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The Generalized Neoclassical Growth Model

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Abstract

We construct and numerically solve a dynamic Heckscher-Ohlin model which, depending on the distribution of production factors in the world and parameter values, allows for worldwide factor price equalization or complete specialization. We explore the dynamics of the model under different parameter values, and relate our theoretical results to the empirical literature that studies the determinants of countries' income per capita growth and levels. In general, the model is capable of generating predictions in accordance with the most important findings in the empirical growth literature. At the same time, it avoids some of the most serious problems of the (autarkic) neoclassical growth model.

Keywords: International Trade, Heckscher-Ohlin, Economic Growth, Convergence, Simulation.

JEL codes: F1, F4, O4.

1 Introduction

This paper revisits neoclassical growth economics from a theoretical perspective. In contrast, however, with its dominant "autarkic" approach, we let countries (that is, Ramsey agents) trade with each other intratemporally as they consume and accumulate capital. Previous work in this area, e.g. Ventura (1997) and Cuñat and Maffezzoli (2001), suggests that the neoclassical growth model open to trade can produce much richer dynamics and cross-sectional results than expected from its autarky versions.[1] This is the line we pursue here: we set up a more general neoclassical growth model along the lines of Cuñat and Maffezzoli (2001),[2] and explore the kinds of results one can obtain with such a model.

A key parameter in determining the dynamics of the neoclassical growth model is the intratemporal elasticity of substitution between factors of production, since it affects the relationship between capital-labour ratios and factor prices. Under autarky, for example, the intratemporal elasticity of substitution affects the extent to which cross-country differences in capital-labour ratios create cross-country differences in rental rates to capital through the importance of diminishing returns to capital; by affecting the rental rate of capital, it also influences the transitional dynamics and the speed of convergence to the steady states.[3] Under factor price equalization obtained through international trade in commodities, the intratemporal elasticity of substitution can make the cross-section of countries' growth rates of capital-labour ratios display a convergence or divergence pattern by affecting the path of factor prices.[4]

Our framework can display these two types of intuitions at the same time: under international trade in commodities, countries may specialize in different ranges of goods and group in so-called diversification cones. This enables us to allow for factor-price differences between diversification cones and for factor price equalization within diversification cones. We analyze the dynamics of such a model in detail: we explore the predictions of the model by letting the intratemporal elasticity of substitution vary over a reasonably wide range.

In general, the model keeps the standard neoclassical growth model's prediction of convergence in growth rates of capital per capita and income per capita: poor countries grow faster than rich countries. The possibility of divergence in growth rates of capital per capita, first pointed out by Ventura (1997), requires very extreme parameter values.

[1] Lucas (1990) and King and Rebelo (1993) discuss some limitations of the autarkic neoclassical growth model.

[2] Cuñat and Maffezzoli (2001) show that the Ramsey models under autarky, factor price equalization and complete specialization can be viewed as particular cases of a more general model, which we (perhaps somewhat pretentiously) call "The Generalised Neoclassical Growth Model" here.

[3] See Jones and Manuelli (1990), King and Rebelo (1993), and Barro and Sala-i-Martin (1995) for related discussions.

[4] This is a "permanent income hypothesis" driven result. See Ventura (1997) for a thorough discussion.

Moreover, divergence in growth rates of income per capita does not generally follow from the former due to the dynamics of factor prices. Finally, the remarkable differences in steady-state income per capita levels obtained in Cuñat and Maffezzoli (2001) for countries that are identically parameterized and only differ in their initial capital-labour ratios are a fairly robust feature of the model.

In the second part of the paper, we interpret the most important results of the empirical growth literature in the light of our model. Our basic conclusion is that the model is capable of generating predictions in accordance with the most important findings in the empirical growth literature. At the same time, it avoids some of the most serious problems of the (autarkic) neoclassical growth model.

The rest of the paper is structured as follows: Section 2 presents the model. Section 3 explores the predictions of the model for a range of parameter values of the intratemporal elasticity of substitution. Section 4 relates theory to empirics, and Section 5 concludes.

2 The Model

This section combines the Ramsey model with a three-good, two-factor version of the Heckscher-Ohlin model. The result is a neoclassical growth model with heterogenous agents that allows for international trade in intermediate goods under two possible trade scenarios: complete specialization and factor price equalization.

2.1 International Trade: Two Regions

Our static trade model is a relatively simple version of the Heckscher-Ohlin model: economies trade in goods and differ only in their relative factor endowments. Let us initially assume that there are two regions in the world (North and South), indexed by $i \in \{N, S\}$. Technologies and preferences are identical across regions, and markets are perfectly competitive. The world has $k = k_N + k_S$ units of capital and $l = l_N + l_S$ units of labor. We assume both regions have positive initial endowments of each factor; in particular, we assume that $l_N = l_S = 1$ for simplicity. Without loss of generality, we also assume that the North is the capital-abundant region, that is, $k_N/l_N > k_S/l_S$.

Regions produce a final good y (which is also the numeraire: $p_y = 1$) out of three intermediate inputs with prices p_j, $j \in \{1, 2, 3\}$, with a CES production function

$$y = \phi \left[\frac{\alpha}{2} x_1^{\rho} + (1 - \alpha) x_2^{\rho} + \frac{\alpha}{2} x_3^{\rho} \right]^{\frac{1}{\rho}}, \tag{1}$$

where $\alpha \in [0, 1]$ and $\rho \leq 1$. Notice that $\zeta = 1/(1 - \rho)$ is the elasticity of substitution among intermediates. Intermediate goods are produced with technologies $x_1 = l_1$, $x_2 =$

$k_2^{\gamma} l_2^{1-\gamma}$, $\gamma \in (0,1)$, and $x_3 = k_3$.

Let us assume that: (*i*) the final good y cannot be traded, whereas intermediates can be traded freely; (*ii*) there is no international factor mobility.[5]

The following results can be shown:

1. The solution to the model is unique: for any pattern of factor endowments, a unique pricing pattern of goods and factors is determined. Also, there is a unique equilibrium pattern of world and regional consumptions of intermediate goods, with consumption ratios the same in every region.[6]

2. The static model can lead to only two scenarios: (*i*) worldwide FPE, in which both North and South produce the three goods; (*ii*) CS, with capital-abundant North producing goods 2 and 3, and capital-scarce South producing goods 1 and 2.[7] What scenario actually takes place depends on the distribution of factor endowments across North and South. If they are 'similar enough', we will have FPE. If they are 'too diverse', we will have CS.

In general, we do not know in how many cones the world sorts itself in the absence of worldwide FPE. Our assumption of only two cones is made first for simplicity. Secondly, it corresponds to the idea that we can roughly divide the world in rich and poor countries.

2.1.1 The Integrated Equilibrium

To understand what we mean by 'similar enough' and 'too diverse', let us review the concept of integrated equilibrium, which is defined as the resource allocation the world would have if both goods and factors were perfectly mobile internationally.[8] The integrated equilibrium's conditions are summarized in the Appendix.

[5] From a technical point of view, given our discrete time framework, we need to assume that capital and labor cannot freely move across countries during each period. In particular, we need the capital stock to be a state variable. This, in principle, does not rule our migration or international financial flows, as long as: (*i*) they influence the capital and labor endowments in the next period only; (*ii*) adjustment costs dampen the cross-country factor flows. Our guess is that introducing these extension would increase the speed of convergence to the steady state, but not change our main qualitative results.

[6] This is a standard result in international trade theory. See, for example, Dixit and Norman (1980) and Dornbusch et al. (1980).

[7] No other scenario is possible for the following reason: first, given that both N and S have positive amounts of capital and labor, full employment of resources implies they cannot specialize completely in good 1 one or good 3. Second, CS in good 2 is not possible either, since a region with comparative advantage in this good would also have a comparative advantage in either of the other goods, due to $(w/r)_N \neq (w/r)_S$. This implies that in the absence of worldwide factor price equalization each region produces two goods. Moreover, in such a scenario we cannot have one region producing goods 1 and 3: with different factor prices across regions, a region cannot have a comparative advantage in the production of both of these two goods.

[8] See Dixit and Norman (1980).

Since the world's integrated equilibrium behaves like a closed economy, factor prices depend on world aggregates. In terms of our model, the wage rate w and the rate of return to capital r depend, respectively, positively and negatively on the world's capital-labor ratio k/l. Subsequently, the relationship between the factor-price ratio $\sigma \equiv w/r$ and the world's capital-labor ratio is positive.

The FPE set is the set of distributions of factors among economies that can achieve the resource allocation of the integrated equilibrium if we allow for free international trade, but no international factor mobility. Intuitively, the FPE set is the set of distributions of factors across economies that enable them to achieve full employment of resources while using the techniques implied by the integrated equilibrium. Thus, if the vector of production factors lies within the FPE set, the trading equilibrium will reproduce the integrated equilibrium's factor prices.

When $\alpha = 1$, only infinitely labor-intensive good 1 and infinitely capital-intensive good 3 are produced in the integrated equilibrium. This would grant FPE for any distribution of factors across regions.[9] The smaller α, the less likely FPE. Finally, when $\alpha = 0$, both regions produce only intermediate good 2, and therefore need not trade with each other. In fact, they behave as if they were closed economies with aggregate production function $y_i = \phi k_i^{\gamma} l_i^{1-\gamma}$. In this sense, the FPE and autarky cases are limiting cases of a more general model that allows for the possibility of CS as the "standard" case. a formal set of FPE conditions are reported in the Appendix.

2.1.2 Complete Specialization

If the factor endowment vectors lie outside the FPE set, that is if $k_N \notin \left[k_N^L, k_N^U\right]$, the trading equilibrium cannot reproduce the integrated equilibrium. This leads factor prices to differ across North and South, which specialize in different ranges of goods according to comparative advantage. As we mentioned above, under our assumptions there is only one equilibrium pattern of CS, which implies implies $x_{N1} = x_{S3} = 0$, and $x_{S1}, x_{S2}, x_{N2}, x_{N3} > 0$. The equilibrium conditions for the two-cone case are again summarized in the Appendix. Notice that the North's factor-price ratio is greater than that of the South: $\sigma_N > \sigma_S$ ($w_N > w_S$ and $r_N < r_S$). Notice that otherwise the North would not have a comparative advantage in the production of good 3, and a comparative disadvantage in the production of good 1.

[9] This is the case analysed in Ventura (1997).

2.2 International Trade: Four Countries

Assume that North and South consist each of two countries, East and West, with identical preferences and technologies. Without loss of generality we assume that in both regions the West is more capital-abundant than the East. Within each region i, production factors are distributed as follows: $k_{iW} = (1/2 + \varepsilon_i)k_i$, $k_{iE} = (1/2 - \varepsilon_i)k_i$, and $l_{iW} = l_{iE} = 1/2$, where $\varepsilon_i \in (0, 1/2)$. Obviously, we need to make sure that ε_i small enough for FPE to hold within each cone. To obtain the FPE condition in this four-country world, we need to consider two cases: (i) First, we study the FPE condition within each cone when North and South have different factor prices. (ii) Second, we study the condition for worldwide FPE in the presence of more than two countries. These conditions are spelled out in detail in the Appendix.

2.3 Consumption and Capital Accumulation

In this section we combine the static model discussed above with the discrete-time Ramsey model. Each region is populated by a *continuum* of identical and infinitely lived households, each of measure zero. Being identical, they can be aggregated into a single country-level representative household. The non-traded final good can be used for both consumption and investment. The representative households' preferences over consumption streams can be summarized by the following intertemporal utility function:

$$U_{i,t} = \sum_{s=t}^{\infty} \beta^{s-t} \ln c_{i,s} \tag{2}$$

where β is a subjective intertemporal discount factor and $c_{j,t}$ the per-capita consumption level in region i at date t.

The representative households maximize (2) subject to the following intratemporal budget constraint:

$$c_{i,t} + \Delta k_{i,t} = w_{i,t} + (r_{i,t} - \delta)\, k_{i,t} \tag{3}$$

where $k_{i,t}$ is the current per-capita stock of physical capital in region i, $w_{i,t}$ the wage rate, $r_{i,t}$ the rental rate, and δ the depreciation rate. Factor prices are taken as given by the representative household. Depending on the distribution of capital across regions, factor prices $w_{i,t}$ and $r_{i,t}$ will be determined in the integrated or complete specialization equilibrium.

The first order conditions

$$\beta c_{i,t}(r_{i,t+1} + 1 - \delta) = c_{i,t+1} \tag{4}$$

$$k_{i,t+1} = w_{i,t} + (r_{i,t} + 1 - \delta) k_{i,t} - c_{i,t} \tag{5}$$

and the transversality condition

$$\lim_{t \to \infty} \beta^t \frac{k_{i,t+1}}{c_{i,t}} = 0 \tag{6}$$

are necessary and sufficient for the representative household's problem.

A recursive competitive equilibrium for this economy is characterized by equations (4)-(6) together with the values of $w_{i,t}$ and $r_{i,t}$, which are obtained from the trade regime that applies (factor price equalization or complete specialization).

The recursive structure of our problem guarantees that the solution can be represented as a couple of time-invariant policy functions expressing the optimal level of consumption in each region as a function of the two state variables, k_N and k_S. These policy functions have to satisfy the following functional equations:

$$\beta c_i (k_N, k_S) \left(r_i^{'} + 1 - \delta \right) = c_i \left(k_N^{'}, k_S^{'} \right) \tag{7}$$

where $k_i^{'} = w_i + (1 - \delta + r_i) k_i - c_i (k_N, k_S)$ and the factor prices w_i and r_i are obtained by numerically solving the appropriate equilibrium conditions. Furthermore, the policy functions have to generate stationary time series in order to satisfy the transversality conditions. To numerically solve equation (7), we apply the Galerkin projection method described in Judd (1992). The Appendix describes our computational strategy in detail.

We solve the dynamic model under the assumption that $\alpha = 0.2$ and $\gamma = 1/3$. We assume the same initial values for the two regions' capital stocks: we arbitrarily choose $k_N = 0.5$ and $k_S = 0.1$. Following Cooley and Prescott (1995), we assume $\beta = 0.949$ and $\delta = 0.048$, both standard values in the quantitative macroeconomics literature.[10] The previous parameterization implicitly assumes that the unit time period is a year.

3 The Generalized Neoclassical Growth Model and the Elasticity of Substitution

This section discusses how the intratemporal elasticity of substitution ζ affects the dynamics generated by our model. We let ζ vary over what we consider a reasonable range: $[0.1, 3]$.[11] ζ influences the way in which changes in factor endowments affect countries'

[10] The scale parameter ϕ is calibrated to reproduce a world steady-state capital stock equal to unity.

[11] The unitary elasticity case was studied in Cuñat and Maffezzoli (2001). Note that for values of the elasticity outside this range either the CS or the FPE case become very difficult to solve from a numerical point of view.

factor prices in two ways:

1. The elasticity of substitution determines the relationship between w/r and factor endowments: ceteris paribus, the higher ζ, the lower the response of w/r to a change in the capital-labour ratio.

2. The elasticity of substitution also affects the likelihood of the trade scenario: the higher ζ, the smaller the relative size of the FPE set with respect to the CS region.

The intuition behind the second effect is as follows: when ζ is close to zero, one wants to consume the three intermediates in almost constant proportions. Given that goods one and three, respectively, are produced with only labour and capital, one needs to allocate labour to good one and capital to good 3 more than proportionally in comparison with the world's endowment of capital and labour. This implies that the allocation of capital and labour to sector two is rather small, yielding a FPE set close to the whole Edgeworth Box. For ζ high, the FPE set is instead very small: with intermediates being good substitutes, y is maximized by allocating most of the world's capital and labour to good 2.[12] Actually, for a high enough elasticity of substitution, only good 2 is produced, and therefore countries behave, in a sense, as if they were autarkic economies.

3.1 Intratemporal Elasticity of Substitution: $\zeta = 0.1$

With $\zeta = 0.1$, the FPE set is large enough for the model to yield FPE from time zero, given our assumption about the initial values of k_N and k_S.[13] Figure 1 displays the behavior of the levels and growth rates of income per capita and capital per capita for both North and South. Notice that the bottom right panel of Figure 1 exhibits divergence in the growth rate of capital per worker in the sense that the rich region's growth rate of capital per capita is higher than that of the poor region.

The intuition for this result can be found in Ventura (1997): North and South behave as permanent-income consumers. Given identical homothetic preferences across the two regions, they spend the same fraction of their wealth in each period, thus exhibiting identical rates of wealth accumulation. The growth rate of wealth is a weighted average of the growth rates of its two components: the stock of capital and the net present value of wages. Under FPE the growth rate of the latter is the same for North and South, since wage growth is independent of domestic conditions and identical for all countries. If diminishing returns to capital at the world level are not strong enough, wage growth

[12]The relationship between the elasticity of substitution and the relative size of the FPE set depends crucially on the production functions we have assumed for the intermediate goods. We leave a more detailed of this issue for future work.

[13]In this sense, this resembles Ventura (1997).

will be slow as the world accumulates capital over labor. The capital-scarce region, the South, will need to accumulate more than the capital-abundant region for both to keep the same rate of wealth accumulation.. In the long run, though, the two regions' growth rates become equal as they approach zero.

The bottom left panel of Figure 1 implies that divergence in the growth rate of capital per worker does not create divergence in the growth rate of income per capita: actually, the South's growth rate of income per capita is higher than that of the North. This is due to the speed at which factor prices change with the world's capital-labour ratio, which is high enough to reverse the divergence in the growth rates of capital per worker. In the long run, the two regions' growth rates become equal as they approach zero.

Concerning the levels, the two top panels of Figure 1 exhibit remarkable differences in steady-state levels of income per capita and capital per capita despite the fact that both countries are ruled by identical parameters. Under FPE, both regions face the same rate of return to capital, which is determined by the integrated equilibrium. Hence, the system's steady state is determined by the rate of return to capital r equalling the rate of time preference. With the North's and South's capital-labor ratios, respectively, above and below that of the world, nothing prevents the system from generating a bimodal steady-state equilibrium if the South catches up with the North more slowly than the integrated equilibrium converges to its steady state. Moreover, the levels display an important degree of divergence in the sense that the output gap between North and South increases over time: cross-country differences in growth rates do not compensate cross-country differences in income per capita levels.

3.2 Intratemporal Elasticity of Substitution: $\zeta = 0.5$

In this case, the FPE set is again large enough for the model to yield FPE from time zero, given our assumption about the initial values of k_N and k_S. In contrast with the previous case, the bottom panels of Figure 5 exhibit convergence in the growth rates of both income per capita and capital per worker. This suggests that the results in Ventura (1997) on divergence are possibly rather implausible regarding the parameter values generating them. In the long run, again, the two regions' growth rates become equal as they approach zero.

As in the previous case, the two top panels of Figures 5 exhibit remarkable differences in steady-state levels of income per capita and capital per capita and an important degree of levels divergence. This phenomenon is a little bit less pronounced than with $\zeta = 0.1$.

3.3 Intratemporal Elasticity of Substitution: $\zeta = 1.5$

With $\zeta = 1.5$, the FPE set is small enough for the model to yield CS at time zero, given our assumption about the initial values of k_N and k_S. The top panel of Figure 9 plots the time path of the rental rates of North and South. Observe that factor price equalization only occurs after more or less 60 years. Notice that the bottom panels of Figure 9 exhibit convergence in the growth rates of both income per capita and capital per worker. In the long run, again, the two regions' growth rates become equal as they approach zero. Notice also that the growth rate differentials obtained in this case, where $r_S > r_N$, are one order of magnitude larger than in the two previous cases, where $r_S = r_N$.

The two top panels of Figure 9 still exhibit remarkable differences in steady-state levels of income per capita and capital per capita. In contrast with the previous cases, however, the model exhibits some degree of levels convergence. This is due to the larger magnitude of the growth rate differentials.

3.4 Intratemporal Elasticity of Substitution: $\zeta = 3$

In this case, the FPE set is small enough for the model to yield CS during most of the transition to the steady state, given our assumption about the initial values of k_N and k_S. Figure 13 exhibits hardly any growth for the North, which is close to its steady-state income per capita level from time zero. Notice that the bottom panels of Figure 13 exhibit convergence in the growth rates of both income per capita and capital per worker. In the long run, again, the two regions' growth rates become equal as they approach zero. Notice that the growth rate differentials obtained with $\zeta = 3$ are higher than those obtained with $\zeta = 1.5$.

The two top panels of Figure 13 exhibit much smaller differences in steady-state levels of income per capita and capital per capita than in previous cases, and a large degree of levels convergence. This is due to the larger magnitude of the growth rate differentials. From a qualitative perspective, the dynamic behavior of our model with $\zeta = 3$ resembles that of the autarkic neoclassical growth model. In this respect, two effects are at work here: first, an increase in the elasticity reduces the size of the FPE set; second, an increase in elasticity speeds up convergence in levels during the CS phase, since growth differentials are higher under. Unfortunately, we cannot identify these effects separately. To conclude, the fact that out model converges to the autarky case as the elasticity increases is an artifact generated by our extreme assumptions on the factor intensities. However, the differences in steady-state levels should likely be reduced (but not eliminated) as the elasticity increases in more general version of our model.

4 Reinterpreting the Evidence

This sections discusses the empirics of growth of the last decade in the light of our results.

4.1 Cross-Country Convergence

Barro (1991) and Barro and Sala-i-Martin (1992) regress the national growth rates of income per capita on initial (log) income per capita and a set of controls. Their findings support the idea of conditional convergence across nations: countries grow faster the further they are from their own steady states. Our setup is compatible with the evidence on conditional convergence. However, it points out that the relationship between countries' growth rates and initial income per capita levels may depend on the sample: within-cone and between-cone distinction.

An additional qualification is necessary: Barro and Sala-i-Martin's analysis implies that countries have got different steady states because of differences in "parameters," whereas this is not necessary in our framework. In this respect, Canova and Marcet (1995) study the issue of income convergence across countries and regions with a Bayesian model which allows them to use information in an efficient and flexible way. Their approach permits the estimation of different convergence rates to different steady states for each cross-sectional unit. When this diversity is allowed, they find that convergence of each unit to (its own) steady-state income level is much faster than previously estimated, but that cross-sectional differences persist: inequalities will only be reduced by a small amount by the passage of time. The cross-country distribution of the steady state is largely explained by the cross-sectional distribution of initial conditions: this result seems to support our theoretical results.

4.2 Income Levels

Mankiw, Romer and Weil (1992) regress income per capita level on the investment share and the growth rate of population.[14] They obtain a positive correlation between income per capita and the investment share (which stands as a proxy for the savings rate), and interpret it in terms of the Solow model's steady state predictions: richer countries benefit from a higher (exogenously given) savings rate.

Our model also yields a positive correlation between income per capita levels and the investment-output ratio in steady state. Moreover, our model predicts that cross-country differences in growth rates are positively correlated with cross-country differences in investment shares over the transition to the steady state. However, these correlations

[14] Bernanke and Gurkaynak (2001) update and criticise the econometric results in Mankiw et al. (1992).

are not caused by differences in parameters across countries. This implies that Mankiw et al.'s results need not support neither a world in steady state nor the idea that there are cross-country differences in the parameters ruling countries' savings rates.

A (theoretical) problem with the Solow model discussed by Mankiw et al. has an easy solution within our framework. As they point out, large (and persistent) differences in capital-output ratios across countries should lead to large interest rate differentials in the Solow model. This might be a problem if one allows for a certain degree of capital mobility. Our framework suggests instead that steady-state differences in capital-output ratios are compatible with identical interest rates across countries. Moreover, over the transition, differences in capital-output ratios generate interest rate differentials smaller than in the Solow model. (See the intuition in Cuñat and Maffezzoli (2001).)

Mankiw et al. and Bernanke and Gurkaynak tend to obtain different results for different samples: in general, they obtain very low R^2 values when they constrain the sample to OECD countries. They interpret these findings in terms of departures from the steady-state, which might be more important for countries that were involved in World War II.[15] A similar argument can be made in terms of our model, with the qualification that shocks to a large group of countries such as those that took part in WWII would drive the whole world far from its steady state.

4.3 Regional Convergence

Barro and Sala-i-Martin (1992) regress the U.S. states' growth rates of income per capita on initial (log) income per capita, finding a negative correlation between these variables. Barro and Sala-i-Martin (1991) perform a similar exercise for regions belonging to a few European countries and obtain similar results. This type of regressions is based on the assumption that all regions within a country have the same steady-state, since their dynamic behavior is subject to identical parameters (unconditional convergence). This is pretty much in line with the predictions of the neoclassical growth model under autarky, given that, and implies the (rather optimistic) view that poor regions are catching up in levels with rich regions.

Boldrin and Canova (2000) find instead convergence in growth rates but no convergence in levels. It is hard to explain the lack of convergence in levels in terms of parameter differences across regions. In this respect, our setup looks more promising than the standard autarkic neoclassical growth model.

[15] Actually, Bernanke and Gurkaynak show that for countries not involved in World War II the Solow model fits remarkably better.

4.4 The Importance of Productivity

In recent years, a strand of the empirical growth literature - e.g., Klenow and Rodriguez-Clare (1997a, 1997b) and Hall and Jones (1999) - has emphasized the importance of productivity for understanding cross-country differences in the growth rates and levels of income per capita. In general, these productivity estimates start with the assumption that all countries have the same aggregate production function $F(k, Al)$, where A is a labor-augmenting measure of productivity, and that differences in its arguments is what explains cross-country differences in the levels and growth rates of income per capita. Based on this setup, this literature concludes that the most important argument explaining cross-country differences is A. Although our model is silent about technical progress, we can still make a couple of observations regarding this empirical literature.

First, as Lucas (1988) pointed out, differences in productivity levels and growth rates may be due to different specialization patterns despite all countries having access to the same technologies. In this sense, our model could be easily extended by reformulating the production functions of intermediates as follows: $x_1 = A_1 l_1$, $x_2 = k_2^{\gamma} (A_2 l_2)^{1-\gamma}$, and $x_3 = k_3$. Cross-sector differences in the levels or growth rates of A_j will induce differences in productivity levels or growth rates across countries in the presence of cross-country differences in capital-labor ratios, since the latter will lead to cross-country differences in production structures.

Secondly, given that production structures are likely to differ across countries, we have a reason to be skeptical about growth accounting exercises based on aggregate production functions. To make our point, let us perform the following "growth accounting" exercise, based on the aggregate production function $y_i = k_i^{\omega} (A_i l_i)^{1-\omega}$ for all countries i. Since in our framework labor is constant over time and equal to one both in the North and the South, let us make the same normalization here. Thus, $A_i = (y_i / k_i^{\omega})^{\frac{1}{1-\omega}}$. Let us assume $\omega = 1/3$ as in Hall and Jones (1999), and let us compute the level and growth rate of A_i with the series of y_i and k_i generated in our simulations (which implicitly assumed $A_{j,t} = 1$ for all j, t.) The corresponding figures are self-explanatory: our "naive" growth accounting exercise unveils apparent cross-country differences in the levels and growth rates of productivity. Notice, for example, that the high-income country (the North) always appears to have a higher productivity level than the low-income country (the South).

Needless to say, we are not trying to downplay the importance of understanding the "exogeneity" of technical progress. However, in the light of our model, we wonder how much one can learn about the relationship between technology and growth by focusing on aggregate production functions.

5 Concluding Remarks

The neoclassical growth model open to international trade is a rich framework to study the economics of capital accumulation and interpret many of the recent empirical findings in this area. Needless to say, ours is still a very stylized framework that should stimulate future work both on theoretical and empirical grounds.

On the theoretical side, making the intermediate goods technologies more general should add to the richness (and realism) of the model. Another major contribution would be the analysis of the case in which a country (a so-called miracle) switches cones, leading to major changes in the country's production structures.

In this respect, on the empirical side it might be interesting to look at historical examples where one can roughly identify large positive or negative shocks to a country's economic performance, and try to assess the extent to which our model can fit these experiences. Finally, it might also be worth exploring the relationship between economic growth, trade and sectoral allocation of resources, since our model has strong predictions about this issue.

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7 Appendix

7.1 The Integrated Equilibrium

The integrated equilibrium's conditions can be summarized as follows:

1. Price equal to unit cost:

$$1 = \phi^{-1}\Pi^{\frac{1}{1-\varepsilon}} \tag{8}$$

$$p_1 = w \tag{9}$$

$$p_2 = \gamma^{-\gamma}(1-\gamma)^{\gamma-1}r^{\gamma}w^{1-\gamma} \tag{10}$$

$$p_3 = r \tag{11}$$

where:

$$\Pi \equiv \left(\frac{\alpha}{2}\right)^{\zeta} p_1^{1-\zeta} + (1-\alpha)^{\zeta} p_2^{1-\zeta} + \left(\frac{\alpha}{2}\right)^{\zeta} p_3^{1-\zeta} \tag{12}$$

2. Goods-market clearing:

$$x_1 = \frac{(\alpha/2)^{\zeta} p_1^{-\zeta}}{\Pi} y \tag{13}$$

$$x_2 = \frac{(1-\alpha)^{\zeta} p_2^{-\zeta}}{\Pi} y \tag{14}$$

$$x_3 = \frac{(\alpha/2)^{\zeta} p_3^{-\zeta}}{\Pi} y \tag{15}$$

where $y = rk + wl$. Here, implicit is the normalization due to our choice of numeraire.

3. Factor-market clearing:

$$x_1 + \left(\frac{\gamma}{1-\gamma}\right)^{-\gamma}\left(\frac{w}{r}\right)^{-\gamma} x_2 = l_N + l_S \tag{16}$$

$$\left(\frac{\gamma}{1-\gamma}\right)^{1-\gamma}\left(\frac{w}{r}\right)^{1-\gamma} x_2 + x_3 = k_N + k_S \tag{17}$$

The unknowns of the problem are p_1, p_2, p_3, x_1, x_2, x_3, w, and r.

The FPE conditions are the following:[16]

1. If $l_N \in (0, l_1]$, $k_N^L = 0 \leq k_N \leq k_3 + \frac{k_2}{l_2} l_N = k_N^U$.
2. If $l_N \in (l_1, l_2)$, $k_N^L = \frac{k_2}{l_2}(l_N - l_1) \leq k_N \leq k_3 + \frac{k_2}{l_2} l_N = k_N^U$.
3. If $l_N \in [l_2, l)$, $k_N^L = \frac{k_2}{l_2}(l_N - l_1) \leq k_N \leq k = k_N^U$.

[16] The variables indexed in numbers denote the integrated equilibrium's resource allocation. $k = k_N + k_S$ is the world's capital stock.

7.2 Complete specialization

The equilibrium conditions for the two-cone case can be summarized as follows:

1. Price equal to unit cost:

$$1 = \phi^{-1}\Pi^{\frac{1}{1-\zeta}} \tag{18}$$

$$p_1 = w_S \tag{19}$$

$$p_2 = \gamma^{-\gamma}(1-\gamma)^{\gamma-1} r_S^{\gamma} w_S^{1-\gamma} = \gamma^{-\gamma}(1-\gamma)^{\gamma-1} r_N^{\gamma} w_N^{1-\gamma} \tag{20}$$

$$p_3 = r_N \tag{21}$$

2. Goods-market clearing:

$$x_1 = x_{S1} = \frac{(\alpha/2)^{\zeta}\, p_1^{-\zeta}}{\Pi}(y_N + y_S) \tag{22}$$

$$x_2 = x_{S2} + x_{N2} = \frac{(1-\alpha)^{\zeta}\, p_2^{-\zeta}}{\Pi}(y_N + y_S) \tag{23}$$

$$x_3 = x_{N3} = \frac{(\alpha/2)^{\zeta}\, p_3^{-\zeta}}{\Pi}(y_N + y_S) \tag{24}$$

where $y_i = r_i k_i + w_i l_i$.

3. Factor-market clearing:

$$x_{S1} + \left(\frac{\gamma}{1-\gamma}\right)^{-\gamma}\left(\frac{w_S}{r_S}\right)^{-\gamma} x_{S2} = l_S \tag{25}$$

$$\left(\frac{\gamma}{1-\gamma}\right)^{-\gamma}\left(\frac{w_N}{r_N}\right)^{-\gamma} x_{N2} = l_N \tag{26}$$

$$\left(\frac{\gamma}{1-\gamma}\right)^{1-\gamma}\left(\frac{w_S}{r_S}\right)^{1-\gamma} x_{S2} = k_S \tag{27}$$

$$\left(\frac{\gamma}{1-\gamma}\right)^{1-\gamma}\left(\frac{w_N}{r_N}\right)^{1-\gamma} x_{N2} + x_{N3} = k_N \tag{28}$$

The unknowns of the problem are p_1, p_2, p_3, x_1, x_{S2}, x_{N2}, x_3, w_S, r_S, w_N, and r_N.

7.3 The four countries case

7.3.1 The Within-Cone FPE Condition

The South's FPE condition can be written as follows:[17]

[17] The variables indexed in ij denote the corresponding cone's resource allocation. $k_i = k_{iW} + k_{iE}$ is the cone's capital stock.

1. If $l_{SW} \leq l_{S1}$, $k^L_{SW} = 0 \leq k_{SW} \leq k_S = k^U_{SW}$.
2. If $l_{SW} > l_{S1}$, $k^L_{SW} = \frac{k_{S2}}{l_{S2}} (l_{SW} - l_{S1}) \leq k_{SW} \leq \frac{k_{S2}}{l_{S2}} l_{SW} = k^U_{SW}$.

The corresponding North's FPE condition is:

For any $l_{NW} \in (0, l_N)$, $k^L_{NW} = \frac{k_{N2}}{l_{N2}} l_{NW} \leq k_{NW} \leq k_{N3} + \frac{k_{N2}}{l_{N2}} l_{NW} = k^U_{NW}$.

7.3.2 The Worldwide FPE Condition[18]

Define (k^n, l^n), $n = 1, 2, 3$, as follows:

$$(k^1, l^1) \equiv (k_{NW}, l_{NW}) \tag{29}$$

$$(k^2, l^2) \equiv (k_{NW} + k_{NE}, l_{NW} + l_{NE}) \tag{30}$$

$$(k^3, l^3) \equiv (k_{NW} + k_{NE} + k_{SW}, l_{NW} + l_{NE} + l_{SW}) \tag{31}$$

Then, a necessary and sufficient condition for FPE is:[19] for $n = 1, 2, 3$,

1. If $l^n \in (0, l_1]$, $0 \leq k^n \leq k_3 + \frac{k_2}{l_2} l^n$.
2. If $l^n \in (l_1, l_2)$, $\frac{k_2}{l_2} (l^n - l_1) \leq k^n \leq k_3 + \frac{k_2}{l_2} l^n$.
3. If $l^n \in [l_2, l)$, $\frac{k_2}{l_2} (l^n - l_1) \leq k^n \leq k$.

7.4 Computational Strategy

7.4.1 Policy functions

Following Judd (1992), we approximate the policy functions for consumption over a rectangle $D \equiv [\underline{k}, \overline{k}] \times [\underline{k}, \overline{k}] \in R^2_+$ with a linear combination of multidimensional orthogonal basis functions taken from a *2*-fold tensor product of Chebyshev polynomials. In other words, we approximate the policy function for cone $j \in \{N, S\}$ with:

$$\widehat{c}_j (k_N, k_S; \mathbf{a}_j) = \sum_{z=0}^{d} \sum_{q=0}^{d} a^j_{zq} \psi_{zq} (k_N, k_S) \tag{32}$$

where:

$$\psi_{zq} (k_N, k_S) \equiv T_z \left(2 \frac{k_N - \underline{k}}{\overline{k} - \underline{k}} - 1 \right) T_q \left(2 \frac{k_S - \underline{k}}{\overline{k} - \underline{k}} - 1 \right) \tag{33}$$

and $\{k_N, k_S\} \in D$. Each T_n represents an n-order Chebyshev polynomial, defined over $[-1, 1]$ as $T_n(x) = \cos(n \arccos x)$, while d denotes the higher polynomial order used in our approximation.

[18] See Cuñat (2001).

[19] k and l denote the world's capital and labor endowments. Variables indexed in numbers refer to the integrated equilibrium's resource allocation.

We defined the residual functions as:

$$R_j\left(k_N, k_S; \mathbf{a}_j\right) \equiv \beta \hat{c}_j\left(k_N, k_S; \mathbf{a}_j\right)\left(1-\delta+r_j^{'}\right)-\hat{c}_j\left(k_N^{\prime}, k_S^{\prime}; \mathbf{a}_j\right) \tag{34}$$

where $k_j^{'} = w_j + (1 + r_j) k_j - \hat{c}_j (k_N, k_S; \mathbf{a}_j)$. The factor prices r_j and w_j are obtained by numerically solving equations (8) and (16) under FPE, and equations (18), (20), (25), and (28) under CS.

The vectors $\mathbf{a}_j$ can be chosen efficiently using the Galerkin projection method. To apply the latter, we need $m > 2(d+1)^2$ points in D where to evaluate the residual functions. The *Chebyshev Interpolation Theorem* suggests these points can be optimally chosen among the zeros of Chebyshev polynomials in $[-1, 1]$. In other words, we should find $\hat{m} = \sqrt{m}$ of these zeros, reverse the normalization and obtain the corresponding values in $[\underline{k}, \overline{k}]$, and organize them into two (identical) vectors $\{k_{N,i}\}_{i=1}^{\hat{m}}$and $\{k_{S,i}\}_{i=1}^{\hat{m}}$. Then, we should numerically solve for the $2(d+1)^2$ elements in $\{\mathbf{a}_N, \mathbf{a}_S\}$ the following system of equations, using a standard Newton-type algorithm:

$$P_{il}^{j}\left(\mathbf{a}_j\right) \equiv \sum_{z=1}^{m} \sum_{q=1}^{m} R_j\left(k_{N,z}, k_{S,q}; \mathbf{a}_j\right) \psi_{il}\left(k_{N,z}, k_{S,q}\right)=0 \tag{35}$$

where $i, l \in \{0, 1, ..., d\}$.

In our case, however, it turns out very difficult to approximate well the policy functions under both trade regimes using the same set of coefficients. Under FPE, the policy functions look like a positively sloped hyperplane in R_+^3, while under CS they assume a slightly curved shape. The standard procedure fails in producing numerically accurate approximations of the policy functions, in particular around the "regime"-switching region. To bypass this problem, we develop a two-step procedure that generates two different sets of coefficients, one for each region.

First of all, we define two proper subsets of D: D_{FPE}, the FPE region, and D_{CS}, the CS region where the North is the capital-intensive cone (the policy functions in the other CS region are perfectly symmetric).

Our first step consists in finding m_{FPE} zeros of Chebyshev polynomials in $[-1, 1]$, map them in $[\underline{k}, \overline{k}]$, and organize the result into $\{k_{N,i}\}_{i=1}^{m_{FPE}}$and $\{k_{S,i}\}_{i=1}^{m_{FPE}}$. Then, we isolate the subset of $\{k_{N,i}\}_{i=1}^{m_{FPE}} \times \{k_{S,i}\}_{i=1}^{m_{FPE}}$ that belongs to D_{FPE} by numerically checking the FPE conditions, obtaining some $\tilde{m}_{FPE} > 2(d+1)^2$ values for $\{k_N, k_S\}$. Finally, we numerically solve for the $2(d+1)^2$ elements of $\{\mathbf{a}_N^{FPE}, \mathbf{a}_S^{FPE}\}$ the following system of equations, using

Broyden's algorithm:

$$P_{il}^{j}\left(\mathbf{a}_{j}^{FPE}\right)\equiv\sum_{z=1}^{m_{FPE}}\sum_{q=1}^{m_{FPE}}R_{j}\left(k_{N,z},k_{S,q};\mathbf{a}_{j}^{FPE}\right)\psi_{il}\left(k_{N,z},k_{S,q}\right)=0 \tag{36}$$

where $i,l=0...d$.

Given the regular shape of the policy functions under FPE, the first step in our approximation procedure reaches a high degree of numerical precision easily.

The second and final step consists in finding m_{CS} zeros of Chebyshev polynomials in $[-1,1]$, and organize the corresponding values in $[\underline{k},\overline{k}]$ into $\{k_{N,i}\}_{i=1}^{m_{CS}}$ and $\{k_{S,i}\}_{i=1}^{m_{CS}}$. Then, we isolate the subset of $\{k_{N,i}\}_{i=1}^{m_{CS}}\times\{k_{S,i}\}_{i=1}^{m_{CS}}$ that belongs to D_{CS}. Furthermore, we include the m_{CS} points lying exactly on the border between D_{CS} and D_{FPE}: given the set $\{k_{S,i}\}_{i=1}^{m_{CS}}$, we numerically find the set $\{k_{N,i}\}_{i=1}^{m_{CS}}$ that lies exactly on the border of the FPE region. This slightly decreases the overall performance of the projection method, but guarantees that the policy functions under CS and FPE coincide along the border. We are left with $\tilde{m}_{CS}>2(d+1)^{2}$ values for $\{k_{N},k_{S}\}$. Again, we numerically solve for the $2\left(d+1\right)^{2}$ elements of $\left\{\mathbf{a}_{N}^{CS},\mathbf{a}_{S}^{CS}\right\}$ the following system of equations:

$$P_{il}^{j}\left(\mathbf{a}_{j}^{CS}\right)\equiv\sum_{z=1}^{\tilde{m}_{CS}}\sum_{q=1}^{\tilde{m}_{CS}}R_{j}\left(k_{N,z},k_{S,q};\mathbf{a}_{j}^{CS}\right)\psi_{il}\left(k_{N,z},k_{S,q}\right)=0 \tag{37}$$

where $i,l=0...d$.

This two-step procedure deals efficiently with the curved shape of the policy functions while maintaining a good degree of numerical precision. Once the approximated policy functions are available, we choose the initial conditions and simulate the system recursively, generating the artificial time series for all variables of interest.

7.4.2 Disaggregation

To further disaggregate each cone into East and West, we start by assuming *ex-ante* that countries in both cones are under FPE. By iterating the country-level intratemporal budget constraint under this assumption and imposing the transversality condition, we obtain (for each cone):

$$\sum_{s=t}^{\infty}R_{t}^{s}c_{j,s}=\left(1-\delta+r_{t}\right)k_{j,t}+\sum_{s=t}^{\infty}R_{t}^{s}w_{s} \tag{38}$$

where $j \in \{W, E\}$, $R_t^s \equiv \prod_{i=t+1}^{s} (1 - \delta + r_i)^{-1}$ and $R_t^t \equiv 1$. By substituting the Euler equation we obtain:

$$c_{j,t} = (1 - \beta) \left[(1 - \delta + r_t) k_{j,t} + \sum_{s=t}^{\infty} R_t^s w_s \right] \tag{39}$$

The previous result implies that:

$$c_{W,t} = \frac{1}{2} \left[(1 - \beta) (1 - \delta + r_t) (k_{W,t} - k_{E,t}) + c_t \right] \tag{40}$$

$$c_{E,t} = c_t - c_{W,t} \tag{41}$$

where c_t is total consumption in each cone.

The time series for c_t and r_t, together with the initial conditions $k_{W,0}$ and $k_{E,0}$ are sufficient to recover the times series for $c_{W,t}$, $c_{E,t}$, $y_{W,t}$, $y_{E,t}$, $k_{W,t}$, and $k_{E,t}$. Once these series are at hand, we check *ex-post* that the static FPE conditions have been satisfied in all periods.

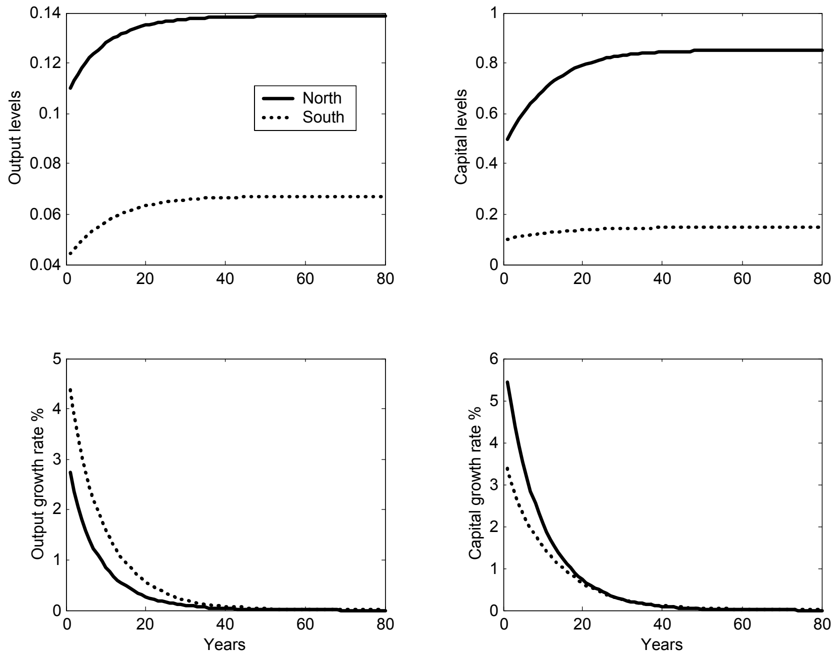


Figure 1: Output and capital (levels and growth rates) for $\zeta = 0.1$.

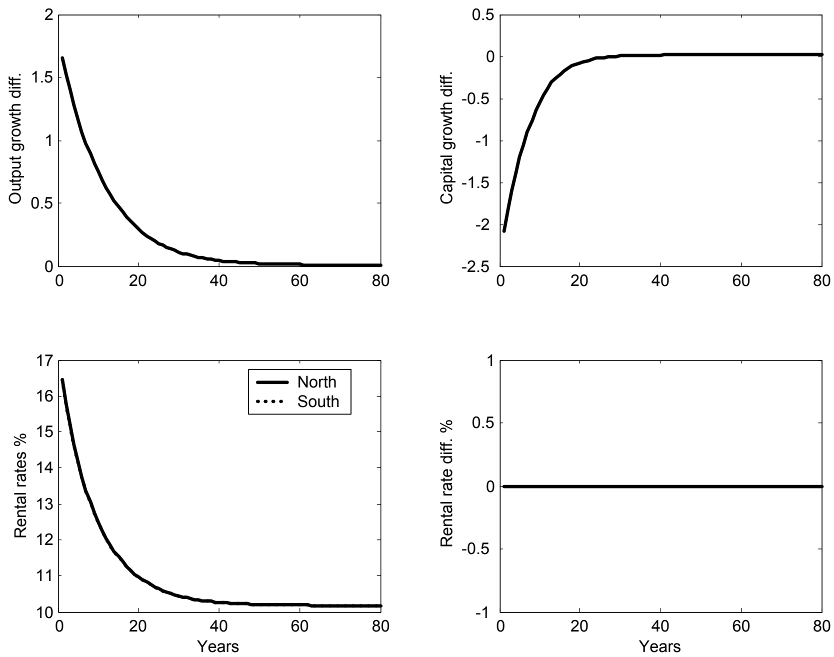


Figure 2: Growth and rental rate differentials for $\zeta = 0.1$.

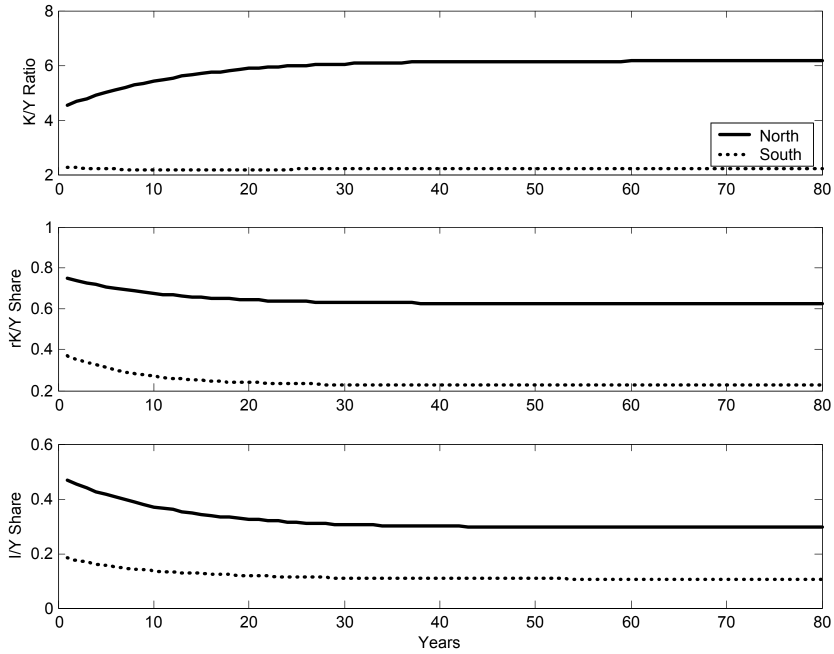


Figure 3: Shares and ratios for $\zeta = 0.1$.

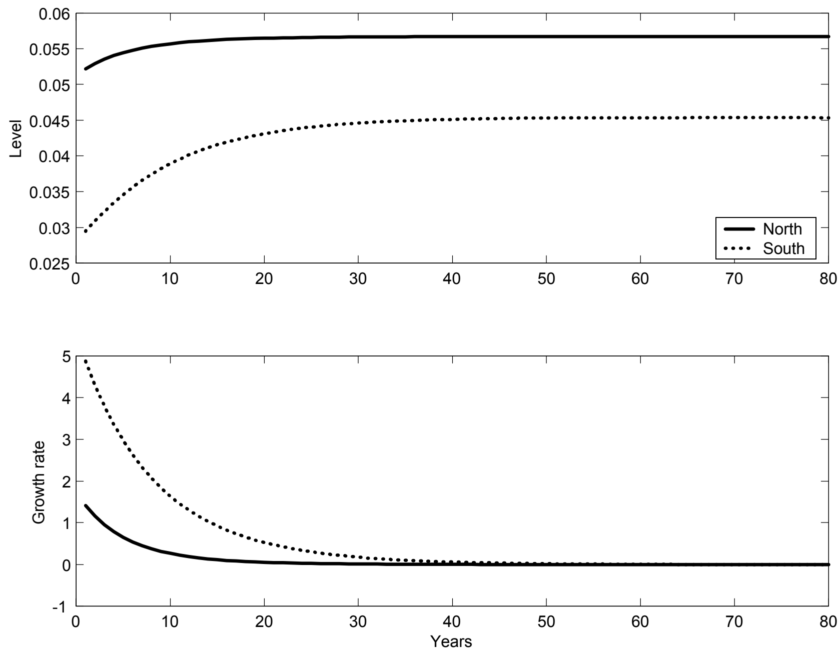


Figure 4: Labor-augmenting "technological progress" for $\zeta = 0.1$.

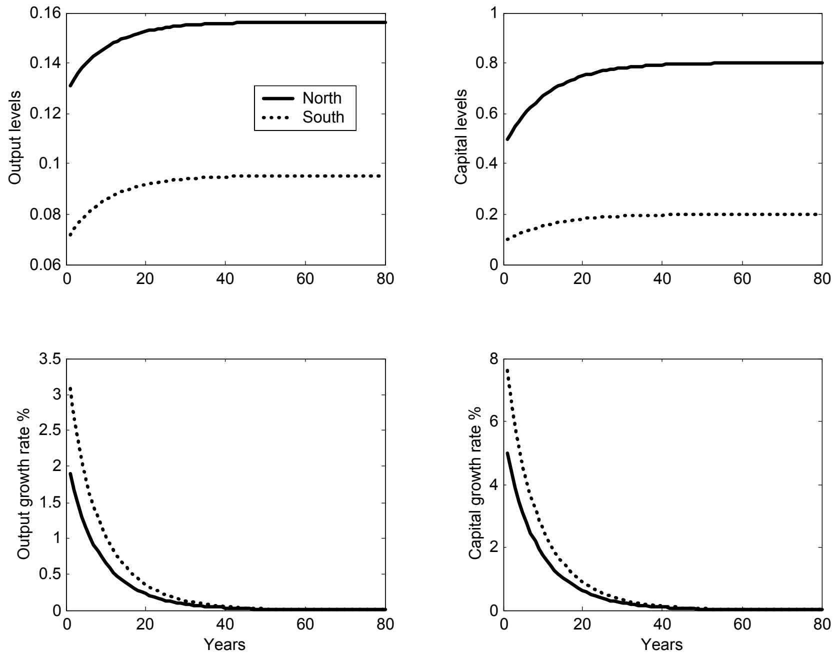


Figure 5: Output and capital (levels and growth rates) for $\zeta = 0.5$.

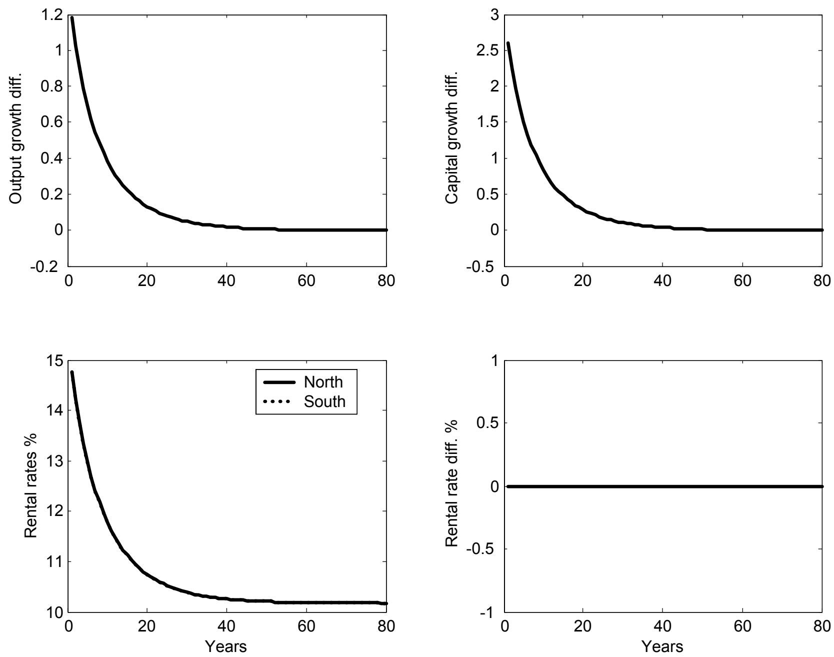


Figure 6: Growth and rental rate differentials for $\zeta = 0.5$.

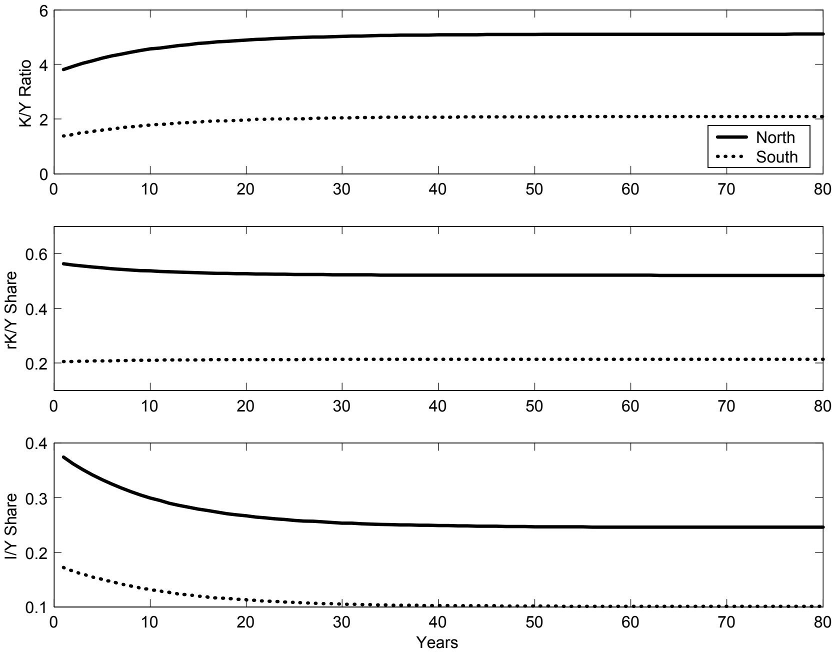


Figure 7: Shares and ratios for $\zeta = 0.5$.

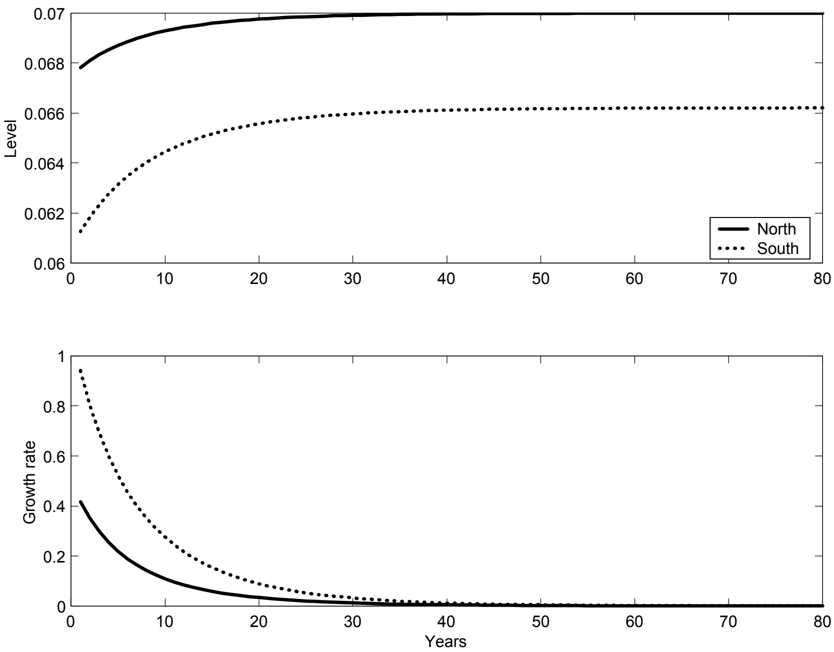


Figure 8: Labor-augmenting "technological progress" for $\zeta = 0.5$.

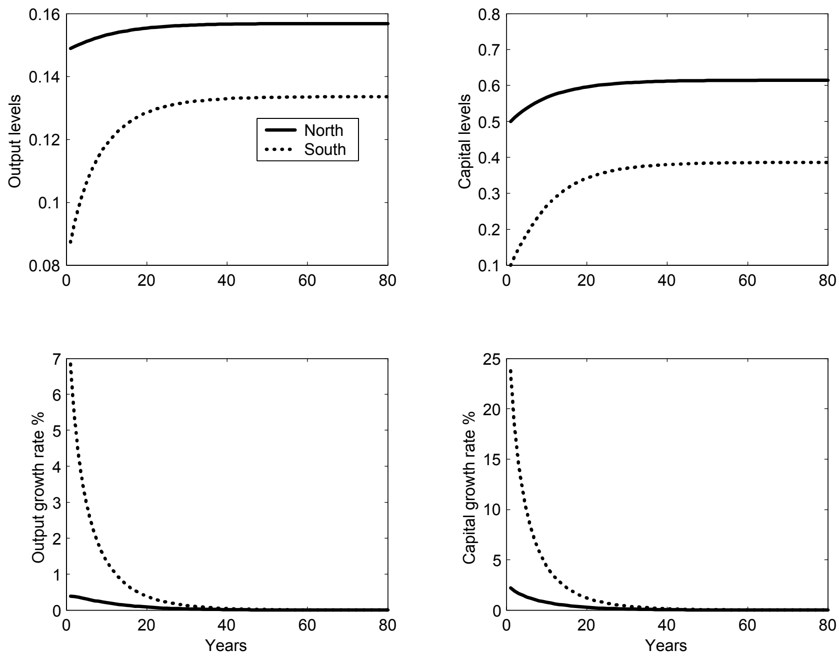


Figure 9: Output and capital (levels and growth rates) for $\zeta = 1.5$.

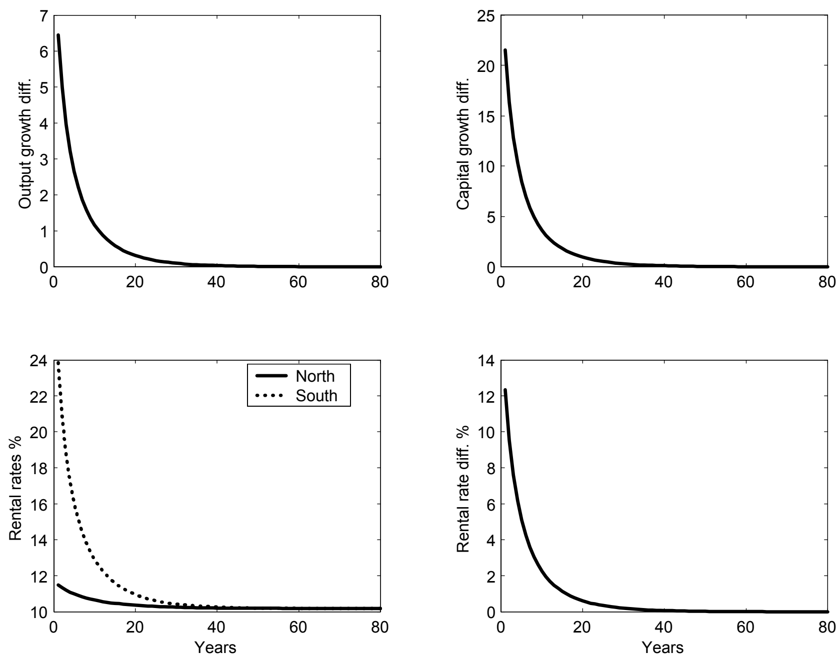


Figure 10: Growth and rental rate differentials for $\zeta = 1.5$.

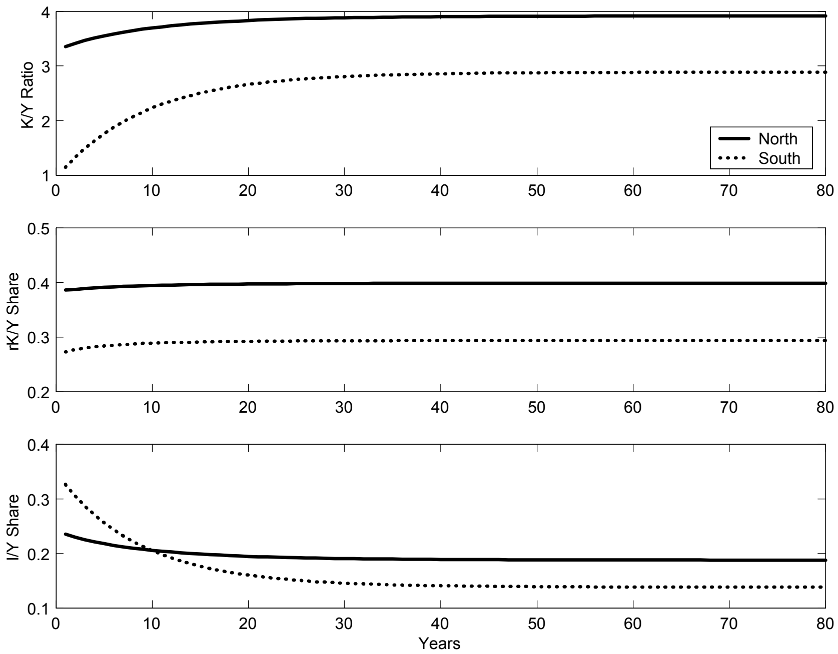


Figure 11: Shares and ratios for $\zeta = 1.5$.

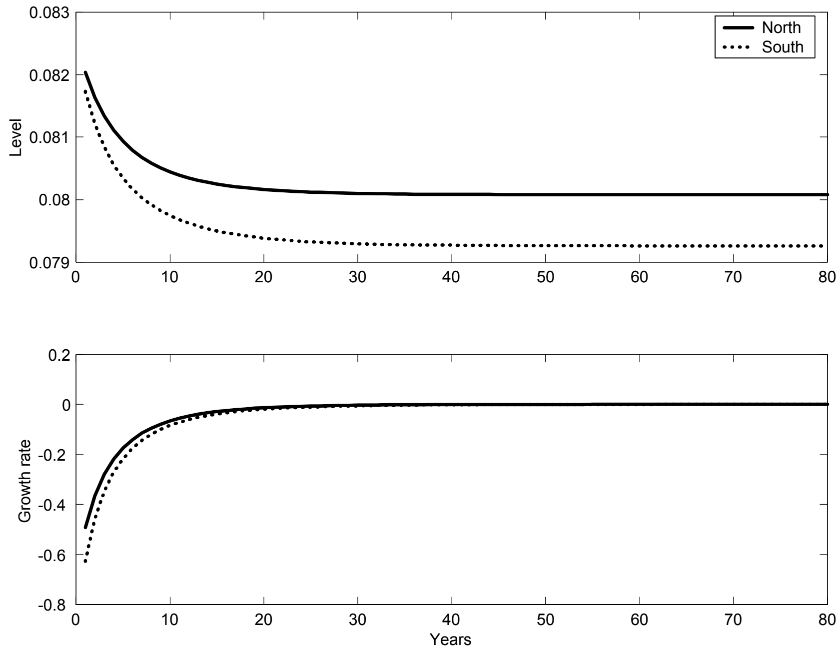


Figure 12: Labor-augmenting "technological progress" for $\zeta = 1.5$.

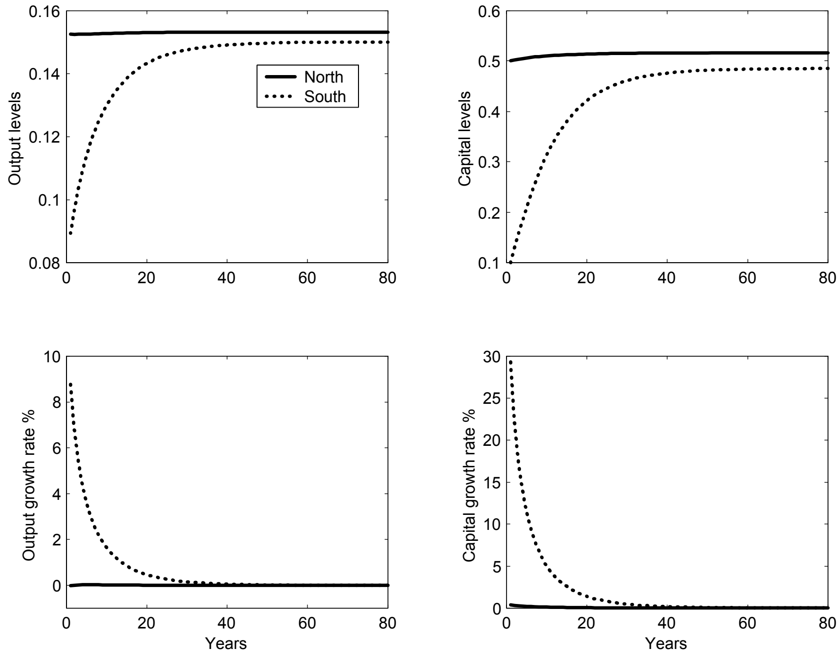


Figure 13: Output and capital (levels and growth rates) for $\zeta = 3$.

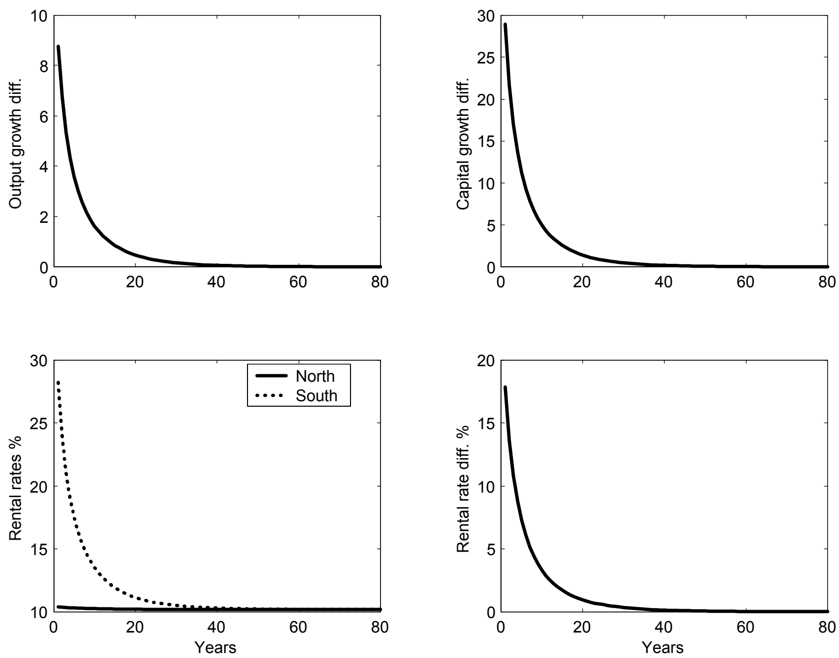


Figure 14: Growth and rental rate differentials for $\zeta = 3$.

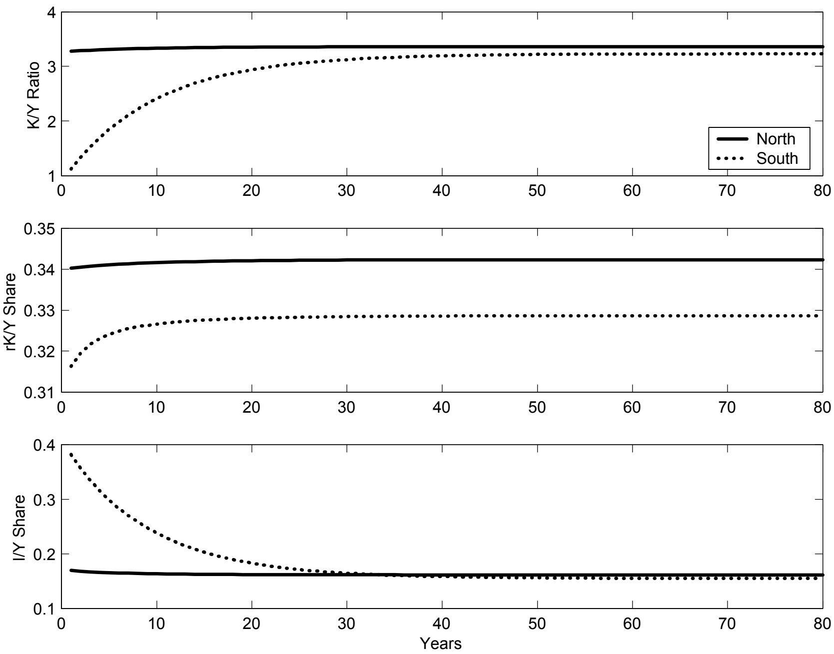


Figure 15: Shares and ratios for $\zeta = 3$.

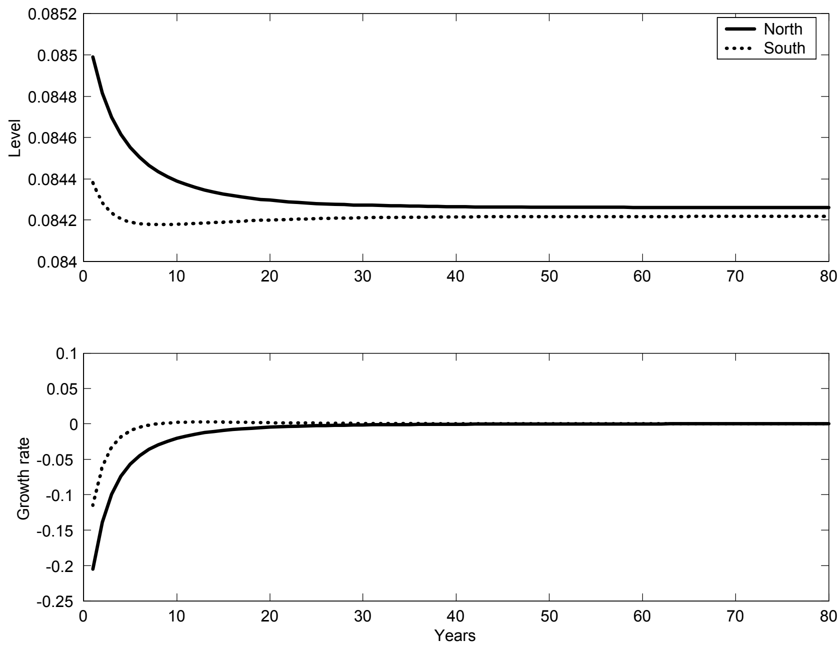
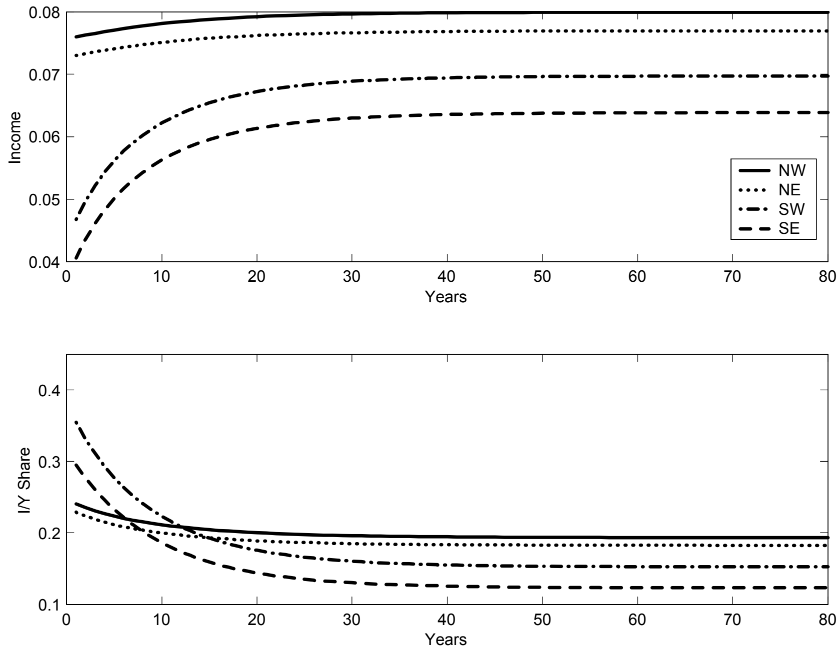


Figure 16: Labor-augmenting "technological progress" for $\zeta = 3$.


0.08
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Years
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SW
SE
0.4
0.3
0.2
0.1
I/Y Share
Years

Figure 17: