Parameters' Instability, Model Uncertainty and Optimal Monetary Policy

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Abstract

Observed policy rates are smooth. Why should central banks smooth interest rates? We investigate if model uncertainty and parameters instability are a valid reason. We do so by implementing a novel "thick recursive modelling" approach within the framework of small structural macroeconomic models. At each point in time we estimate all models generated by the combinations of a base-set of k observable regressors. Our econometric procedure delivers 2^k models for aggregate demand and supply at any point in time. We compute optimal monetary policies for each of these specifications and then take their average as our benchmark optimal monetary policy. We then compare observed policy rates with those generated by the traditional "thin modelling" approach to optimal monetary policy and to our proposed "thick modelling" approach. Our results confirms the difficulty of recovering the deep parameters describing the preferences of the monetary policy makers from their observed behaviour. However, they also show that thick recursive modelling can, at least partially, explain the observed interest rate smoothness.

Keywords: model uncertainty, optimal monetary policy, interest rate smoothing

JEL classification: E44, E52, F41

1 Introduction

The analysis of monetary policy in the framework of small macroeconomic models[1] recently developed in the literature, points clearly towards the importance of interest rate smoothing. Forward-looking Taylor rules, according to which policy rates are made linearly dependent on the deviation of expected inflation from target inflation and on the output gap, need to include the lagged dependent variable in the specification to match the apparent very slow partial adjustment of the policy interest rates. Some preference for interest rate smoothing by central banks, and therefore the explicit inclusion of policy rates volatility in their loss function, is the quick-fix commonly adopted to match this evidence with the parameterization of the economy provided by small models of aggregate demand and supply.

However, why should central banks smooth interest rates?

The direct inclusion of interest rate smoothing in central banks' preference with loose theoretical foundations obviously makes the profession unease. In a recent survey of the empirical literature Sack and Wieland(2000) have discussed three main motives for interest rate smoothing which do not require the direct inclusion of volatility of interest rates in the loss functions of the monetary policy makers.

The first motive is forward-looking expectations. In models with forward expectations, estimated policy rules with inertia are more effective in stabilizing output and inflation for a given level of volatility in the policy instrument. If policy features an high degree of partial adjustment, then forward-looking market participants will expect an initial policy move to be followed by additional moves in the same direction. Such expectations effect

[1] see, for example, Rudebusch-Svensson(1999), Sack(2000), Clarida, Gali and Gertler(2000).

increases the impact of policy on output and in‡ation. Smoothing is then induced by the structure of the economy and there is no need to include some cost in the preferences of central banks to generate the observed behaviour of interest rates.

The second motive is data-uncertainty. According to this motive a moderate responsiveness of interest rate to initial data releases is optimal when the data are measured with errors. In fact, an aggressive policy response would induce unnecessary ‡uctuations in policy rates resulting in unintended ‡uctuations in output and in‡ation.

The third motive is uncertainty about the parameters. This is a revisiting of the classical argument o¤ered by Brainard(1967). When policy-makers are uncertain on the key parameters which determine the transmission of monetary policy in the adopted structural model of the economy, aggressive policy moves are more likely to have unpredictable consequences on output and in‡ation, then gradual policy is optimal to minimize ‡uctuations of output and in‡ation around their targets.

Rudebusch(2000) pushes the argument even further to label monetary policy inertia as an illusion, re‡ecting the episodic unforecastable persistent shocks that central banks face. His views are supported by the empirical evidence from the term structure of interest rates, which does not indicate the large amount of forecastable variation in interest rates at horizons of more than three months that monetary policy inertia would imply.

In this paper we consider the omitted consideration of model uncertainty and parameters instability as the potential source of the observed persistence in interest rates. Model uncertainty has already been considered in a number of papers[2] applying robust control techniques to design interest rate

[2] See Hansen and Sargent(2000), Onatski and Stock(2000), and Tetlow and von zur Muehlen(2001)

policies capable of performing well in presence of model mis-speci…cation. Interestingly, this research …nds that model uncertainty might call for a more aggressive policy stance when policy-makers are guarding against worst-case scenarios.

We consider a di¤erent approach based on "thick recursive modelling" to simultaneously deal with the two problems.

At any point in time we mimic the decision of a monetary policy-maker who sets policy rates on the basis of the available data.

To this end at each point in time, t, we search over a base set of observable k regressors to construct a small structural model of the economy. In each period we estimate a set of regressions spanned by all the possible combinations of the k regressors. We estimate our system equation by equation and we keep the number of regressors k constant for all equations. This gives a total of 2^k di¤erent models for each structural equation. We keep the sample size constant and all models run are based on a sample of twenty-two years of quarterly data. As we keep a …xed window of 88 observations, our method amounts to running a number of rolling regressions, an alternative could be to proceed to a series of recursive regressions[3], in which case at any point in time the size of the sample used for estimation is increased by one observation.

Our econometric procedure delivers 2^k models for aggregate demand and supply at any point in time, therefore the decision of monetary policy requires us to take a stand on model, or speci…cation, uncertainty.

A traditional approach taken in the literature is to proceed to 'thin' modelling by specifying a selection criteria and therefore by selecting the best model in each period. We follow Granger (2000) and label this approach

[3] The use Rolling regressions for forecasting allows more parameters'variability over time than recursive regressions .

‘thin’ modelling in that the optimal monetary policy is described over time by a thin line.

Thin modelling needs to be based on a selection criterion which weights goodness of fit against parsimony of the specification. The literature typically considers BIC, AKAIKE, and the adjusted R^2 as selection criteria.

The advantage of this approach is that a process potentially non-linear is modeled by applying recursively a selection procedure among linear models. The specification procedure mimics a situation in which the specification of aggregate demand and supply is chosen in each period from a pool of potentially relevant regressors.

The main limit of thin modelling is that model, or specification, uncertainty is not considered. In each period the information coming from the discarded $2^k - 1$ is ignored for the design of optimal monetary policy. The explicit consideration of estimation risks naturally generates ‘thick’ modelling, where optimal monetary policy is described by a thick line to take account of the multiplicity of models estimated. The thickness of the line is a direct reflection of the estimation risk. Given the range of all optimal monetary policies, we consider their average to evaluate comparatively the behaviour of policy rates implied by thick and thin modelling.

The paper is structured in four sections. The first section discusses the relevance of parameters instability and model uncertainty in small macroeconomic models of the monetary transmission mechanism. The second section illustrates the differences in the calculation of optimal monetary policy when thin modelling, recursive thin modelling and recursive thick modelling are adopted. The third sections contains the empirical results for the US case, to show to what extent model uncertainty and parameters instability can explain the observed degree of smoothness in monetary policy. The last section concludes.

2 Parameters instability and model uncertainty in small structural models

Recent studies of optimal monetary policy in closed economies have adopted a simple two-equation framework. An aggregate supply equation relates inflation to its lagged values and to current and/or lagged output gap, and an aggregate demand equation relates the output gap to lags of itself and to past real interest rates.

A typical model in this class is the one estimated by Rudebusch and Svensson,1999, who represent the aggregate supply and demand of the economy as follows:

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_2 \pi_{t-1} + \alpha_3 \pi_{t-2} + \alpha_4 \pi_{t-3} + \alpha_5 y_t, \tag{1}$$

$$y_{t+1} = \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 (i_t - \pi_t). \tag{2}$$

The authors estimate the equations using quarterly data over the sample 1961:1 to 1996:4. Inflation, π_t,is calculated as $100 \ast (\log(p_t) - \log(p_{t-4}))$ where p_t is the GDP implicit price deflator, the output gap y_t is obtained as $100 \ast (\log(Q_t) - \log(Q_t^\ast))$ with Q_t, which is the actual GDP (in chained 1996 dollars), and $Q_t^\ast$ the potential GDP, and i_t represents the federal funds rate (our nominal interest rate).

This small structure delivers the constraints under which the reaction function of the central bank is derived by minimizing an intertemporal loss function. Optimal setting of interest rates delivers in general a functional specification resembling a forward-looking Taylor rule. The parameters in the central bank's reaction function are convolutions of the parameters in the structure of the economy and on the parameters describing the preferences of the monetary policy maker. Hence, joint estimation of the simple structure for the economy and the interest rate settings equation allows to evaluate which structure of central bank's preferences delivers a path for policy rates closest to that observed in the data. As discussed in the introduction, the implementation of this framework usually forces the researcher to insert interest rate smoothing among central banks' preferences in order to replicate the observed persistence in the data.

We shall use this simple structural representation to illustrate the importance of parameters instability and model uncertainty for the determination of optimal monetary policy.

2.1 Parameters Instability

We start by replicating the Rudebusch and Svensson results using quarterly data over the period 1961:1-2000:3. Our estimated equations are as follows[4]:

$$\pi_{t+1} = \underset{(0.080)}{0.632}\pi_t + \underset{(0.093)}{0.005}\pi_{t-1} + \underset{(0.094)}{0.214}\pi_{t-2} + 0.149\pi_{t-3} + \underset{(0.033)}{0.140}y_t + \hat{u}_{1,t+1}, \quad (3)$$

$$y_{t+1} = \underset{(0.075)}{1.237}y_t - \underset{(0.075)}{0.309}y_{t-1} - \underset{(0.026)}{0.060}(i_t - \pi_t) + \hat{u}_{2,t+1}. \quad (4)$$

To evaluate potential parameters instability we re-estimate the system by considering two sub-samples.

The first sub-period goes from 1961:1 to 1983:4; estimation delivers the following results:

[4] All the specifications for the supply equation impose the restrictions that the coefficients on the lags of the dependent variable add up to unity.

$$\pi_{t+1} = \underset{(0.106)}{0.705}\pi_t - \underset{(0.129)}{0.018}\pi_{t-1} + \underset{(0.129)}{0.186}\pi_{t-2} + 0.127\pi_{t-3} + \underset{(0.042)}{0.136}y_t + \hat{u}_{1,t+1} \quad (5)$$

$$y_{t+1} = \underset{(0.099)}{1.212}y_t - \underset{(0.099)}{0.300}y_{t-1} - \underset{(0.037)}{0.089}(i_t - \pi_t) + \hat{u}_{2,t+1}. \quad (6)$$

Concentrating instead on the last sub-period 1984:1-2000:2, we obtain:

$$\pi_{t+1} = \underset{(0.117)}{0.331}\pi_t + \underset{(0.119)}{0.043}\pi_{t-1} + \underset{(0.122)}{0.315}\pi_{t-2} + 0.311\pi_{t-3} + \underset{(0.053)}{0.142}y_t + \hat{u}_{1,t+1} \quad (7)$$

$$y_{t+1} = \underset{(0.117)}{1.267}y_t - \underset{(0.117)}{0.300}y_{t-1} + \underset{(0.031)}{0.000332}(i_t - \pi_t) + \hat{u}_{2,t+1}. \quad (8)$$

We take these results as an indication of parameters' instability of a clear economic importance. Consider inflation persistence and the effect of monetary policy on the output gap, two crucial parameters for the design of optimal monetary policy. Although the sum of the coefficients on the lagged dependent variables in the supply equation is restricted to one in all sub-samples, the weight on shorter lags decreases across periods, and consequently, the weight on longer lags increases. Similarly the effect of real interest rates on the output gap in the aggregate demand equation features an important shift from being significantly negative in the first sub-period, with a sizeable long-run effect of about one, to being insignificant in the second sub-period.

Recently, Pesaran and Timmermann(1995) have proposed recursive modelling as an appropriate approach to deal with parameters instability and non-linearity in the context of small models. Consider a monetary policy maker who believes that demand and supply equations can be modelled by projecting output and inflation on macroeconomic indicators but does not know the "true" form of the underlying specification and the "true" parameter values. To keep the macro structure simple and comparable to that of Rudebusch and Svensson, consider a situation in which there is uncertainty only on the specification of the lags with which the relevant variables enter the supply and demand equations. The best option for the policy-maker is to search for a suitable model specification among the set of models believed a-priori appropriate to describe supply and demand. As time elapses, in the presence of potential parameters' instability, such specification might change in the sense that different variables might enter the two equations for demand and supply or the same variables might enter the specification with different coefficients. An open minded policy maker with no strong a-priori belief on the specification of lags in the demand and supply equations would probably like to update the econometric model to base monetary policy on the best possible representation of the unknown Data Generating Process. Therefore, at each point in time, t, the policy maker searches over a base set of k factors or regressors to obtain the best possible specification for output and inflation based on information available at that time. Recursive modelling mimics such decision process by assuming that the policy maker estimates, at each point in time, the entire set of regression models spanned by all the possible permutations of the k regressors and chooses the best one, according to some statistical criteria, to generate optimal monetary policy. Hence in each period the decision is based on the best specification for inflation and output, out of 2^k models for each variable. Given that variables and parameters entering the best chosen specification are allowed to vary over time, recursive modelling is capable of accommodating parameters instability and non-linearity in the effect of some factors on output and inflation. In practice, recursive modelling is implemented by considering the following specification for aggregate demand and supply:

$$M^{AS}_{i,t} \; : \; \pi_t = \beta_0 + \beta_1 \pi_{t-1} + \beta_i' X^1_{t,i} + u^1_{t,i} \tag{9}$$

$$M^{AD}_{i,t} \; : \; y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_i' X^2_{t,i} + u^2_{t,i} \tag{10}$$

where $X^1_{t,i}, X^2_{t,i}$ are $(k_i x1)$ vectors of regressors under model $M^{AS}_{i,t}$,$M^{AD}_{i,t}$ obtained as a subset of the base set of regressors X^1_t, X^2_t:

$$X^{1\prime}_t = \begin{bmatrix} \pi_{t-2} & \pi_{t-3} & \pi_{t-4} & y_t & y_{t-1} & y_{t-2} & y_{t-3} & y_{t-4} \end{bmatrix};$$

$$X^{2\prime}_t = \begin{bmatrix} y_{t-2} & y_{t-3} & y_{t-4} & rr_{t-1} & rr_{t-2} & rr_{t-3} & rr_{t-4} & rr_{t-5} \end{bmatrix}$$

$k_i = e'v_i$; where e is a (kx1) vector of ones and v_i is a (kx1) selection vector, composed of zeros and ones where a one in its j-th element means that the j-th regressor is included in the model. All variables are defined as above and $rr_t = i_t - \pi_t$: The constant and the lagged dependent variable are always included in all specifications. Uncertainty on the specification of lags implies that the policy maker searches over $2^8 = 256$ specifications to select in each period the relevant demand and supply equations. The selection is based on traditional criteria such as adjusted R^2, Akaike Information Criterion, or Schwarz's Bayesian Information Criterion.

2.2 Model Uncertainty

We follow Granger (2000) and label the approach described above as 'thin' recursive modelling in that optimal monetary policy is described over time by a thin line.

As we have already seen, this approach allows to model a process potentially non-linear by applying recursively a selection procedure among linear models and is also capable of accommodating parameters instability. Moreover, keeping track of the selected variables helps the reflection on the economic significance of the 'best' regression.

The main limit of thin modelling is that model, or specification, uncertainty is not considered. In each period the information coming from the discarded $\left(2^k - 1\right) * 2$ models for aggregate demand and supply is ignored for the determination of optimal monetary policy.

This choice seems to be particularly strong.

First, the distance among models, measured by the chosen selection criterion is small. Moreover, the ranking of models according to a within sample performance criterion does not match that obtained by using an out-of-sample forecasting performance criterion. Figure 1-2 make this point by showing the cross-plot of the Adjusted R^2 and the Theil's U for the 256 models of aggregate demand and supply at each possible sample after initialization.

Insert Figures 1-2 here

Clearly the ranking of models according to the adjusted R^2 is not only different but also little correlated with the ranking of models based on the Theil's U. Given that, in the face of the lags with which the policy instruments affect the output gap and inflation, optimal monetary policy has to be based on forecasts for the relevant variables it is not clear at all that the best thin model selected by the adjusted R^2 is the most appropriate to design monetary policy. The first two figures show how hard is to decide among different models of demand and supply. To evaluate the importance of this choice we need to measure the potential relevance of model uncertainty. To this aim we consider the distribution over time of some key parameters in our small structural model, across the di¤erent (256) models of aggregate demand and supply.

Figure 3-4 report the distribution across models of the two crucial parameters determining the e¤ect of monetary policy on in‡ation in our model economy: the e¤ect of the output gap on in‡ation and the e¤ect of real interest rates on the output gap.

Insert Figures 3-4 here

We consider total e¤ects, given by the sum of the coe¢cients on all lags of the relevant variable. Note that a policy-maker, who bases on thin modelling would measure the impact of an interest rate move to real activity and to in‡ation respectively at ¡0:113 and 0:046.

Allowing instead for the potential model uncertainty we end up with the distributions, reported in the graphs.

A natural way to interpret model uncertainty is to refrain from the assumption of the existence of a "true" model and attach instead probabilities to di¤erent possible models. This approach has been labelled 'Bayesian Model Averaging', see, for example, Hoeting J.et al.(1999), and Raftery et al.(1997).

The main di¢culty with the application of Bayesian Model Averaging to problems like ours lies with the speci…cation of prior distributions for parameters in all $2*2^k$ equations to our interest. Recently, Doppelhofer et al.(2000) have proposed an approach labelled 'Bayesian Averaging of Classical Estimates'(BACE) which overcomes the need of specifying priors by combining the averaging of estimates across models, a Bayesian concept, with classical OLS estimation, interpretable in the Bayesian camp as coming from the assumption of di¤use, non-informative, priors.

In practice BACE averages parameters across all models by weighting them proportionally to the logarithm of the likelihood function corrected for the degrees of freedom, using then a criterion analogous to the Schwarz model selection criterion. The results reported in Figure 1-2 show clearly that the ranking of models in terms of their within sample performance does not match the ranking of models in terms of their out-of-sample forecasting performance. In the face of the risk involved in choosing a weighting scheme we opted for the selection method proposed by Granger(2000) of using a '... procedure [which] emphasizes the purpose of the task at hand rather than just using a simple statistical pooling...'. Therefore we derive the optimal monetary policy associated to each specification for the simple aggregate demand-supply system and we then consider the average monetary policy obtained by giving equal weights to each of the alternative monetary policies.

3 Optimal Monetary Policy

To assess the impact of recursive thick modelling we calculate the optimal federal funds rate paths, by applying dynamic optimization techniques, considering the following model choices:

- Thin modelling: Rudebusch, Svensson model;
- Recursive thin modelling: best adjusted R^2 model;
- Recursive thin modelling: best forecasting model (lowest Theil U);
- Recursive thick modelling: average monetary policy.

The central bank minimizes an intertemporal loss function of the form:

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} L_{t+\tau}; \qquad (11)$$

where δ is the discount factor and E_t is the usual expectations' operator. The central bank, thus, minimizes the expected discounted sum of future values of a loss function, L_t, given in each period by:

$$L_t = \lambda_{\pi}\pi^2 + \lambda_y y^2 + \lambda_R (i_t - i_{t-1})^2; \qquad (12)$$

which is quadratic in the deviations of output and inflation from their targets and includes an additional term reflecting a penalty for an excessive volatility of the policy instrument. The parameters λ_{π}, λ_y and λ_R represent the relative weights of inflation stabilization, output gap stabilization and interest rate smoothing objective; these different weights sum to 1.

When the discount factor δ approaches unity, the intertemporal loss function approaches the unconditional mean of the period loss function, which can be also expressed as

$$E[L_t] = \lambda_{\pi} Var[\pi_t] + \lambda_y Var[y_t] + \lambda_R Var[i_t - i_{t-1}]: \qquad (13)$$

We shall solve the optimization problem taking different values for the weights to evaluate which weighting scheme has the best performance in replicating the observed data.

In practice we shall calculate the optimal monetary policy rule in the different described frameworks (Rudebusch-Svensson, "thin" and "thick" modelling), under five alternative specifications for preferences:

- CASE 1. Pure (strict) inflation targeting: $\lambda_\pi = 1$, $\lambda_y = 0$, $\lambda_r = 0$.
- CASE 2. Pure inflation targeting with interest rate smoothing (strong): $\lambda_\pi = 0.8$, $\lambda_y = 0$, $\lambda_r = 0.2$.
- CASE 3. Flexible inflation targeting: $\lambda_\pi = 0.5$, $\lambda_y = 0.5$, $\lambda_r = 0$.
- CASE 4. Flexible inflation targeting with interest rate smoothing: $\lambda_\pi = 0.4$, $\lambda_y = 0.4$, $\lambda_r = 0.2$.
- CASE 5. Pure inflation targeting with interest rate smoothing (weak): $\lambda_\pi = 0.95$, $\lambda_y = 0$, $\lambda_r = 0.05$.

Before going into the details of each case, two problems, relevant when recursive modelling is implemented, are worth mentioning. First, there are specifications in which the question of optimal monetary policy is not worth addressing because monetary policy has no effect on target variables. We then dropped all the specifications featuring a zero effect of interest rates on the output gap and/or a zero effect of the output gap on inflation. Second, thick modelling delivers 256 specifications for aggregate demand and 256 specifications for aggregate supply. When demand and supply are combined in a model the curse of dimensionality is relevant and the total number of possible models becomes $256^2 = 65536.$ To keep the number of models limited we ordered specifications for aggregate demand and supply in terms of performance and generated models by considering aggregate demand and supply equations with the same position in their respective ranking. We therefore considered a number of models equal to the number of specifications for aggregate demand and aggregate supply.

3.1 Thin Modelling

Under thin modelling the optimization problem is solved subject to the dynamics of the economy, which is given by a constant parameter specification of the two stochastic difference equations for demand and supply. We first make use of a standard representation of the economy such as the one adopted by Rudebusch, Svensson (1999) and consisting of two simple empirical relations for inflation and output gap:

$$M^{AS} : \pi_t = \beta_0 + \beta_1 \pi_{t-1} + \beta' X_t^1 + u_t^1 \quad (14)$$

$$M^{AD} : y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma' X_t^2 + u_t^2 \quad (15)$$

$$X_t^{1'} = \left[\pi_{t-2} \quad \pi_{t-3} \quad \pi_{t-4} \quad y_{t-1} \right]$$

$$X_t^{2'} = \left[y_{t-2} \quad r_{t-1} - \pi_{t-1} \right]$$

where the parameters are estimated on the whole available sample and kept constant over time.

As shown in the Appendix, the optimal policy rule, computed by rewriting the model in state-space form and by solving the relevant optimal control problem, can be written as

$$i_t = f \left[\pi_{t-1} \quad y_{t-1} \quad X_t^{1'} \quad X_t^{2'} \right]$$

where f is the optimal feedback vector which depends both on the parameters describing the preferences of the central bank and on the parameters describing the stochastic difference equations for aggregate demand and supply.

All estimated parameters are assumed to be stable over time; the only uncertainty entering the economy consists of additive uncertainty, in the form of additive disturbances entering the model equations. In this case, the certainty-equivalence principle holds: additive uncertainty has no effect over the optimal rule.

Within this framework, the policy-maker is sure about the true model representing the economy. Neither parameter uncertainty nor model uncertainty are relevant within this framework.

3.2 Recursive Thin Modelling

Recursive thin modelling implies that the policy maker investigates much more deeply the constraints under which optimal policy is designed. At any point in time all possible models are estimated and the best, according to some criterion, is chosen. As a new observation becomes available the process is iterated, thus allowing for a different specification of the demand and supply equations. We assume that a rolling window of fixed length is chosen for estimation.

Recursive modelling is implemented by considering the following specification for aggregate demand and supply:

$$M^{AS}_{i;t} \ : \ \pi_t = \beta_0 + \beta_1 \pi_{t-1} + \beta_i' X^1_{t;i} + u^1_{t;i} \tag{16}$$

$$M^{AD}_{i;t} \ : \ y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_i' X^2_{t;i} + u^2_{t;i} \tag{17}$$

where $X^1_{t;i}; X^2_{t;i}$ are $(k_i \times 1)$ vectors of regressors under model $M^{AS}_{i;t}$,$M^{AD}_{i;t}$ obtained as a subset of the base set of regressors $X^1_t; X^2_t$:

$$X^{1\prime}_t = \begin{bmatrix} \pi_{t-2} & \pi_{t-3} & \pi_{t-4} & y_t & y_{t-1} & y_{t-2} & y_{t-3} & y_{t-4} \end{bmatrix};$$

$$X^{2\prime}_t = \begin{bmatrix} y_{t-2} & y_{t-3} & y_{t-4} & rr_{t-1} & rr_{t-2} & rr_{t-3} & rr_{t-4} & rr_{t-5} \end{bmatrix}$$

$k_i = e'v_i$; where e is a (kx1) vector of ones and v_i is a (kx1) selection vector, composed of zeros and ones where a one in its j-th element means that the j-th regressor is included in the model. The constant and the lagged dependent variable are always included in all specifications, the uncertainty on the specification of lags implies that the policy maker searches over $2^8 = 256$ specifications to select in each period the relevant demand and supply equations. All estimated models are then ranked in accordance to a selection criteria and the best model is then chosen. In the light of the evidence proposed in the previous section on the differences in ranking of models when within sample or out-of-sample performance are considered we shall consider ranking models using the adjusted R^2 and the Theil's U as selection criteria. When the best model has been chosen optimal policy is then derived by solving the usual optimal control problem.

As shown in the Appendix, the optimal monetary policy rule takes now the following form:

$$i_t = f_t \left[\pi_{t-1} \quad y_{t-1} \quad X_{t;i}^{1\prime} \quad X_{t;i}^{2\prime} \right]$$

Such optimal rule is time-varying along two dimensions: the size of the coefficients and the set of variables to which monetary policy responds.

3.3 Recursive Thick Modelling

So far optimal monetary policy has been designed at each sample point by estimating all possible models but by optimizing just once, taking the best model as the relevant constraint. As we have discussed, this procedure does not retain information from the other non-selected models.

To implement thick modeling we consider a situation in which the central banker not only estimates all possible models but also derives all the associated optimal monetary policies. Then the adopted monetary policy is the average of all the possible optimal policies.

$$
\begin{aligned}
i_t^* &= \frac{1}{n}\sum_{j=1} i_t^j \\
i_t^j &= f_t^j\left[\pi_{t-1} \quad y_{t-1} \quad X_{t;j}^{1\prime} \quad X_{t;j}^{2\prime}\right]
\end{aligned}
$$

Our effort to take model uncertainty on account differs from the traditional two solutions adopted in the literature to insert the uncertainty into the policy-maker's decision problem, i.e. adding multiplicative (parameter) uncertainty and using robust control techniques.

The consideration of multiplicative (parameter) uncertainty, introduced first by Brainard(1967), implies that the optimal policy rule is also affected by the variances of the estimated parameters, not only by their first moments. The traditional result achieved by this approach is that uncertainty about the model parameters causes attenuation of the central bank's optimal response.[5]

Importantly, the impact of uncertainty on the optimal policy rule is heavily affected by the over-parameterization of the adopted model. In fact uncertainty is empirically much more important in VAR specifications of aggregate demand and supply than in parsimonious small structural models a-la-Rudebusch-Svensson. We feel rather uneasy with these empirical results, in that any match between observed and optimal policy rates could be achieved by augmenting the specification of the relevant constraints with the necessary number of statistically insignificant factors.

Robust control (see for example Onatski and Stock (2000)) assumes that the policy-maker plays a game against a malevolent Nature and tries to minimize the maximum possible loss (minimize the loss in the worst-case state), whereas his opponent, Nature, tries to maximize his loss.

[5] For a recent revisitation of this result see Sack (2000) and Söderström (1999a, 1999b).

In the Onatski, Stock paper, there is a model M, which is known to be an approximation of the true model of the economy, with an unknown deviation from the true model Δ, belonging to the set of perturbations D. Being K the set of policy rules and $R(K; M + \Delta)$, the risk of policy K when the real model is $M + \Delta$, the robust control problem is given by:

$$\min_{\{K\}} \sup_{\Delta \in D} R(K; M + \Delta).$$

The robust solution to this problem is very different from the multiplicative uncertainty case: now the consideration of uncertainty can induce the policy-maker to a more aggressive policy than in the perfect certainty state, in order to minimize the welfare loss in the worst case alternative.

Our approach concentrates on model uncertainty, in its simplest possible form, i.e. uncertainty on the specification of the relevant dynamics, to evaluate its potential for explaining interest rate smoothness.

4 Empirical Results.

Our empirical results are summarized in Table 1.

We consider five possible parameterizations of the loss function and four modelling strategies: thin modelling (adopting the parameterization in Rudebusch-Svensson), recursive thin modelling using a within sample performance selection criterion (best adjusted R^2), recursive thin modeling using an out-of-sample performance selection criterion (Theil's U), and recursive thick modeling based on the choice of the average optimal policy across all different possible models.

Therefore, we end up with 20 optimal federal funds rate series, to be compared with the observed one. Table 1 reports the first two moments of the simulated and observed series.

The results clearly show that observed monetary policy is nowhere near to optimal monetary policy when no weight is attached to interest rate smoothing in the loss function of the policy maker.

When some weight to interest rate smoothing is allowed three optimal policy rate series feature first two moments comparable to those shown by the actual policy rates.

In fact the mean of 6.26 with a standard deviation of 1.98 featured by the actual policy rates are most closely replicated by the optimal policy rates obtained with (i) thin modelling and weights $\lambda_{\pi} = 0.8; \lambda_y = 0; \lambda_r = 0.2$; (mean 7.82, standard deviation 2.85), (ii) thin modelling and weights $\lambda_{\pi} = 0.4; \lambda_y = 0.4; \lambda_r = 0.2$;(iii) thick modelling with weights $\lambda_{\pi} = 0.95; \lambda_y = 0; \lambda_r = 0.05$.

On the negative side these results confirm the difficulty of recovering the deep parameters describing the preferences of the monetary policy makers from their observed behaviour. This is because optimal policy depends both on the parameters describing the preferences of the policy maker and on those defining the structure of the economy. Model uncertainty and parameters' instability imply very low precision in the estimation of the structure of the economy and therefore the observational equivalence of optimal policy rates generated by different preference parameters.

On the positive side thick recursive modelling is capable of rationalizing the observed interest rate smoothness allowing for a much smaller weight on interest rate smoothing in the central bank preferences. In other words, model uncertainty and parameters instability are capable of explaining a sizeable portion of the degree of interest rate smoothing observed in actual policy rates.

5 Conclusions

Observed policy rates are smooth. The derivation of observed rates as optimal by solving the intertemporal optimization of the policy makers under the constraints given by small structural models of aggregate demand and supply requires the attachment of some weight to interest rate smoothing in central bank's preferences.

This paper starts from the observation that parameters' instability and model uncertainty are very relevant in the speci…cation of the constraints under which the monetary policy maker operates. We then analyze explicitly if an optimal control methodology which takes these two aspects on accounts can deliver the observed degree of interest rate smoothing without including interest rate volatility in the central bank loss function.

We implemented "thick recursive modelling" to simultaneously deal with the two problems.

At any point in time we mimic the decision of a monetary policy-maker who sets policy rates on the basis of the available data.

To this end at each point in time, t, we search over a base set of observable k regressors to construct a small structural model of the economy. In each period we estimate a set of regressions spanned by all the possible permutations of the k regressors. We estimate our system equation by equation and we keep the number of regressors k constant for all equations.

Our econometric procedure delivers 2^k models for aggregate demand and supply at any point in time, therefore the decision of monetary policy requires us to take a stand on model, or speci…cation, uncertainty. We do so by computing all optimal monetary policies, and by then taking their average as our benchmark optimal monetary policy.

We then compare observed policy rates with those generated by the traditional "thin modelling" approach to optimal monetary policy and to our proposed "thick modelling" approach.

Our results con…rm the di¢culty of recovering the deep parameters describing the preferences of the monetary policy makers from their observed behaviour. However, they also show that thick recursive modelling is capable of rationalizing the observed interest rate smoothness allowing for a much smaller weight on interest rate smoothing in the central bank preferences.

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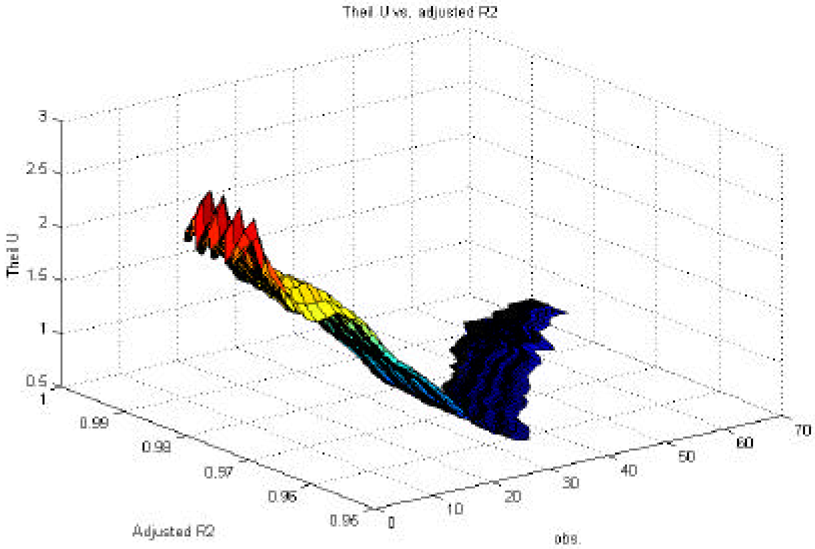

Figure 1: Theil U vs. adjusted R2: cross-plot across every possible model of aggregate supply at each sample period.

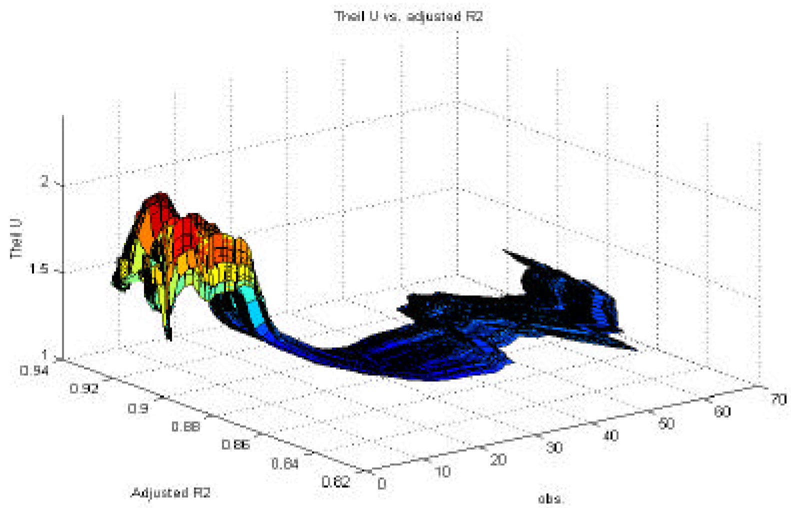

Figure 2 - Theil U vs. adjusted R2: cross-plot across every possible model of aggregate demand at each sample period.

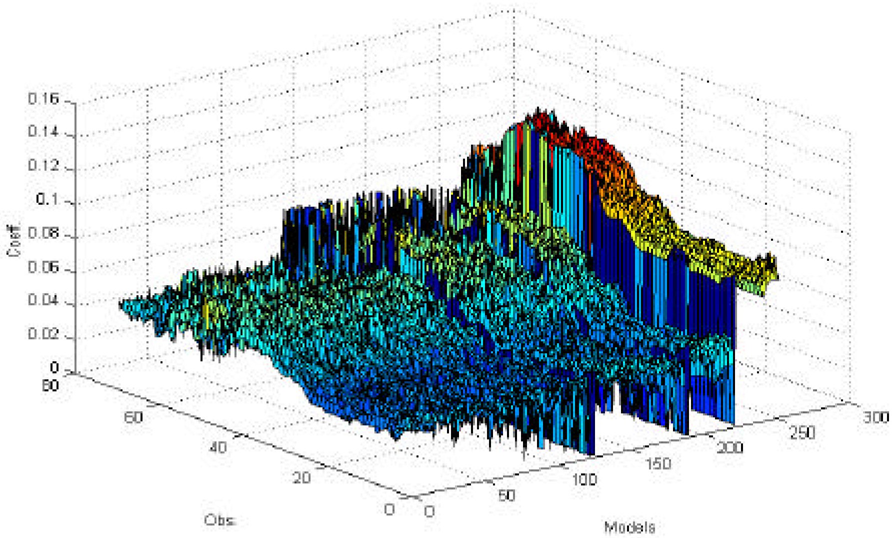

Figure 3 - Short term output gap e¤ect over in‡ation across models and observations.

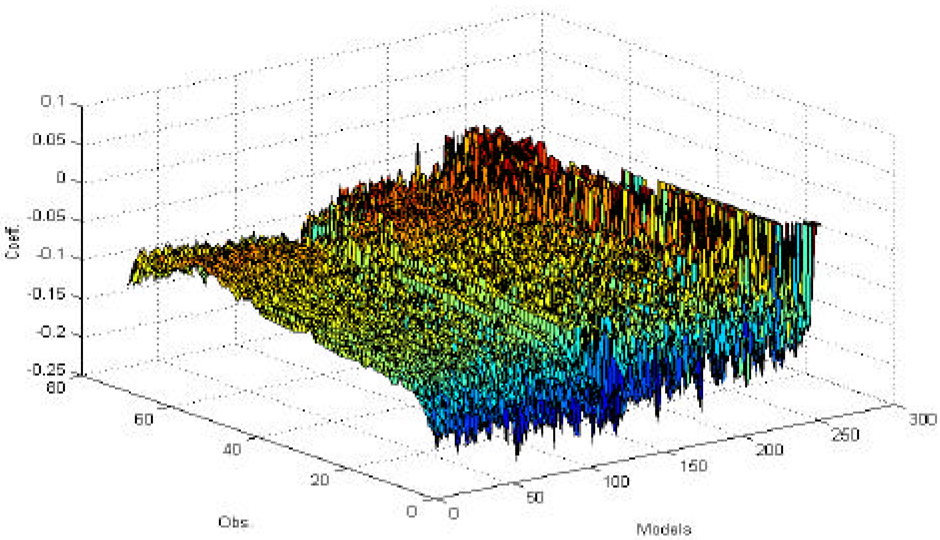

Figure 4 - Short term real interest rate e¤ect over output across models and observations.

Table 1 - Optimal and actual federal funds rate paths: descriptive statistics.					
Loss Function	Thin	Best R^2	Best U	Thick	actual FF
$L_t = \lambda_\pi \pi^2 + \lambda_y y^2 + \lambda_R (i_t - i_{t-1})^2$	Mean / Std	Mean / Std	Mean / Std	Mean / Std	Mean / Std
$\lambda_\pi = 1; \lambda_y = 0; \lambda_r = 0$	337.36	228.70	548.77	521.56	6.26
	101.57	148.98	602.52	237.71	1.98
$\lambda_\pi = 0.8; \lambda_y = 0; \lambda_r = 0.2$	7.82	3.77	2.47	2.62	6.26
	2.85	1.39	3.17	1.36	1.98
$\lambda_\pi = 0.5; \lambda_y = 0.5; \lambda_r = 0$	43.73	6.83	35.81	6.79	6.26
	30.61	7.73	95.64	14.56	1.98
$\lambda_\pi = 0.4; \lambda_y = 0.4; \lambda_r = 0.2$	6.51	2.48	1.02	1.26	6.26
	2.76	2.02	2.98	1.56	1.98
$\lambda_\pi = 0.95; \lambda_y = 0; \lambda_r = 0.05$	12.88	8.21	5.90	6.24	6.26
	4.52	2.88	6.08	3.02	1.98

The Table reports the first two moments of observed interest and optimal policy rates derived by implementing four alternative modelling strategies: thin modelling(thin), thin modelling by implementing a within-sample performance as a selection criterion(Best R^2), recursive thin modelling by implementing an out-of-sample performance as a selection criterion(Best U);and thick modelling by taking the average optimal rate across all possible models.

A Appendix: The optimal control problem

In this appendix we illustrate explicitly the derivation of the solution of the central bank's optimization problem under all the different modelling strategies adopted in the paper.

A.1 Thin modelling

Assume that the central bank minimizes an intertemporal loss function of the form:

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} L_{t+\tau}; \tag{18}$$

where β is the discount factor and E_t is the usual expectations' operator. The central bank, thus, minimizes the expected discounted sum of future values of a loss function, L_t, given in each period by:

$$L_t = \lambda_{\pi}\pi^2 + \lambda_y y^2 + \lambda_R (i_t - i_{t-1})^2; \tag{19}$$

which is quadratic in the deviations of output and inflation from their target values and includes an additional term reflecting a penalty for an excessive volatility of the policy instrument. The parameters λ_{π}, λ_y and λ_R represent the relative weights of inflation stabilization, output gap stabilization and interest rate smoothing objective; they sum to 1.

When the discount factor β approaches unity, the intertemporal loss function approaches the unconditional mean of the period loss function, which can be also expressed as

$$E[L_t] = \lambda_{\pi} Var[\pi_t] + \lambda_y Var[y_t] + \lambda_R Var[i_t - i_{t-1}]: \tag{20}$$

The discussed optimization problem is then solved subject to the dynamics of the economy, which is usually given by stochastic difference equations. We first make use of a standard representation of the economy like the one employed by Rudebusch, Svensson (1999) and consisting of two simple empirical relations for inflation and output gap:

$$\pi_{t+1} = \beta_0 + \beta_1 \pi_t + \beta' X^1_{t+1} + u^1_{t+1} \tag{21}$$

$$y_{t+1} = \gamma_0 + \gamma_1 y_t + \gamma' X^2_{t+1} + u^2_{t+1}, \tag{22}$$

where π, y stand for the inflation rate and the output gap, respectively, and X^1_{t+1}, X^2_{t+1} correspond to the following regressors

$$X^{1'}_{t+1} = [\pi_{t-1}\ \pi_{t-2}\ \pi_{t-3}\ y_t] \tag{23}$$

$$X^{2'}_{t+1} = [y_{t-1}\ \ i_t - \pi_t], \tag{24}$$

β', γ' are vectors of parameters which we can express as

$$\beta' = [\beta_2\ \ \beta_3\ \ \beta_4\ \ \beta_5] \tag{25}$$

$$\gamma' = [\gamma_2\ \ \gamma_3]. \tag{26}$$

Finally, u^1_{t+1}, u^2_{t+1} are iid shocks with variances $\sigma^2_{u^1_{t+1}}$, $\sigma^2_{u^2_{t+1}}$.

In order to calculate the optimal policy rule, it is convenient to rewrite the model in state-space form, as

$$X_{t+1} = AX_t + Bi_t + \varepsilon_{t+1}. \tag{27}$$

X_t is the vector of state variables $[\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_t, y_{t-1}]$, i_t is the policy instrument (the federal funds rate) and ε_{t+1} is a vector of structural shocks. A and B are parameters matrices, with dimensions 6×6 and 6×1 respectively, given by:

$$A = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \gamma_3 & 0 & 0 & 0 & \gamma_1 & \gamma_2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\gamma_3 \\ 0 \end{bmatrix} \tag{28}$$

The loss function can now be rewritten as:

$$L_t = X_t' Q X_t; \tag{29}$$

where Q is the 6×6 weights matrix, with λ_π, λ_y as elements $(1;1)$ and $(5;5)$, respectively, and zeros elsewhere. The central bank solves the optimal control problem

$$J(X_t) = \min_{i_t} \{X_t' Q X_t + \phi E_t J(X_{t+1})\}; \tag{30}$$

subject to the laws of evolution of the economy (21) and (22). After deriving the first-order condition for the minimization problem, we have that the solution for the optimal interest rate is

$$i_t = f X_t; \tag{31}$$

where f is the optimal feedback vector given by

$$f = -(R + \phi B' V B)^{-1} \phi B' V A; \tag{32}$$

and the matrix V is obtained as the solution of the following Riccati equation:

$$V = Q + \phi (A + Bf)' V (A + Bf) + f' R f; \tag{33}$$

where R incorporates the interest rate smoothing objective. We obtain that the central bank sets the optimal policy instrument value in every period as a function of the current and lagged values of the state variables as well as lagged values of the instrument itself.

Given this optimal policy rule, the dynamics of the relevant variables is defined as follows:

$$X_{t+1} = M X_t + \varepsilon_{t+1}; \tag{34}$$

with the matrix M given by

$$M = A + Bf: \tag{35}$$

A.2 Recursive thin modelling

We consider now the following representation for aggregate supply and demand equations:

$$\pi_{t+1} = \beta_0 + \beta_1 \pi_t + \beta_i' X^1_{t+1,i} + u^1_{t+1,i}, \tag{36}$$

$$y_{t+1} = \gamma_0 + \gamma_1 y_t + \gamma_i' X^2_{t+1,i} + u^2_{t+1,i}. \tag{37}$$

where $X^1_{t+1} = [\pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_{t+1}, y_t, y_{t-1}, y_{t-2}, y_{t-3}]$,
$X^2_t = [y_{t-1}, y_{t-2}, y_{t-3}, rr_t, rr_{t-1}, rr_{t-2}, rr_{t-3}, rr_{t-4}]$.
In each period only a subset of regressors is selected. The parameters' vectors are given by

$$\beta_i' = [\beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9] \tag{38}$$

$$\gamma_i' = [\gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7, \gamma_8, \gamma_9]. \tag{39}$$

Re-writing the system in state-space form, we have:

$$i^1_{t+1} X_{t+1} = M^1_{t+1} X_t + W^1_{t+1} i_t + \varepsilon_{t+1}, \tag{40}$$

where

$$X_t = [c, \pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_{t-4}, y_t, y_{t-1}, y_{t-2}, y_{t-3}, i_{t-1}, i_{t-2}, i_{t-3}, i_{t-4}] \tag{41}$$

X_t is the 14×1 vector of state variables including a constant, current and lagged values of inflation, current and lagged values of the output gap and lagged values of the nominal interest rate (the federal funds rate). The central bank's policy instrument is denoted by i_t, whereas ε_{t+1} is the vector of shocks.

Here, the matrices M and W are, not invariant over time. They are, in fact, characterized by the subscript t, $t = 1, ..., 70$, which indicates the period to which they refer. The superscript 1 stands for the ranking of the selected model. In each period models are ranked in accordance to some selection criterion and the best model is selected.

As the economy is recursively estimated, the parameter matrix M^1_t, with dimension 14×14, contains the coefficients obtained for the corresponding period t. This matrix has the second and the seventh rows in period t, $t = 1, ..., 70$, given by:

$$\left[\beta_0^{t,1}, \beta_1^{t,1}, \beta_2^{t,1}, \beta_3^{t,1}, \beta_4^{t,1}, 0, \beta_6^{t,1}, \beta_7^{t,1}, \beta_8^{t,1}, \beta_9^{t,1}, 0, 0, 0, 0\right]; \tag{42}$$

$$\left[\gamma_0^{t,1}, 0, 0, 0, 0, 0, \gamma_1^{t,1}, \gamma_2^{t,1}, \gamma_3^{t,1}, \gamma_4^{t,1}, \gamma_6^{t,1}, \gamma_7^{t,1}, \gamma_8^{t,1}, \gamma_9^{t,1}\right] \tag{43}$$

with zeros and occasional ones in the other places; the βs represent the parameters of the inflation equation, whereas the γs are those in the output gap relation. W_t^1 is a 14×1 parameter vector with elements:

$$\left[0, 0, 0, 0, 0, 0, \gamma_5^{t,1}, 0, 0, 0, 1, 0, 0, 0\right]. \tag{44}$$

The matrix Γ_{t+1}^1 is inserted to account for the simultaneity between output gap and inflation, it has ones on the diagonal and zeros in every place other than position $(2, 7)$ where we have the parameter $-\beta_5^{t,1}$.

Then, we find $A_t^1 = (\Gamma_t^1)^{-1} M_t^1$ and $B_t^1 = (\Gamma_t^1)^{-1} W_t^1$ obtaining the usual representation:

$$X_{t+1} = A_{t+1}^1 X_t + B_{t+1}^1 i_t + \varepsilon_{t+1}. \tag{45}$$

We thus have that the parameters are allowed to change over time and, as a consequence, also the derived optimal rule has varying optimal coefficients over time.

We end up with an optimal monetary policy rule of the form:

$$i_t = f_t^1 X_t, \tag{46}$$

with the superscript 1 as we are considering the best model, $t = 1, \ldots, 70$, and the feedback vector f expressed as

$$f_t^1 = -(R + \delta B_t^{1\prime} V B_t^1)^{-1} \delta B_t^{1\prime} V A_t^1, \tag{47}$$

which is now a 70×14 matrix since the 14 optimal coefficients are recalculated in every period.

A.3 Recursive thick modelling

Here we derive, as usual, the optimal policy rule, characterized by recursive optimal coefficients, for each possible model.

The minimization problem is subject to the constraint given by the dynamics of the economy

$$X_{t+1} = A^j_{t+1} X_t + B^j_{t+1} i_t + \varepsilon_{t+1}, \tag{48}$$

with t indicating the observations from 1983:01 to 2000:02 and where j is the superscript relative to the model employed. We estimate 255 models coming from every possible combination of the different regressors; however, we exclude from this set of models those not incorporating an effect of monetary policy on output gap and inflation. We end up with a set of 241 relevant models; thus we are considering $j = 1, \ldots, 241$. The matrices A^j_{t+1} and B^j_{t+1} are calculated as $A^j_{t+1} = (\Gamma^j_{t+1})^{-1} M^j_{t+1}$ and $B^j_{t+1} = (\Gamma^j_{t+1})^{-1} W^j_{t+1}$.

The matrix M^j_t has the second and the seventh rows in period t, $t = 1, \ldots, 70$, and for every estimated model j, $j = 1, \ldots, 241$, given by:

$$\left[\beta^{t,j}_0, \beta^{t,j}_1, \beta^{t,j}_2, \beta^{t,j}_3, \beta^{t,j}_4, 0, \beta^{t,j}_6, \beta^{t,j}_7, \beta^{t,j}_8, \beta^{t,j}_9, 0, 0, 0, 0\right], \tag{49}$$

$$\left[\gamma^{t,j}_0, 0, 0, 0, 0, 0, \gamma^{t,j}_1, \gamma^{t,j}_2, \gamma^{t,j}_3, \gamma^{t,j}_4, \gamma^{t,j}_6, \gamma^{t,j}_7, \gamma^{t,j}_8, \gamma^{t,j}_9\right] \tag{50}$$

with zeros and occasional ones in the other places; the βs represent the parameters of the inflation equation, whereas the γs are those in the output gap relation. W^j_t is a 14×1 parameter vector with elements:

$$\left[0, 0, 0, 0, 0, 0, \gamma^{t,j}_5, 0, 0, 0, 1, 0, 0, 0\right]. \tag{51}$$

The matrix Γ^j_{t+1} accounts for the simultaneity between output gap and inflation and has parameter $-\beta^{t,j}_5$ in position $(2, 7)$.

The optimal policy rule is:

$$i^j_t = f^j_t X_t, \tag{52}$$

where f_t^j is now a $70 \times 14 \times 241$ matrix, as it reports parameters resulting from every specification.

We implement thick modelling by calculating the average optimal monetary policy:

$$i_t^* = \frac{1}{241}\sum_{j=1}^{241} i_t^j. \qquad (53)$$

Thus the optimal federal funds rate selected in every period by the central bank is the average of all the possible optimal decisions, which would have been taken under the several possible models of the economy.