Product differentiation and endogenous mode of competition*

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Abstract

There exists a continuum of prices between Bertrand and joint-profit maximization prices which can be interpreted as the outcome of a two-stage game where firms first invest to increase product differentiation and then compete in prices. The lower the costs of differentiating their products from each other the more relaxed competition in the product market and the closer firms will be to the collusive outcome of the one-shot game for given degree of differentiation. The higher the costs the harsher competition in the market and the closer to the Bertrand solution of the one-shot game with given degree of differentiation.

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1 Introduction

In industrial organization the concepts of Bertrand competition, Cournot competition and joint profit maximization are often used as benchmark cases which indicate, in decreasing order, the toughness of competition in the market.[1] In this note, we argue that these benchmark concepts, as well as all the possible intermediate cases, can be interpreted as outcomes of a two-stage game where firms first invest to differentiate its product from the rival, and then compete in prices. The lower the investment costs for the firms the higher product differentiation and the more relaxed price competition in the product market. Consider for instance the case where, in the absence of any investment, firms would sell homogenous goods. If firms can costlessly differentiate their products in the first stage they will be able to sell goods which are perceived as independent, and equilibrium prices will be identical to those arising in a situation of joint profit maximization. At the other extreme, if investments require infinite costs, firms will be unable to differentiate their products and the outcome of the game will coincide with the Bertrand solution of the one-shot game. Any intermediate case, including Cournot competition, can arise as the reduced form of this two-stage game, depending on the cost of investing in product differentiation.

This note is related to the literature which interprets the mode of competition in the market as the result of a richer game. The best known example probably lies in the work of Kreps and Scheinkman (1983), who show that competition in quantities gives the same outcome as a model where firms choose capacities and then compete in prices given the capacity constraint determined by the first stage choices.[2]

Klemperer and Meyer (1989) consider a model where firms choose a supply function (i.e., a price-quantity schedule) under demand uncertainty. Different factors might affect the steepness of the equilibrium supply functions. The steeper (flatter) these functions are the more similar the model is to the case of Cournot (Bertrand) competition.[3]

A related branch of the literature includes d'Aspremont, Dos Santos Ferreira and Gerard-Varet (1991), who have showed that the Cournot equilibrium can be interpreted as equivalent

[1] See for instance Sutton (1991).

[2] See Tirole (1988) for a discussion and qualifications.

[3] See also Laussel (1992) and Grant and Quiggin (1996).

to a situation where firms choose quantities and price signals which are converted in the market price by a pricing scheme (such as the "min-pricing"). Holt and Scheffman (1987) have found that the Cournot equilibrium is the result of a game where price changes are notified in advance but price rebates are possible.

The closest works to this paper are Vives (1986) and Maggi (1996).[4] In their papers, firms choose capacity in the first stage of the game but they can extend output beyond capacity in the second stage of the game at an additional cost. If this cost is prohibitive, like in Kreps and Scheinkman (1983), then capacity choices made in the first stage have complete commitment force and fully constrain outputs in the last stage: the Cournot result is found again. At the other extreme, if increasing output beyond capacity does not bring about any additional cost, capacity decisions do not have any commitment value and firms can expand output at will in the last stage: the outcome will be the same as in a Bertrand model. Between these two cases, a continuum of intermediate outcomes is found according to the degree of flexibility in outputs. There exists a close link between these works and the framework proposed by Klemperer and Meyer (1989), because the former can be seen as one where firms compete in supply functions whose slope is determined by the exogenously given flexibility coefficient.

Our approach here is reminiscent of work by Vives and Maggi in at least two respects. First, because the strength of competition depends on (first-stage) investment decisions. Second, because a continuum of results can be obtained as the cost parameter changes. However, with respect to the first point it should be noted that the crucial variable here is not the ability of the firms to commit to their capacity choices but rather their ability to create product differentiation; with respect to the second point, a substantial difference exists because we find that all the intermediate cases between Bertrand pricing and joint profit maximization can be found as equilibrium situations.

This note is also clearly related to a number of other papers which have shed light on the fact that product differentiation relaxes price competition[5]. Although these results are by now well known, it is only fair to remember that the basic insight of this note can be found in those works. Further, Tirole (1988, pp. 216-17) has hinted at a similarity between product differentiation and capacity constraints in affecting the mode of competition: "...when choosing

[4] See also Dixon (1985) for similar formalizations.

[5] d'Aspremont, Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) among the first.

a location, firms try to differentiate themselves from other firms so as to avoid the intense Bertrand competition associated with perfectly substitutable products (in the same way that firms [..] avoid accumulating "too much capacity" in order to soften price competition)." Finally, Klemperer and Meyer (1989) had showed that the higher the degree of product differentiation the steeper the equilibrium supply functions (which in turn makes competition resemble the Cournot competition case).

This paper continues with the description of a simple linear model and the analysis of the results in section 2. Section 3 generalises the results both on the demand side (by considering a whole class of demand functions for differentiated products) and on the cost side. Section 4 concludes the paper.

2 A simple model

Consumers have the following utility function:

$$U = a(q_i + q_j) - (b - g)(\frac{q_i^2}{2} + \frac{q_j^2}{2}) - gq_iq_j + z; \quad a > 0, \quad b > 2g \geq 0, \quad i, j = 1, 2; i \neq j, \quad (1)$$

where z is an outside composite good and q_i, q_j are the quantities of a differentiated good whose industry we want to analyze. We assume that firms 1 and 2 are the only sellers in this industry.[6] Restrictions on parameter values ensure that demand has a positive intercept and that the own price effect is stronger than the cross price effect. Although not explicitly written in the utility function above, the substitution parameter $g = g(x_i, x_j)$ is exogenous to consumers' choice but is an endogenous variable in the model, since it is determined by the investments x_i, x_j of the firms in a way we shall specify below.

Utility maximization implies: $p_i = a - (b - g)q_i - gq_j$, from which we can write the (direct) demand functions as:

$$q_i = \frac{a(b - 2g) - (b - g)p_i + gp_j}{b(b - 2g)}; \qquad i, j = 1, 2; i \neq j. \quad (2)$$

The degree of product differentiation in this model can be indexed by $((b - 2g)/g) \in (0, \infty)$. When g tends to 0 the goods become independent, and product differentiation is the highest. As

[6] We consider a duopoly for ease of exposition, but extending the model to n firms would give the same qualitative results.

g tends to $b/2$, goods become perfect substitutes and product differentiation is nil. These demand functions have the property that aggregate demand in the industry does not change when the substitutability parameter g varies. Therefore, investments by the firms which decrease the value of g increase differentiation between the products but do not affect the size of the market.[7]

We can now turn to the game played by the two firms. In the first stage, firms simultaneously decide on their investment levels x_i, which affect the value of the parameter g as follows:

$$g = \bar{g} - x_i - x_j, \tag{3}$$

where $\bar{g} \in (0, b/2)$ is the ex-ante degree of substitution between the products. Note that there exists a complete positive externality in investments' levels. In this model, investment by a firm only increases the degree of product differentiation and this one is equally beneficial to both firms. This is similar to what would happen in a Hotelling horizontal product differentiation model if a firm moved along the line away from its rival.[8]

The cost of such investments is the same for both firms and is given by $C(x_i) = (kx_i^\gamma)/\gamma$, with $\gamma > 1$ and $k \geq 0$.

In the second stage of the game, firms simultaneously decide on their price. We assume that they have (zero) constant marginal costs of production.

2.1 Benchmark cases: Bertrand and Cournot

To solve the model, we start as usual from the last stage of the game, which will also give the benchmark Bertrand solution for the case where no investments are made.

Firms' profits are given by $\Pi_i = p_i q_i(p_i, p_j)$. By substituting the demand functions given above and maximizing with respect to p_i, one finds the FOCs: $ab - 2ag - 2(b-g)p_i + gp_j = 0$. The last stage reaction functions $R_i(p_j)$ and $R_j(p_i)$ are depicted in Figure 1. Stability and uniqueness are satisfied by $R_i' = 2(b-g)/g > 1 > g/(2(b-g)) = R_j'$.

[7] If market size changed with g then we could still find, say, the equivalence between the equilibrium price of the two-stage game analyzed here and the Cournot price outcome, but we could not extend the equivalence to quantity and profits. This is because with identical prices but a lower g, the firms would sell larger quantities than in the Cournot case. See also section 3 on this point.

[8] The effect is not identical though. In the Hotelling model, moving away from the center of a segment would give an additional advantage to the rival, since it would give it a more central location.

It is also easy to verify that an increase in the value of g raises (lowers) the intercept of R_i (R_j) and decreases (increases) the slope of R_i (R_j). In particular, when g is close to 0, the best reply function R_i tends to the vertical line passing at $(a/2, 0)$, whereas the function R_j tends to the horizontal line passing at $(0, a/2)$. The intersection of the best reply functions moves to $p_i = p_j = a/2$. For higher values of g, the curves shift as indicated above until they reach the extreme case where $g = \bar{g}$, where R_i (R_j) meets the horizontal (vertical) axis in $a(b - 2\bar{g})/(2(b - \bar{g}))$.

The last stage Nash equilibrium prices and profits are given by:[9]

$$p_i^* = \frac{a(b - 2g)}{2b - 3g}, \qquad \Pi_i^* = \frac{a^2(b - 2g)(b - g)}{b(2b - 3g)^2}, \tag{4}$$

where $g = \bar{g} - x_i - x_j$. For the specific case where no investments are made, the solutions above give the benchmark Bertrand case after substitution of $g = \bar{g}$. Therefore, the Bertrand price is: $p_B = (a(b - 2\bar{g}))/(2b - 3\bar{g})$.

Note also that we are using the term "Bertrand" in a broad sense, that is to indicate the outcome of price competition for any given values of $\bar{g}$ and b, and therefore for any given ex-ante degree of product differentiation. The standard Bertrand outcome would correspond to the particular case where $b \to 2g$. In this case, goods are homogeneous and prices tend to zero at equilibrium.

The Cournot solution for any given values of parameters $\bar{g}$ and b can also easily be found by using the inverse demand functions given above and maximizing with respect to q_i. The equilibrium prices and profits for $g = \bar{g}$ are:

$$p_C = \frac{a(b - \bar{g})}{2b - \bar{g}}, \qquad \Pi_C = \frac{a^2(b - \bar{g})}{(2b - \bar{g})^2}. \tag{5}$$

Note that for the case of homogenous good ($b \to 2g$), the solutions above tend to the familiar expressions $p_c = a/3$ and $\Pi_c = a^2/(9b)$.

Finally, the last benchmark case, represented by the joint-profit maximization is given by: $p_M = a/2$ and $\Pi_M = a^2/(4b)$.

[9] Second order conditions are satisfied.

2.2 The investment game

At the first stage of the game, firms maximize their profit $\pi_i(x_i, x_j) = \Pi_i^*(x_i, x_j) - C(x_i)$. Replacing Π_i^* and C_i, recalling that $g = \bar{g} - x_i - x_j$ and taking first derivatives gives the following first order conditions:

$$\frac{\partial \pi_i}{\partial x_i} = \frac{a^2(\bar{g} - x_i - x_j)}{(2b - 3\bar{g} + 3x_i + 3x_j)^3} - kx_i^{\gamma-1} = 0. \tag{6}$$

These expressions implicitly define the first-stage reaction functions $r_i(x_j)$ and $r_j(x_i)$. By differentiating we can find the slopes r_i' and r_j' in the plane (x_i, x_j) as:

$$r_i' = \frac{dx_j}{dx_i}|_{R_i} = -\frac{1 + (k/a^2)x_i^{\gamma-1}F^2(9 + ((\gamma - 1)F/x_i))}{1 + 9(k/a^2)x_i^{\gamma-1}F^2}, \tag{7}$$

$$r_j' = \frac{dx_j}{dx_i}|_{R_j} = -\frac{1 + 9(k/a^2)x_j^{\gamma-1}F^2}{1 + (k/a^2)x_j^{\gamma-1}F^2(9 + ((\gamma - 1)F/x_j))}, \tag{8}$$

where $F \equiv 2b - 3\bar{g} + 3x_i + 3x_j$.

Both reaction functions are negatively sloped [10] and for $k > 0$ we have $r_i' > 1 > r_j'$, which guarantees stability and uniqueness. Figure 2 offers a graphical representation of the reaction functions of the first stage of the game. To draw the picture, we have also made use of the facts that r_i passes through the point $(0, \bar{g})$, that r_j passes through $(\bar{g}, 0)$, and that $r_i = r_j$ for $x_i = x_j$.

It is straightforward to check that $\frac{dr_i'}{dk} > 0$ and that $\frac{dr_j'}{dk} < 0$. This effect is illustrated in Figure 2. When the efficiency of the investment decreases (that is, when it is more costly to differentiate) the intersection of the reaction functions move along the diagonal towards the origin and the firms will select a lower level of investment at equilibrium (x_i, x_j will decrease). This implies that the degree of product differentiation decreases (g rises) which in turn lowers prices at the last stage of the game (see Figure 1).

Note what happens for extreme values taken by the parameter k. When $k = 0$ both reaction functions collapse to the same curve having slope -1. As a result, equilibrium in the investment stage is given by all the points x_i, x_j such that $x_j = \bar{g} - x_i$, whence the ex-post substitution parameter is given by $g = \bar{g} - x_i - (\bar{g} - x_i) = 0$. We know from the analysis of the last stage reaction functions that this entails the equilibrium price $p_i = p_j = p_M = a/2$. This amounts to

[10] Therefore, investments are strategic substitutes, whereas prices are strategic complements.

saying that when firms can costlessy differentiate their product from each other, they will select the highest degree of differentiation and they will manage to get the same profit that they would get if they could collude.

At the other extreme, when $k \to \infty$, both first stage reaction functions r_i and r_j rotate inwards toward the origin so that r_i becomes vertical and r_j horizontal. Their intersection tends to $x_i = x_j = 0$ so that $g = \bar{g}$. This will result in the last stage reaction functions intersecting at the point $p_i = p_j = p_B = (a(b - 2\bar{g}))/(2b - 3\bar{g})$. Again, the intuition is obvious: if firms are infinitely inefficient in their investments, they will not be able to increase product differentiation in the industry beyond its ex-ante level and the equilibrium price of the whole game will coincide with the Bertrand price.[11]

We have therefore established a relationship between k, x (which uniquely determines g) and p which is summarised by Figure 3. Any price outcome at the last stage of the game can therefore be interpreted as if it were determined by a certain investment cost parameter. As an example, consider the Cournot outcome. To find the level of the cost parameter k which would bring about a Cournot price at the equilibrium of the whole game proceed in two steps. First, find the investment values x_C which satisfies $p^*(x_i, x_j) = p_C$. Second, look for the parameter k_c which solves the FOCs of the first stage, namely: $\frac{\partial \pi_i}{\partial x_i}(k, x_C, x_C) = 0$.[12] The first step amounts to solving:

$$\frac{a(b - 2\bar{g} + 4x)}{2b - 3\bar{g} + 6x} = \frac{a(b - \bar{g})}{2b - \bar{g}}. \tag{9}$$

Some algebra shows that the result is given by:

$$x_C = \frac{\bar{g}^2}{2(b + \bar{g})}. \tag{10}$$

The second step of the problem involves solving the following equation:

$$\frac{a^2(\bar{g} - 2x_C)}{(2b - 3\bar{g} + 6x_C)^3} - kx_C^{\gamma - 1} = 0. \tag{11}$$

By replacing the value of x_C and rearranging one obtains:

$$k_C = \frac{a^2 \bar{g}^{3-2\gamma}(b + \bar{g})^3}{b^2(2b - \bar{g})^3(2(b + g))^{2-\gamma}}. \tag{12}$$

[11] Recall we use the "Bertrand" term in a broad sense. Only if goods are ex-ante homogenous, that is if $b \to 2\bar{g}$, do we have that the Bertrand price equals zero.

[12] Equivalently, we could say that we look for the ex-post substitutability parameter g_C which satisfies $p^*(g_C) = p_C$, and then which satisfies the FOC $\frac{\partial \pi_i}{\partial x_i}(k, g_C) = 0$. The two approaches are equivalent.

3 A generalization

In this section we keep the same modeling structure as in the previous example but we generalise the analysis to a system of demand functions which are not necessarily linear, and to a class of functions which describe the cost of increasing product differentiation.

Consider the following system of demand functions for differentiated products:

$$q_i = q_i(p_i, p_j, g) \qquad i, j = 1, 2 \quad i \neq j \tag{13}$$

where q_i is the quantity demanded, p_i, p_j are prices and g a parameter which defines the degree of substitutability between the two goods, with $g \in [\underline{g}, \overline{g}]$ [13]. The following assumptions characterise the restrictions on the demand system:

$$(A1) \qquad q_i^i = \frac{\partial q_i}{\partial p_i} < 0 \quad q_i^j = \frac{\partial q_i}{\partial p_j} > 0 \quad \mid q_i^i \mid \geq \mid q_i^j \mid$$

$$(A2) \qquad q_i^g = \frac{\partial q_i}{\partial g} \leq 0 \quad q_i^{gg} = \frac{\partial^2 q_i}{\partial g^2} \leq 0$$

$$(A3) \qquad q_i^{ig} = \frac{\partial^2 q_i}{\partial p_i \partial g} < 0 \quad q_i^{jg} = \frac{\partial^2 q_i}{\partial p_j \partial g} > 0$$

$$(A4) \qquad q_i^{ii} = \frac{\partial^2 q_i}{\partial p_i^2} \leq 0 \quad q_i^{ij} = \frac{\partial^2 q_i}{\partial p_i \partial p_j} \geq 0 \quad \mid q_i^{ii} \mid \geq \mid q_i^{ij} \mid$$

$$(A5) \qquad If \;\; p_i = p_j, then \;\; q_i = q_j \;\; q_i^i = q_j^j \;\; q_i^{ii} = q_j^{jj} \;\; q_i^g = q_j^g \;\; q_i^{ig} = q_j^{jg} \;\; q_i^{gg} = q_j^{gg}$$

Hence, A1 defines the products as substitutes, A2 refers to the degree of substitutability, A3 guarantees that equilibrium prices rise with product differentiation, A4 is relevant for the concavity of the demand and profit function and implies that the price game is in strategic complements; finally, A5 imposes symmetry on the demand system[14].

[13] The assumptions on the demand side are stated for simplicity directly on the system of demand functions. However we can imagine that the representative consumer's utility is quasi-linear with $U = V(q_i, q_j, g) + I - \sum_i p_i q_i$, where I is income. In this case $q_i(\cdot) = \partial V / \partial q_i$ and the assumptions below can be referred to the features of the function $V(\cdot)$ in the utility function.

[14] Notice that as firms differentiate, symmetry is preserved, as in a Hotelling model and our example in section 2.

If we assume zero (constant) marginal cost, the profit function is:

$$\Pi_i = p_i q_i(p_i, p_j, g)$$

The Nash equilibrium in prices, (p_i^*, p_j^*), is implicitly defined by $\Pi_i^i(p_i^*, p_j^*) = 0, \Pi_j^j(p_i^*, p_j^*) = 0$; the second order conditions hold given A4. Firm i's reaction function has a slope $dp_j/dp_i = -\Pi_i^{ii}/\Pi_i^{ij} > 0$, by A4 (products are strategic complements). Moreover, from A1 and A4 it can be easily established that $\Pi_i^{ii} = 2q_i^i + p_i q_i^{ii} < 0$ and $\Pi_i^{ij} = q_i^j + p_i q_i^{ij} > 0$. Therefore $\mid \Pi_i^{ii} \mid > \mid \Pi_i^{ij} \mid$ and the reaction functions are contraction mappings ensuring uniqueness. Finally, given A5, the unique equilibrium is symmetric.

Consider now the effects of a variation in g on the equilibrium prices. Totally differentiating the first order condition for a Nash equilibrium we get, using the previous notation for partial derivatives:

$$\Pi_i^{ii} dp_i + \Pi_i^{ij} dp_j + \Pi_i^{ig} dg = 0 \tag{14}$$

$$\Pi_j^{ji} dp_i + \Pi_j^{jj} dp_j + \Pi_j^{jg} dg = 0 \tag{15}$$

Comparative statics of the equilibrium prices can be performed by evaluating

$$\frac{dp_i^*}{dg} = -\frac{\Pi_i^{ig}\Pi_j^{jj} - \Pi_i^{ij}\Pi_j^{jg}}{\Pi_i^{ii}\Pi_j^{jj} - \Pi_i^{ij}\Pi_j^{ji}} = -\frac{\Pi_i^{ig}}{\Pi_i^{ii} + \Pi_i^{ij}} < 0 \tag{16}$$

using symmetry (A5), the fact that own effects are stronger than cross effects (A1 and A4) and being $\Pi_i^{ig} = q_i^g + p_i q_i^{ig} < 0$ given A2 and A3. Hence, an increase in product substitutability ($g \uparrow$) reduces equilibrium prices (and vice versa).

Differentiating the equilibrium profits with respect to g and using the envelope theorem we obtain

$$\frac{d\Pi_i}{dg} = \frac{\partial \Pi_i}{\partial g} + \frac{\partial \Pi_i}{\partial p_j}\frac{\partial p_j^*}{\partial g} = p_i q_i^g + q_i p_i^{*g} + p_i q_i^j p_j^{*g} < 0 \tag{17}$$

Hence, the equilibrium profits are decreasing in the degree of substitutability.

We can now turn to the first stage in which the degree of substitutability is chosen by firms through a sunk investment x_i. In line with the symmetric feature of differentiation of our model, we assume that the investment of both firms contribute equally to determine the overall degree of differentiation. Hence, $g = g(x_i + x_j)$ with $g(0) = \overline{g}$ and $g' < 0$; as in section 2, if no firm invests the degree of substitutability is set at the highest (ex-ante) level, while investing both firms are able to increase differentiation, with a complete externality. The cost of the investment is $C_i = C(x_i, \gamma)$ where γ is a parameter that increases the marginal cost of the investment. We assume

$$(A7) \quad C_i^x > 0 \;\; C_i^{xx} > 0 \;\; C_i^{x\gamma} > 0 \;\; C_i(0) = C_i^x(0) = 0$$

The subgame perfect equilibrium in the first stage game requires

$$\frac{d\Pi_i}{dg}\frac{\partial g}{\partial x_i} - \frac{\partial C_i}{\partial x_i} = 0 \tag{18}$$

The comparative statics of the equilibrium investment x_i^* with respect to parameter γ can be studied totally differentiating the system of first order conditions and obtaining:

$$\frac{dx_i^*}{d\gamma} = \frac{C_i^{xx}}{2(g^x)^2(\frac{d^2\Pi_i}{dg^2}) - C_i^{xx}}. \tag{19}$$

A sufficient condition for this expression to be negative is that equilibrium profits are concave in g: $\frac{d^2\Pi_i}{dg^2} < 0$. This seems a reasonable assumption, as it amounts to saying that the marginal impact of an investment in product differentiation is the smaller the more differentiated products are already.[15]

Another sufficient condition for $\frac{dx_i^*}{d\gamma} < 0$ could be expressed by requiring that costs are "convex enough", so as to guarantee that the denominator of the above expression be negative.

Either of these conditions is enough to guarantee that as the marginal cost of differentiation γ increases, the equilibrium investment x_i^* decreases and the substitutability g between products rises. Allowing for a sufficiently wide range of values of parameter γ we can therefore obtain any value of g in equilibrium, replicating the equilibrium prices from the non-cooperative Nash

[15] In turn, a sufficient condition for concavity of the equilibrium profit function is that the equilibrium price function is concave in g (which means that the marginal effect on prices of more differentiation decreeases with product differentiation itself). By differentiating further equation (18) one obtains:

$$\frac{d^2\Pi_i}{dg^2} = \Pi_i^{gg} + p_i^g \Pi_i^{jg} + \Pi_i^j p_i^{gg}.$$

Since the first two terms are negative due to A2 and A3, $p_i^{gg} \leq 0$ guarantees the negative sign of the derivative. In turn, it can be proved that concavity of prices in g would require additional assumptions which also involve the third derivatives of the demand functions.

equilibrium with the ex-ante $\overline{g}$ to the one corresponding to full collusive prices, exactly as showed in the example studied in section 2. Notice that the equilibrium profits follow the same pattern as the equilibrium prices, falling as products become more similar: therefore, shifting γ we would be able to replicate also the pattern of equilibrium profits corresponding to different degrees of competition (Bertrand, Cournot, collusion, etc.). However, in general we cannot replicate through a single parameter γ the overall market configurations (price, quantity, profits) associated with different equilibria (price or quantity setting, collusion, etc.). In order to do that, we need to reduce the dimensions of the problem: if $q_i^g = 0$, as occurs in the linear example of section 2, the total and individual quantity for given prices do not change when the degree of substitutability varies, although the elasticity of demand does vary. In this case the equilibrium quantity is always the same and a single parameter γ is sufficient to map the marginal cost of differentiation into different equilibrium market outcomes, by replicating exactly prices, quantities and profits of the one-shot game with given product differentiation (as in section 2 example).

4 Conclusions

The extent to which firms can differentiate their products (or choose a location away from each other) determines the toughness of price competition at the last stage of the game, in much the same way as the degree of commitments on capacity determined the mode of competition in Vives (1986) and Maggi (1996). In those papers, it is the cost of producing above capacity which is crucial in determining the mode of competition. The lower the parameter which measures that cost the lower the commitment value of capacities chosen in the first period and in turn the closer the prices to the Bertrand solution. Such a parameter determines a continuum of situations which are intermediate cases between Bertrand and Cournot.

In the present paper, it is the cost of product differentiation which determines the mode of competition at the last stage of the game. Firms can invest to increase the degree of differentiation between their products, to relax price competition which occurs in a later stage. The lower the cost of these investments the closer the firms will be to the outcome of a one-shot game where firms maximisize their joint profits. The higher the investment costs, the closer they will be to the outcome of a one-shot game where they choose prices (and sell products whose degree of differentiation is exogenously given). There exists a continuum of solutions between Bertrand pricing and joint profit maximization pricing that can be associated to the cost of investing in product differentiation.

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Appendix for the referees: Conditions to guarantee concavity of the equilibrium profit function in g

To see the conditions for the concavity of the equilibrium profits with respect to g; differentiating further equation (18) to obtain:

$$\frac{d^2\Pi_i}{dg^2} = \Pi_i^{gg} + p_i^g \Pi_i^{jg} + \Pi_i^j p_i^{gg}. \tag{20}$$

While the first two terms are negative due to A2 and A3, we have to evaluate p_i^{gg} in order to sign the derivative. Differentiating (16):

$$\frac{d^2 p_i^*}{dg^2} = -\frac{\Pi_i^{igg}(\Pi_i^{ii} + \Pi_i^{ij}) - \Pi_i^{ig}(\Pi_i^{iig} + \Pi_i^{ijg})}{(\Pi_i^{ii} + \Pi_i^{ij})^2} \tag{21}$$

Consider the terms in the expression above.

$$\Pi_i^{igg} = q_i^{gg} + p_i q_i^{ig} + p_i^g q_i^{ig} \tag{22}$$

$$\Pi_i^{iig} + \Pi_i^{ijg} = 2q_i^{ig} + q_i^{jg} + p_i(q_i^{iig} + q_i^{ijg}) + p_i^g(q_i^{ii} + q_i^{ij}) \tag{23}$$

which cannot be signed using the assumptions above. Sufficient conditions for the concavity of the profit function require therefore to further assume that:

$$(A6) \quad \Pi_i^{igg} < 0 \quad \Pi_i^{iig} + \Pi_i^{ijg} < 0$$

If A6 holds, then the equilibrium price and the profit function are concave in g.

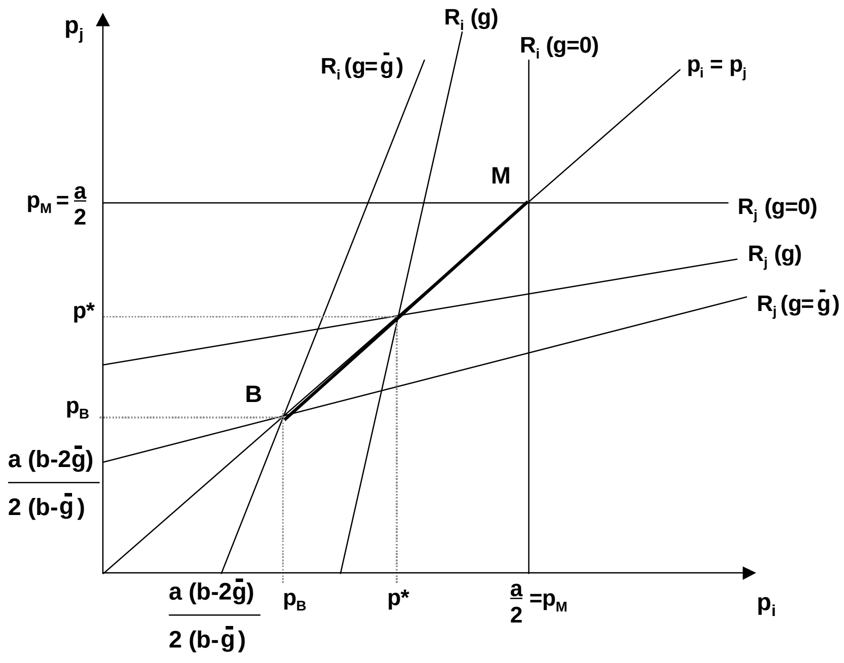

Fig. 1 - Last stage reaction functions R_i and R_j .
Equilibrium prices are comprised bewteen p_B and p_M according to the value of g.

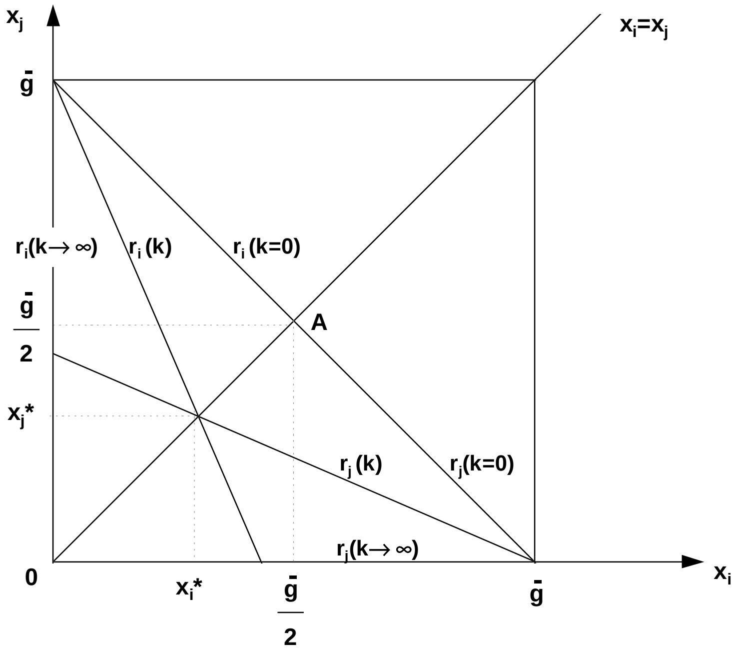

Fig. 2 - First stage reaction functions r_i and r_j .
Equilibrium investments x_i and x_j are the images of the locus OA.

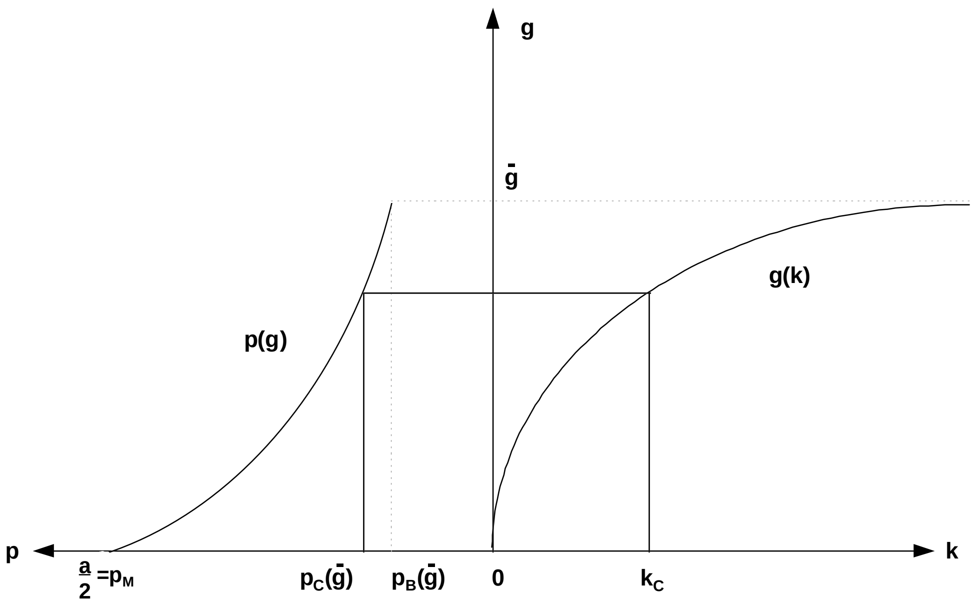

Fig. 3 - When k changes, this modifies investment values X and hence "ex-post differentiation" g. In turn, this modifies equilibrium prices.