Mortality, Fertility and Saving in a Malthusian Economy*

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1 Introduction

In this paper, we develop and analyze a simple model of fertility choice by utility maximizing households. Following the work of Barro and Becker (1988 and 1989),[1] our model is based on an explicit notion of intergenerational external effects. In contrast to the Barro and Becker model however, we assume that the external effects run from children to parents. That is, parents consumption when old directly enters the utility function of the children whereas in the Barro and Becker model, utility of children enters directly into the utility function of the parent. This gives rise to a fundamentally different reason for bearing of children from what is present in the Barro and Becker model. This is that parents expect to be cared for, at least partially, by their children in their old age when their labor productivity is low. Thus, children are an investment in own old age consumption from the point of view of parents.

Moreover, in our model, parents understand that the amount of support that they can expect from their children in old age is itself dependent on the number of children that are born. We consider two alternative formulations of the decision a child has in providing old age care for his parents, one is cooperative among the siblings and the other is non-cooperative. We find that other things being equal, cooperation among the children gives rise to a substantially higher equilibrium level of support for parents, and hence leads to a much higher fertility choice. The difference of behavior is so substantial that, in reasonably parameterized versions of the model (discussed in Section 5) the non-cooperative solution induces a level of fertility which does not even guarantee a constant population, whereas the cooperative behavior induces, in the same circumstances, levels of population growth that are high by historical standards. We discuss the empirical and positive relevance of these findings later on.

There is an abundant demographic and anthropologic literature studying the direction of intergenerational family transfers, the motives to which they can be attributed and the institutional and cultural arrangements supporting their existence. Caldwell (1978, 1982) is an important contribution to this literature. He proposes a theory of the fertility transition based on the idea that transfers from children to parents are the reason behind high fertility. Changes in the social fabric that reduce or eliminate such transfers bring about a reduction in fertility. In fact,

[1] This approach dates back to Becker (1960). Razin and Ben-Zion (1975) contains an early dynamic formulation of the dynastic model of fertility choice.

Caldwell advances the hypothesis that the third stage of the fertility transition, which we discuss below, is to be attributed mostly to such change in socio-economic norms within the familial unit. Willis (1982) provides an interesting discussion of Caldwell's theory and analyzes a stylized model of decision making in the settings envisioned in Caldwell's work. In related empirical work, Lillard and Willis (1997) find that there is little support for the parental altruism motivation for child-bearing in Malaysian data on intergenerational transfers. Rather, they conclude, that these transfers are consistent with either the investment point of view, or the view that children are repaying parents for implicit loans made when the children were young. Similar results are reported in Jensen (1990), while an even earlier assessment of the empirical evidence supporting the so called "old-age security" hypothesis can be found in Nugent (1985).

Our approach to modeling fertility choice is roughly consistent with Caldwell's hypothesis, i.e., that children are investment goods from the view point of parents and that the desired number of children depends, among other things, on how much they transfer to old parents in relation to the cost of rearing them to adult life. We provide a formalization of the "change in social norms" by comparing cooperative to non-cooperative behavior. We also study the interaction between land/capital accumulation, infant mortality and fertility choices. These themes are absent in the earlier theories.

One of the key features of our model is the comparative statics with respect to infant mortality that it possesses. In the Barro and Becker model, a reduction in the infant mortality rate both decreases the cost of creating surviving children and increases the expected benefits since more children survive to consume (see Barro and Becker (1989) and Fernandez-Villaverde (2001)). Because of these effects, reductions in the infant mortality rate give rise to increased fertility in the standard formulation of this class of models.[2] As argued in Section 2, this appears to be strongly counterfactual. In our model, we find that the opposite is true. Since, from a parents perspective, children are simply an investment in their own future consumption, decreases in infant mortality decrease fertility.

The dynamics of the response to changes in infant mortality are also interest-

[2] A more detailed discussion of this issue can be found is Section 5. It should be noted that Sah (1991) contains a qualitative analysis of a simplified, static, stochastic, discrete choice version of the "utility from children" model, in which reductions in infant mortality do bring about reduction in fertility.

ing. Fertility falls along the path of adjustment when infant mortality is steadily decreased, but this reduction occurs with a lag and is less than proportional. Thus, along the transition path, a decrease in infant mortality brings about both a decrease in fertility and an increase in the growth rate of the population. As we argue below, this pattern corresponds to the fundamental stylized facts of the first two stages of what is defined as the "demographic transition", which we discuss in more details in the next Section.

The model also has interesting implications for fertility from an increasing access to financial markets and an increase in social security pensions. Both of these modifications to the model provide channels through which older people obtain income independently from the support of their children. In the cooperative case, increases in "independent" late age income are offset by decreased support from children, leaving parents roughly neutral with respect to fertility. In the non-cooperative case, increases in independent income are more than offset by reductions in support and hence, fertility falls.

These properties of the model give rise to interesting possibilities for understanding some features of the fertility data that we see. First, there is a high and positive correlation between infant mortality rates and total fertility rates in cross sectional observations (care must be taken here in interpreting causality of course). Second, the historical timing of the reduction in infant mortality in almost all countries gives rise to a predicted reduction in fertility in the model that is in some ways similar to that seen in the demographic transition. Third, the much lower levels of fertility induced, ceteris paribus, by the non-cooperative model point to an important regularity across countries and historical periods if we hypothesize that urbanization is related to a reduction in cooperation among siblings. That is, urbanization and the dissolution of the dynastic family are accompanied by a rapid and dramatic drop in fertility rates, even in the absence of changes either in income per capita or in other relevant economic determinants of fertility. This is reminiscent of the earlier, informal discussion in the demographic and anthropological literature we quoted above. Finally, the properties of the model with respect to changes in social security are suggestive of the recent observed reductions in fertility across many developed countries.

We speculate that the last two properties of the model may help explain also the third phase of the demographic transition in which, in face of low and basically constant rates of child mortality, fertility continues to drop leading to a halting (or even a reverse) in the process of population growth. We do not pursue this line of investigation here, though. The potential of the model to be properly calibrated to historical data to provide a full, quantitative account of the main features of the historical process of demographic transition is therefore left for future research.

The demographic literature on both the old age support aspect of child bearing and the effects of infant mortality on fertility is extremely large and cannot be properly discussed here; Kirk (1996) and Preston (1996) are good starting points for reading about it. On the economic side there is a small but significant previous literature on the role played by these two aspects in the onset of the demographic transition. Surprisingly, at least from our vantage point, seldom, if ever, can one find infant mortality and old age support modeled together as the driving forces behind long run demographic movements until the dawn of the XX century. A large portion of this literature is set in a partial equilibrium framework and/or does not make any attempt to endogenize the size of the transfer from middle age people to old parents and its linkage with saving behavior, arable land expansion and capital accumulation. Those contributions which are closer to ours are discussed in detail in the next section.

Section 2 contains a summary of the relevant data and of previous theoretical work in this area. Sections 3 and 4 contain the specifications and development of the models that we analyze. Section 5 contains some simple and purely qualitative computational results for the basic model. Finally section 6 offers some concluding remarks and provides directions for future work.

2 Empirical Evidence and Previous literature

2.1 Summary of Historical Evidence

There are several historical regularities in the data on population dynamics that are well established. These include the historical response of fertility to the switch from hunter-gatherer societies to agricultural ones, the short term response of fertility to sudden changes in the ratio of population to available and productive land (as determined, e.g., by plagues, famines and wars) and the Demographic Transition that began to occur in Europe in the 18th and 19th centuries.

The switch from hunter-gatherer to agriculture is equivalent to the transition from a technology in which the amount of land controlled by a family is essentially fixed (for given territorial mobility and hunting-gathering technique) to one in which the amount of land controlled by a family unit is incremented through investment both in land and in children. The availability of additional land was accompanied by an increase in fertility (the total fertility rate rose from about 5 to around 7 or 8 according to most archaeological evidence) and a small but visible reduction of overall standards of living. All along recorded history, the short term responses to temporary changes in the ratio of population to arable land were to counter these changes with movements in fertility. Thus, plagues and famines were followed by bursts of fertility of approximately the same size. A similar pattern is observed after significant military conflicts. These facts are well documented in the demographics literature (see e.g. Livi-Bacci (1989))and they should be treated as basic consistency requirements for any dynamic model of population behavior.

The expression "Demographic Transition", as used by demographers, refers to the large scale change from a state of high fertility and high mortality to a state of low fertility and low mortality, that has taken place in very many countries around the world. These transitions first began in the early part of the XVIII century in a subset of European countries and in China (see, e.g., Chesnais (1992) and Livi-Bacci (1989)). Briefly, this was a period of rapid change marked by a drastic improvement in public health followed by a quick reduction in mortality rates. The initial equilibrium is one of slowly growing population with very high mortality and fertility rates. Mortality rates are not only high but also extremely volatile, with spikes occurring more or less in correspondence with wars, famines and epidemics. As mentioned earlier, fertility rates respond to sudden increases in mortality, rapidly bringing the population level back to where it was before, to resume then a pattern of slow secular population growth. When a long run reduction in mortality sets in, it is invariably accompanied by a rapid growth in population. In most countries, this reduction in mortality was followed, after a lag of from 20 to 60 years, by an equally large drop in fertility rates. Demographers have come to identify (see e.g. Chesnais (1992) and Kirk (1996)) three phases of the demographic transition: (I) in the initial stage both mortality and fertility are very high, population growth is very low (on the order of half of a percentage point or less per year); (II) in the intermediate stage both mortality and fertility are dropping but the first drops more than the second, at least initially, hence population growth increases, reaching levels of 2 or even 3 percent per year in some cases; (III) in the final phase mortality reaches a lower bound together with fertility, population stops growing and sometimes, like in many western countries during the second half of the XX century, declines. Quantitatively, this transition was very large. Before the transition began, both crude birth and crude death rates were in the range of 30 to 40 per thousand per year. By the time the transition was over, these were on the order of 10 to 15.

In England, the most widely studied case, the reduction in mortality was followed by a short period of increased fertility with a reduction following about a century later. A similar pattern was observed in France although the length of the period of increased fertility was shorter and later. In most countries in Europe, no such increase in the fertility rate occurred. Rather, the reduction in the crude death rate was initially met with little or no response on the part of fertility. This caused the birth and death rates to spread apart temporarily giving rise to a rapid increase in population before the reduction in fertility occurred. As we argue below, the drop in fertility follows, almost in every country, a drop in the mortality of infants and children. Due to the availability of modern health techniques, the drop in mortality rates has been much faster in the demographic transitions of the second half of the XX century than in those of the previous two centuries. Even in these more recent transitions however, the most common pattern has been one of constant fertility for about one to two generations. This has brought about dramatic increases in the total population of the developing countries that have undergone the demographic transition during the second half of the XX century.

Figure 1 shows the typical pattern of crude birth and death rates that occurs in a demographic transition. This was constructed as an average of the actual data from eleven European countries[3] over the period from 1740 to 1991. As such, it mimics the overall pattern of each of the transitions while being less sensitive to

[3] These are: France and the United Kingdom since 1740, Finland, Norway and Sweden since

individual country variations in the patterns of births and deaths. We have also superimposed the time series of average Infant Mortality Rates (IMR) on this figure as well. As can be seen, this follows a similar pattern, but the timing is different and should be noted. In particular, the drop in IMR is after that of the death rate, but precedes or is almost contemporary to that of the birth rate. In fact, the figure suggests that overall birth rates were roughly constant for a century or more while crude death rates were dropping and that fertility starts to decrease at roughly the same time that IMR's do. This observation is the crucial motivating factor behind the model we develop in Section 3, which concentrates on the impact of IMR on fertility and abstracts from changes in the total death rate. This behavior of IMR, crude death and birth rates gives rise to a pattern of population which is shown in Figure 2, which reports the total population for the European countries included in Figure 1. Notice also a second, important characteristic of the demographic data which we ignore in our subsequent analysis for the sake of simplicity. The variability of the death rates, both total and for infant, is much higher in the earlier than in the later period. Extending the time series backward to the XVII and XVI centuries would make the drop in volatility of death rates even more striking. Clearly, the variance of children's rates of survival must be a powerful factor behind the fertility choice of parents; an increase in the expected survival rate and a simultaneous reduction in its variance converge in pushing fertility rates down for a given expected value of "desired" children. Bringing uncertainty in death rates back into our model would therefore strengthen our conclusions.

There have been several explanations proposed for this transition. The most relevant of these, from our point of view, are the increase in income that came with the Industrial Revolution and the reduction in infant mortality that occurred in most European countries somewhat later. In the paradigmatic case of England 1750-1900, the more than doubling of the population came together with the onset of the Industrial Revolution. This has lead many economists (e.g. Becker, Murphy, and Tamura (1990), Ehrlich and Lui (1991), Galor and Weil (1998), Hansen and Prescott (1999), Lucas (1998)) to establish a causal link between the adoption of modern production technologies, the sustained increase in living standards they bring about and the demographic transition.

While we cannot dismiss such a link, we believe its existence and direction are far from obvious. The reduction in mortality rates takes place in many countries

1755, Spain since 1797, Denmark since 1800, Belgium, Germany, Italy and the Netherlands since 1816.

around the world in roughly the same period it takes place in England (see, e.g., Livi Bacci (1989)). An example of a country in which mortality drops during the XVIII century and population growth takes off is China. No Industrial Revolution occurred there during this time period. Demographic transitions have been occurring since then in almost every country around the globe with little if any correlation with the spread of the industrial mode of production. For example, Sub-Saharan African countries are now in the middle of their demographic transition, with infant mortality and fertility rates dropping despite very little economic development. Another startling example is that of Cuba, which, with a stagnating income per capita orders of magnitude smaller than the one in the USA, appears to have reached crude and infant mortality rates and fertility rates that are indistinguishable from those in the USA. In these and other examples, income per capita progresses little if at all; the reduction in mortality rates is accompanied by a reduction in fertility, however.

Further evidence questioning the link between the income expansion generated by the Industrial Revolution and the onset of the demographic transition comes from the northern European countries, where the impact of the Industrial Revolution was felt earlier than in the rest of the World but fertility rates typically decline much later, well after per capita incomes had started to grow. This leaves open whether or not there is some causal link between these two transitions. For example, although most researchers date the beginning of the increase in GDP per capita in England sometime in the late 1700's, the real reduction in fertility does not begin to occur until someplace between 1820 and 1890. As noted above, the increase in fertility rates that began in England sometime around 1700 and lasted until 1820 makes England a particularly hard case to 'date.' A more prototypical country might be Denmark (or, for that matter, any of the Scandinavian countries) where the reduction in fertility seems to begin around 1880 or 1890, well after the Industrial Revolution began. (Denmark is more prototypical because there is no sustained increase in fertility before the demographic transition began.) Sweden and the Netherlands follow a similar pattern with the reduction in fertility first starting in about 1860 to 1870. In all these cases, though, the strong correlation between mortality rates, especially IMR's, and fertility is replicated. Further, the historical and demographic literature (see e.g. Livi Bacci (1990)) has convincingly documented that a causal relation between income level, nutrition and mortality seems to be present only in those very extreme circumstances in which low nutrition leads to famine and massive mortality. For all intermediate ranges, covering the average nutritional levels which characterized European countries during the last millenium, the causality running from income and nutrition to mortality (and from the latter to fertility) appears dubious at best. To put it plainly: historical research seems to reject the idea that a pure increase in income levels and living standards would lead, or has historically lead, to a reduction in fertility rates.

More important, this view has caused difficulties for quantitative researchers trying to develop models based on the hypothesis that the Industrial Revolution is at the root of the demographic transition (see Fernandez-Villaverde (2001) and Doepke (2000)). In short, the basic idea is to get an increase in income to generate the reduction in fertility. There are two problems. First, as noted above, any model that does this faces difficulty because of the timing. Second, as income has continued to grow in industrial countries, there has not been a continued proportional decline in fertility. Third, fertility rates have decreased dramatically in very many countries where income per capita has increased only slightly or not increased at all.

What is uncontroversial is the set of quantitative facts describing the demographic transition, including the variable delay between mortality and fertility drop. A reduction in mortality rates is followed, perhaps with a lag to be explained, by a reduction in fertility. After a transition, fertility and mortality rates converge to a new, much lower plateau with almost zero or even negative natural growth of the population.[4]

The other key variable of interest from our point of view is the time series behavior of infant mortality rates. Infant deaths also went through a dramatic transition over this period, falling from levels around 200 deaths per thousand in most countries in 1800 (and as high as 300 in some like Germany) to less than 20 in modern times, to reach a level below 10 per thousand in the most advanced countries during the 1990's. The bulk of this reduction occurred between 1880 and 1930 in most countries. For example, in England, it was 150 per 1000 in 1890 and 50 in 1930. There has been a more gradual, but continuing decline since that time. Since the timing of this drop coincides with the reduction in fertility in many countries, we take this as an interesting potential explanation.

The cross sectional evidence on these data is also of interest. There is a strong positive correlation between total fertility rates at the country level and the infant

[4] Schultz (1966) is an earlier study poiting out the strong correlation displayed by the data. Since then, both micro and macro evidence has been mouting. For a, partially, dissenting view see van de Walle (1986).

mortality rate in that country. This is evidenced in Figure 3. This is cross sectional data on IMR's and the Total Fertility Rate in a collection of countries in 1997. As can be seen from the figure, there is a strong positive relationship between the Total Fertility Rate of a country in a year and the Infant Mortality Rate of that country in the same year. However, care must be used in interpreting these data. IMR and income per capita are also highly correlated (the correlation coefficient for 1997 is -0.814). Because of this, it is natural to question whether or not this relationship just represents a spurious relationship with income being the basic causal variable. Regression results offer some insight with the effect of IMR on TFR being more statistically significant than log income.

For the 1997 data, we have:

$$TFR_{97} = 3.24 + 0.0356 IMR_{97} - 0.386 \log_{10} GDPPC_{97},$$

with an adjusted R^2 of .763. The t-statistics for IMR_{97} and $\log_{10} GDPPC_{97}$ are, respectively 11.7 and -1.57.

While the p-value associated to the IMR variable is practically zero, the one for per capita GDP is 12 percent.

Moreover, this relationship seems to be remarkably stable over time. For the same data in 1962, there is a non-linearity in the relationship, with what seems to be a biological maximum TFR of 7.5 or 8.0 at IMR's above 150 per thousand per year. Below this level, the estimated slope is 0.0387, virtually identical to the estimate from the 1997 data.[5]

So far, all countries that have undergone these demographic transitions were essentially agricultural, at least when the transition started. The key feature of our modeling effort, that differentiates it from previous work in a similar vein, is the view that in poor, agricultural societies, the impetus for having children in the first place is as an investment rather than as consumption. That is, children are borne out of a need to man the farm when the parents grow older. In this arrangement, parents provide nurture to children when they are young, feed and clothe them, and provide them land to work when older. In exchange for this, parents are cared for when they are old. This is a better arrangement than just

[5] Ehrlich and Lui (1991) have also investigated this issue using a cross section of data from various countries for the period 1960-85. They find that a decrease in mortality rate of the younger group (age 0-25) has a significant negative effect on fertility, while measures of longevity for older age group have no impact.

selling the land on the market when old, because, as argued by Rosenzweig and Wolpin (1985), children have learned to work the parents' land better than people from outside the family. Although there are many mechanisms one can imagine for implementing such an arrangement (see Guner (1998) for a recent study), in this first set of models, we concentrate on a particularly simple one. This is an external effect running from parents to children. That is, children care about the well being of their parents.[6] This is in stark contrast to much of the previous literature in this area, where it is assumed that parents care about their children's well being, but not vice versa (Becker (1960) and Barro and Becker (1988) are the basic references). This fundamental difference in motivations and modeling gives rise to some interesting and important differences in results, and is at the heart of our research strategy.

2.2 Summary of Earlier Literature

The standard model in the economic literature is presented in two papers by Barro and Becker (1988) and (1989). In the general equilibrium version of the model, they assume that there is exogenous, labor augmenting technological change and assume that the cost of child rearing is made up of two components, one part consumption goods, one part time. They assume that the motivation for having children in the first place is altruism on the part of parents. That is, parents get utility from both their own consumption and from the utility of their children. They analyze the balanced growth path of the model along with off steady state dynamics. The character of this BGP is that fertility is constant so that population grows at a steady rate while income per capita grows. This is important from the point of view of the explanations for the Demographic Transition outlined above. That is, although income is growing in this model, fertility does not decline. For this reason, this model, as it is, does not provide a foundation for the story outlined above in which income growth (i.e., the onset of the Industrial Revolution) provides the basis for the Demographic Transition.

[6] In Appendix A we present the case in which there is no external effects and children are providing old parents with a flow of income equal to the total return on the land, while parents have the choice of setting the labor to land ratio in the family farm by picking fertility and investment in the previous period. The qualitative predictions are remarkably similar.

Barro and Becker also analyze some simple comparative statics exercises across steady states. In particular, they look at the effect of a reduction in the costs of child rearing on fertility. In the general equilibrium version of the model, they show that across balanced growth paths, fertility increases when the cost of children falls. This is because of the effect on interest rates of the cost in raising children. That is, since interest rates rise when the cost of children falls, it pays to invest more in the future through a larger family.

One of the interesting and relevant interpretations of this comparative statics exercise is a change in the Infant Mortality Rate. That is, if they reinterpret their cost of children as a cost of producing a surviving child, then a reduction in the cost of children is like a reduction in the Infant Mortality Rate. (See below for more on this.) They argue that therefore, a reduction in the IMR would give rise to an increase in the number of surviving children. This could be consistent with a reduction in the fertility rate as long as fertility did not fall as much as the probability of death.

Fernandez-Villaverde (2001) brings a quantitative analysis into the discussion by studying a calibrated version of the Barro and Becker model. He conducts three quantitative experiments with the model. These are an increase in productivity, a reduction in the infant mortality rate and a fall in the relative price of capital. As expected due to Barro and Becker's balanced growth path results, the first does not give rise to a demographic transition. Reducing the relative price of capital, while holding the infant mortality rate fixed does give rise to reduced fertility and population growth which in some ways is similar to the historical experience, but reducing infant mortality holding the relative price of capital fixed moves both fertility and population growth rates in the opposite direction. He does not consider the experiment of changing both simultaneously. It is difficult to tell which of these effects is larger and hence what the overall prediction of the model would be. However, one thing that unambiguously does come out of the model is that even a small reduction in IMR's increases both fertility and population growth rates. Further, the endogenous increase in fertility is quantitatively large for the drop in IMR considered by Villaverde, at least from a historical point of view. Finally, the model cannot provide any explanation of why fertility drops to such a low level that population growth comes to a full halt in the second half of the last century.

The paper by Doepke (2000) is in a similar vein. He analyzes a quantitative version of the Barro and Becker model and considers the effects of child labor laws and compulsory, subsidized education. He finds that each of these policy experiments has the potential to have big effects on equilibrium fertility and population growth rates. The key thing in the model that is driving the fertility decision is the relative cost of producing a new skilled vs. a new unskilled child. It is through this avenue that policies have a bearing on the outcomes in the model. Thus, to match the time series observations in his model, the key thing is that relative cost of skilled and unskilled are changing over time due to the dynamics of the policies in different countries.

The Becker, Murphy and Tamura paper (1990), contains a model in which there is an external effect from the accumulation of human capital. They do not have an explicit model of the transition. They show that, in their model, there are multiple steady states with different fertility and GNP growth rates. One of the steady states has high fertility and low GNP per capita growth (a "Malthusian" steady state) and the other has low fertility and a high GNP per capita growth (a "modern" steady state). They conclude that 'luck' must play an important role in the transition, since it requires a 'move' from one steady state to another. Interestingly enough, one does not need either external effects or accumulation of human capital to produce a model of fertility with multiple steady states. In Appendix A we sketch a most simple OLG model, *without any external effect* in either direction, in which there are two equilibrium fertility rates for each initial condition. The high fertility rate is associated with low capital/labor ratio and low per capita income and vice versa for the other one.

In addition to that literature, there is also a group of papers that concentrates on models of fertility and the demographic transition but are not based on microfoundations through external effects between members of a family. Rather, they use reduced form representations of preferences (e.g., utility for either expected future family income or the number of children), that do not correspond to either the Barro and Becker formulation or the one analyzed here. Some of these papers do construct models with the possibility of a Demographic Transition (see Gal-Or and Weil (1998)), while others study the effects of infant mortality on fertility (e.g., Kalemeli-Ozcan, Ryder and Weil (2000)). None of these papers contains quantitative implementations of the models however, and they often obtain results that are at odds with models based on microfoundations making them difficult to interpret. Finally, other authors have estimated reduced form representations of dynamic models of fertility choice using data from various historical episode of the fertility transition (see e.g. Eckstein, Mira and Wolpin (1999), Haynes, Phillips and Votey (1985)).

The older paper we are aware of adopting the "children as investment" approach to endogenous fertility is Neher (1971). This is a three period OLG model in which middle aged agents care for their current consumption, their consumption when old and that of their children two periods later, when the children are in their last period of life. The economy faces a neoclassical technology in which labor (of middle age people) is the only variable factor. A share alike ethics is assumed, according to which the output of each period is shared equally among the three generations alive. Neher concentrates on deriving a Golden Rule for fertility and on the fact that this rate of fertility is highly sensitive to how much middle aged agents discount their own future utility and that of their children. He argues that in general, one should expect "overpopulation" as parents underestimate the full social cost of bearing new children to the extent that this is realized two periods from now, when the generating parents are dead. Neher does point out something which is also apparent in our analysis. This is that the establishment of more efficient private financial markets and the introduction of pension schemes that do not depend upon one own fertility, should tend to reduce the demand for children. Since Neher, this theme has been taken up by a number of other authors, e.g. Azariadis and Drazen (1993), Chakrabarti (1999), Ehrlich and Lui (1991), Nishimura and Zhang (1993), Raut and Srinivansan (1994) among other. Interestingly enough, the very simple model we study here in which the external effect runs from parents to children, intergenerational transfers are endogenous and capital accumulation is possible, has not, to the best of our knowledge, been considered. More often than not researchers have assumed that the portion of labor income being transferred as a "pension" to the old parents is an exogenous parameter of the model. Nishimura and Zhang (1993) are an exception, as they endogenize this donation in a form which is similar to our non-cooperative solution. Their analysis concentrates on the existence of endogenous oscillations in the fertility and saving rate and are not concerned with matching stylized historical facts or with evaluating the impact that changes in mortality have on fertility rates.

The Ehrlich and Lui (1991) paper is particularly close to ours, at least in the direction of the external effect within the family. They look at the family as a mutual insurance mechanism in which both intergenerational transfers and altruism are at play. They concentrate not on the role of land in traditional societies but, instead, on human capital accumulation and its relationship with persistent growth. Parents invest in children's human capital and, by assumption, are entitled to an exogenous fraction of their children's earnings. Thus, they receive a transfer, when old, which is, by assumption, strictly proportional to the wage bill of their, then working, offsprings. To the extent that human capital can be accumulated without bounds, this generates a model of endogenous growth driven by inter-familial arrangements. Their model does not induce an equilibrium demographic transition however. Hence, they enrich their model by introducing a specific version of the "children as consumption" hypothesis. In particular, they assume a "companionship function," according to which parents receive utility from the number and quality of the children. The quantitative properties of these models are not studied in the paper.

Our work is therefore complementary with that of the aforementioned papers. We endogenize the degree of support that parents get from their children, and consider both cooperative and non-cooperative mechanisms for determining its level. This allows us to study the effects of a lack of commitment (by children for the level of support that they will offer parents) and compare the effects of alternative transfer arrangements. Thus, we can outline a formal model which may be able to capture the transition from tight to loose family arrangements that many anthropologists and sociologists say occurred during the same period in which the demographic transition took place.

3 The Basic Model

The basic version of the model concentrates on the impact of infant and child mortality on fertility choices. The set-up is meant to capture the crucial features of a traditional agricultural society, with competitive markets for land and labor but without any form of technological progress. Adding exogenous growth in labor productivity does not alter any of the substantive conclusions about fertility in the model and hence, it is not included here.

Agents live for a maximum of 3 periods, young (y), middle age (m) and old (o). At birth, individuals have an endowment of productive time equal to $(0, 1, 0)$.

3.1 Fertility

People born in period t are capable of reproducing during period $t+1$. They choose the number of per-capita children, which we denote by f_{t+1}. If there are N^m_{t+1} middle age people alive, $f_{t+1}N^m_{t+1} = N^y_{t+1}$ children are born in period $t+1$.

Denote by θ_t the amount of resources (current consumption) needed to take care of one child. Given the level of fertility, total amount of resources needed for rearing children is $N_t^m \cdot \theta_t \cdot f_t$.

A general specification for c_t^y is

$$\theta_t = a + bw_t$$

where w_t is the wage rate in period t.

3.2 Mortality

A fraction of individuals alive in period t die at the end of that period. For old people this fraction is always equal to one. For middle age people, the mortality rate is assumed to be zero in this version of the model, hence we abstract from changes in life expectancy of adult individuals. For the young it is given by $m_t^y \in [0, 1)$. Because of these assumptions, it follows that, old and middle age people in period $t+1$ are equal to 1 and $f_t(1 - m_t^y)$ times the number of middle age people in the previous period, respectively. Notice that m_t^y should be interpreted as the total mortality rate between birth and the reaching of working age.

In the illustrative simulations reported in Section 5 we assume the length of a period to be approximately twenty years. This is obviously a gross simplification, which is made particularly inaccurate by our assumption of a constant survival rate of one between the second and the third period of life. European demographic data show that, since the end of the XVI until the middle of the XIX century, most gains in life expectancy are concentrated among living adults as opposed to children. This change in both life expectancy, population age structure and incentive to have children cannot be captured by this simple version of our model and will have to be the object of further research.

Denote by

$$\pi_t = 1 - m_t^y$$

the exogenous process for the probability of survival of the youngsters. Then π_t is the probability that someone born in period t reaches middle age in period $t+1$.

For future reference, total population at time $t+1$ is

$$N_{t+1} = N_{t+1}^y + N_{t+1}^m + N_{t+1}^o,$$

and hence, the growth rate of population is

$$1 + g_t^n = \frac{N_{t+1}}{N_t} = f_{t-1}\pi_{t-1} \cdot \frac{1 + (1 + f_{t+1})f_t\pi_t}{1 + (1 + f_t)f_{t-1}\pi_{t-1}}.$$

Thus, along a balanced growth path associated with any constant survival probability, we have

$$1 + g^n = \pi \cdot f.$$

3.3 Production

In period t total consumable output is

$$Y_t = F(L_t, K_t)$$

where $K_t = K_{t-1} + N_{t-1}^m s_{t-1}$ is the total stock of land/capital (we assume zero depreciation) and $L_t = N_t^m$ is total labor supply from middle age people. We make the simplifying assumption that the resources saved and invested to acquire new land or in accumulating new capital (s_t) translate into new land/capital at a constant one-to-one rate. Notice that this implies both an innocuous choice of units in which land/capital is measured and the less innocuous assumption that returns to investment are constant and land/capital are homogenous and can be accumulated indefinitely. Introducing decreasing returns in the investment technology would probably make the model more realistic but add little new insights to our analysis. The main difference would be that, with decreasing returns, the size of the economy could not grow forever and a steady state should be reached in the absence of some form of technological progress. With our assumption of constant returns, instead, a balanced growth path is reached. Assume that $F(K, L)$ is concave, homogeneous of degree one, and increasing in both arguments. As usual, set $f(k) = F(K/L, 1)$.

The aggregate resource constraint is

$$Y_t \geq N_t^o \cdot c_t^o + N_t^m \cdot [c_t^m + s_t] + N_t^y \cdot \theta_t$$

Other definitions and accounting identities, used throughout the paper are

$$k_t = \frac{K_t}{L_t} = \frac{K_t}{N_t^m}$$

$$x_t = \frac{K_t}{N_t^o} = \frac{K_{t-1} + N_{t-1}^m s_{t-1}}{N_{t-1}^m} = k_{t-1} + s_{t-1} = \pi_{t-1} f_{t-1} k_t$$

Along a balanced growth path the stock of capital per worker, individual saving and fertility must satisfy

$$k^* = \frac{s^*}{\pi f^* - 1}$$

3.4 Utility Function and Budget Constraints

Individuals receive utility from their own consumption and from that of their parents. When young, parental attention is needed to consume and survive. For simplicity, we assume that consumption when young does not affect life-time utility. We assume that there is no utility from leisure; that labor supply of both the young and old is zero and normalize by setting labor supply of the middle age individual at one. Given these assumptions, the utility function of an individual born in period $t-1$ (middle age in period t) is given by

$$U_{t-1} = u(c_t^m) + \eta u(c_t^o) + \delta u(c_{t+1}^o)$$

where $u(\cdot)$ has all the standard properties of a concave utility function. The parameters δ and η are in $(0,1)$. δ is the discount factor, while η reflects the value that children place on the consumption of their parents.

There is no budget constraint in the first period of life, as the youngsters are taken care of by their parents. Denote by w_t the wage rate per unit of labor time and let d_t^i denote the per-capita donation from middle aged individuals to their parents. We have

$$d_t^i + c_t^m + s_t + \theta_t f_t \leq w_t,$$

when middle ages, and

$$c_t^o \leq \sum_{i=1}^{n_t^m} d_t^i + R_t x_t,$$

when old. Recall that $n_t^m = N_t^m / N_t^o = f_{t-1}\pi_{t-1}$ is the number of surviving children per old person and R_t is the rate of return on capital. Notice that the rate of return R_t measures the consumable output due to land/capital and is net of the (undepreciated) stock of land/capital K_t. Because of zero depreciation, we also assume that old people inherit the existing stock of capital (land) from their dead parents.

3.5 Games of Giving

In period t, each one of the $i = 1, 2, \ldots, N_t^m$ surviving middle age agents donate some consumption to their parents. We consider two different solutions to the "gift-giving" game: (a) the *cooperative* one, in which middle age individuals maximize the simple sum of their (equally weighted) utilities by choosing the total consumption of their parents and sharing the burden equally; (b) the *non-cooperative* one, in which each middle age takes the gift of his siblings as given and the equilibrium concept is Nash with symmetry.

We associate the cooperative solution to environments in which the organization of economic activity and social norms are such that both parental "authority" over own children and reciprocal "control" among siblings are strong enough to enforce cooperative behavior. What we have in mind are traditional agrarian communities where patriarchal families constitute the backbone of society. In such environments, family land and other possessions (animals, tools, seeds) are transferred from parents to children in a relatively egalitarian form. Further, children, by living nearby and cultivating the land left them by the parents, are able to monitor each other behavior and enforce cooperative rules. When such traditional structures breaks down, people move apart and engage in economic activities other than working the family land, both parental authority over children and the extent to which the latter can monitor each other are greatly reduced. Non-cooperative behavior appears as a much more reasonable assumption in such circumstances.

In the *cooperative equilibrium*, middle age agent i chooses donation d_t^i to solve

$$\max_{d_t^i} \quad u\left(w_t - \theta_t f_t - d_t^i - s_t\right) + \eta u\left(n_t^m d_t^i + R_t K_t / N_t^o\right)$$

First order conditions yield

$$u'(c_t^m) = n_t^m \eta u'(c_t^o)$$

Note that this is the same first order condition that would be obtained through joint maximization of the sum of utilities of the siblings.

In the *non cooperative equilibrium*, middle age agent i chooses donation d_t^i to solve

$$\max_{d_t^i} \quad u(w_t - \theta_t f_t - d_t^i - s_t) + \eta u\left(\sum_{j \neq i, j=1}^{n_t^m} d_t^j + d_t^i + R_t K_t / N_t^o\right)$$

First order conditions yield

$$u'\left(c_t^m\right) = \eta u'\left(c_t^o\right)$$

Whenever $\pi_{t-1} f_{t-1} = n_t^m > 1$, the cooperative solution entails a higher individual donation and, ceteris paribus, higher consumption for the old age people.

3.6 Life Cyle Problem and First Order Conditions

The planning problem for a middle age individual is

$$\max_{s_t, f_t, d_t^i} u(c_t^m) + \delta u(c_{t+1}^o) + \eta u(c_t^o)$$

$$\text{s. to} \quad c_t^m + \theta_t f_t + d_t^i + s_t \leq w_t$$

$$c_{t+1}^o \leq \sum_{i=1}^{n_{t+1}^m} d_{t+1}^i + R_{t+1} x_{t+1}$$

First order conditions with respect to s_t, f_t and d_t^i yield:

$$u'(c_t^m) = \delta u'(c_{t+1}^o) \frac{\partial c_{t+1}^o}{\partial s_t}, \quad (FOC1)$$

$$u'(c_t^m)\theta_t = \delta u'(c_{t+1}^o) \frac{\partial c_{t+1}^o}{\partial f_t}, \quad (FOC2)$$

$$u'(c_t^m) = \eta u'(c_t^o) n_t^m, \quad (FOC3C)$$

$$u'(c_t^m) = \eta u'(c_t^o). \quad (FOC3NC)$$

$(FOC1)$ and $(FOC2)$ are traditional intertemporal conditions for investment decisions. They equate the ratio of discounted marginal utilities to the rate of return on the particular investment project at hand. Notice, though, that a strategic component is introduced here. This is because donations from children to parents take place within individual families. To the extent they are aware of the strategy (cooperative or not) children will follow in determining d_{t+1}, it is rational for parents to take into account the impact that a variation in the amount of savings and in the number of siblings may have on the total donation they will receive from their children when old. Hence, the terms $\frac{\partial c_{t+1}^o}{\partial s_t}$ and $\frac{\partial c_{t+1}^o}{\partial f_t}$ appearing on the right hand sides of $(FOC1)$ and $(FOC2)$ instead of the "competitive" rates of return R_{t+1} and $\pi_t d_{t+1}$. Finally, equations $(FOC3C)$ and $(FOC3NC)$ are, respectively, the first order condition for the choice of donation in the cooperative and non-cooperative case.

4 Log Utility and Cobb-Douglas Production

We now solve the model explicitly for the case of logarithmic utility and Cobb-Douglas production functions.

Set $u(c) = \log(c)$ and $F(K, L) = AK^{\alpha}L^{1-\alpha}$. Then

$$\max_{f_t, s_t, d_t^i} \log(c_t^m) + \delta \log(c_{t+1}^o) + \eta \log(c_t^o)$$

$$\text{s. to} \quad c_t^m + \theta_t f_t + d_t^i + s_t \leq w_t$$

$$c_{t+1}^o \leq \sum_{i=1}^{n_{t+1}^m} d_{t+1}^i + R_{t+1} x_{t+1}$$

4.1 Cooperative Equilibrium

¿From the first order condition ($FOC3C$) and the budget constraint, per-capita donation and consumption when old are, respectively,

$$d_t = \frac{\eta}{1+\eta}(w_t - \theta_t f_t - s_t) - \frac{R_t x_t}{n_t^m(1+\eta)}$$

and

$$c_t^o = \frac{\eta}{1+\eta}\Big[n_t^m(w_t - \theta_t f_t - s_t) + R_t x_t\Big]$$

Hence, the relevant derivatives are

$$\frac{\partial c_t^o}{\partial s_{t-1}} = \frac{R_t \eta}{1+\eta}$$

$$\frac{\partial c_t^o}{\partial f_{t-1}} = \frac{\pi_{t-1}\eta}{1+\eta}(w_t - \theta_t f_t - s_t)$$

The "equality of rate of returns" condition yields

$$\theta_t R_{t+1} = \pi_t(w_{t+1} - \theta_t f_{t+1} - s_{t+1}). \tag{RR}$$

This economy has one endogenous, the capital/labor ratio $k_t = K_t/N_t^m$, and one exogenous state variable, the survival probability π_t. Hence, the equilibrium dynamics should be summarized by some function $k_{t+1} = h_{\pi_t}(k_t)$. If, given initial conditions, there is only one such h_π for any given π, the competitive equilibrium is unique. Notice that the (RR) condition involves only tomorrow's capital/labor ratio and exogenous parameters. This means that, given the initial conditions k_t and π_t, tomorrow's capital/labor ratio is uniquely determined and equilibria are unique if (RR) has a unique solution. In such circumstances and for a constant π

the economy converges, in one period, to a constant capital/labor ratio $k(\pi)$ compatible with (RR). Associated with each such constant $k(\pi)$ there is a balanced growth rate $\pi \cdot f(\pi)$ where $f(\pi)$ is the fertility rate associated to π.

First order conditions for the determination of saving and fertility in period t can be manipulated by replacing in either of them the optimal amount of donation and the equality of rates of return condition, to get

$$s_t + \theta_t f_t = \frac{1}{1+\delta+\eta}\Big[\delta(w_t + R_t k_t) - (1+\eta)k_t\Big].$$

Hence, aggregate investment I_t (the sum of investment in capital and in children) is

$$s_t + \theta_t f_t = \frac{1}{1+\delta+\eta}\Big[\delta f(k_t) - (1+\eta)k_t\Big] = g(k_t) \qquad (AI)$$

Notice that, in principle, $g(k_t)$ is a non monotone function of k_t, as it satisfies $g(0) = g(\overline{k}) = 0$ for some finite value $\overline{k} > 0$. Per capita saving is a non-monotone function of per capita stock of capital. An economy with a very large endowment of capital relative to labor will accumulate little or no additional capital and expand the population to bring down k toward the long-run efficient ratio. This seems to fit well with observed historical experience.

In practice, the intertemporal equilibrium is computed this way. Use K_t, L_t to determine $y_t = f(k_t)$ and $g(k_t) = I_t$. Given I_t and k_t, the three equations

$$I_t = s_t + \theta_t f_t,$$

$$k_{t+1} = \frac{k_t + s_t}{\pi_t f_t},$$

and

$$f'(k_{t+1}) = \frac{\pi_t}{\theta_t}\Big[f(k_{t+1}) - k_{t+1}f'(k_{t+1}) - g(k_{t+1})\Big] \qquad (RR)$$

determine f_t, s_t and k_{t+1}. The latter, together with mortality rates and initial conditions, determine the stock of capital and the structure of population tomorrow. Hence, for given initial conditions, this system is determinate and there exists a unique equilibrium path as long as the (RR) condition has only one solution.

We next check if the rate of return condition can be satisfied by more than one value of k^*. Algebraic manipulation of (RR) gives

$$f'(k^*)[1 + \frac{\pi}{\theta}k^*] = \frac{\pi(1+\eta)}{\theta(1+\delta+\eta)}\left[f(k^*) + k^*\right].$$

The left hand side is a non monotone function: decreasing near zero until a minimum, then increasing again without bound. The right hand side is monotone increasing. So, in principle, for each set of exogenous parameters there could exist more than one desired capital/labor ratio. This means that, potentially, the system has multiple equilibria as, for a given exogenous sequence of π_t, it may randomly jump back and forth between these two balanced growth paths. But functional forms for which more than one solution to (RR) exist are not easy to find. In the Cobb-Douglas case we have

$$A\alpha k^{\alpha-1} = \frac{\pi}{\theta}\left[A\left(1-\alpha-\frac{\delta}{1+\delta+\eta}\right)k^{\alpha} + \frac{1+\eta}{1+\delta+\eta}k\right].$$

Here the LHS is monotone decreasing. The RHS is monotone increasing for reasonable parameter values, i.e. for $(1-\alpha) > \frac{\delta}{1+\delta+\eta}$. Hence in the Cobb Douglas case we expect only one, asymptotically stable, balanced growth path and a unique equilibrium for every initial condition and sequence of exogenous survival probabilities π_t.

Population Growth in a Malthusian Economy

We call our economy "Malthusian" because of the hypotheses upon which it is built. There is no technological progress and aggregate production displays constant returns to scale on land/capital and labor. Production of children also displays constant returns, and children are seen as an investment good by their parents insofar as they provide future labor to work the land. These were Malthus original assumptions (see Malthus (1798)). To understand the "Malthusian" predictions of this simple model, begin by noticing that along a balanced growth equilibrium, income per-capita remains constant as the population reproduces at the same rate as the stock of capital, and there is no technological progress. On the other hand, an increase in π (reduction in mortality rates) always *decreases* the capital/labor ratio. This is because the desired capital/labor ratio is determined by the (RR) condition and any increase in π increases the rate of return on fertility. Higher fertility means a larger middle age group next period and a lower capital/labor ratio. To each $k^*(\pi)$ the model associates a balanced growth rate $g^*(\pi) = \pi f^*(\pi)$ which is increasing in π, as one may verify by taking derivatives. Notice that while $g^*(\pi)$ is increasing in π, $f^*(\pi)$ is not. In fact, one can show numerically that it is decreasing. Hence, as mortality decreases, population expands at higher and higher rates. The capital stock expands likewise, but at a lower rate. Consequently, per capita labor productivity decreases (as technological change is absent) as population increases.

In summary, intertemporal equilibria constructed by moving from one balanced growth path to another with π_t increasing have the following Malthusian properties.

1. Fertility decreases but the growth rate of population increases.
2. The capital/labor ratio decreases, together with per capita income and consumption.
3. The wage rate decreases and the rate of return on capital increases.

These properties are illustrated numerically in the calibration exercise we report in Section 5.

4.2 Non Cooperative Equilibrium

Next, we examine the properties of the model when siblings play the non cooperative game as outlined above. We have already discussed the different first order conditions affecting the level of donations from children to parents. Using those first order conditions and the budget constraint, donation of middle age and consumption when old can be computed to be

$$d_t^i = \frac{1}{n_t^m + \eta}\left[\eta(w_t - \theta_t f_t - s_t) - R_t x_t\right]$$

and

$$c_t^o = \frac{\eta}{n_t^m + \eta}\left[n_t^m(w_t - \theta_t f_t - s_t) + R_t x_t\right]$$

Hence, the relevant derivatives are

$$\frac{\partial c_t^o}{\partial s_{t-1}} = \frac{R_t \eta}{n_t^m + \eta},$$

and

$$\frac{\partial c_t^o}{\partial f_{t-1}} = \frac{\pi_{t-1}\eta}{(n_t^m + \eta)^2}\left[\eta(w_t - \theta_t f_t - s_t) - R_t x_t\right].$$

Notice that, contrary to the cooperative case, the impact of increased fertility on consumption when old may now be negative. This occurs when, relative to the donations coming from the children, capital income is a large share of old age consumption.

Equality of rates of return yields, first

$$R_{t+1} = \frac{\pi_t}{\theta_t(\eta + n^m_{t+1})} \left[\eta(w_{t+1} - \theta_t f_{t+1} - s_{t+1}) - R_{t+1} x_{t+1} \right]$$

and then, after simplification,

$$R_{t+1} = \frac{\eta \pi_t}{\theta_t(\eta + n^m_{t+1}) + \pi_t x_{t+1}} \left(w_{t+1} - \theta_t f_{t+1} - s_{t+1} \right) \qquad (RR)$$

Next we use $(FOC1)$ and (RR) to obtain an explicit solution for aggregate investment $I_t = \theta_t f_t + s_t$. This is

$$I_t = \frac{\delta\eta(w_t n^m_t + R_t x_t) - (\eta + n^m_t)(\eta + n^m_{t+1})k_t}{(\eta + n^m_t)(\eta + n^m_{t+1}) + \delta\eta n^m_t}$$

Notice that $w_t n^m_t + R_t x_t = n^m_t f(k_t)$ where $f(k_t)$ is output per middle age worker. So this is output per member of the old generation. Define the weight $\gamma_t \in (0,1)$ as

$$\gamma_t = \frac{\delta\eta n^m_t}{(\eta + n^m_t)(\eta + n^m_{t+1}) + \delta\eta n^m_t}$$

Per capita total investment of the middle age portion of the population can be written as

$$I_t = \gamma_t[f(k_t) + k_t] - k_t \qquad (AI)$$

Inspection of (AI) shows that, given the current population ratio n^m_t, the capital/labor ratio k_t and the saving rate s_t, there is only one positive level of fertility f_t that satisfies it with equality. This is because the left hand side is increasing and the right hand side is decreasing in f_t. Hence (AI) can be solved for f_t as a function of the state variables and s_t. Replacing this in (RR), the latter together with $k_{t+1} = (k_t + s_t)/\pi_t f_t$ can be solved for s_t and k_{t+1}. Notice that (under essentially the same conditions we adopted for the cooperative model) it is still true that, given a fertility rate, there is only one saving rate solving the (RR) equation. In particular, this implies that for any level of the survival rate π_t there is only one steady state (balanced growth) capital/labor ratio and, as in the cooperative equilibrium, convergence to such a capital/labor ratio takes place in just one period when the survival rate is constant.[7] Hence, equilibria are unique in the non

[7] Appendix B contains a brief summary of the algebra for the non-cooperative case. The cooperative one is altogether similar.

cooperative model under the same restrictions for which they are unique in the cooperative one.

A word on why we are not interested in studying multiple equilibria in this paper is in order. While we find multiple equilibria an interesting theoretical twist[8] we cannot avoid seeing them as a very weak explanation for the existence of fertility differentials that are persistent over time, and for the demographic transition in particular. The latter, which is our main concern here, is a dynamic phenomenon, common to almost all countries in the world and with remarkably similar patterns of evolution. It appears somewhat implausible to interpret it as the outcome of a world wide coordination in jumping, just at the right time, from one equilibrium level of fertility to another, following some not well identified "sunspot" signal.

While the model cannot be solved analytically, a numerical investigation of its properties can be carried out. Simulations reported in the following sections concentrate, in particular, on the dependence of the fertility rate, the population growth rate and the capital/labor ratio on the survival probability π_t. Our main findings are summarized next.

1. The qualitative predictions of the model are the same as in the cooperative case. In particular, as π increases, fertility decreases, k decreases and the growth rate of population increases.

2. The quantitative properties are remarkably different. At the same parameter values the equilibrium level of fertility is substantially lower and that of the capital labor ratio higher than in the cooperative case.

3. Similarly, fertility drops much faster as the survival probability increases yielding a growth rate of the population that is barely increasing over the historically relevant range of values for π.

4. In particular, the parameter values at which the non-cooperative model replicates the fertility behavior of the period 1561-1661 are substantially different from those for the cooperative one.

[8] The presence of which, in this model and in the even simpler one summarized in the appendix, shows that one does not need to adopt complicated production externalities if all one wants are multiple equilibria in competitive model of endogenous fertility.

5 Calibrating and Simulating the Model

In this section we present a calibration of the model outlined above and use it to run a few, simple numerical experiments. One word of caution seems appropriate on the interpretation of the results to be presented. The simple model discussed in this paper is meant to focus on two points, which seem left at the margin of the economic literature on the demographic transition. These are: the impact of exogenous changes in the IMR upon, and the relevance of intergenerational sharing agreements within the family for the determination of aggregate fertility. To properly concentrate attention upon these issues, we abstract from the overall evolution of life expectancy[9], from the impact of technological progress (either endogenous or exogenous), from intergenerational transfer agreements implemented outside the family (e.g., pensions, education, public health) and, finally, from the development of capital markets. Our numerical experiments are therefore to be read as possible answers to three simple questions. From an historical standpoint, do the "comparative static" responses of fertility and population growth to increases in young age survival rates appear reasonable? In the absence of any increase in per capita income, can a persistent drop in the infant mortality rate, similar to the one experienced in Europe in the second half of the XIX century, bring about an historically reasonable drop in fertility and increase in population growth rate? Does a shift from cooperative to non cooperative behavior entail a significant quantitative difference in the model's response to these perturbations? We interpret the numerical results as implying a positive answer in each case.

We calibrate the model in such a way that, when the survival probability π is constant and equal to its average, for England, during the century from 1561 to 1661, the model yields a growth rate of population equal to the observed average in the same country over the same period. This implies a fertility rate that is slightly lower than the one historically observed between 1561 and 1661, as we assume a mortality rate of zero between the second and the third period of life, which, together with the mortality rate of one hundred percent at the end of the third period, yields a Crude Death Rate (CDR) slightly lower than the average historical value. Next we use these calibrated version of the model to carry out some numerical experiments.

First, we carry out a "comparative static" exercise by computing the value of

[9] In fact, with a three-period model in which each period equals twenty years, we are foregoing any ability to capture the demographic evolution of the XX century.

fertility, population growth, capital/labor ratio and average labor productivity as π increases from its historical lower bound (about 0.4) to its maximum of 1.0. Second, we compute what the model predicts the dynamics of response to a one time, unexpected increase in infant mortality will be. This might correspond to the response of a Malthusian economy to the onset of a plague or famine similar to those for which historical records are available. These computations are done for both the cooperative and non cooperative versions of the model. For comparative purposes, we also include here a version of the first simulation for a similarly calibrated version of the Barro and Becker model.

Calibrating the Malthusian Economy

To avoid repetitions, we use the cooperative model to describe the procedure used for calibration. We have adopted the same logic for the non-cooperative model, with the obvious changes in the form of the equilibrium conditions. For the cooperative model we have

$$s_t + \theta_t f_t = \frac{1}{1+\delta+\eta}\Big[A\delta k_t^{\alpha} - (1+\eta)k_t\Big], \tag{5.1}$$

$$A\alpha k_{t+1}^{\alpha-1} = \frac{\pi_t}{\theta_t}\left[A\Big(1-\alpha-\frac{\delta}{1+\delta+\eta}\Big)k_{t+1}^{\alpha} + \frac{1+\eta}{1+\delta+\eta}k_{t+1}\right], \tag{5.2}$$

$$k_{t+1} = \frac{k_t + s_t}{pi_t f_t}. \tag{5.3}$$

Given k_t, π_t and a set of parameters α, η, δ, A and $\theta_t = a + bw_t$, equations (5.1) – (5.3) determine the 3 endogenous variables f_t, k_{t+1}, s_t. To determine the appropriate values for the six parameters, we need to make some assumptions linking our model to observable data. A natural benchmark is the case in which the cost of rearing children consists mainly of parents' time, i.e. $a = 0$. All the results reported in what follows have been computed under this assumption.

Fairly detailed population, fertility and mortality data are available for England since 1541, at intervals of either five or ten years from Wrigley and Schofield (1981)) and Mitchell (1978). Data on agricultural productivity, wages and factor shares are available from Clark (2000) and Hoffman (1996). We use this data to calibrate our model. We proceed as follows.

1. We assume that each period of our model corresponds to twenty calendar years.

2. From data we compute the probability of survival π_t at age twenty for each intermediate period $t = 1541 - 1561, 1561 - 1581, \ldots, 1961 - 1981$.

3. From data in Clark and Hoffman we compute $\alpha = .5$ and $A = 74.00$.

4. We set $\delta = (1.05)^{-20} = .3768$.

5. From data, we compute the average survival probability ($\pi = 0.523$) and the average population growth factor ($\pi f = 1.1$) during the five periods comprised within the calendar years 1541-1641.

6. We calibrate b and η in such a way that our model generates a steady state growth rate of population equal to 1.1 when the survival probability is kept constant at $\pi = 0.523$. This procedure gives $b = .105$, and $\eta = .185$.

7. We set initial conditions according to three criteria. Total population in the first period is equal to its historical value in 1541 (about 4.2 milion people). Its composition is such that $N^y_{1541} = 1.1/(1 - 0.523)N^m_{1541}$ and $N^m_{1541} = 1.1N^o_{1541}$. We pick the aggregate capital stock K_{1541} in such a way that $k_{1541} = 100$.

The first experiment that we run with the model is to compute the changes in steady state (balanced growth) values of fertility, population growth rate, capital/labor ratio and labor productivity as a function of the survival probability, π, in the interval $[0.4, 1.0]$. These are reported in Figures 4a and 4b.[10] The interpretation of these figures is straightforward. Fertility decreases rapidly as π increases, from around an average of 4.6 children per woman, when π is at its historical level in 1500, to about 3.8 children per woman when it reaches a value of one. In spite of this decrease in fertility, the population growth rate increases continuously, reaching levels around 80 per cent per period (20 years) at very high levels of π. Capital per worker, and hence income per capita, decreases throughout this range even as the total stock of capital/land increases. As mentioned earlier, these are the basic Malthusian properties of our model. Notice that the population growth rate keeps increasing with π even if per capita fertility decreases and reaches very high values when infant mortality is near zero. This second prediction is highly counterfactual. Population growth rate first increased and then eventually decreased as infant and youth mortality decreased during the second half of the last century.

[10] Similar, in fact, quantitatively better, results obtain when $b = 0$.

The comparative static properties of the Barro and Becker model are less satisfactory. First, the population growth rate at high levels of π is extremely high in all examples we calculated. More importantly: fertility is also increasing (and strongly so) as a function of infant mortality. [11] Figures 4c and 4d show a typical set of calculations for a calibrated version of the Barro and Becker model when $a = 0$. For this choice of parameters, fertility increases very rapidly as a function of the infant survival rate.

Figures 5a and 5b report the results from the same exercise for the non-cooperative model. The same parameters are used here as in Figures 4a and 4b. While the qualitative behavior of the model follows that of the cooperative version, the quantitative results are strikingly different. Fertility is barely at one child per woman when the survival rate is $\pi = .4$ and decreases to .5 as the survival rate goes to one. As a consequence of this, the population growth rate is practically constant (and negative) as $\pi \cdot f$ ranges from .4 to 1. Similarly, the capital/labor ratio is not only larger than in the cooperative case (1200 versus 100), it also decreases a lot less. Moreover, in the cooperative case the balanced growth ratio K/L drops of more than 50 per cent as π increases from .4 to 1 while it drops of about 10 per cent when people behave non cooperatively.

For the sake of comparison, we report in Figures 5c and 5d the results for a different calibration of the non-cooperative model. In this case we have calibrated it in such a way that the non-cooperative model replicates the historically observed fertility and population growth rates when $\pi = .52$. The new set of parameter values is $\alpha = .42$, $\delta = .36$, $A = 304.00$, $\eta = .92$ and $b = .005$. In this calibration the drop in fertility is much more substantial than what is seen in the cooperative case, and the increase in the population growth rate almost negligible. In fact, when the survival probability reaches contemporary values, fertility per woman is about 2.3 children and the population growth rate is 16 per cent every twenty years. For the sake of comparison, over the 20 years from 1965 to 1985, European population (as defined in the data of Figures 1 and 2) grew about 9%, while from 1970 to 1990 it grew about 7%. Thus, this version of the model mimics the beginning and end of the Demographic Transition quite well. What is missing in the model, however, is the non-monotone behavior of the population growth rate

[11] When the cost of rearing children is fixed, that is $b = 0$, fertility is increasing at some parameter values, decreasing at others and even non-monotone some other times. Population growth rates are always very high and increasing with π.

(first increasing then decreasing) which we observe in the demographic transition.

The second experiment that we run with the model is to calculate the predicted response to a one time, short term increase in the infant mortality rate. This mimics what would happen in a plague or a famine in our model economy if these shocks were unexpected and affected only the children's mortality rate. For this, we held π at it's steady state value of 0.523 and then decreased it for one period (i.e., for 20 years). This was followed by a return to the steady state level. We assume the drop is unexpected so that, in the period of the shock, fertility cannot react to the sudden increase in mortality. From the data from the 16th to 18th century, a large famine or epidemic increased the infant mortality rate between 15 and 20%. For example, the infant mortality rate rose from 168 for the period from 1661 to 1671 to 202 for the period from 1671 to 1681, and then fell back to 174 from 1681 to 1691. Hence, we have carried out our experiment assuming the unexpected shock increases children's mortality by 20 percent. Results are reported in Figures 6a and 6b.

The overall response of fertility in the model is consistent with historical records: when mortality increases fertility reacts with a lag and then increases more than proportionally causing the Crude Birth Rate to rise significantly. Parallel to this, the capital/labor ratio increases leading to several consecutive decades of high labor productivity, high wages and high income per capita. Subsequently, the increase in population that follows the shock brings the system back to the initial condition. Notice that the amount by which real wages first increase and then decrease in our model is small (about 10-12 percent) relative to historical values observed in Europe in the immediate aftermath of a plague. Historical variations were of the order of 25-40 percent with peaks of 50 percent in certain instances (see, e.g. Phelps-Brown and Hopkins (1981) for detailed data). Still, historical plagues and famines affected every age group and brought about reductions in the total population of between 20 and 40 percent. In our case the reduction in total population is much smaller (about 6 or 7 percent) since only the young generation is assumed to be affected. Hence, our model seems to incorporate an elasticity of wages to aggregate employment which is stronger than that observed in historical data.

As can be seen in the figure, although mortality is increased dramatically, the crude birth rate is only slightly affected after the initial surprise. This is because the offsetting effects on fertility and mortality are almost exactly equal in size. In the model, the increase in the CBR is quite modest, rising by about 5% over a 40 year period. In the data, a similar size change is seen. The CBR per year in

England rose from 28.91 per thousand in 1671 to 32.06 (about 10%) before falling back to 28.48 by 1706. Thus, this simple model accounts for about 50% of the changes over this period in the data. No doubt, including the effect of deaths in other age groups would improve this dramatically. The crude death rate increases from 26.25 to 32.14 per year in the data, or about 20% and only from .48 to .53, or about 11% in the model.

6 Conclusions

We have studied a simple dynamic general equilibrium model of fertility and saving in an economy with overlapping generations and capital accumulation. To simplify, we have abstracted from technological progress and government intervention. From a theoretical perspective, the distinguishing feature of our model is the assumption that parents reproduce only to guarantee themselves some economic support in late age. Children, on the other hand, because of altruism (or, under an alternative interpretation, because of a contract which is binding within the family), transfer resources to the old parents (possibly in exchange for the inheritance of the family land). How much is transferred is determined endogenously in accordance with different forms of interaction within the family. We look at two sets of social norms which may regulate such interactions. According to the first norm, siblings choose donations to maximize the sum of their utilities. We call this behavior cooperative. We call non cooperative the case in which each child maximizes his own utility taking the behavior of the other siblings as given.

The exogenous driving force behind the model's dynamics is the rate of infant mortality. (Here "infant" has to be interpreted extensively, as each of our model's three periods of life last for twenty years.) As the probability of survival until age twenty changes, parents adjust their fertility decisions and, correspondingly, their land/capital accumulation decisions.

In general, when survival rates are low, fertility is high but, since children are a fairly expensive way of saving for late age consumption, capital/labor intensity is high. As survival rates increase, fertility decreases, population increases and the capital/labor ratio decreases. These qualitative predictions hold for both the cooperative and non cooperative versions of the model.

Of particular interest is the prediction that fertility falls as infant survival rates increase. This is in agreement with the evidence, both time series and cross sectional and hence represents an improvement over simple versions of the Barro and Becker model. Moreover, simple calibrated examples of the non-cooperative version of the model have the property that the population growth rate is roughly independent of the infant mortality rate. This is in contrast with both the cooperative version of the model and the Barro and Becker model which both generate large increases in population growth across steady states when the IMR is lowered significantly.

Although the qualitative properties of the cooperative and non-cooperative versions of the model are similar, quantitatively the two versions are strikingly different. For example, given parameter values such that the cooperative version of the model generates historically reasonable fertility and population growth rates, the non cooperative solution predicts fertility so low that population shrinks. This is because, other things equal, the donations a parent can expect from his children are much lower under non-cooperative behavior, giving rise to much lower fertility levels. On the other hand, when each model is calibrated to match population growth rates of England in the 1600's, the size of the reduction in fertility in the steady state from a given reduction in the infant mortality rate is much larger in the non-cooperative version of the model. Indeed, in the non-cooperative version of the model, across steady states given historical levels of IMR's for both the 1600's and the 1990's, the overall population growth rate is unchanged.

The model also seems to be able to replicate other important features of the historical data as well. In particular, the delayed fertility response to either a plague or a famine, the parallel increase in the real wage and the subsequent return of population, per capita productivity and fertility to their initial levels after the shock has passed, are all well captured by the model.

Where our current framework fails most dramatically is in modeling the hump-shaped, or non monotone behavior of the growth rate of the population as the demographic transition unravels. As shown, fertility rates decrease monotonically but never enough to eventually reduce the growth rate of population. In the data, a reduction in population growth rates sets in (quite rapidly) after the period of high growth. European and North American evidence suggest that zero population growth may be the final outcome of the demographic transition. We are not aware of any intertemporal economic model based on microfoundations and capable of mimicking this last stage together with the two first ones. In fact, we are not aware of any model which can reasonably replicate the quantitative dynamic features of even the first two stages.

The model presented here replicates the first two stages reasonably well. In future research we plan to study the effect of lowering over time the percentage of the population behaving cooperatively and increasing the percentage behaving non-cooperatively as the IMR evolves according to its historical pattern. This switch from cooperative to non-cooperative behavior would follow the pattern of urbanization that is seen in the historical record. The idea here is that family ties are weakened when the population moves from agriculture to manufacturing. If the basic quantitative properties of the models that we have uncovered here continue to hold in this more complex environment, it is possible that this transition could successfully mimic the third stage of the demographic transition as well. We also plan to study the quantitative impacts on fertility of the increase in social security systems that have been seen in developed countries in recent years, and address the question of the extent to which this can be used as an explanation of the recent reductions in fertility seen in European countries.

To show that our conjecture holds in a full-blown dynamic model of fertility and capital accumulation in which social norms and mortality rates are both changing over time, is an important task for our future research.

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Appendix A: Purely Selfish Farmers

We briefly consider a "toy" model, without altruism in either direction in this Appendix. The model, while extremely simple, replicates many of the stylized facts captured by the more elaborate model analyzed in the paper.

We make the same assumptions as in Section 3, but drop the explicit intergenerationa altruism from children to parents. We assume instead that parents, in order to consume during the third period of their life, sell the land to their offsprings at a value equal to its total contribution to aggregate production. In choosing the level of savings and fertility, parents behave strategically in the following sense. They take into account that their children will work their land, hence they will be operating at a K/L ratio determined by the saving and fertility choices of the parents and will pay them a rate of return $R(k)$ which also depends on the K/L ratio chosen by their parents. Hence, in picking s and f parents will factor in the impact that s and f have on the expected value of $R \cdot x$ for next period. This leads to the following life-cycle optimization problem

$$\max_{s_t, f_t} u(c_t^m) + \delta u(c_{t+1}^o)$$

$$\text{s. to} \quad c_t^m + \theta_t f_t + s_t \leq w_t$$

$$c_{t+1}^o \leq R_{t+1} x_{t+1}$$

First order conditions with respect to s_t and f_t yield:

$$u'(c_t^m) = \delta u'(c_{t+1}^o)[R_{t+1} + x_{t+1} \frac{\partial R_{t+1}}{\partial s_t}] \qquad (FOC1)$$

$$u'(c_t^m)\theta_t = \delta u'(c_{t+1}^o)[x_{t+1} \frac{\partial R_{t+1}}{\partial f_t}] \qquad (FOC2)$$

Equality of rates of return implies

$$\theta_t[R_{t+1} + x_{t+1} \frac{\partial R_{t+1}}{\partial s_t}] = x_{t+1} \frac{\partial R_{t+1}}{\partial f_t}. \qquad (RR)$$

In the Cobb Douglas case the latter boils down to

$$\frac{1}{(1-\alpha)} - \pi_t f_t = \frac{k_t + s_t}{\theta_t f_t}. \qquad (RR)$$

This determines f_t as a function of s_t, albeit not uniquely. In fact, simple algebra shows that, for a large set of parameter values, the former equation has two interior solutions with $f_t > 0$ for given values of π_t, k_t and s_t.

The remaining first order condition can be simplified to read

$$f_t = \frac{\delta(1-\alpha)}{\theta_t[1+\delta(1-\alpha)]}[w_t - s_t],$$

which we solve for s_t. The law of motion $k_{t+1} = (k_t + s_t)/(\pi_t f_t)$ yields the capital/labor ratio next period. Hence, in this simple model, there are multiple equilibria: one with high fertility, low saving and a low level of k, and another with low fertility, high savings and a high capital/labor ratio.

Appendix B: Uniqueness of Balanced Growth Paths

We consider only the non cooperative case, the cooperative one can be solved along similar but simpler lines. Equation (AI) at a balanced growth path can be written as:

$$\gamma A k^{\alpha} - (1-\gamma)k = \theta_t f + s.$$

The law of motion of capital gives a value for steady state s such that

$$\gamma A k^{\alpha} - (1-\gamma)k = \theta_t + k(\pi f - 1).$$

Hence, the value of γ along a balanced growth path is

$$\gamma = \frac{\delta\eta\pi f}{(\eta+\pi f)^2 + \delta\eta\pi f}.$$

This allows us to write (AI) along a balanced growth path as

$$\frac{\delta\eta\pi f A k^{\alpha}}{(\eta+\pi f)^2 + \delta\eta\pi f} = \left(1 - \frac{\delta\eta\pi f}{(\eta+\pi f)^2 + \delta\eta\pi f}\right) k + \theta_t f + k(\pi f - 1).$$

Note that when $f = 0$, this equation holds for any value of k. Finally, equation (RR) along a balanced growth path simplifies to

$$\alpha A k^{\alpha-1} = \frac{\eta\pi(1-\alpha)A k^{\alpha} - \theta_t f - k(\pi f - 1)}{\theta_t(\eta+\pi f) + k\pi^2 f},$$

which, at $f = 0$, yields

$$\alpha A k^{\alpha-1} = \frac{\eta\pi(1-\alpha)A k^{\alpha} + k}{\theta_t \eta}.$$

The left hand side of this equation is monotone decreasing, its value is ∞ at $k = 0$ and 0 at $k = \infty$. The right hand side is monotone increasing, its value is 0 at $k = 0$ and ∞ at $k = \infty$. Thus this equation must have a unique positive solution for k.

¿From the observation that when $f = 0$ the balanced growth version of equation (AI) holds for any value of k, it follows that there is always another degenerate balanced growth path with $f = 0$ and $k > 0$.