Self-Control and Consumption-Saving Decisions: Cognitive Perspectives

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September 2002; PRELIMINARY DRAFT*

1 Introduction

Consider the standard economic approach to dynamic decision making: At each time t an agent must choose a feasible consumption plan c_t to maximize the present (exponentially) discounted utility from time t. When applied for example to consumption-saving choices, this decision problem underlies both Friedman (1957)'s Permanent Income and Modigliani's Life-Cycle Hypothesis (Modigliani-Brumberg (1954)).

A vast empirical evidence in experimental psychology indicates though that agents might have a preference for present consumption that cannot be rationalized by preferences with exponential discounting.[1] Motivated by such evidence behavioral economists have suggested an alternative specification of discounting, *quasi-hyperbolic discounting.*[2] Such a specification rationalizes the preference for present consumption documented by the experimental studies as a form of time inconsistency of preferences.[3]

When preferences are time inconsistent, agents' decisions are not only determined by rationality: At each time agents make decisions based on the expectations regarding their own decisions in the future, which will be based on different preference orderings. Such expectations must therefore be determined at equilibrium. The behavioral economics literature models dynamic decisions as a sequential game between different 'selves', each one choosing at a different time, and it restricts the analysis to Markov perfect Nash equilibria.[4]

By considering only Markovian strategies of a game between present and future selves the behavioral economics literature implicitly models agents as lacking any form of internal psychological commitment or self-control. This is hardly justified. First of all, the experimental evidence which contradicts exponential discounting has no implications for dynamic choice: these experiments are explicitly designed to induce agents to reveal truthfully their subjective

*Thanks to Antonio Rangel, Colin Camerer, Alessandro Lizzeri, Ariel Rubinstein, Aldo Rustichini, Andy Schotter, Giorgio Topa.

[1]See, e.g., Ainsle (1992), (2001), Ainsle-Haslam (1992), Frederick-Loewenstein-O'Donoghue (2002) for comprehensive surveys.

[2]See Laibson (1996), O'Donoghue-Rabin (1999). Psychologists favor a related specification, *hyperbolic discounting;* see e.g., Herrnstein (1961), de Villiers-Herrnstein (1976), and Ainsle (1992).

[3]Of course, quasi-hyperbolic discounting (or even, more generally, time inconsistency) is not the only possible way to rationalize the experimental evidence. Rubinstein (2000) makes this point very effectively, by showing how such evidence can be produced by relaxing rationality in the direction of a well-specified form of procedural rationality. Also, the experimental evidence is consistent in principle with preferences over sets of actions, under standard rationality assumptions; see Gul-Pesendorfer (2000).

[4]See the special issue of the *Journal of Economic Perspectives*, 2001, on the topic, and the references therein.

discounting preferences without facing dynamic choice problems, and therefore are designed to abstract from any dynamic commitment strategy which might result in self-control.[5] Moreover, a vast theoretical and experimental literature in psychology in fact studies the problem of dynamic choice, and identifies the various dynamic commitment strategies that agents use to implement their objectives and goals.[6] Finally, cognitive scientist have also made great progress in the identification of the mechanisms of the human brain which shape high-level cognition, cognitive control in general and self-control in particular.[7]

It is our contention therefore that dynamic choice of agents with time inconsistent preferences cannot be properly understood without an explicit analysis of the agents' dynamic commitment strategies that take the form of will power and self-control. Moreover it is our understanding that the experimental and the cognitive evidence on cognitive control overwhelmingly supports a theory of dynamic decision making in which self-control arises from the competition and the strategic interaction of different 'functions' within the 'self'. This is in contrast with the modelling strategy used in behavioral economics in which a single self controls all decisions at each moment, but cannot commit future selves.

We first introduce a simple model of dynamic decision-making that is a variant of the basic models used in the theoretical and empirical literature in the cognitive sciences studying 'delayed responses' and, more generally, cognitive control and high level cognition processes. In such models cognitive control is a function of the pre-frontal cortex, the most recently evolved component of the human brain, and arises from the *competition* between *automatic* and the *controlled* processing pathways. Automatic processes are the basic mechanisms for processing inputs or stimuli in humans as well as animals, and underlie classical conditioning and Pavlovian responses. In a self-control environment, the automatic process would induce the agent towards 'impulsive', 'uncontrolled' actions. Controlled processes are instead based on the activation, maintenance, and updating of active goal-like representations in order to influence cognitive procedures. In a self-control environment, the controlled process would induce the agent to implement a set of goals, determined independently of impulses or temptations associated with the specific choice problem. The outcome of the competition between the automatic and controlled processing pathways depends on the future expected rewards for the actions induced by these two processing pathways. Automatic processes may be inhibited by the expectation of future regret that would result from letting 'temptations' determine the consumption-saving choice today.

Based on such a model of cognitive control, we develop a simple theory of dynamic decision making for an environment in which agents face a conflict between present and future utility, and which may require self-control for its resolution. We apply this model to a standard consumption-saving problem to derive its economic implications in terms of consumption patterns. Agents trade-off 'excessive' immediate consumption with a saving rule requiring the exercise of self-control for its implementation. In particular, the present bias in the model derives from stochastic 'temptations' that affect the agents' consumption-saving choice each period. Self-control requires the conscious maintenance of attention on a specific goal, e.g., an optimal consumption-saving rule, that is unaffected by temptations. Such a consumption-saving rule, to be implemented, requires inhibitory connections that become stronger the higher is the cognizance of expected regret for 'impulsive' and immediate consumption.

The behavior of an agent facing conflicting preference representations over his consumption-saving choice in our model can be quite simply summarized: He actively maintains a simple

[5] The design of these experiments aims to 'uncover natural spontaneous preferences' (Ainsle, (2001), pg. 33), that is, to 'observe situations where the subject is not challenged to exercise self-control' (Ainsle (1992), pg. 70).

[6] See e.g., Kuhl-Beckmann (1985) for a survey, and Gollwitzer-Bargh (1996) for a collection of essays on the topic).

[7] See Miller-Cohen (2001), O'Reilly-Munakata (2000) for illuminating surveys.

consumption-saving goal, a propensity to consume out of wealth which is independent of any realized temptation; at times he allows temptations to affect his consumption-saving behavior by letting the automatic choice prevail, if this choice does not perturb his underlying consumption-saving plan too much and therefore does not have large permanent effects on his prescribed wealth accumulation pattern. When evaluating the effects on their prescribed wealth accumulation patterns of a deviation from their consumption-saving goal to accomodate a temptation, agents do anticipate that such a temptation will in fact be followed by others, as will the deviation, and the consumption-saving rule representing their goal will reflect such anticipation. We claim that such behavior is quite naturally consistent with several common qualitative descriptions in the psychological literature, which stress e.g., the importance of goal setting, mental rules, and self-regulation in saving behavior[8] .

We also derive some implications of our model of self-control with the aim of understanding how changes in the external environment, e.g., the distribution of temptation, and differences in the internal psychological characteristics of an agent, e.g., cognitive abilities like setting goals and controlling attention, affect consumption-saving behavior. For instance, we show that an environment with larger temptations on average is characterized by a higher probability that self-control is exercised and temptations inhibited. On the other hand, in such an environment, agents set for themselves less ambitious saving goals, that is they consume a larger fraction of the accumulated wealth each time self-control is exercised and temptations inhibited. We also show that an agent with lower cognitive control abilities, or, equivalently, an agent whose attention is drawn by other important cognitive tasks, for instance an agent attempting to control expenditures while exiting an important personal relationship, exercises instead self-control less frequently, and furthermore, sets less ambitious savings goals for the times when he does inhibit temptations: Stressful times are not conducive of self-control.

Finally, we study the issue of the 'complexity' of the consumption-saving goal agents set for themselves. Psychologists in fact constantly remark that the 'complexity' of goals affects in a fundamental manner the agents' ability in self-regulating tasks in general and in self-control tasks in particular.[9] Suppose, according to this view, that a cognitive task is simpler to implement the simpler are the goals that the task requires to be maintained active; e.g., because simple goals do not require exclusive attention. In such an environment, we aim at characterizing under which conditions an agent would gain from setting a simpler consumption-saving goal, e.g., a constant saving rule, as opposed to a 'complex' goal, e.g., one that is contingent on the rate of return on savings. We show that the simple consumption-saving plan is preferred to the complex plan, not surprisingly, if it is cheap to maintain active, and much cheaper then the complex plan. More interestingly, the simple plan tends to be preferred if rate of return is small enough, as in this case self-control is of little use, and it is dominant choice for the agent to consume a large fraction of his wealth each period. The simpler plan will also tend to be preferred, for instance, if temptations grow large on average. This is because when temptations are large enough the complex plan will optimally induce inhibition of the automatic processing most of the times, and independently of the rate of return, and this behavior can be induced also by a simple plan.

A detailed discussion of the related literature on self-control, addiction, and visceral states in behavioral economics is contained in Section 4.

2 A cognitive model of dynamic choice and control

In this section we introduce a simple model of a cognitive control task to outline the theoretical and empirical literature in the cognitive sciences that will form the foundations of our analysis.

[8]See Thaler-Shefrin (1981) for a survey of the psychological evidence on such issues.
[9]See for instance Baumeister-Heatherton-Tice (1994), and Gollwitzer-Bargh (1996).

Variants of such a model underlie many of the studies of 'delayed response' in neurobiology and cognitive psychology, and have been used extensively in behavioral, psychopharmacological, and neuroimaging studies (e.g., by T. Braver and J. Cohen and co-authors; see Braver-Cohen (2000) for an introduction and detailed references). More generally, this class of models is at the core of recent developments in neurobiology aimed at developing a general integrated theory of cognitive high level control based on the function of the prefrontal cortex (see Braver-Cohen-Servan Schreiber (1995), Desimone-Duncan (1995), O'Reilly (1999), O'Reilly-Munakata (2000)).[10] This theory is supported by a large and fast growing evidence on the biological mechanisms by which the pre-frontal cortex controls high level cognition, as in the clinical studies of brain lesioned patients (see Bechara-Damasio-Damasio-Anderson (1994), Bechara-Tranel-Damasio-Damasio (1996), (1997)), and in functional imaging studies of the human brain in experimental tasks (see Miller-Cohen (2001), Schultz (1998)).

Consider an agent facing a task consisting of a repeated series of decision trials. At the beginning of the trial at t the agent is presented with a *cue stimulus,* $a_t \in A$. The cue is, after a delay, followed by a *probe,* which requires an action x_t in a set X which possibly depends on x_{t-1}. During the interval between the cue an the probe the agents faces random appearances of *distractor stimuli,* $s_t \in S$. The utility payoff associated with the decision trial at t is received by the agent at the end of the period, after the action choice x_t. The payoff is independent of distractions, s_t, and is denoted $U(a_t, x_t)$.

The interest of such decision task lies in considering the distractor stimulus as "tempting" for the agent. The agent exercises self-control if he overrides the temptation, that is if his choice coincides with the utility maximizing element in the choice set (that is independent of the distraction). In other words, the decision problem is interesting only if the decision procedure is not independent of s_t.

In the model, cognition and control arise from the *competition* of *automatic* and *controlled* processing pathways activated by the cue a_t and the distractor stimulus s_t. Automatic processes are performed by various sub-cortical areas of the brain which have been extensively and successfully mapped by neurobiologists.[11] In the model, after observing the distractor s_t, the automatic pathway computes a "distorted" payoff map $U^I(a_t, x_t, s_t)$, associating, for example, a positive payoff to succumbing to temptation. The cue a_t and the distractor stimulus s_t also activate a controlled processing pathway. Controlled processes are based on the activation, maintenance, and updating of active goal-like representations in order to influence cognitive procedures. Such representations are maps from expected future rewards (or future negative rewards, e.g., regret) into actions. They are actively maintained in the prefrontal cortex, the component of the brain that is also responsible for active memory representations.[12] In our model, the controlled processing pathway computes the expected utility payoff map $U(a_t, x_t)$, for any choice $x_t \in X$, and maintains the map active and ready in order to influence action. Finally, a selective updating mechanism controls the activation of goals: if the representation of the automatic pathway is successful in replacing the map $U(a_t, x_t)$ in active memory, the choice of the agent will be $x^I \in \arg\max U^I(a_t, x_t, s_t)$; otherwise the agent will choose some $x \in \arg\max$

[10] The pre-frontal cortex is the most recently evolved area of the mammalian brain, and it is much more developed in humans than in any animal species; see Krasnegor-Lyon-Goldman Rakic (1997), Finlay-Darlington-Nicastro (2001).

[11] See e.g., Schultz-Dayan-Montague (1997).

[12] See Monsell-Driver (2000); also, see Miyake-Shah (1999) for related activation mechanisms in working memory.

Also, the areas of the brain specialized in representing and predicting future rewards and punishments are the midbrain nuclei the ventral tegmental area (VTA) and the substantia nigra (see Schultz-Apicella-Romo-Scarnati (1995) for neural recording studies and Bechara-Tranel-Damasio-Damasio (1996) for clinical studied of patients with brain lesions).

$U(a_t, x_t)$.

How does the brain determine which process is prevalent in decision making ? An 'executive function,' or 'supervisory attention system' modulates the activation levels of the different competing processing pathways. The biological processes which constitute the supervisory attention system modulating the competition of automatic and controlled pathways are well documented at the level of cognitive experiments. They rely on the action of a neuro-transmitter, dopamine, in the frontal cortex which activates and inhibits different competing processing pathways. Inhibitory inter-neurons are in fact an important component of the human brain, roughly 20% of the total neurons; see Gabbot-Somogyi (1986).[13]

The updating mechanism regulates the content of representations in active memory. Inhibitory connections are activated depending on expected future rewards, $U(a_t, x_t) - U(a_t, x^I)$, and updating occurs if

$$U(a_t, x) - U(a_t, x^I) \leq b$$

for some parameter b.[14] If we interpret $U(a_t, x_t)$ as the agent 's preferences, while $U^I(a_t, x_t, s_t)$ as his 'temporary' preferences, then the activation of the updating mechanism is governed by $U(a_t, x) - U(a_t, x^I)$, the expected 'regret' from failing to self control and giving in to the temptation. The parameter b is a measure of the costs of maintaining a representation in active memory. Activation costs are induced by the severe biological limitations of the activation capacity of the cortex.[15] If $b = 0$, "temporary" preferences have no effect on decision making, and control is automatically induced.[16]

Suppose the utility function U is concave in x, for any a_t. Then the model has the general straightforward implication that automatic processing is inhibited if $x - x^I$ is large enough in absolute value, that is if the temptation has a large enough effect on choice. Perhaps not surprisingly, given attention costs, small temptations are let go, and large ones are inhibited. In summary then, in this model cognitive control is the active representation of goals for the inhibition of distractions and temptations.[17]

Some of the implications of the self-control model described above can be matched with results in the experimental psychology literature. Since activation costs b are related to the activation capacity of the cortex, cognitive control should be harder to exercise and temptations harder to inhibit when performing unrelated cognitive (hence attention demanding) tasks simultaneously. Mischel and coauthors have studied in great detail techniques that children use to

[13] See Braver-Cohen (2000) for computational models of the frontal cortex around the modulatory role of dopamine; and Schultz (1998) for a survey of the imaging studies on the issue.

Striking evidence of the importance of the dopamine gating system in modulating the activation of different controlling pathways can also be derived from the clinical and neurological observations of the behavior of different psychiatric pathologies, like, e.g., Attention Deficit Hyperactivity Disorder (Barkley (1997)), schizophrenia (Cohen-Servan Schreiber (1992)), and various forms of drug addiction, non-chemical behavioral addictions, cocaine dependence, and in general a large class of addictive, impulsive, and compulsive disorders (Gardner (1999)).

[14] About 20% of the neurons of the human brain are inhibitory interneurons; .

[15] The operation of the prefrontal cortex by means of active representations is supported by the evidence of sustained neural activity during delayed response task in the frontal lobes; see Cohen-Perlstein-Braver-Nystrom-Noll (1997) and Prabhakaran-Narayanan-Zhao-Gabrieli (2000); for the limits in the activation capacity, see Engel-Kane-Tuholski (1999), Just-Carpenter (1992).

[16] As a task becomes more practiced its reliance on controlled processing and on the prefrontal cortex is reduced, and b can change over time; see e.g., Petersen-van Mier-Fiez-Raichle (1998), and Cohen-Dunbar-McClelland (1990) for experimental evidence on the Stroop task (described later).

[17] Such a view of cognitive control, and self-control in particular, is also consistent with the classical view in psychology. For instance, William James, concluding the analysis of 'will' in *The Principles of Psychology*, Holt, 1890, states: 'effort of attention is thus the essential phenomenon of will', and 'the difficulty [of self-control] is mental: it is that of getting the idea of the wise action to stay before your mind at all' (pg. 1167; cited in Shefrin-Thaler (1992)).

block temptations and maintain a focus on the rewards of delayed gratification and have documented the difficulties arising with multiple simultaneous cognitive tasks (see Mischel-Mischel (1983) and especially Mischel-Ebbeson (1970); but also Nisan-Koriat (1984)); relatedly, it is a well documented fact that people find it harder to quit smoking or drinking during stressful times, e.g., when needing exceptional concentration on a working or studying activity (see Baumeister-Heatherton-Tice (1994). The model also implies that cognitive control, and self-control in particular, should be easier to implement if associated with a simple and direct goal. This implication is also supported by the evidence in experimental psychology; once again the work of Mischel with children is a rich source of interesting examples (see Mischel-Patterson (1978); see also Schweitzer-Sulzer Azaroff (1988)).[18]

The fundamental role of the prefrontal cortex in delayed response tasks and especially in self-control is also supported by studies which have shown that lesions in the prefrontal cortex are often associated to a "syndrome of behavioral disinhibition" whereby patients act impulsively and often in socially inappropriate manners (see Fuster (1989) and Stuss-Benson (1986)).

The simple self-control model introduced here can also be experimentally implemented in a purely *connectivist* (that is, loosely, biologically founded) context, with a simple distributed neural network.[19] The model generates a wealth of empirical implications which have been tested at the neurobiological level on agents performing a complex cognitive control task. While a survey of this literature is outside of the scope of the present paper, we will briefly discuss one such study as an example. Cohen-Dunbar-McClelland (1990) have simulated a version of this model to match neuroimaging data of the activation pattern of the pre-frontal cortex of subjects during a *Stroop task* experiment. The Stroop task (after the experiments by Stroop in the 30's) is the paradigmatic experiment in the study of delayed responses. The task consists in naming the ink color of either a conflicting word or a non-conflicting word (e.g., respectively, saying 'red' to the word 'green' written in red ink; and saying 'red' to the word 'red' written in red ink). The standard pattern which is observed in this experiment is a higher reaction time for conflicting than non-conflicting words. Moreover the reaction time is higher, in either case, than the reaction time of a simple reading task; and the reading task reaction time is unaffected by the ink color). The accepted interpretation of this results is that automatic processing (reading) competes with and is dominated by the activation-based controlled processing (naming the ink color) at a cost in terms of reaction time.[20]

3 Consumption-saving decisions

Self-control plays an important role in agents' consumption-savings decisions. Ameriks-Caplin-Leahy (2002) for instance identify different 'propensities to plan' across TIAA-CREF participant

[18] See also, e.g., Hayes Roth-Hayes Roth (1979), and Gollwitzer (1993) for evidence of the kind of personal rules that agents rely upon during cognitive task; and Baumeister-Heatherton-Tice (1994) for a special emphasis on self-regulation to avoid addictions.

[19] Moreover, a learning mechanism which is consistent with the biology of the human brain can be simulated which learns to accurately predict $U(a_t, x) - U(a_t, x^I)$ in a repeated sequence of trials. The class of reinforcement learning algorithms which successfully predicts expected rewards is called "temporal differences," and denoted TD(λ); see Sutton-Barto (1998). TD(λ) algorithms are in fact consistent with neuroimaging data on firing of VTA neurons in a delayed-response experiments; see Schultz–Apicella-Ljungberg (1993) for instance for data on monkeys. This evidence more generally indicates that the prediction of future rewards is central in activation-based processing in delayed response task: while in the first learning trials the VTA neurons fire when the reward is delivered, after acquisition of the task VTA neurons fire when the stimulus is received, thereby consistently predicting the reward; see Schultz (1998).

[20] Also, and consistently with this whole approach to cognition and delayed response, patients with frontal impairment have difficulties with the Stroop task; see Cohen-Servan Schreiber (1992) and Vendrell-Junque-Pujol-Jurado-Molet-Grafman (1995).

households saving for retirement. Also, in Angeletos-Laibson-Repetto-Tobacman (2001)'s calibration exercises, hyperbolic discounting models match better than a standard exponential discounting model the empirically observed large holdings of illiquid assets and costly liabilities, like real estate and credit card debt. (The argument is straightforward: agents discounting the future hyperbolically will commit the future selves, whenever possible, to their present consumption plans; holding illiquid assets and costly liabilities, would serve this purpose and operate as *external commitment* mechanisms.)[21]

As noted in the Introduction, in hyperbolic discounting models dynamic decisions are modelled as a sequential game between different 'selves', each one choosing at a different time, and it restricts the analysis to Markov perfect Nash equilibria; thereby implicitly modelling agents as lacking any form of internal psychological commitment or self-control. It follows that, in this class of models, agents can limit their preference bias for present consumption only by relying on external commitment devices.

We are instead interested in studying the consumption-saving behavior induced by an agent's psychological, or *internal commitment* ability. In other words, we aim at identifying self-control strategies in agents' consumption-saving behavior. We model self-control as arising from the competition of cognitive functions, and therefore as a specific cognitive control task of the kind introduced in the previous section. We study the consumption-saving behavior in an environment in which agents face a conflict between immediate consumption and a saving rule which requires exercising self-control to be implemented. We do not endow the agents with any external commitment mechanism, so that their consumption-saving behavior is governed exclusively by self-control strategies.

Consider a dynamic economy, with time indexed by $t = 0, 1, ..., \infty$. Let the consumer's momentary utility for c_t units of the good at time t be denoted $U(c_t)$. The agent faces a linear production technology, and the wealth accumulation equation is

$$k_{t+1} = a_t k_t - c_t \tag{1}$$

where k_t and c_t denote respectively the agent's wealth and consumption at time t; and a_t is the productivity parameter at t. Since such technology is linear, it is optimal for the consumer to adopt a linear consumption plan:

$$c_t = \lambda_t a_t k_t$$

where λ_t is the propensity to consume at time t, the consumer's choice variable; the implied accumulation equation for capital becomes

$$k_{t+1} = (1 - \lambda_t) a_t k_t$$

The productivity a_t is in general stochastic.

Assumption 1 *The productivity a_t is i.i.d., takes values in $(0, \infty])$, and has well-defined mean, $E(a) > 0$.*

The agent's "baseline" preferences for consumption at time t are represented by a utility function $U(c_t)$. At any time t the agent observes a "temptation", z_t. The effect of the temptation is to generate a representation of "temporary" preferences at time t of the form:

$$U(z_t c).$$

[21]'Christmas Clubs' or specialized accounts, like 'education accounts', are other devices known to serve this purpose (see Elster (1979)). Relatedly, a large experimental evidence has accumulated in psychology which documents that subjects in self control experiments rely when possible upon external commitment devices; see Gollwitzer-Bargh (1996) for a survey of such evidence; even pigeons learn to use such commitment devices, see Ainsle (1974).

To be interpreted as a temptation, such representation is characterized, under our assumptions, by a higher perceived marginal utility of consumption at time t with respect to "baseline" preferences.

Assumption 2 *The consumer's utility for consumption,* $U(c)$, *is CES:*

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

with $\sigma < 1$.

In fact, with CES preferences, $U(z_t c) = (z_t)^{1-\sigma} U(c)$, and $\sigma < 1$ guarantees that the marginal utility of consumption increases with $z_t \geq 1$.[22]

Assumption 3 *The temptation* z_t *is i.i.d., takes values in* $[1, \infty)$, *and has well-defined mean,* $E(z) > 1$.

An agent in such an environment faces in each period a decision regarding consumption and saving. He also faces an internal, or psychological conflict, in the sense that he is hit by temptations which modify his underlying evaluation of the trade-off between consuming today and in the future. The agent's choice in the face of such conflict is the result of a cognitive procedure which involves letting the conflicting preference representations compete in terms of activation of different processing pathways. Activation is governed by the difference in expected future rewards evaluated with respect to the underlying preference ordering that is independent of temptations. It is the expected onset of regret that would result from giving in to temptations that governs self-control. From a psychological viewpoint, an agent's consumption-saving choice in the face of conflicting preference representations is the result of his ability to formulate and keep his attention centered on goals that are independent of temptations.

An agent at t observes a_t, his productivity parameter, which represents the characteristics of the decision problem the agent faces at t. He also observes z_t, the temptation he is facing at t, which determines the marginal utility of present consumption of temporary preferences: a_t and z_t represent, respectively, the cue and the distraction that set the consumption-saving decision procedure in motion at any time t.

An agent facing self-control problems does not solve a complete direct maximization problem. Decision making arises from the interaction of different processing pathways in the brain. The automatic processing pathway computes the desired consumption-saving rule, given a_t and z_t, as the propensity to consume λ_t^I which solves the following recursive problem:

$$V(a_t, k_t, z_t) = \max_{\lambda} \ (1-\sigma)^{-1} (z_t \lambda a_t k_t)^{1-\sigma} + \beta E V(a_{t+1}, (1-\lambda) a_t k_t, z_{t+1}) \tag{2}$$

The solution is denoted $\lambda^I(a_t, k_t, z_t)$.

Following the inputs, that is the cue and the disturbance a_t and z_t, the controlled pathway is also initialized. Such a pathway disregards the temporary preference representations induced by z_t and also produces a consumption-saving rule in the form of a propensity to consume λ_t. Such a consumption saving rule optimally trades off immediate consumption and future consumption

[22] Alternatively, and without major qualitative differences in our results, we could work with $\sigma > 1$ and temptations $z_t < 1$.

Also, we model temptations as a shock to the utility function rather than as a shock to the discount rate. With CES preferences and a single commodity, as in our case, this hardly makes a difference, but the distinction is important in more general models in which temptations can hit differently different goods, e.g., models with addictive and normal goods.

but in general depends on the results of the competition which determines the active pathway at each future time $t+\tau$, given $a_{t+\tau}$ and $z_{t+\tau}$. In particular, we assume that the controlled pathway correctly anticipates the stochastic properties of temptations and the results of the interaction and competition with automatic processes of consumption-saving in the future.[23]

We proceed by formally deriving the consumption-saving rule resulting from the activation of the controlled processing pathway. Such pathway first computes the future value of the consumption-saving plan, $D(a_{t+1}, k_{t+1}, z_{t+1})$ which depends on the active pathway at each future time $t+\tau$, given $a_{t+\tau}$ and $z_{t+\tau}$. If the temptation $z_{t+\tau}$ were to be inhibited at all times in the future, then the automatic pathway would have no role in the determination of the value of the consumption plan, so that $D(a_t, k_t, z_t) = D(a_t, k_t)$, and we simply would have

$$D(a_t, k_t) = \max_{\lambda} \ (1-\sigma)^{-1} (\lambda a_t k_t)^{1-\sigma} + \beta E D(a_{t+1}, (1-\lambda) a_t k_t) \tag{3}$$

The resulting consumption-saving choice in this case coincides with the optimal consumption-saving rule of an agent with exponential discounting at rate β.

In general however, temptation will not be inhibited at all future times, and at some future times t, $\lambda_t^I = \lambda^I(a_t, k_t, z_t)$ will be chosen. Suppose that, as in the previous section, the results of the competition between pathways are determined by expected rewards. Suppose in particular that the automatic pathway is only active if the future regret associated with the temptation is smaller than an exogenous activation cost $b(a,k)$, with the following simple functional form: $b(a,k) = b(a_t k_t)^{1-\sigma}$. (We adopt this functional form to guarantee the stationarity of the consumption-saving decision in order to simplify the problem.) In this case,, $D(a_t, k_t, z_t)$ is given by:

$$D(a_t, k_t, z_t) = \max \begin{bmatrix} U(\lambda_t^I a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}\left(1-\lambda_t^I\right) a_t k_t, z_{t+1}\right)\right], \\ Max_{\lambda} U(\lambda a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda) a_t k_t, z_{t+1}\right)\right] - b(a_t k_t)^{1-\sigma} \end{bmatrix} \tag{4}$$

$$\text{and } \lambda_t^I = \lambda^I(a_t, k_t, z_t) \ \text{ is the policy function of program (2)} \tag{5}$$

Given the future value of the consumption plan, $D(a_{t+1}, k_{t+1}, z_{t+1})$, the controlled processing pathway computes the desired consumption-saving rule as the propensity to consume λ_t which solves:

$$\max_{\lambda} U(\lambda a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda) a_t k_t, z_{t+1}\right)\right]$$

The resulting propensity to consume is independent of z_t; let it be denoted $\lambda^E(a_t, k_t)$.

As we noted earlier, the expected rewards determine the results of the competition between the automatic and the controlled processing pathways. This process, implicit in the determination of $D(a_t, k_t, z_t)$ in (3-5), can be represented simply as follows. Given $\lambda_t^I = \lambda^I(a_t, k_t, z_t)$ and $D(a_t, k_t, z_t)$, the expected regret associated with the temptation z_t at time t, is

$$\begin{aligned} R(a_t, k_t, z_t) &= Max_{\lambda} U(\lambda a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda) a_t k_t, z_{t+1}\right)\right] \\ &\quad - U(\lambda_t^I a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}\left(1-\lambda_t^I\right) a_t k_t, z_{t+1}\right)\right] \end{aligned}$$

Inhibitory controls activate the controlled processing pathway if

$$R(a_t, k_t, z_t) > b(a_t k_t)^{1-\sigma}$$

[23]These anticipations rely on reinforcement learning procedures which have proven quite effective on similar tasks in simulations (see footnote 19); see on the contrary Loewenstein-O'Donaghue-Rabin (2002) for evidence from survey data regarding a 'cold-to-hot empathy gap,' that is a projection bias in predicting future utility.

In summary, the present bias in the model derives from the stochastic temptation which affects the computations of the automatic processing pathway. Self-control at time t coincides with disregarding temptation z_t in the decision process, and it requires the maintained activation in memory of goals, that is of preferences independent of the temptation z_t. Such a representation is maintained by the force of the inhibitory connections linking the reward predictions and active representation. These inhibitory connections therefore modulate the updating of the active representations, and are stronger, the higher is the prediction of regret given (a_t, k_t, z_t).

3.1 Characterization and identification

In this section we characterize the behavior of an agent facing conflicting preference representations over his consumption-saving choice. We first study the consumption saving plan associated with the automatic pathway, $\lambda^I(a_t, , k_t, z_t)$. Given $\lambda^I(a_t, , k_t, z_t)$ we solve for the future value of the consumption-saving plan, $D(a_t, k_t, z_t)$, and the consumption-saving plan associated with the controlled processing pathway, $\lambda^E(a_t, k_t)$. The agent's behavior is then determined at each time t by the competition between pathways: the agent uses $\lambda^I(a_t, , k_t, z_t)$ when he expects a limited future regret, $R(a_t, k_t, z_t) \leq b(a_t k_t)^{1-\sigma}$, while he inhibits the temptation z_t and uses $\lambda^E(a_t, , k_t)$ if $R(a_t, k_t, z_t) > b(a_t k_t)^{1-\sigma}$.

We proceed with the characterization of $\lambda^I(a_t, , k_t, z_t)$. Let $\tilde{z}_{t+\tau} = (a_{t+\tau} z_{t+\tau})^{\frac{\sigma-1}{\sigma}}$, for any $\tau \geq 1$, $\tilde{z}_t = (z_{t+\tau})^{\frac{\sigma-1}{\sigma}}$, and $\gamma \equiv \beta^{\frac{-1}{\sigma}}$; finally, let $E(\tilde{z}) = E(\tilde{z}_{t+\tau})$.

Proposition 1 *For any a_t, k_t,*

$$\lambda^I(a_t, k_t, z_t) = \lambda^I(z_t)$$

Moreover, $\lambda^I(z_t)$ is increasing in z_t, and can be solved for in closed form:

$$\lambda^I(z_t) = \frac{E(\tilde{z})^{-1}}{1 + \frac{\tilde{z}_t}{\gamma} + \frac{\tilde{z}_t}{\gamma}\left[\frac{E(\tilde{z})}{\gamma - E(\tilde{z})}\right]} \tag{6}$$

Given $\lambda^I(a_t, k_t, z_t) = \lambda^I(z_t)$, each agent's consumption-saving plan is characterized by the policy function of dynamic programming problem (5), which we now characterize.

Proposition 2 *The value function $D(a_t, k_t, z_t)$ defined by problem (5) exists. The consumption-saving rule associated to the controlled processing pathway, $\lambda^E(a_t, k_t)$, is in fact a constant, λ^E.*

Moreover, there exist a unique policy function of problem (5), $\lambda(a_t, k_t, z_t)$, which has the following properties: i) *it is independent of (a_t, k_t), that is $\lambda(a_t, k_t, z_t) = \lambda(z_t)$;* ii) *it has a cut-off property, that is, there exists a $\underline{\lambda}$ such that*

$$\lambda(z_t) = \begin{cases} \lambda^I(z_t) & \text{for} \quad \lambda^I(z_t) \leq \underline{\lambda} \\ \lambda^E & \text{for} \quad \lambda^I(z_t) > \underline{\lambda} \end{cases} \tag{7}$$

An alternative related representation in terms of realized temptation can be derived as follows. Automatic choice is inhibited at a time t for large enough realized temptation z_t.

Proposition 3 *There exist a $\underline{z}$ such that*

$$\lambda(z_t) = \begin{cases} \lambda^I(z_t) & \text{for} \quad z_t \leq \underline{z} \\ \lambda^E & \text{for} \quad z_t > \underline{z} \end{cases} \tag{8}$$

The cut-off rule in terms of realized temptation follows from (7) by noting that $\lambda^I(z_t)$ increases with z_t, that is the propensity to consume associated with an automatic choice increases with the intensity of the realized temptation (Proposition 1).

The behavior of an agent facing conflicting preference representations over his consumption-saving choice in our model can be quite simply summarized: He actively maintains a simple consumption-saving goal, a propensity to consume out of wealth which is independent of any realized temptation, and is equal to λ^E; at times he allows temptations to affect his consumption-saving behavior by letting the automatic choice $\lambda^I(z_t)$ prevail, if this choice does not perturb his underlying consumption-saving plan too much and therefore does not have large permanent effects on his prescribed wealth accumulation pattern.

In fact such behavior is quite naturally consistent with several common qualitative descriptions in the psychological literature, which stress e.g., the importance of goal setting, mental rules, and self-regulation in saving behavior (see Thaler-Shefrin (1981) for a survey of this literature). Also, it is important to notice that our agents, when evaluating the effects on their prescribed wealth accumulation patterns of a deviation from their consumption-saving goal to accomodate a temptation, do anticipate that such a temptation will in fact be followed by others, as will the deviation. This is in accordance with the many accounts in popular psychology of agents' motivations for rejecting temptations of the sort "if a fall for this I will fall for many others;" in our terminology, the agent understands that he must impose to himself a stricter cut-off rule.[24]

We now compare the consumption-saving plans which characterize behavior in our model of self-control with the optimal consumption-saving plan of an agent who never faces temptations $\{z_t\}_0^\infty$, or that can inhibit them at no cost, $b = 0$. The consumption-saving plan chosen by such an agent coincides with the policy function of a rational agent who discounts the future exponentially, as in the standard economic theory of dynamic choice, and is determined by the policy function of dynamic programming problem (3). It is straightforward to show that such policy function is in fact a constant, independent of (a_t, k_t); let it be denoted λ^*.

Proposition 4 *Both automatic and controlled choice necessarily result in a bias favoring present consumption over saving with respect to the consumption-saving decision of an agent who never faces temptations*

$$\lambda^I(z_t) > \lambda^E > \lambda^*, \quad \textit{for any } z_t.$$

We can furthermore compare the behavior induced by our formulation of self-control with the behavior induced by the Markov perfect Nash equilibrium of the game of multiple successive selves, as in Laibson (1996) or O'Donogue-Rabin (1999), in the same consumption-saving environment.

Formally, a Markov perfect Nash equilibrium of the game of successive selves with stochastic temptations is represented by a consumption-saving rule $\lambda^M(z_t)$ solving the following fixed point condition:

$$\lambda^M(z_t) = argmax_\lambda (1-\sigma)^{-1} \left(z_t \lambda a_t k_t\right)^{1-\sigma} + EV_{\lambda^M(z)}\left((1-\lambda)a_t k_t, a_{t+1}, z_{t+1}\right), \ \forall z_t \qquad (9)$$

where $V_{\lambda(z)}\ (a_t, k_t, z_t)$, the value at t of present and future consumption induced by an arbitrary

[24]Of course the wording of these accounts are most often interpreted in the psychological literature, and in the economic literature as well (see Benabou-Tirole (2000)), as indicating a sort of reputation mechanism which agents uses to enforce self-control rules. Our interpretation has the advantage that it does not require to postulate a mind composed of different selves endowed with asymmetric information. We discuss more in detail in Section 4 these different approaches to decision making.

consumption-saving rule $\lambda(z)$, is defined by

$$V_{\lambda(z)}\left(a_t,k_t,z_t\right)=(1-\sigma)^{-1}\left(z_t\lambda(z_t)a_tk_t\right)^{1-\sigma}+E\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\left(z_\tau\lambda(z_\tau)a_\tau k_\tau\right)^{1-\sigma} \tag{10}$$

In this formulation, an agent at each time t chooses a consumption-saving rule which depends on the realization of the temptation z_t: he never exercises self-control and always lets go of the temptation, that is, the higher the temptation the more he consumes; he anticipates though the same behavior at equilibrium of all his future selves and sets his consumption-saving rule in the present accordingly (such forward looking aspect of the equilibrium concept in facts exacerbates his present bias in consumption). Formally,

Proposition 5 *The Markov perfect Nash equilibrium consumption-saving plan, $\lambda^M(z_t)$ is increasing in z_t, and results in a bias favoring consumption over savings with respect to the plan of an agent exercising self-control and, under our preference specification, even with respect to the plan associated to automatic processing:*

$$\lambda^M(z_t)>\lambda^I(z_t)>\lambda^E,\ \forall z_t$$

It is interesting to note that the behavior of an agent who as in our model of self-control can be formally distinguished from the behavior associated to the Markov perfect Nash equilibrium of the game of multiple selves, even if the stochastic process driving temptations is not directly observed, that, is with only data on consumption and wealth. This identification result follows simply by exploiting the cut-off property of the consumption-saving plan which characterizes self-control, and under minimal assumptions on the distribution of temptations z_t. The behavior both of an agent in our model of self-control and of an agent who plays the Markov perfect Nash equilibrium of the game of successive selves can in fact be represented by a stochastic process for the consumption-saving plan λ_t, induced by the stochastic process governing temptations. If for instance the distribution of temptation is non-atomic, the equilibrium distribution of λ_t associated to the Markov perfect Nash equilibrium will also in general be non-atomic, while the distribution associated to behavior in our self-control model has a positive probability mass associated to the lower bound of its support (which corresponds to λ^E). In other words, if the agent behaves as we postulate in our model of self-control we should observe a positive fraction of times a low consumption-saving plan λ^E, as opposed to a different value of λ_t at any t.[25]

3.2 Comparative statics

We turn now to some implications of our model of self-control in terms of comparative statics, with the aim of understanding how changes in the external environment, e.g., the distribution of temptation or the distribution of productivity shocks, and differences in the internal psychological characteristics of an agent, e.g., cognitive abilities like setting goals and controlling attention, affect consumption-saving behavior.[26]

We turn first to a series of comparative statics results regarding the dependence of $\lambda^I(z_t)$, λ^E and $\underline{\lambda}$ on first order dominance changes of the distribution of z_t, and a_t and on changes in

[25] This is of course just an illustration of a possible identification procedure, but different assumptions on the distribution of z_t give rise to different possible strategies to obtain the identification result from consumption and wealth data.

[26] As already noted, different 'propensities to plan' have been documented by Ameriks-Caplin-Leahy (2002) with survey data on retirement savings. The measure of 'propensity to plan' they introduce is designed to identify psychological and cognitive aspects of behavior of the respondents, and is constructed from answers to questions related to planning behavior, but not in self-control situations.

b.[27] We will then discuss the implications of these results for the general qualitative properties of consumption-saving behavior.

We impose the following independence assumption which allows us to separate the productivity and temptation effects.

Assumption 4 *The random variables a_t and z_t are independent.*

Expected future temptations as well as expected future productivity shocks affect automatic choice, and the propensity to consume today when temptation is not inhibited, $\lambda^I(z_t)$.

Proposition 6 $\lambda^I(z_t)$, *increases with increases, in the first order dominance sense, in the distribution of z_t and in the distribution of a_t.*

In other words, $\lambda^I(z_t)$, increases as a consequence of shifting probability weight into larger values of z_t or a_t.

Consider again the effect of a first order stochastic dominance change in the distribution of z_t, and of a_t, now on the propensity to consume in the event temptations are inhibited, λ^E.

Proposition 7 *The propensity to consume associated with controlled processing, λ^E, increases with an increase in the first order dominance sense, in the distribution of z_t, and with an increase in b; while it decreases with an increase in the first order dominance sense in the distribution of a_t.*

The intuition for the effect of an increase in the first order dominance sense in the distribution of z_t hinges on the change in the expected future value of the consumption-saving program, which represents at the margin the value of savings. If a change in the distribution of temptations has the effect of decreasing the expected future value of the consumption-saving program, then at the margin an agent exercising self-control will save less and consume more in the present. This is in fact the effect of an increase in the first order sense of the distribution of z_t: the value of the program is weakly decreasing in z_t and hence an increase in the distribution of z_t in the first order sense shifts probability mass from realizations of temptations associated to higher values of the program into realizations associated to lower values of the program, thereby decreasing its expected value. Similarly, an increase in attention costs b reduces the expected future value of the consumption-saving program, by making it more costly to exercise self-control, and hence it reduces the marginal value of saving.

The case of a change in the distribution of productivity shocks is the opposite: an increase in the first order dominance sense in the distribution of a_t increases the expected value of the consumption-saving program, by improving the technological opportunities of the economy on average in the future, and therefore has the effect of increasing the marginal value of saving and saving itself in the present.

To identify more precisely the determinants of the stochastic process which regulates an agent's exercise of his cognitive self-control inhibitions we study also the comparative statics properties of the cut-off $\underline{\lambda}$. (Note that the properties of the cut-off $\underline{z}$ are qualitatively equivalent.)

[27] Consider two probability densities f and f' on a compact subset of $\Re$, X, and the associated cumulative function, F and F'. The density f' dominates in the first order stochastic sense the density f if

$$F'(x) \leq F(x), \ \forall x \in X.$$

An increase, in the first order dominance sense, of the distribution of x represents therefore a shift of probability mass from the smaller realizations of $x \in X$ to the larger (and, as a consequence, it increases the mean of the distribution).

We are able in this case to consider only *infinitesimal* changes in the first order stochastic sense of the distributions of z_t and a_t.[28]

Proposition 8 *The cut-off $\underline{\lambda}$ is increasing in b, decreasing in a local (infinitesimal) increase in the first order dominance sense in the distribution of z_t, and increasing in a local (infinitesimal) increase in the first order dominance sense in the distribution of a_t.*

Other things equal, it is not surprising that an increase in the cost of inhibiting automatic processing at any time t, and hence in the cost of exercising self-control, has the effect of increasing the cut-off $\underline{\lambda}$, that is, of rendering it less stringent. Perhaps it is more surprising that an increase in the first order stochastic sense in the distribution of temptations makes instead the cut-off more stringent. The intuition is that the cost of inhibiting automatic processing is unvaried and equal to b, while the value of inhibiting is on average higher, since the distribution of z_t is shifted towards higher realizations of z_t.[29] Interestingly, the value of inhibiting automatic processing decreases by shifting the probability mass towards more productive technology shocks, and the cut-off $\underline{\lambda}$ is reduced as a consequence of such a change.

How then, in summary, are the general qualitative properties of consumption-saving behavior affected e.g., by an increase in the first order stochastic sense of the distribution of temptation ? And in particular, is the probability that self-control is exercised and automatic choice inhibited in the course of the consumption-saving decision problem higher or smaller due to such a change in the distribution of temptations ?

Proposition (3) implies that such probability coincides with the mass of the distribution of z_t on $z > \underline{z}$. An increase in the first order dominance sense in the distribution of z_t increases therefore by definition the mass of the distribution of z_t on $z > \underline{z}$, for any $\underline{z}$. Furthermore, an increase in the first order dominance sense in the distribution of z_t *i)* decreases the cut-off $\underline{\lambda}$ by Proposition (7), and *ii)* increases $\lambda^I(z)$, for any z. But both *i)* and *ii)* have the effect of decreasing $\underline{z}$. We conclude then that an increase in the first order dominance sense in the distribution of z_t increases the probability that self-control is exercised and automatic choice inhibited. On the other hand, by Proposition 7 a local (infinitesimal) increase in the first order dominance sense in the distribution of z_t increases λ^E, i.e., the consumption when self-control is exercised and automatic processing inhibited. We conclude that an agent facing larger temptations in the future reacts by exercising self-control more often but at the same time by consuming a higher fraction of his wealth even while controlling himself.

It is straightforward to examine also the effects of an increase in attention costs, b, on the general qualitative properties of consumption-saving behavior. In this case in fact, the cut-off $\underline{z}$ increases ($\lambda^I(z_t)$ is unaffected) and hence the probability that self-control is exercised and automatic choice inhibited is lowered as an effect. Furthermore, an increase in b has, by Proposition 7, the effect of increasing λ^E. An increase in b, therefore, has negative effects on the agent's ability to self-control and reduces the present consumption bias in all dimensions.

A local (infinitesimal) increase in the first order dominance sense in the distribution of a_t increases the cut-off $\underline{\lambda}$ by Proposition (7), and it also increases $\lambda^I(z)$, for any z; and therefore it

[28]Consider once more two probability densities f and f' on a compact subset of $\Re$, X, and the associated cumulative function, F and F'. Fix a density f' which dominates f in the first order stochastic sense, and consider the distribution obtained by mixing $f(x)$ with $f'(x)$:

$$g(x) = (1 - \alpha)f(x) + \alpha f'(x)$$

By an infinitesimal increase in the first order dominance sense in the distribution of x we mean an infinitesimal increase $d\alpha > 0$ evaluated at $\alpha = 0$.

[29]In fact, a countervailing effect must be taken into account: the value of inhibiting automatic processing is reduced by the increase in λ^E, see Proposition 7. But this effect is second order for infinitesimal changes in the distribution of z_t by the Envelope Theorem.

decreases $\underline{z}$; it has therefore an ambiguous effect on the probability that self-control is exercised and automatic choice inhibited. The effect of such a change in the distribution of productivity shocks on the consumption-saving plan in the event that automatic processing is in fact inhibited is to increase savings in the present period (by Proposition 7).

3.3 Complexity of goals

The behavior of an agent facing conflicting preference representations over his consumption-saving choice in our model, as we noted, involves maintaining actively a simple consumption-saving goal. Such goal consists, in our analysis, in a propensity to consume out of wealth which is independent of any realized temptation. Psychologists constantly remark that the determination of the 'complexity' of the goals individuals set for themselves affects in a fundamental manner their ability in self-regulating tasks in general and in self-control tasks in particular.[30] But the simple formulation the agent problem we have adopted, with linear production technology and CES preferences, implies that the consumption-saving goal is in fact extremely simple: it is constant over time, as it is in fact independent of the realization of the production shock a_t. It can actually be expressed verbally as simple constant percent rule: "Save $\lambda^E \times 100$ percent of your wealth each period."

To study the issue of complexity of the goal agents set to themselves, we need to examine a different formulation of the model, which gives rise potentially to more complex consumption-savings plans in the event of self-control. As a way of illustration consider the following formulation of technology, leaving preferences unchanged:

$$k_{t+1} = a_t \left(k_t - c_t\right) \tag{11}$$

In this formulation, the shock a_t acts on net wealth $k_t - c_t$, rather than on initial period wealth, k_t, and therefore takes the interpretation of a rate of return on saving at t, rather than of a productivity shock, as in the case of the technology studied in the previous section, equation (1). The important difference is that in this formulation the value of controlling any temptation is random, and proportional to the realization of a_t : If the return from saving is small, a_t is small, and self-control is of little use. As a consequence, the consumption-saving plan depends on a_t; let it be denoted $\lambda(z_t, a_t)$. Let also $\lambda^I(z_t, a_t)$ and $\lambda^E(a_t)$, denote the propensities to consume associated, respectively, to the automatic and the controlled pathways; let finally $\underline{\lambda}(a_t)$ denote the cut-off which characterizes $\lambda(z_t, a_t)$.

Proposition 9 *The propensity to consume $\lambda^I(z_t, a_t)$, and $\lambda^E(a_t)$, and the cut-off $\underline{\lambda}(z_t, a_t)$ are all decreasing in a_t, for any z_t.*

We can therefore in this environment study the issue of the complexity of the goal $\lambda^E(a_t)$, with respect to any simpler goal represented by a constant consumption-saving plan over time, that is a plan independent of a_t. Suppose in fact that the activation cost parameter, b, decreases with the complexity of the goal to be maintained active. In particular, we interpret this to mean that activation costs are lower to maintain active a constant consumption-saving, $\lambda^{E,simple}$, than a fully contingent plan $\lambda^E(a_t)$; and we take the constant plan $\lambda^{E,simple}$ to coincide with the optimal consumption-saving plan associated to cognitive control under the restriction that it be independent of a_t at any time t. Our objective is to characterize under which conditions

[30] The books by Baumeister-Heatherton-Tice (1994), and Gollwitzer-Bargh (1996), for instance, discuss enormous rich literatures on the topic. In the specific context of consumption-saving environment, Thaler-Shefrin (1981) discuss evidence for the effectiveness of mental rules and other internal psychological mechanisms which have the objective of reducing the complexity of the consumption-saving goals agents rely on to exercise self-control.

in the parameters an agent would gain from setting the simpler constant goal as opposed to the 'complex' goal, contingent on the state of the technology, a_t.[31]

Let the activation cost associated to the simple plan be denoted b^{simple}, and let the difference in the cost parameters, that is the saving in activation costs per unit of wealth to maintain the simpler plan active, be denoted Δb.

Proposition 10 *A simpler constant consumption-saving plan $\lambda^{E,simple}$ tends to be preferred to the complex plan $\lambda^E(a_t)$ if in the limit, and other things equal,* i) *b^{simple} is small and Δb large enough,* ii) *the mean of a_t is small enough, and finally if* iii) *the mean of z_t as well as Δb are large enough.*

The simple consumption-saving plan is preferred to the complex plan, not surprisingly, if it is cheap to maintain active, and much cheaper then the complex plan. More interestingly, the simple plan is preferred if the mean of the stochastic rate of return, $E(a_t)$, is small enough, that is close to 0. In this case, since the support of the rate of return shocks is $[0, \infty)$, the variance of a_t also tends to 0 and hence rate of return in the limit is degenerate, and concentrated on 0. But in this case self-control is useless, and it is a dominant choice for the agent to consume all of his wealth each period. Therefore, the utility gain of conditioning the consumption-saving plan on the realization of a_t vanishes.

The simple plan is also preferred if the mean of the stochastic process of temptations grows large. This is because when temptations are large enough, in the limit, the complex plan will optimally induce inhibition of the automatic processing all the times, independently of a_t, and this behavior can be induced also by a simple plan. (The condition on Δb is required as the savings in terms of attention costs associated to the simple plan must of course also more than compensate the loss of utility due to the non contingent plan by itself, once inhibition is guaranteed at all times.)

4 Related literature, extensions, and discussion

The view that decision making arises from the interaction of automatic and cognitive processes, or visceral and rational states, hot and cold states, is at least as old as the Bible; and it has taken different forms and conceptualizations over the centuries, perhaps most notably in recent times in psychoanalytic theory where it takes the form of the *Ego* and the *Id* (see Freud (1927).

A formal model exploiting such dichotomy in decision making has been introduced in economics by Loewenstein (1996) in his pioneering analysis of the psychology of visceral influences on behavior; Bernheim-Rangel (2001) also study the visceral/rational dichotomy in a model of addiction which is more directly motivated by neurobiological evidence. The identification and the modelling of the neural processes responsible for cognitive control, and especially of the mechanism which modulates the competition of such processes, is the recent contribution of

[31] The constant consumption-saving plan is defined formally as follows:

$$\lambda^{E,simple} = argmax_\lambda E \left(U\left(\lambda a_t k_t\right) + \beta D^{simple}\left(a_{t+1}, a_{t+1}\left(1-\lambda a_t\right) a_t k_t, z_{t+1}\right)\right)$$

where

$$D^{simple}(a_t, k_t, z_t) = \max \begin{bmatrix} U(\lambda_t^I a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}\left(1-\lambda_t^I a_t\right) a_t k_t, z_{t+1}\right)\right], \\ Max_\lambda E\left(U\left(\lambda a_t k_t\right) + \beta D\left(a_{t+1}, a_{t+1}\left(1-\lambda a_t\right) a_t k_t, z_{t+1}\right) - b(a_t k_t)^{1-\sigma}\right) \end{bmatrix} \tag{12}$$

$$\text{and } \lambda_t^I = \lambda^I(a_t, k_t, z_t) \text{ is the policy function of program (2)} \tag{13}$$

cognitive sciences which we are introducing to study economic decision making and which characterizes our approach. The foundations of our model of self-control lie in the explicit modelling of cognitive processes introduced in Section 2 rather than in visceral/rational dichotomy per se.

Regret plays an important role in our cognitive model of self-control. In our formulation in fact future expected regret, through a dopamine gating mechanism, regulates the competition between automatic and controlled processing pathways, and essentially controls behavior. Regret plays an important role also in Gul-Pesendorfer (2001)'s axiomatic analysis of self-control: the preference representation they obtain contains a term which represents the preference for present consumption and a term which limits such preference, which can be interpreted as psychological regret. A more precise comparison of our work with Gul-Pesendorfer's is though difficult because of the different methodological foundations: in terms of cognitive processes in our study and in terms of classical axiomatic theory in theirs.

Our approach to decision making in self-control environments, by exploiting the interaction of different controlling pathways, could appear related to a multiple selves approach, in which decisions result from the strategic interaction, at each time t, of different selves. In Thaler-Shefrin (1981), for instance, choice results from the interaction of a *planner* self and a *doer* self. The doer is impulsive and the planner exercises self-control, modelled as an explicit costly effort which has the effect of changing the preferences of the doer, e.g., by building guilt. However, a problem with all specifications of multiple selves is that they postulate a subdivision of the brain into active symbolic and logic modules which have no parallel in the actual neurobiology of the human brain. In the words of *connectivist* cognitive science, such specifications represent *homunculus-based* theories of the mental process.[32]

Such critique of multiple selves models applies also to the standard behavioral specifications of hyperbolic discounting models which postulate a game between multiple successive selves and adopt an equilibrium concept, Markov perfect Nash equilibrium, which is conceptually well-founded in principle only for interpersonal strategic interactions. Even more explicitly homunculus-based are those theories of behavior, like Benabou-Tirole (2000), which exploit information asymmetries across different selves and Bayesian inference methods in the strategic interaction between the selves. In our set-up, on the contrary, the modulatory activity of the executive function, it has been shown by the literature in cognitive sciences we cite in Section 2, may arise endogenously from the activation of the neural network; it does not require the executive function to operate explicitly as a maximizing agent, an homunculus.

Several studies in economics have directly extrapolated evidence pertaining to the consumption of addictive substances and to the behavior of addicted individuals to formulate a general theory of consumption (examples are Loewenstein (1996) and Laibson (2001)). While addictive behavior might be more properly considered a pathological phenomenon, and it is therefore probably best studied distinctly from consumption,[33] addictions do in fact reflect lack of self-control, and as a consequence a theory of self-control should be naturally tested in terms of its ability to understand addictions. In fact, we claim that the general model of cognitive control we have adopted could have important implications in the study of addiction: different forms of addiction would be associated to a specific pathology concerning the mechanism controlling the competition of the different processing pathways, e.g., in the form of a difficulty in inhibiting distractions and temptations. In our framework, and consistently with such an explanation of addiction, we could model dependence from addictive substances as follows. Let A_t denote the

[32]Examples include also Fodor (1983)'s *executive function*, Minski (1985)'s *society of minds*, as well as Tooby-Cosmides (92)'s *specialized modules*; see Monsell-Driver (2000) for a clear methodological discussion on the issues involved in modelling controlled processes, and in particular on the homunculus fallacy.

[33]Even Baumeister-Heatherton-Tice (1996) conclude their encyclopedic analysis of self-regulation failures by reminding the reader that such failures are "the exception, not the rule", pg. 263.

indicator function of the activation of automatic choice, that is

$$A_t = \begin{cases} 1 & \text{for} \quad \lambda_t = \lambda_t^I \\ 0 & \text{for} \quad else \end{cases}$$

We can now construct an index Ψ_t which measures how often an agent lets automatic choice overcome self control:

$$\Psi_t = \frac{\sum_{\tau < t} A_t}{t}$$

Addiction in the model is then captured, e.g., if we let the activation cost b decrease with Ψ_t.

This would represent a complementary but different analysis than Bernheim-Rangel (2001)'s, who study addiction as resulting from the interaction of a rational and visceral self. They support their model with evidence from neurobiology, and identify in the misrepresentation of hedonic experience when the brain operates in the *hot mode* the rationale of consumption patterns which might lead to addiction. While the hot mode operation of the brain is triggered by external cues, in their formulation, they do not consider any internal neurobiological mechanisms which govern the operation mode of the brain (or, in our terminology, which modulate the activation of different controlling pathways and hence the inhibition of temptations). Relatedly, Laibson (2001) models addiction as arising from decisions automatically triggered by external cues, with no space for cognitive control. While we do not doubt the relevance of cues or temptations in inducing impulsive behavior and possibly leading to dependence on addictive substances, it is our contention that the study of the brain mechanisms processing and possibly inhibiting the representations that such cues induce is necessary to understand many important phenomena of decision making and addictions e.g., why the same cues have different effects in different persons and why the same cues have different effects on the same person in different circumstances.

In fact the view that addictive substances operate directly on the mechanisms which regulate the competition of different processing pathways in the brain, that is, on the dopamine gating system, has received recent empirical support in cognitive sciences. In particular, recent neurobiological studies have postulated the existence of a possible generalized dopamine hypofunctionality syndrome subsuming drug addiction, non-chemical behavioral addictions, cocaine dependence, and in general a large class of addictive, impulsive, and compulsive disorders (see Gardner (1999)). Indirect evidence of the importance of regulation of different processing pathways can also be derived from the clinical and neurological observations regarding various psychiatric pathologies. For instance, attention disorders (e.g., Attention Deficit Hyperactivity Disorder, ADHD, in school-aged children) have been associated with dysfunctions of the dopamine-activated brain reward neuronal circuits (Barkley (1997)); and , Cohen-Servan Schreiber (1992) have conceptualized the behavioral deficits associated with schizophrenia as failures of cognitive control 'due to the impaired ability to represent, maintain, and update context information,' and have shown that such deficits can be explained by a form of 'noise' in the dopamine gating system.

In our model, self-control arises from the activation of controlled processing pathways. Why aren't the controlled processing pathways always active ? After all, this would allow the agent never to give way to temptations which motivate behavior he will later regret. In our set-up the controlled processing pathway is in fact always active if $b = 0$. On the contrary, when $b > 0$, consumers save on the controlled processing pathway, by allowing at times automatic processing to govern choice. We could speculate that the reason why the controlled processing pathway is not always active during decision making, and an executive function modulates the activation of different pathways, must represent an evolutionary adaptation in the face of limitations in the computational ability of the brain and especially of active memory. For instance, consider the production-saving model with rate of return shocks, a_t; if the distribution of such shocks has been characterized by a mean close to 0 (and hence by a small variance) in early evolutionary times, brain connections in the prefrontal cortex to guarantee self-control would have been of little fitness gain. We could therefore envision a sequence of adaptive evolutionary steps whereby fast automatic processing, e.g., classical conditioning and Pavlovian responses, evolved first in an environment in which storage naturally represented a second order effect in terms of adaptiveness; and cognitive control evolved later in evolutionary history and subject to the constraints imposed by the architecture of the mammal brain, possibly exploiting the neurological connections for the operation of working memory in the prefrontal cortex. This view, while speculative, is at least consistent with the developmental structure in brain evolution (see e.g., Finlay-Darlington-Nicastro (2001)).

5 Conclusions

By considering only Markovian strategies of a game between successive selves, or by extrapolating from the behavior of addicted individuals, the behavioral economics literature implicitly models agents as lacking any form of internal psychological commitment or self-control in consumption. But only when their frontal cortex is lesioned do agents display no self-control in consumption. Yet patients with lesions in the frontal lobes display odd and impulsive behavior; e.g., they might react to seeing a hammer and nails by hanging a picture on the wall, independently of the social context in which they find themselves (see Lhermitte (1986)); or they might eat enormous quantity of food immediately after claiming of not being hungry (see Pribram (1984); quoted by Loewenstein (1996))).

We interpret our analysis of dynamic choice and self-control as introducing the functions of the frontal cortex in behavioral economics. We conclude that, once neurological and psychological pathologies are excluded, human behavior might be generally much more consistent with standard dynamic choice theory than behavioral economists postulate.

Appendix: Proofs

Proposition 1: Proof. The first order conditions of the maximization problems include:

$$z_t (z_t c_t)^{-\sigma} = \beta E V_1 (a_t k_t - c_t, z_{t+1})$$

$$V_1 (k_t, z_t) = a_t \beta E V_1 (a k_t - c_t, z_{t+1}) = a_t (z_t c_t)^{-\sigma} z_t$$

$$z_t (z_t c_t)^{-\sigma} = \beta E (z_{t+1} c_{t+1})^{-\sigma} a_{t+1} z_{t+1}$$

Let

$$c_t = \lambda_t^I a_t k_t$$

then

$$z_t \left(z_t \lambda_t^I a_t k_t\right)^{-\sigma} = \beta E \left(z_{t+1} \lambda_{t+1}^I a_{t+1} k_{t+1}\right)^{-\sigma} a_{t+1} z_{t+1}$$

$$z_t \left(z_t \lambda_t^I\right)^{-\sigma} = \beta E \left(a_{t+1} z_{t+1} \lambda_{t+1}^I \left(1 - \lambda_t^I\right)\right)^{-\sigma} a_{t+1} z_{t+1}$$

$$z_t^{\frac{-1}{\sigma}} \left(z_t \lambda_t^I\right) = \beta^{\frac{-1}{\sigma}} E \lambda_{t+1}^I \left(1 - \lambda_t^I\right) (a_{t+1} z_{t+1}) (a_{t+1} z_{t+1})^{\frac{-1}{\sigma}}$$

$$\begin{aligned}
\lambda_t^I &= \left(\frac{\beta^{\frac{-1}{\sigma}}}{z_t^{\frac{\sigma-1}{\sigma}}}\right) E\left(\lambda_{t+1}^I \left(1 - \lambda_t^I\right)\right) (a_{t+1} z_{t+1})^{\frac{\sigma-1}{\sigma}} \\
\lambda_t^I &= \gamma z_t^{\frac{1-\sigma}{\sigma}} \left(E \lambda_{t+1}^I \left(1 - \lambda_t^I\right) (a_{t+1} z_{t+1})^{\frac{\sigma-1}{\sigma}}\right)
\end{aligned}$$

where $\gamma = \beta^{\frac{-1}{\sigma}}$

$$\lambda_t^I = \frac{\gamma_t z_t^{\frac{1-\sigma}{\sigma}} \left(E\left(\lambda_{t+1}^I\right) (a_{t+1} z_{t+1})^{\frac{\sigma-1}{\sigma}}\right)}{1 + \gamma z_t^{\frac{1-\sigma}{\sigma}} \left(E\left(\lambda_{t+1}^I\right) (a_{t+1} z_{t+1})^{\frac{\sigma-1}{\sigma}}\right)}$$

$$\begin{aligned}
\lambda_t^I &= \frac{1}{1 + z_t^{\frac{\sigma-1}{\sigma}} \gamma^{-1} \left(E\left(\lambda_{t+1}^I\right) (a_{t+1} z_{t+1})^{\frac{\sigma-1}{\sigma}}\right)^{-1}} = \\
&\frac{1}{\left(1 + z_t^{\frac{\sigma-1}{\sigma}} \gamma^{-1} \left(E\left(\left(1 + z_{t+1}^{\frac{\sigma-1}{\sigma}} \gamma^{-1} \left(E\left(\lambda_{t+2}^I\right) (a_{t+2} z_{t+2})^{\frac{\sigma-1}{\sigma}}\right)^{-1}\right)^{-1}\right) (a_{t+1} z_{t+1})^{\frac{\sigma-1}{\sigma}}\right)^{-1}\right)}
\end{aligned}$$

Define the random variable $\tilde{z}_{t+\tau} = (\alpha_{t+\tau} z_{t+\tau})^{\frac{\sigma-1}{\sigma}}$, for any $\tau \geq 0$, where the sequence $\alpha_{t+\tau}$ satisfies $\alpha_t = 1$, $\alpha_{t+\tau} = a_{t+\tau}$, for any $\tau \geq 1$. Let $E(\tilde{z}_{t+\tau}) = E(\tilde{z})$.

We then guess for a solution of the form:

$$\begin{aligned}
\lambda_t^I &= \frac{E(\tilde{z})^{-1}}{1 + E \sum_{s=t} \Pi_t^s \tilde{z}_s \gamma^{t-s-1}} = \frac{E(\tilde{z})^{-1}}{1 + \gamma^{-1} \tilde{z}_t + \gamma^{-1} \tilde{z}_t E \sum_{s=t+1} \Pi_{t+1}^s \gamma^{t+1-s-1}} \\
&= E(\tilde{z})^{-1} \left(1 + \gamma^{-1} \tilde{z}_t + \gamma^{-1} \tilde{z}_t E \sum_{s=t+1} \Pi_{t+1}^s \gamma^{t+1-s-1}\right)^{-1}
\end{aligned}$$

If the guess is correct,

$$
\begin{aligned}
E\lambda_{t+1}^{I}\tilde{z}_{t+1} &= E\frac{E(\tilde{z})^{-1}\tilde{z}_{t+1}}{1+E\sum_{s=t+1}\Pi_{t+1}^{s}\tilde{z}_{s}\gamma^{t+1-s-1}} = E\left(1+E\sum_{s=t+1}\Pi_{t+1}^{s}\tilde{z}_{s}\gamma^{t-s}\right)^{-1}E(\tilde{z})\ E(\tilde{z})^{-1} \\
&= \left(1+E\sum_{s=t+1}\Pi_{t+1}^{s}\tilde{z}_{s}\gamma^{t-s}\right)^{-1}
\end{aligned}
$$

Substitute into λ_t^I to check:

$$
\begin{aligned}
\lambda_t^I &= \frac{E(\tilde{z})^{-1}}{1+\tilde{z}_t\gamma^{-1}E\left(1+E\sum_{s=t+1}\Pi_{t+1}^{s}\tilde{z}_s\gamma^{t-s}\right)} \\
&= \frac{E(\tilde{z})^{-1}}{1+\tilde{z}_t\gamma^{-1}+\gamma^{-1}\tilde{z}_tE\sum_{s=t+1}\Pi_{t+1}^{s}\tilde{z}_s\gamma^{t-s}}
\end{aligned}
\tag{14}
$$

We conclude that the guess is in fact correct. It follows that $\lambda_t^I = \lambda^I(z_t)$. ■

Proposition 2: Proof. Existence of the value function $D(a_t, k_t, z_t)$ follows by Blackwell's Theorem by a standard argument. Moreover, it is straightfoward to show that $D(a_t, k_t, z_t)$ is increasing in (a_t, k_t). Let the policy function be denoted $\lambda(a_t, k_t, z_t)$. We will first show that the policy function satisfies a cut-off rule, that is:

$$
\lambda(a_t, k_t, z_t) =
$$

$$
\begin{cases}
\lambda^I(z_t) & \text{for} \quad \lambda^I(z_t) \leq \underline{\lambda}(a_t, k_t) \\
\arg\max_\lambda U(\lambda a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda)a_t k_t, z_{t+1}\right)\right] & \text{for} \quad \lambda^I(z_t) > \underline{\lambda}(a_t, k_t)
\end{cases}
$$

Then, we will show that the cut-off, hence the policy function, are independent of (a_t, k_t).

The cut-off rule follows if we can show concavity of $U(\lambda a_t k_t)+\beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda)a_t k_t, z_{t+1}\right)\right]$ with respect to λ. Fix $(a_t,\ k_t)$. Concavity guarantees that

$$
\max_\lambda U(\lambda a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda)a_t k_t, z_{t+1}\right)\right]
$$

has a unique solution, λ^E, independent of the realization z_t. It is also straightforward to see that $\lambda^I > \lambda^E$, for $z_t > 1$. It follows that there exists either a unique $\lambda \geq \lambda^E$ which solves

$$
(1-\sigma)^{-1}\left(\lambda^E a_t k_t\right)^{1-\sigma} + \beta E\left[D\left(a_{t+1}, a_{t+1}\left(1-\lambda^E\right)a_t k_t, z_{t+1}\right)\right] - b(a_t k_t)^{1-\sigma} =
$$

$$
(1-\sigma)^{-1}(\lambda a_t k_t)^{1-\sigma} + \beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda)a_t k_t, z_{t+1}\right)\right]
$$

Let it be denoted $\underline{\lambda}$. By construction, therefore,

$$
\frac{\partial}{\partial\lambda}\left[(1-\sigma)^{-1}(\lambda a_t k_t)^{1-\sigma} + \beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda)a_t k_t, z_{t+1}\right)\right]\right] \leq 0 \text{ at } \lambda = \underline{\lambda}
$$

and $\underline{\lambda}$ is the cut-off for given (a_t,k_t). Since (a_t,k_t) are arbitrary in the argument, we can construct in fact the cut-off $\underline{\lambda}(a_t, k_t)$ of the statement.

We turn now to show the concavity of

$$
U(\lambda a_t k_t) + \beta E\left[D\left(a_{t+1}, a_{t+1}(1-\lambda)a_t k_t, z_{t+1}\right)\right]
$$

with respect to λ. It requires $U'' a_t k_t + \beta E\left[a_{t+1}a_t k_t \frac{\partial^2}{\partial(k_{t+1})^2}D(a_t, k_t, z_t)\right] < 0$, and hence, in turn, $\frac{\partial^2}{\partial(k_{t+1})^2}D(a_t, k_t, z_t) < 0$.

Let $q_t = a_t k_t$. Choose arbitrary concave functions $h, U : R_+ \times R_+ \to R_+$ where $R_+ = [0, \infty)$, that is h, U take non-negative values. In particular, we can choose $U = (1-\sigma)^{-1} c^{(1-\sigma)}$, $0 < \sigma < 1$. Let the operator T defined as follows:

$$(Th)(q_t; z_t) = \max \left[\begin{array}{c} U(\lambda_t^I(a_t, z_t) q_t) + \beta E \left[h \left(a_{t+1} \left(1 - \lambda_t^I(a_t, z_t) \right) q_t, z_{t+1} \right) \right], \\ \max_\lambda U(\lambda q_t) + \beta E \left[h \left(a_{t+1} (1-\lambda) q_t, z_{t+1} \right) \right] - b(q_t)^{1-\sigma} \end{array} \right] \tag{15}$$

To show that $D(a_t, k_t, z_t)$ is concave, it suffices to show that the operator T preserves the concavity of the map h. Let $q = v q_t^1 + (1-v) q_t^2$. From concavity of U and h, it follows that:

$$\begin{aligned} (Th)(q_t; z_t) &\geq \max \left[\begin{array}{c} v \left[U(\lambda_t^I(a_t, z_t) q_t^1) + \beta E \left[h \left(a_{t+1} \left(1 - \lambda_t^I(a_t, z_t) \right) q_t^1, z_{t+1} \right) \right] \right] \\ + (1-v) \left[U(\lambda_t^I(a_t, z_t) q_t^2) + \beta E \left[h \left(a_{t+1} \left(1 - \lambda_t^I(a_t, z_t) \right) q_t^2, z_{t+1} \right) \right] \right], \\ v \left[\max_\lambda U \left(\lambda q_t^1 \right) + \beta E \left[h \left(a_{t+1} (1-\lambda) q_t^1, z_{t+1} \right) \right] - b(q_t)^{1-\sigma} \right] \\ + (1-v) \left[\max_\lambda U \left(\lambda q_t^2 \right) + \beta E \left[h \left(a_{t+1} (1-\lambda) q_t^2, z_{t+1} \right) \right] - b(q_t)^{1-\sigma} \right] \end{array} \right] \\ &\geq \max \left[\begin{array}{c} \max \left[\begin{array}{c} v U(\lambda_t^I(a_t, z_t) q_t^1) + \beta E \left[h \left(a_{t+1} \left(1 - \lambda_t^I(a_t, z_t) \right) q_t^1, z_{t+1} \right) \right], \\ v \left[\max_\lambda U \left(\lambda q_t^1 \right) + \beta E \left[h \left(a_{t+1} (1-\lambda) q_t^1, z_{t+1} \right) \right] - b(q_t)^{1-\sigma} \right] \end{array} \right], \\ \max \left[\begin{array}{c} (1-v) U(\lambda_t^I(a_t, z_t) q_t^2) + \beta E \left[h \left(a_{t+1} \left(1 - \lambda_t^I(a_t, z_t) \right) q_t^2, z_{t+1} \right) \right], \\ (1-v) \left[\max_\lambda U \left(\lambda q_t^2 \right) + \beta E \left[h \left(a_{t+1} (1-\lambda) q_t^2, z_{t+1} \right) \right] - b(q_t)^{1-\sigma} \right] \end{array} \right] \end{array} \right] \end{aligned}$$

The latter follows from $\max(a+b, c+d) \geq \max(a, c, b, d) = \max(\max(a, c), \max(b, d)) \geq 0$ if $a, b, c, d \geq 0$. Therefore,

$$(Th)(q; z_t) \geq \left[v (Th) (q_t^1; z_t) \right] + (1-v) (Th) (q_t^2; z_t) \tag{16}$$

and $(Th)(q_t; z_t)$ is concave.

We turn now to the independence of the policy function from (a_t, k_t). The cut-off $\underline{\lambda}(a_t, k_t)$. solves equation

$$\begin{aligned} & \max_\lambda U(\lambda a_t k_t) + \beta E \left[D \left(a_{t+1}, a_{t+1} (1-\lambda) a_t k_t, z_t \right) \right] - b(a_t k_t)^{1-\sigma} \\ = \; & U(\lambda a_t k_t) + \beta E \left[D \left(a_{t+1}, a_{t+1} (1-\lambda) a_t k_t, z_t \right) \right] \end{aligned}$$

in λ. Consider

$$D(a_t, k_t, z_t) = \max \left[\begin{array}{c} U(\lambda_t^I a_t k_t) + \beta E \left[D \left(a_{t+1}, a_{t+1} \left(1 - \lambda_t^I \right) a_t k_t, z_{t+1} \right) \right], \\ \max_\lambda U(\lambda a_t k_t) + \beta E \left[D \left(a_{t+1}, a_{t+1} (1-\lambda) a_t k_t, z_{t+1} \right) \right] - b(a_t k_t)^{1-\sigma} \end{array} \right]$$

Guess the following functional form for $D(a_t, k_t, z_t)$:

$$D(a_t, k_t, z_t) = M(z_t) (a_t k_t)^{1-\sigma}$$

Then,

$$M(z_t) (a_t k_t)^{1-\sigma} = \max \left[\begin{array}{c} (\lambda_t^I a_t k_t)^{1-\sigma} + \beta E M(z_{t+1}) \left(a_{t+1} \left(1 - \lambda_t^I \right) a_t k_t \right)^{1-\sigma} \\ \max_\lambda (\lambda a_t k_t)^{1-\sigma} + \beta E M(z_{t+1}) \left(a_{t+1} (1-\lambda) a_t k_t \right)^{1-\sigma} - b(a_t k_t)^{1-\sigma} \end{array} \right]$$

$$M(z_t) (a_t k_t)^{1-\sigma} = \max \left[\begin{array}{c} (\lambda_t^I)^{1-\sigma} + \beta E M(z_{t+1}) \left(a_{t+1} \left(1 - \lambda_t^I \right) \right)^{1-\sigma} \\ \max_\lambda (\lambda)^{1-\sigma} + \beta E M(z_{t+1}) \left(a_{t+1} (1-\lambda) \right)^{1-\sigma} - b \end{array} \right] (a_t k_t)^{1-\sigma}$$

$$M(z_t) = \max \left[\begin{array}{c} (\lambda_t^I)^{1-\sigma} + \beta EM(z_{t+1})\left(a_{t+1}\left(1-\lambda_t^I\right)\right)^{1-\sigma} \\ \max_\lambda (\lambda)^{1-\sigma} + \beta EM(z_{t+1})\left(a_{t+1}(1-\lambda)\right)^{1-\sigma} - b \end{array} \right] \tag{17}$$

It follows that the policy function $\lambda(z_t)$ associated with the dynamic program (17) is also the policy function associated with the program (5), and hence is independent of (a_t, k_t). Furthermore, then, the cut-off is also independent of (a_t, k_t), $\underline{\lambda}(a_t, k_t) = \underline{\lambda}$. ■

The **proof of Proposition 3** follows as a simple corollary of Propositions 1 and 2, and is therefore left to the reader.

Proposition 4: Proof. Consider problem (3): its argmax is λ^*. By construction, $\lambda^I(z_t) = \lambda^*$ when $z_t = 1$ and $E(\tilde{z}) = 1$. Assumption 3 guarantees then that $z_t \geq 1$ and $E(\tilde{z}) < E(\tilde{a})$. To conclude then that $\lambda^I(z_t) \geq \lambda^*$, for any a_t, z_t, it is sufficient to prove that $\lambda^I(z_t)$ is increasing in z_t and decreasing in $E(\tilde{z})$. That this is in fact the case can be shown from the closed form solution for $\lambda^I(z_t)$ in Proposition (1).

By Proposition (2),

$$\lambda^E = \arg\max_\lambda \lambda^{1-\sigma} + \beta EM(z_{t+1})\left(a_{t+1}(1-\lambda)\right)^{1-\sigma} \tag{18}$$

The first order conditions of this maximization problem readily imply that λ^E decreases with an increase of $E\left[M(z_{t+1})(a_{t+1})^{1-\sigma}\right]$. Moreover, it is straightforward to show that $E\left[M(z_{t+1})(a_{t+1})^{1-\sigma}\right]$ decreases with b. But λ^* equals λ^E for $b = 0$. We conclude that, for any $b > 0$, $\lambda^E > \lambda^*$.

Finally, $\lambda^I(z_t) > \lambda^E$, for any $z_t > 1$, follows from the characterization of $M(z_t)$ in Proposition 2. ■

Proposition 5: Proof.

We prove the statement, to simplify notation, for an economy with a deterministic technology, $a_t = a$, for any t; the proof generalizes. Given an exogenous process $\lambda_t = \lambda(z_t)$, and letting

$$V(k_t, t) = m_t k_t^{1-\sigma} = (1-\sigma)^{-1}(\lambda_t a k_t)^{1-\sigma} + \beta_t m_{t+1}\left((1-\lambda_t) a k_t\right)^{1-\sigma}$$

we can solve for m_t as follows:

$$\begin{aligned} m_t &= (1-\sigma)^{-1}(\lambda_t a)^{1-\sigma} + E\left((1-\sigma)^{-1}(\lambda_{t+1} a)^{1-\sigma}\right)\left[\beta\left((1-\lambda_t) a\right)^{1-\sigma}\right] \\ &\quad + E\sum_{s=t+2}^{\infty}\left((1-\sigma)^{-1}(\lambda_s a)^{1-\sigma}\right)\left[\prod_{j=t+1}^{s-1}\beta\left((1-\lambda_j) a\right)^{1-\sigma}\right] \\ &= (1-\sigma)^{-1}(\lambda_t a)^{1-\sigma} + \left((1-\sigma)^{-1}(a)^{1-\sigma}\right)\left[\beta\left((1-\lambda_t) a\right)^{1-\sigma}\right] E(\lambda_{t+1})^{1-\sigma} \\ &\quad + E\sum_{s=t+2}^{\infty}\left((1-\sigma)^{-1}(\lambda_s a)^{1-\sigma}\right)\left[\prod_{j=t+1}^{s-1}\beta\left((1-\lambda_j) a\right)^{1-\sigma}\right] \end{aligned}$$

The maximization problem of the agent at time t is therefore:

$$\max \frac{(\lambda_{t-1} z_{t-1} a k_{t-1})^{1-\sigma}}{(1-\sigma)} + \beta E m_t\left((1-\lambda_{t-1}) a k_{t-1}\right)^{1-\sigma}$$

and the first order conditions for the maximization include:

$$(\lambda_{t-1} z_{t-1} a k_{t-1})^{-\sigma} z_{t-1} a k_{t-1} = (1-\sigma)^{-1}\beta E m_t\left((1-\lambda_{t-1}) a k_{t-1}\right)^{-\sigma} a k_{t-1}$$

which can be written as:

$$\lambda_{t-1} = \left(1 + \left((z_{t-1})^{\sigma-1}(1-\sigma)^{-1}\beta E m_t\right)^{\frac{1}{\sigma}}\right)^{-1}. \tag{19}$$

As a consequence, from (19),

$$\frac{d\lambda_{t-1}}{dz_{t-1}} = -\left(1 + \left((z_{t-1})^{\sigma-1}(1-\sigma)^{-1}\beta E m_t\right)^{\frac{1}{\sigma}}\right)^{-2}\frac{1}{\sigma}\left((z_{t-1})^{\sigma-1}(1-\sigma)^{-1}\beta E m_t\right)^{\frac{1-\sigma}{\sigma}}$$
$$\left(-(z_{t-1})^{\sigma-2}\beta E m_t\right) > 0,$$

that is, $\lambda^M(z_t)$ is increasing in z_t.

Condition (19) is of course has the same form as the first order condition of the maximization problem associated to automatic processing, (2), which defines $\lambda^I(z_t)$; the only difference is that the expected future value of the program, Em_{t+1} is different. Let Em^I_{t+1} and Em^M_{t+1} denote, respectively, the expected future value of the program evaluated at the solution of automatic processing and at the Markov perfect Nash equilibrium. Note that $Em^M_{t+1} < Em^I_{t+1}$, since $\lambda^I(z_t)$ by definition maximizes $m_t = m(z_t)$. But, (19) implies that

$$\frac{d\lambda_t}{dEm_{t+1}} < 0,$$

and therefore, $\lambda^M(z_t) > \lambda^I(z_t)$, for any z_t. ■

Proposition 6: Proof. An increase in the first order dominance sense in the distribution of z_t (respectively, in the distribution of a_t) increases by definition $E(z)$ (resp. $E(a)$), and therefore it decreases $E(\tilde{z})$. The statement then follows directly from the closed form solution for $\lambda^I(z_t)$ in Proposition 1. ■

Proposition 7: Proof. Consider dynamic program (17) that, we have shown in the proof of Proposition (2), characterizes $\lambda(z_t)$:

$$M(z_t) = \max\begin{bmatrix} (\lambda^I_t)^{1-\sigma} + \beta E M(z_{t+1})(a_{t+1}\left(1-\lambda^I_t\right))^{1-\sigma} \\ \max_\lambda \lambda^{1-\sigma} + \beta E M(z_{t+1})(a_{t+1}(1-\lambda))^{1-\sigma} - b \end{bmatrix} \tag{20}$$

The characterization of the cut-off rule in Proposition (2) implies that $M(z_t)$ is independent of z_t, for $z_t > \underline{z}$. Moreover $M(z_t)$ is decreasing in z_t, for $z_t \leq \underline{z}$. This is because

$$\lambda^I(z_t) > \arg\max_\lambda \lambda^{1-\sigma} + \beta E M(z_{t+1})(a_{t+1}(1-\lambda))^{1-\sigma}, \text{ for any } z_t > 1$$

and

$$\lambda^{1-\sigma} + \beta E M(z_{t+1})(a_{t+1}(1-\lambda))^{1-\sigma}$$

is concave in λ.

We conclude then that, other things equal, a first order stochastic dominance increase in the distribution of z_t, has the effect of decreasing $EM(z_t)$; an effect which cannot be undone by a change in the cut-off without contradicting the definition of $M(z)$ as a value function, equation (20).

We pass now on to analyze the following problem

$$\arg\max_\lambda(\lambda)^{1-\sigma} + \beta E M(z_{t+1})(a_{t+1}(1-\lambda))^{1-\sigma} \tag{21}$$

which, by Proposition (2) is equivalent to problem

$$\arg\max_{\lambda} U\left(\lambda a_t k_t\right) + \beta E\left[D\left(a_{t+1}, a_{t+1}\left(1-\lambda\right) a_t k_t, z_{t+1}\right)\right]$$

which appears in the statement.

Assumption (4) guarantees the independence of a_{t+1} and z_{t+1}. Therefore problem (21) can be written equivalently,

$$\arg\max_{\lambda}(\lambda)^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda\right)^{1-\sigma}$$

The first order conditions of this maximization problem readily imply that λ increases with a decrease of $EM\left(z_{t+1}\right)$, that is with a first order stochastic dominance increase in the distribution of z_t.

The same first order conditions imply that λ is increasing in of $E\left(a\right)^{1-\sigma}$, that is, with a first order stochastic dominance increase in the distribution of a_t. ■

Proposition 8: Proof. Given b and $EM\left(z_{t+1}\right)$, and using Assumption 4, the cut-off $\underline{\lambda}$ is determined by the following equation:

$$(\underline{\lambda})^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\underline{\lambda}\right)^{1-\sigma} = (\lambda^E)^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda^E\right)^{1-\sigma} - b \tag{22}$$

where $\lambda^E = \arg\max_\lambda \lambda^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda\right)^{1-\sigma}$ depends on b only through $EM(z_{t+1})$. From the definition of $M(z_t)$, equation (20), it follows in a straightforward manner that $EM\left(z_{t+1}\right)$ is decreasing in b. Finally, since $\underline{\lambda} > \lambda^E$ by construction of the cut-off, in Proposition (2), and $(\lambda)^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda\right)^{1-\sigma}$ is concave in λ, it follows that $(\lambda)^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda\right)^{1-\sigma}$ is in fact decreasing in λ at $\lambda = \underline{\lambda}$. The Implicit Function theorem on (22) now implies that $\underline{\lambda}$ is locally increasing in b.

We pass now to characterize the effect of a infinitesimal increase in the first order dominance sense in the distribution of z_t. Let $F(z_t)$ denote the cumulative distribution of z_t. Take a distribution $G(z_t)$ which dominates $F(z_t)$ in the first order stochastic sense, and consider the distribution obtained by mixing $F(z_t)$ with $G(z_t)$:

$$H(z_t) = (1-\alpha)F(z_t) + \alpha G(z_t)$$

Recall that, by an infinitesimal increase in the first order dominance sense in the distribution of z_t we mean an infinitesimal increase $d\alpha > 0$ at $\alpha = 0$.

Since $M(z_{t+1})$ is a continuous function, $d\alpha > 0$ has an infinitesimal negative effect on $EM\left(z_{t+1}\right)$, that is $dEM\left(z_{t+1}\right) < 0$.

Given b and $EM\left(z_{t+1}\right)$ the cut-off $\underline{\lambda}$ is determined by equation (22), where $\lambda^E = \arg\max_\lambda \lambda^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda\right)^{1-\sigma}$. By the Envelope theorem, $(\lambda^E)^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda^E\right)^{1-\sigma}$ is unaffected by any infinitesimal change $dEM\left(z_{t+1}\right)$.

Once again, since $\underline{\lambda} > \lambda^E$ by construction of the cut-off, in Proposition (2), and $\lambda^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda\right)^{1-\sigma}$ is concave in λ, it follows that $\lambda^{1-\sigma} + \beta EM\left(z_{t+1}\right) E(a)^{1-\sigma}\left(1-\lambda\right)^{1-\sigma}$ is in fact decreasing in λ at $\lambda = \underline{\lambda}$. The Implicit Function Theorem on (22) now implies that $\underline{\lambda}$ is locally decreasing in $EM(z_{t+1})$.

Finally, a similar argument proves that an increase in $E(a\ )^{1-\sigma}$ increases the the cut-off $\underline{\lambda}$. But, since the function $f(x) = x^{1-\sigma}$ is increasing in x for $\sigma < 1$, it follows then that an infinitesimal increase in the first order dominance sense in the distribution of a_t has a positive infinitesimal effect on $E(a\ )^{1-\sigma}$. ■

Proposition 9: Proof. We first solve in closed form for $\lambda^I(a_t, z_t)$. conditions of the maximization problem are:

$$z_t\left(z_t c_t\right)^{-\sigma} = \beta a_t E V_1\left(a_t\left(k_t - c_t\right), z_{t+1}\right)$$

$$V_1\left(k_t, z_t\right) = \beta a_t E V_1\left(a_t\left(k_t - c_t\right), z_{t+1}\right) = a_t\left(z_t c_t\right)^{-\sigma} z_t$$

$$z_t\left(z_t c_t\right)^{-\sigma} = \beta a_t E\left(z_{t+1} c_{t+1}\right)^{-\sigma} a_{t+1} z_{t+1}$$

Let

$$c_t = \lambda_t^I a_t k_t$$

then

$$\begin{aligned}
z_t\left(z_t \lambda_t^I a_t k_t\right)^{-\sigma} &= \beta a_t E\left(z_{t+1}\lambda_{t+1}^I a_{t+1} k_{t+1}\right)^{-\sigma} a_{t+1} z_{t+1} \\
z_t\left(z_t \lambda_t^I a_t k_t\right)^{-\sigma} &= \beta a_t E\left(z_{t+1}\lambda_{t+1}^I a_{t+1} a_t\left(k_t - \lambda_t^I a_t k_t\right)\right)^{-\sigma} a_{t+1} z_{t+1} \\
‘z_t\left(z_t \lambda_t^I a_t k_t\right)^{-\sigma} &= \beta a_t E\left(z_{t+1}\lambda_{t+1}^I a_{t+1} a_t k_t\left(1 - \lambda_t^I a_t\right)\right)^{-\sigma} a_{t+1} z_{t+1} \\
z_t\left(z_t \lambda_t^I\right)^{-\sigma} &= \beta a_t E\left(a_{t+1} z_{t+1}\lambda_{t+1}^I\left(1 - a_t \lambda_t^I\right)\right)^{-\sigma} a_{t+1} z_{t+1} \\
z_t^{\frac{-1}{\sigma}}\left(z_t \lambda_t^I\right) &= \left(\beta a_t\right)^{\frac{-1}{\sigma}} E\lambda_{t+1}^I\left(1 - a_t \lambda_t^I\right)\left(a_{t+1} z_{t+1}\right)\left(a_{t+1} z_{t+1}\right)^{\frac{-1}{\sigma}}
\end{aligned}$$

$$\begin{aligned}
\lambda_t^I &= \left(\frac{\left(\beta a_t\right)^{\frac{-1}{\sigma}}}{z_t^{\frac{\sigma-1}{\sigma}}}\right) E\left(\lambda_{t+1}^I\left(1 - a_t \lambda_t^I\right)\right)\left(a_{t+1} z_{t+1}\right)^{\frac{\sigma-1}{\sigma}} \\
\lambda_t^I &= \gamma a_t^{\frac{-1}{\sigma}} z_t^{\frac{1-\sigma}{\sigma}}\left(E\lambda_{t+1}^I\left(1 - a_t \lambda_t^I\right)\left(a_{t+1} z_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\right)
\end{aligned}$$

where $\gamma = \beta^{\frac{-1}{\sigma}}$

$$\begin{aligned}
\lambda_t^I &= \frac{\gamma_t a_t^{\frac{\sigma-1}{\sigma}} z_t^{\frac{1-\sigma}{\sigma}}\left(E\left(\lambda_{t+1}^I\right)\left(a_{t+1} z_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\right)}{1 + \gamma a_t^{\frac{\sigma-1}{\sigma}} z_t^{\frac{1-\sigma}{\sigma}}\left(E\left(\lambda_{t+1}^I\right)\left(a_{t+1} z_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\right)} \\
&= E\frac{\gamma_t a_t^{\frac{\sigma-1}{\sigma}} z_t^{\frac{1-\sigma}{\sigma}}\left(\left(\lambda_{t+1}^I\right)\left(a_{t+1} z_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\right)}{1 + \gamma a_t^{\frac{\sigma-1}{\sigma}} z_t^{\frac{1-\sigma}{\sigma}}\left(\left(\lambda_{t+1}^I\right)\left(a_{t+1} z_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\right)}
\end{aligned}$$

$$\begin{aligned}
\lambda_t^I &= E\frac{1}{1 + z_t^{\frac{\sigma-1}{\sigma}}\gamma^{-1} a_t^{\frac{1-\sigma}{\sigma}}\left(\left(\lambda_{t+1}^I\right)\left(a_{t+1} z_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}} = E\frac{1}{1 + \left(\left(\lambda_{t+1}^I\right)\gamma\left(a_t a_{t+1} z_t^{-1} z_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}} \\
&= E\frac{1}{\left(1 + E\left(1 + \left(\left(\lambda_{t+2}^I\right)\gamma\left(a_{t+1} a_{t+2} z_{t+1}^{-1} z_{t+2}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\right)\gamma^{-1}\left(a_t a_{t+1} z_t^{-1} z_{t+1}\right)^{\frac{1-\sigma}{\sigma}}\right)} \\
&= \frac{1}{\left(1 + E\left(1 + \left(E\left(1 + \left(E\left(\lambda_{t+3}^I\right)\gamma\left(a_{t+2} a_{t+3} z_{t+2}^{-1} z_{t+3}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\right)\gamma^{-1}\left(a_{t+1} a_{t+2} z_{t+1}^{-1} z_{t+2}\right)^{\frac{1-\sigma}{\sigma}}\right)\right)\right.}
\end{aligned}$$

Define the random variable $\tilde{z}_{t+\tau-1} = \gamma^{-1}\left(a_{t+\tau-1}a_{t+\tau}z_{t+\tau-1}^{-1}z_{t+\tau}\right)^{\frac{1-\sigma}{\sigma}}$, for any $\tau \geq 1$. Let $E(\tilde{z}_{t+\tau-1}) = E(\tilde{z})$ for $\tau \geq 2$.We then guess for a solution of the form:

$$\lambda_t^I = \frac{1}{1+E\sum_{s=t}\Pi_t^s\left(\tilde{z}_s\right)^{t+1-s}} = \frac{1}{1+E\left(\tilde{z}_t\right)+E\left(\tilde{z}_t\right)\sum_{s=t+1}\Pi_{t+1}^s\left(E(\tilde{z})\right)^{t+2-s}}$$

where $E\left(\tilde{z}_t\right) = \gamma^{-1}\left(a_t z_t\right)^{\frac{1-\sigma}{\sigma}} E\left(a_{t+\tau}z_{t+\tau-1}^{-1}\right)^{\frac{1-\sigma}{\sigma}}$.If the guess is correct,

$$\lambda_{t+1}^I = \frac{1}{1+E\left(\tilde{z}_{t+1}\right)+E\left(\tilde{z}_{t+1}\right)\sum_{s=t+2}\Pi_{t+2}^s\left(E(\tilde{z})\right)^{t+3-s}} = \left(1+E\left(\tilde{z}_{t+1}\right)+E\left(\tilde{z}_{t+1}\right)\sum_{s=t+2}\Pi_{t+2}^s\left(\tilde{z}_s\right)^{t+3-s}\right)$$

Substitute into λ_t^I to check:

$$\begin{aligned}
\lambda_t^I &= E\frac{1}{1+\left(\left(\lambda_{t+1}^I\right)\gamma\left(a_t a_{t+1}z_t^{-1}z_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}} = E\frac{1}{1+\left(\left(\lambda_{t+1}^I\right)\tilde{z}_t^{-1}\right)^{-1}} \\
&= E\frac{1}{1+\left(\frac{1}{\left(1+E(\tilde{z}_{t+1})+E(\tilde{z}_{t+1})\sum_{s=t+2}\Pi_{t+2}^s(E(\tilde{z}))^{t+3-s}\right)\tilde{z}_t}\right)^{-1}} \\
&= E\frac{1}{1+\left(\left(1+E\left(\tilde{z}_{t+1}\right)+E\left(\tilde{z}_{t+1}\right)\sum_{s=t+2}\Pi_{t+2}^s\left(E(\tilde{z})\right)^{t+3-s}\right)\tilde{z}_t\right)} \\
&= E\frac{1}{1+\left(\tilde{z}_t+\tilde{z}_t E\left(\tilde{z}_{t+1}\right)+\tilde{z}_t E\left(\tilde{z}_{t+1}\right)\sum_{s=t+2}\Pi_{t+2}^s\left(E(\tilde{z})\right)^{t+3-s}\right)} = E\frac{1}{1+\tilde{z}_t++\tilde{z}_t E\sum_{s=t}\Pi_t^s\tilde{z}_s^{-1}}
\end{aligned}$$

We conclude that the guess is in fact correct. It follows that, since $\tilde{z}_t$ contains a_t,that $\lambda_t^I = \lambda^I(z_t, a_t)$.

In this economy, $\lambda(a_t)$ solves:

$$M\left(z_t, a_t\right) = \max\left[\begin{array}{l} (\lambda_t^I)^{1-\sigma}+\beta EM\left(z_{t+1}, a_{t+1}\right)\left(a_t\right)^{1-\sigma}\left(1-\lambda_t^I\right)^{1-\sigma} \\ \max_\lambda \lambda^{1-\sigma}+\beta EM\left(z_{t+1}\right)\left(a_t\right)^{1-\sigma}\left(1-\lambda\right)^{1-\sigma}-b \end{array}\right]$$

and $\underline{\lambda}_t = \underline{\lambda}(a_t)$ solves

$$(\underline{\lambda}_t)^{1-\sigma}+\beta EM\left(z_{t+1}\right)\left(a_t\right)^{1-\sigma}E(a)^{1-\sigma}\left(1-\underline{\lambda}\right)^{1-\sigma} = (\lambda^E)^{1-\sigma}+\beta EM\left(z_{t+1}\right)\left(a_t\right)^{1-\sigma}E(a)^{1-\sigma}\left(1-\lambda^E\right)^{1-\sigma}-b$$

The statements follow from the analysis of these problems along the lines of Propositions 3-7. ■

We leave to the reader the straightforward **proof of Proposition 10**.

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