (1 + x_a)}{\partial \alpha}}{(1 + x_a)^2} \]
\[ = \frac{\partial (x_a)}{(1 + x_a)^2} \quad (48) \]

Now, differentiate (46) w.r.t. \( \alpha \) and substitute (47) and (48):

\[ \frac{\partial b(\alpha)}{\partial \alpha} = \left( \frac{\partial \left( \frac{1 + x_a}{x_a} \right)}{\partial \alpha} + \frac{\partial \left( \frac{x_a}{1 + x_a} \right)}{\partial \alpha} \right) \frac{1}{1 - \varphi_a^2} \]
\[ = \left( \frac{\partial (x_a)}{x_a^2} + \frac{\partial (x_a)}{(1 + x_a)^2} \right) \frac{1}{1 - \varphi_a^2} \]
\[ = \frac{\partial (x_a)}{\partial \alpha} \left( \frac{1}{(1 + x_a)^2} - \frac{1}{x_a^2} \right) \frac{1}{1 - \varphi_a^2} \]

<0
Hence, \( \text{Sign} \left( \frac{\partial \alpha}{\partial x} \right) = - \text{Sign} \left( \frac{\partial (\alpha)}{\partial x} \right) \) and \( \text{Sign} \left( \frac{\partial (\alpha)}{\partial x} \right) = \text{Sign} \left( \frac{\partial (\alpha)}{\partial x} \right) > 0 \). Therefore, from (45), we have that \( \frac{\partial \text{Corr}(\Delta c_t, \Delta c_{t+1})}{\partial x} < 0 \). Similar derivations show that \( \frac{\partial \text{Corr}(\Delta c_t, \Delta c_{t+1})}{\partial \tau} > 0 \) and \( \frac{\partial \text{Corr}(\Delta c_t, \Delta c_{t+1})}{\partial \pi > 0} \).

\( \square \)

**Proof of Result 2:** The conditional correlation between realized and expected consumption growth is

\[
\text{Corr}_{t-1}(\Delta c_t, E_t(\Delta c_{t+1})) = \frac{\text{Cov}_{t-1}(\Delta c_t, E_t(\Delta c_{t+1}))}{\sigma_{t-1}(\Delta c_t) \sigma_{t-1}(E_t(\Delta c_{t+1}))} = \frac{(1 + x_a) g_a \sigma_a^2}{1 + x_a |g_a| \sigma_a^2} = +1 ,
\]

since \( x_a > 0 \) and \( g_a > 0 \). \( \square \)

### F A Model with a Labor Supply Shock

Consider a model in which the intratemporal utility function \( U(C_t, L_t) \) is given by (29), where the shock to the disutility of labor evolves according to (30). The labor supply shock introduces a wedge that distorts the static relationship between real wages and the marginal rate of substitution between consumption and labor (equation 31). This distortion makes the problem faced by the monetary authority non-trivial in the sense that it introduces a trade-off between optimal inflation and optimal output.

Strictly speaking, this distortion is not bad, in the sense that, while it moves away the equilibrium allocation from the one that would be obtained with flexible prices and constant disutility of labor, it does not introduce any additional welfare inefficiencies other than the ones generated by price stickiness. However, as pointed out by Chari, Kehoe, and McGrattan (2007), in the model studied in this paper, like in the model of Smets and Wouters (2007), one cannot identify what structural shock is the actual source of such a wedge. For instance, a similar distortion could be obtained with the introduction of a time-varying price markup shock, which instead is a bad shock in the sense that it moves the competitive equilibrium allocation away from the efficient allocation, even further away than what is generated by price stickiness only. Here, I use a labor supply shock for analytical tractability of the firms’ maximization problem (see Appendix A.2.2), and analyze the case in which the monetary authority erroneously believes that the labor wedge comes from a bad shock and therefore has an incentive to offset it.

I conduct the following exercise. I tweak the baseline calibration in Section 4 to account for the additional variability in the model coming from the introduction of an i.i.d. labor supply shock with an annualized volatility of 1.73%. Compared to the baseline calibration of Table 1, I increase the Calvo parameter \( \alpha \) to 8/9 and the Taylor coefficient to inflation \( \tau \) to 1.5. This is due to the fact that the introduction of the labor shock increases the overall variability in the economy. While I do not attempt to provide a quantitative measure of the welfare costs associated with different monetary policy reactions to inflation and output, I show that the equilibrium that would be obtained with flexible prices is no longer the efficient allocation and that therefore the monetary authority does not have an obvious incentive to achieve it.

The efficient output \( Y^E_t \) is defined as the output that would be produced in the case of flexible prices and constant disutility of labor. From Appendix A.2.1, efficient output growth evolves following the dynamics of technology, that is \( \Delta y^E_t = \Delta \alpha_t \), while flexible output growth is \( \Delta y^F_t = \Delta \alpha_t - \frac{1}{\psi + \omega} \Delta \chi_t \). 

47
Notice that the monetary authority cannot influence the dynamics of $\chi_t$, so that the efficient allocation cannot be achieved. Here, I do not perform a quantitative analysis of the welfare losses associated with different interest rate rules (see Woodford (2003) for a general treatment and Levin, Lopez-Salido, Nelson, and Yun (2008) for the specific case of recursive preferences). Rather, the results in Table 15 show that the monetary authority does not have an obvious interest in offsetting the effects of price stickiness. Indeed, the dynamics of flexible consumption growth largely differ from the dynamics of efficient consumption growth. In particular, flexible consumption growth is more volatile than efficient consumption growth, with an annualized volatility of 1.79% (versus 1.04%), and also shows very little autocorrelation (−0.02) compared to the efficient allocation.
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.999</td>
</tr>
<tr>
<td>EIS of consumption</td>
<td>$1/\psi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\gamma$</td>
<td>6.5</td>
</tr>
<tr>
<td>Elasticity of labor supply</td>
<td>$1/\omega$</td>
<td>1</td>
</tr>
<tr>
<td>Degree of price rigidity</td>
<td>$\alpha$</td>
<td>7/9</td>
</tr>
<tr>
<td>Elasticity of substitution of goods</td>
<td>$\theta$</td>
<td>7</td>
</tr>
<tr>
<td>Inflation target</td>
<td>$\pi^*$</td>
<td>$2.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Average technology growth</td>
<td>$\theta_a$</td>
<td>$1.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Conditional volatility of technology growth</td>
<td>$\sigma_a$</td>
<td>$0.95 \times 10^{-3}$</td>
</tr>
<tr>
<td>Autocorrelation of technology growth</td>
<td>$\varphi_a$</td>
<td>0.95</td>
</tr>
<tr>
<td>Constant in the policy rule</td>
<td>$\tau$</td>
<td>$2.27 \times 10^{-3}$</td>
</tr>
<tr>
<td>Response to inflation in the policy rule</td>
<td>$\tau_\pi$</td>
<td>1.1</td>
</tr>
<tr>
<td>Response to output gap in the policy rule</td>
<td>$\tau_x$</td>
<td>0.5/12</td>
</tr>
<tr>
<td>Leverage</td>
<td>$\phi_d$</td>
<td>2.6</td>
</tr>
<tr>
<td>Sensitivity of dividends to consumption innovations</td>
<td>$\xi_d$</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 1: Baseline parameter values. See Section 4 for details.
### Table 2: Moment conditions of macro variables and asset prices

Means are annualized by multiplying by 12 the monthly observation. Volatilities are annualized by multiplying by $\sqrt{12}$ the monthly observation. The Sharpe Ratio is defined as the ratio between the annualized excess return and the annualized volatility of the return on the dividend claim. The autocorrelation moments refer to the monthly autocorrelations, with the exception of the correlation between consumption growth and inflation, which is obtained after aggregating the monthly observations to annual frequency. The Jensen’s inequality term is $J.I. = \text{Var}_r(r_{t+1}^d)/2$. Panel A reports the moments that were calibrated to match the data, and Panel B reports other moments. The empirical moments for consumption growth are taken from Bansal and Yaron (2004). Data on nominal interest rate and inflation is from CRSP.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: calibrated moments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\Delta c_t)$</td>
<td>1.80</td>
<td>1.80</td>
</tr>
<tr>
<td>$\sigma(\Delta c_t)$</td>
<td>2.72</td>
<td>2.72</td>
</tr>
<tr>
<td>$E(i_t)$</td>
<td>6.42</td>
<td>6.42</td>
</tr>
<tr>
<td>$\sigma(i_t)$</td>
<td>3.72</td>
<td>3.72</td>
</tr>
<tr>
<td>$E(r_t^d - r_f) + J.I.$</td>
<td>7.02</td>
<td>7.02</td>
</tr>
<tr>
<td>$\sigma(r_t^d) + J.I.$</td>
<td>20.60</td>
<td>20.60</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>Panel B: other moments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr($\Delta c_t, \Delta c_{t+1}$)</td>
<td>n.a.</td>
<td>0.12</td>
</tr>
<tr>
<td>Corr($E_t \Delta c_{t+1}$)</td>
<td>n.a.</td>
<td>0.63</td>
</tr>
<tr>
<td>Corr($E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2}$)</td>
<td>n.a.</td>
<td>0.95</td>
</tr>
<tr>
<td>$E(\pi_t)$</td>
<td>4.34</td>
<td>4.72</td>
</tr>
<tr>
<td>$\sigma(\pi_t)$</td>
<td>1.32</td>
<td>0.74</td>
</tr>
<tr>
<td>Corr($\pi_t, \pi_{t+1}$)</td>
<td>0.60</td>
<td>0.95</td>
</tr>
<tr>
<td>Corr($i_t, i_{t+1}$)</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>$E(r_t)$</td>
<td>0.86</td>
<td>1.44</td>
</tr>
<tr>
<td>$\sigma(r_t)$</td>
<td>0.97</td>
<td>1.46</td>
</tr>
<tr>
<td>Corr($\Delta c_t, \pi_t$)</td>
<td>$-0.32$</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Table 2: Moment conditions of macro variables and asset prices.
<table>
<thead>
<tr>
<th>Baseline Model</th>
<th>Power utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>γ = 6.5</td>
</tr>
<tr>
<td></td>
<td>ψ⁻¹ = 1.5</td>
</tr>
<tr>
<td>σ(Δcₜ)</td>
<td>2.72</td>
</tr>
<tr>
<td>σ(πₜ)</td>
<td>0.74</td>
</tr>
<tr>
<td>E(rₜ)</td>
<td>1.44</td>
</tr>
<tr>
<td>σ(rₜ)</td>
<td>1.45</td>
</tr>
<tr>
<td>E(iₜ)</td>
<td>6.42</td>
</tr>
<tr>
<td>σ(iₜ)</td>
<td>3.72</td>
</tr>
<tr>
<td>E(ᵣₜ⁻ µ) + J.I.</td>
<td>1.55</td>
</tr>
<tr>
<td>σ(ᵣₜ⁻ µ)</td>
<td>3.95</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.34</td>
</tr>
<tr>
<td>of which: short-run risk</td>
<td>0.17 (50%)</td>
</tr>
<tr>
<td>long-run risk</td>
<td>0.17 (50%)</td>
</tr>
</tbody>
</table>

Table 3: The role of recursive preferences. Moment conditions for the baseline calibration and for an alternative model with standard power utility (γ = ψ = 6.5). All other parameters remain unchanged (see Table 1). The Jensen’s inequality term is $J.I. = \text{Var}(r_{t+1})/2$.

<table>
<thead>
<tr>
<th>φₐ</th>
<th>γ = 6.5, ψ = 1/1.5</th>
<th>γ = 6.5, ψ = 6.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>1.381</td>
<td>-7.340</td>
</tr>
<tr>
<td>0.91</td>
<td>0.163</td>
<td>-8.946</td>
</tr>
<tr>
<td>0.92</td>
<td>1.964</td>
<td>-11.127</td>
</tr>
<tr>
<td>0.93</td>
<td>2.406</td>
<td>-14.197</td>
</tr>
<tr>
<td>0.94</td>
<td>3.020</td>
<td>-18.719</td>
</tr>
<tr>
<td>0.95</td>
<td>3.913</td>
<td>-25.789</td>
</tr>
<tr>
<td>0.96</td>
<td>5.303</td>
<td>-37.792</td>
</tr>
<tr>
<td>0.97</td>
<td>7.694</td>
<td>-60.794</td>
</tr>
<tr>
<td>0.98</td>
<td>12.562</td>
<td>-114.601</td>
</tr>
<tr>
<td>0.99</td>
<td>26.794</td>
<td>-303.584</td>
</tr>
</tbody>
</table>

Table 4: Sensitivity $A_a$ of the price-consumption ratio to the technology shock. Comparative statics for different values of autocorrelation in technology growth, $φ_a$. Baseline calibration ($γ = 6.5, ψ = 1/1.5$) and standard power utility ($γ = ψ = 6.5$).
Table 5: Consumption growth dynamics and consumption risk premium as a function of the degree of price stickiness. Average price duration is 1 month ($\alpha = 0$, flexible prices), 2 months ($\alpha = 1/2$), 4.5 months ($\alpha = 7/9$, baseline calibration), and 9 months ($\alpha = 8/9$). The Jensen’s inequality term is $J.I. = \text{Var}_t(r^c_t)/2$.

<table>
<thead>
<tr>
<th>Moment Data Baseline Model</th>
<th>$\tau_\pi = 2.17$</th>
<th>$\tau_x = 2.55/12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta c_t)$</td>
<td>2.72</td>
<td>2.72</td>
</tr>
<tr>
<td>$\text{Corr}(\Delta c_t, \Delta c_{t+1})$</td>
<td>n.a.</td>
<td>0.12</td>
</tr>
<tr>
<td>$\sigma(E_t \Delta c_{t+1})$</td>
<td>n.a.</td>
<td>0.63</td>
</tr>
<tr>
<td>$\text{Corr}(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$</td>
<td>n.a.</td>
<td>0.95</td>
</tr>
<tr>
<td>$E(r^c_t - r_f) + J.I.$</td>
<td>4.34</td>
<td>4.72</td>
</tr>
<tr>
<td>$\text{Corr}(\pi_t, \pi_{t+1})$</td>
<td>1.32</td>
<td>0.74</td>
</tr>
<tr>
<td>$E(i_t)$</td>
<td>6.42</td>
<td>6.42</td>
</tr>
<tr>
<td>$\sigma(i_t)$</td>
<td>3.72</td>
<td>3.72</td>
</tr>
<tr>
<td>$\text{Corr}(i_t, i_{t+1})$</td>
<td>0.98</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 6: Moment conditions of macro variables and asset prices. Annualized means are obtained multiplying by 12 the monthly observations. Volatilities are annualized by multiplying monthly volatilities by $\sqrt{12}$. Autocorrelations are not annualized. Two policy exercises are conducted. In both exercises, the unconditional mean of the one-month nominal interest rate is reduced by 1%. The first exercise requires $\tau_\pi$ to move from 1.1 to 2.17. The second exercise requires $\tau_x$ to move from 0.5/12 to 2.55/12.
Table 7: Regression coefficients, t statistics and $R^2$ statistics for predictive regressions of consumption, dividend, wage growth and inflation on the price-dividend ratio. The baseline model uses the coefficients reported in Table 1. With the exclusion of real wages, the statistics for the model are mean values for 10,000 simulations each with $85 \times 12$ monthly observations that are aggregated to an annual frequency. For real wages, the statistics for the model are mean values for 10,000 simulations each with $64 \times 12$ monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with $2(j-1)$ lags. The last six columns report summary statistics for alternative models where $\alpha = 6/9$, and $\alpha = 7/9$, respectively, leaving all other parameters unchanged. With the exception of wage growth, data is monthly for the period 1926-2010. Wage growth is real compensation per hour from BLS, for the period 1947-2010.
Table 8: Regression coefficients, $t$ statistics and $R^2$ statistics for predictive regressions of consumption, dividend, wage growth and inflation on the price-dividend ratio. The baseline model uses the coefficients reported in Table 1. With the exclusion of real wages, the statistics for the model are mean values for 10,000 simulations each with $85 \times 12$ monthly observations that are aggregated to an annual frequency. For real wages, the statistics for the model are mean values for 10,000 simulations each with $64 \times 12$ monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with $2(j-1)$ lags. The last six columns report summary statistics for alternative models where $\tau_\pi = 1.50$, and $\tau_\pi = 1.01$, respectively, leaving all other parameters unchanged. With the exception of wage growth, data is monthly for the period 1926-2010. Wage growth is real compensation per hour from BLS, for the period 1947-2010.
Table 9: Regression coefficients, t statistics and $R^2$ statistics for predictive regressions of consumption, dividend, wage growth and inflation on the one-period nominal interest rate. The baseline model uses the coefficients reported in Table 1. With the exclusion of real wages, the statistics for the model are mean values for 10,000 simulations each with 85 x 12 monthly observations that are aggregated to an annual frequency. For real wages, the statistics for the model are mean values for 10,000 simulations each with 64 x 12 monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with 2($j$ − 1) lags. The last six columns report summary statistics for alternative models where $\alpha = 6/9$, and $\alpha = 7/9$, respectively, leaving all other parameters unchanged. With the exception of wage growth, data is monthly for the period 1926-2010. Wage growth is real compensation per hour from BLS, for the period 1947-2010.
Table 10: Regression coefficients, t statistics and $R^2$ statistics for predictive regressions of consumption, dividend, wage growth and inflation on the one-period nominal interest rate. The baseline model uses the coefficients reported in Table 1. With the exclusion of real wages, the statistics for the model are mean values for 10,000 simulations each with 85 x 12 monthly observations that are aggregated to an annual frequency. For real wages, the statistics for the model are mean values for 10,000 simulations each with 64 x 12 monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with 2(j - 1) lags. The last six columns report summary statistics for alternative models where $\tau_\pi = 1.50$, and $\tau_\pi = 1.01$, respectively, leaving all other parameters unchanged. With the exception of wage growth, data is monthly for the period 1926-2010. Wage growth is real compensation per hour from BLS, for the period 1947-2010.
Moment Data Model

Panel A: calibrated moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta c_t)$</td>
<td>2.72</td>
<td>2.72</td>
</tr>
<tr>
<td>$E(i_t)$</td>
<td>6.42</td>
<td>6.42</td>
</tr>
<tr>
<td>$Corr(i_t)$</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.34</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Panel B: other moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\Delta c_t)$</td>
<td>1.80</td>
<td>1.74</td>
</tr>
<tr>
<td>$Corr(\Delta c_t, \Delta c_{t+1})$</td>
<td>n.a.</td>
<td>0.12</td>
</tr>
<tr>
<td>$\sigma(E_t\Delta c_{t+1})$</td>
<td>n.a.</td>
<td>0.63</td>
</tr>
<tr>
<td>$Corr(E_t\Delta c_{t+1}, E_{t+1}\Delta c_{t+2})$</td>
<td>n.a.</td>
<td>0.93</td>
</tr>
<tr>
<td>$\sigma(i_t)$</td>
<td>3.72</td>
<td>0.33</td>
</tr>
<tr>
<td>$E(\pi_t)$</td>
<td>4.34</td>
<td>3.14</td>
</tr>
<tr>
<td>$\sigma(\pi_t)$</td>
<td>1.32</td>
<td>0.74</td>
</tr>
<tr>
<td>$Corr(\pi_t, \pi_{t+1})$</td>
<td>0.60</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 11: Moment conditions of macro variables and asset prices for the model with interest rate inertia in Section 6. Means are annualized by multiplying by 12 the monthly observation. Volatilities are annualized by multiplying by $\sqrt{12}$ the monthly observation. The Sharpe Ratio is defined as the ratio between the annualized excess return and the annualized volatility of the return on the dividend claim. The autocorrelation moments refer to the monthly autocorrelations. Panel A reports the moments that were calibrated to match the data, and Panel B reports other moments. The empirical moments for consumption growth are taken from Bansal and Yaron (2004). Data on nominal interest rate and inflation is from CRSP.
Table 12: Moment conditions of macro variables and asset prices for the model with interest rate inertia and an i.i.d monetary policy shock in Footnote 10 in Section 6. The introduction of the policy shock allows the model to return sensible volatilities for the nominal interest rate and the inflation rate. Means are annualized by multiplying by 12 the monthly observation. Volatilities are annualized by multiplying by $\sqrt{12}$ the monthly observation. The Sharpe Ratio is defined as the ratio between the annualized excess return and the annualized volatility of the return on the dividend claim. The autocorrelation moments refer to the monthly autocorrelations. Panel A reports the moments that were calibrated to match the data, and Panel B reports other moments. The empirical moments for consumption growth are taken from Bansal and Yaron (2004). Data on nominal interest rate and inflation is from CRSP.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: calibrated moments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta c_t)$</td>
<td>2.72</td>
<td>2.72</td>
</tr>
<tr>
<td>$E(i_t)$</td>
<td>6.42</td>
<td>6.42</td>
</tr>
<tr>
<td>$\text{Corr}(i_t)$</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>Panel B: other moments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\Delta c_t)$</td>
<td>1.80</td>
<td>1.74</td>
</tr>
<tr>
<td>$\text{Corr}(\Delta c_t, \Delta c_{t+1})$</td>
<td>n.a.</td>
<td>0.11</td>
</tr>
<tr>
<td>$\sigma(E_t \Delta c_{t+1})$</td>
<td>n.a.</td>
<td>0.64</td>
</tr>
<tr>
<td>$\text{Corr}(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$</td>
<td>n.a.</td>
<td>0.93</td>
</tr>
<tr>
<td>$\sigma(i_t)$</td>
<td>3.72</td>
<td>3.32</td>
</tr>
<tr>
<td>$E(\pi_t)$</td>
<td>4.34</td>
<td>3.17</td>
</tr>
<tr>
<td>$\sigma(\pi_t)$</td>
<td>1.32</td>
<td>1.14</td>
</tr>
<tr>
<td>$\text{Corr}(\pi_t, \pi_{t+1})$</td>
<td>0.60</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 13: Persistence in expected and realized consumption growth and volatility of realized consumption growth for various level of interest rate inertia.
Table 14: Persistence in expected consumption growth and consumption risk premium components as a function of the degree of price stickiness. The level of interest rate inertia is fixed at $\tau_i = 0.98$. The Jensen’s inequality term is $J.I. = Var_t(r_{t+1})/2$. See Section 6 for the functional forms of the coefficients $A_i$ and $B_a$. 

<table>
<thead>
<tr>
<th>Moment and Coefficient</th>
<th>$\alpha = 1/9$</th>
<th>$\alpha = 7/9$</th>
<th>$\alpha = 8.5/9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$</td>
<td>0.628</td>
<td>0.925</td>
<td>0.972</td>
</tr>
<tr>
<td>$E(r^c - r_f) + J.I.$</td>
<td>0.721</td>
<td>0.934</td>
<td>0.949</td>
</tr>
<tr>
<td>$A_i$</td>
<td>0.434</td>
<td>4.637</td>
<td>11.645</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>19.404</td>
<td>19.750</td>
<td>19.849</td>
</tr>
<tr>
<td>$B_a$</td>
<td>1.529</td>
<td>1.943</td>
<td>1.971</td>
</tr>
<tr>
<td>Moment</td>
<td>Data</td>
<td>Model</td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>Panel A: Calibrated moments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\Delta c_t)$</td>
<td>1.800</td>
<td>1.800</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta c_t)$</td>
<td>2.720</td>
<td>2.720</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(\Delta c_t, \Delta c_{t+1})$</td>
<td>n.a.</td>
<td>0.118</td>
<td></td>
</tr>
<tr>
<td>$E(i_t)$</td>
<td>6.420</td>
<td>6.420</td>
<td></td>
</tr>
<tr>
<td>$\sigma(i_t)$</td>
<td>3.720</td>
<td>3.720</td>
<td></td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.340</td>
<td>0.340</td>
<td></td>
</tr>
<tr>
<td>Panel B: Other moments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(E_t \Delta c_{t+1})$</td>
<td>n.a.</td>
<td>0.635</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$</td>
<td>n.a.</td>
<td>0.931</td>
<td></td>
</tr>
<tr>
<td>$E(\Delta c_t^E)$</td>
<td>n.a.</td>
<td>1.800</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta c_t^E)$</td>
<td>n.a.</td>
<td>1.790</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(\Delta c_t^E, \Delta c_{t+1}^E)$</td>
<td>n.a.</td>
<td>-0.020</td>
<td></td>
</tr>
<tr>
<td>$E(\pi_t)$</td>
<td>4.340</td>
<td>4.780</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\pi_t)$</td>
<td>1.320</td>
<td>0.708</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(\pi_t, \pi_{t+1})$</td>
<td>0.600</td>
<td>0.945</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(i_t, i_{t+1})$</td>
<td>0.980</td>
<td>0.940</td>
<td></td>
</tr>
<tr>
<td>$E(r_t)$</td>
<td>0.862</td>
<td>1.351</td>
<td></td>
</tr>
<tr>
<td>$\sigma(r_t)$</td>
<td>0.969</td>
<td>1.466</td>
<td></td>
</tr>
</tbody>
</table>

Table 15: Moment conditions of macro variables and asset prices for the model with an i.i.d. labor supply shock of Appendix F. Means are annualized by multiplying by 12 the monthly observation. Volatilities are annualized by multiplying by $\sqrt{12}$ the monthly observation. The Sharpe Ratio is defined as the ratio between the annualized excess return and the annualized volatility of the return on the dividend claim. The autocorrelation moments refer to the monthly autocorrelations. Panel A reports consumption moments, and Panel B reports other moments of interest. The empirical moments for consumption growth are taken from Bansal and Yaron. Data on nominal interest rate and inflation is from CRSP.
Figure 1: Simulated path of 360 months of consumption growth, expected consumption growth, output gap, inflation, the real risk free rate, the one-month nominal interest rate, the return on the consumption stream and the price-consumption ratio. All variables, except the price-consumption ratio, are annualized by multiplying by 12 the monthly observation.
Figure 2: Consumption risk premium, price of technology risk $\lambda_a$, sensitivity $A_a$ of the price-consumption ratio to technology growth, and $B_a$ coefficient as a function of the persistence of the technology shock, $\varphi_a$. Risk aversion $\gamma$ is fixed at 6.5. The solid line refers to the baseline calibration with $\psi = 1/1.5$, the dashed line refers to the case of power utility ($\psi = 6.5$), and the dashed-dotted line refers to the case of late resolution of risk ($\psi = 10$).
Figure 3: Consumption risk premium (on the left, solid line), autocorrelation of realized consumption growth (on the right, solid line), and autocorrelation of expected consumption growth (on the right, dashed line) as a function of the parameter $\alpha$. 
Figure 4: The two top graphs show the consumption risk premium (on the left, solid line), the autocorrelation of realized consumption growth (on the right, solid line), and autocorrelation of expected consumption growth (on the right, dashed line) as a function of the Taylor rule sensitivity to inflation, $\tau\pi$. The two bottom graphs are a zoom of the two top graphs for values of $\tau\pi$ that are common in the literature.
Figure 5: The two top graphs show the consumption risk premium (on the left, solid line), the autocorrelation of realized consumption growth (on the right, solid line), and autocorrelation of expected consumption growth (on the right, dashed line) as a function of the Taylor rule sensitivity to inflation, $\tau_x$. The two bottom graphs are a zoom of the two top graphs for values of $\tau_x$ that are common in the literature.
Figure 6: Consumption risk premium and price of technology risk $\lambda_a$ as a function of the elasticity of intertemporal substitution $\psi^{-1}$ for the model with interest rate inertia considered in Section 6. Risk aversion is fixed at 10.
Figure 7: Impulse response functions to a one-percent permanent technology shock. Consumption growth is $\Delta c_t$, expected consumption growth is $E_t \Delta c_{t+1}$, labor growth is $\Delta l_t$, and real wages growth is $\Delta w_t$. The red line refers to the baseline case with no interest rate inertia ($\tau_i = 0$), the green line refers to the intermediate case of $\tau_i = 0.50$, and the blue line refers to the case of strong interest rate inertia ($\tau_i = 0.98$).
Figure 8: Impulse response functions to a one-percent permanent technology shock. Detrended consumption is \( \tilde{c}_t \), labor is \( \tilde{l}_t \). The red line refers to the baseline case with no interest rate inertia \( \tau_i = 0 \), the green line refers to the intermediate case of \( \tau_i = 0.50 \), and the blue line refers to the case of strong interest rate inertia \( \tau_i = 0.98 \).
Figure 9: Impulse response functions to a one-percent permanent technology shock. The one-period nominal interest rate is $i_t$, the output gap is $x_t$, and inflation is $\pi_t$. The red line refers to the baseline case with no interest rate inertia ($\tau_i = 0$), the green line refers to the intermediate case of $\tau_i = 0.50$, and the blue line refers to the case of strong interest rate inertia ($\tau_i = 0.98$).
Figure 10: Impulse response functions to a one-percent permanent technology shock. Expected labor growth is $E_t \Delta l_{t+1}$, expected real wages growth is $E_t \Delta w_{t+1}$, detrended (log) real wage is $\log \tilde{W}_t P_t$, and (log) real marginal cost is $\log \frac{W_t}{P_t A_t}$. The red line refers to the baseline case with no interest rate inertia ($\tau_i = 0$), the green line refers to the intermediate case of $\tau_i = 0.50$, and the blue line refers to the case of strong interest rate inertia ($\tau_i = 0.98$).
Figure 11: Coefficients $x_a$ and $g_a$ as a function of the Calvo parameter $\alpha$, and the Taylor coefficients $\tau_\pi$ and $\tau_x$. All other parameters are from the baseline calibration in Table 1.