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Working Paper n. 291

June 2005

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http://www.igier.uni-bocconi.it

Consumption, Wealth, the Elasticity of Intertemporal Substitution and Long-Run Stock Market Returns*

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Abstract

Consumption is striking back. Some recent evidence indicates that the well-known asset pricing puzzles generated by the difficulties of matching fluctuations in asset prices with high frequency fluctuations in consumption might be solved found by considering consumption in the long-run. A first strand of the literature concentrates on multiperiod differences in log consumption, a second concentrates on the cointegrating relation for consumption. Interestingly, only the (multiperiod) Euler Equation for the consumer optimization problem is considered by the first strand of the literature, while the cointegration-based literature concentrates exclusively on the (linearized) intertemporal budget constraint. In this paper, we show that using the first order condition in the linearized budget constraint to derive an explicit long-run consumption function delivers an even more striking strike back.

JEL Classification Numbers: E2, E44, G12

Keywords: Cointegrating Consumption function, lon-run stock market returns, elasticity of intertemporal substitution.

*The idea for this paper was developed while I was preparing the discussion of Hansen,Heaton and Li(2004) for the conference "Macroeconomics and Reality, 25 years after" held at UPF in Barcelona, 1-2 April 2005. I am grateful to conference participants, and in particular to Tom Sargent, for stimulating me to write this paper. Michele D'Ambrosio provided excellent research assistance.

1 Introduction

Some of the best known puzzles in the macro-finance literature depend on the fact that single period Euler equation for US consumption generates implausible and imprecise estimates of the taste parameters: a CRRA utility function requires a risk aversion coefficient of about 40 to match asset price fluctuations and short-term fluctuations in consumption. Even if that implausibly high coefficient is accepted, the unconditional first two moments of the distribution of one-period consumption growth observed in the data require a negative discount rate to generate plausible values for the risk-free rate[1].

The recent literature has produced some hope for matching consumption growth and asset pricing fluctuations by concentrating on long-run consumption growth.

This literature has analyzed multiperiod Euler equations and cointegrating relations for consumption.

In the first strand, Parker and Julliard(2005) use the multiperiod moment condition, which they consider a moment condition robust to measurement error in consumption and simple "mistakes" by consumers,. to find that this model accounts for the value premium, i.e. the difference in average returns of value vs.growth stocks. Bansal-Yaron(2005) also argue that average returns of value vs. growth stocks can be understood by their different covariance with long-run consumption growth. In fact, they examine long-run covariances of earnings with consumption, rather than the covariance of returns with consumption. Hansen, Heaton and Li(2005) show that the recursive Epstein-Zin-Weil utility variety produces a model in which asset returns at date $t + 1$ are priced by their exposure to the long-run consumption risk. Importantly, clear microeconomic foundations are provided to the empirical evidence in Bansal-Yaron(2005). However, the title of the Hansen et al. paper ends with a question mark, which is justified by the empirical evidence that the results on the differences between value and growth stocks depend crucially on whether one includes a time-trend in the regression of earnings on consumption.

The second strand of the literature examines long-run consumption and asset prices from the perspective of a cointegrating relation.

[1] See, for example, Campbell, Lo and McKinlay(1997) chapter 7, for an excellent discussion of the equity premium puzzle and of the risk-free rate puzzle.

Lettau and Ludvigson(1991), LL from now on, observe that the linearized intertemporal consumer's budget constraint implies that excess consumption with respect to wealth, its long-run equilibrium value, should be positively related to future returns from the market portfolio and negatively to future expected consumption growth. LL observe that aggregate wealth—specifically the human capital component of it—is unobservable. They then argue that the important predictive components of the consumption–aggregate wealth ratio for future market returns may be expressed in terms of observable variables, namely in terms of consumption, asset holdings, and current labor income. Their model implies that the log of consumption, labor income, and asset holdings share a common stochastic trend. They are cointegrated. The parameters of this shared trend are the average shares of human capital and asset wealth in aggregate wealth. Under the maintained assumption that expected consumption growth is not too volatile, stationary deviations from the shared trend among these three variables produce movements in the consumption–aggregate wealth ratio and predict future asset returns. LL estimate a relation between *cay* (the excess consumption with respect from its long-run target) and future stock market returns to find that *cay* is a good predictor of future stock market returns.

Somewhat surprisingly, nobody, to the best of our knowledge, has brought together the first order conditions for the solution of the consumer problem with the linearized intertemporal budget constraint to assess the empirical performance of a long-run consumption function. This is the objective of this paper.

The next section derives an explicit long-run consumption function by using the first order conditions for the consumer optimization problem in the linearized budget constraint. The following section assesses the empirical performance of the derived model along two dimensions: the precision with which relevant deep parameters are estimated and its performance in predicting stock market returns.

The last section concludes.

2 Theory: the derivation of a long-run consumption function.

Consider a representative agent economy in which all wealth, including human capital, is tradable. Let W_t be aggregate wealth,i.e. , human capital plus asset holdings in period t,. C_t is consumption and $R_{m,t+1}$ is the net return on aggregate wealth, i.e. the market portfolio. The accumulation equation for aggregate wealth may be written as:

$$W_{t+1} = (1 + R_{m,t+1})(W_t - C_t) \tag{1}$$

Define $r_{m,t+1} = \log(1 + R_{m,t+1})$, and use lowercase letters to denote log variables throughout. As LL we follow Campbell and Mankiw (1989) and assume that the consumption–aggregate wealth ratio is stationary. In this case the budget constraint may be approximated by taking a first-order Taylor expansion of equation (1) , to obtain

$$\begin{aligned} \Delta w_{t+1} &= r_{m,t+1} + k + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) \qquad (2) \\ \rho &= 1 - \exp\left(\overline{c - w}\right) \end{aligned}$$

where k, is a constant of normalization, not relevant for the problem at our hands.

By solving (2)forward, we have :

$$c_t - w_t = E_t \left[\sum_{j=1}^{\infty} \rho^j (r_{m,t+j} - \Delta c_{t+j}) \right] + \frac{\rho k}{1 - \rho} \tag{3}$$

Equation (3)constitutes the conclusion of the theoretical analysis in LL. The two authors point out that (3) shows that the consumption–wealth ratio is a function of expected future returns to the market portfolio in a broad range of optimal consumption models, so they concentrate in finding a proxy for $c_t - w_t$ and in assessing its performance for forecasting market returns. However (3)is almost an identity, so the predictive evidence on LL is only partially informative on consumer's behavior: it tells us that expected consumption growth does not fluctuate too much and that the proxy derived by LL, using cointegration analysis, for $c_t - w_t$ is not a bad one. Moreover, given that the predictive regressions in LL relate ex-post realized stock market returns at long-horizons[2] and excess consumption, their results tell us also that ex-post realized long-run returns are somewhat correlated with ex-ante expected long-run returns.

It could be interesting to go beyond this limited use of theory to see if the information in the predictive regressions could be enhanced by some structural interpretation.

To do so, we follow the recent literature concentrating on multiperiod Euler equation for consumption and consider the Epstein-Zin-Weil objective function[3], defined recursively by:

$$\begin{aligned} U_t &= \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\delta}} + \delta \left(E_t \left(U_{t+1}^{1-\gamma} \right) \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ \theta &= \frac{1-\gamma}{1-\frac{1}{\psi}} \end{aligned}$$

When $\theta = 1$ we have the usual recursion, ψ is the elasticity of intertemporal substitution, which can be different from the reciprocal of the coefficient of relative risk aversion γ.

The utility function and the budget constraint imply an Euler equation of the form:

$$1 = E_t \left[\left\{ \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right\}^{\theta} \left\{ \frac{1}{(1+R_{m,t+1})} \right\}^{1-\theta} ((1+R_{i,t+1})) \right] \tag{4}$$

Where $R_{i,t+1}$ is the return of the generic asset i. If asset returns and consumption are homoscedastic and jointly lognormal, then we can derive expression for the riskless real rate $r_{f,t+1}$ and for the return of any generic asset $r_{i,t+1}$:

$$r_{f,t+1} = -\log \delta + \frac{\theta - 1}{2} \sigma_m^2 - \frac{\theta}{2\psi^2} \sigma_c^2 + \frac{1}{\psi} E_t (\Delta c_{+1}) \tag{5}$$

$$E_t (r_{i,t+1}) - r_{f,t+1} + \frac{\sigma_i^2}{2} = \theta \frac{\sigma_{ic}}{\psi} + (1-\theta) \sigma_{im} \tag{6}$$

[2] Note that long-horizons returns are computed in LL just by cumulating period returns, in other words by assuming that $\rho = 1$.Such assumption, as we will show in the next section, is counter-factual.

[3] The following derivation is standard in the literature, see Campbell et al. Ch.8, pages 319-320.

where σ_i^2 is the variance of the return on the generic asset i,σ_m^2 is variance of the return on the market portfolio, σ_c^2 is the variance of consumption growth, σ_{ic} is the covariance between the return on asset i and consumption growth and σ_{im} is the covariance between the return on asset i and the return on the market portfolio. By using (6)[4] in equation (5)to solve out for future expected consumption growth in the intertemporal budget constraint we obtain:

$$c_t - w_t = (1 - \psi) E_t \left[\sum_{j=1}^{\infty} \rho^j r_{m,t+j} \right] + \frac{\rho (k - \mu_m)}{1 - \rho} \tag{7}$$

The solved-out consumption function (7)shows that the log consumption-wealth ratio is a constant plus $(1 - \psi)$ times the discounted value of expected future returns on invested wealth. Values of the EIS ψ lower than one imply that the income effect of higher returns dominates the substitution effect, while when ψ is greater than one, then the substitution effect dominates and the consumption-wealth ratio falls when expected returns rise. The combination of the intertemporal budget constraints with the first order condition of the consumer optimization problem under Eptein-Zin-Weil preferences makes the relation between excess consumption and expected long-term returns tighter than in the intertemporal budget constraints. Moreover, it is now explicit that the correlation between consumption and long-horizon returns depends on the combined effect of income and substitution effects. A positive relation implies that the income effect dominates, this what Lettau and Ludvigson meant when stating "...**If expected consumption growth is not too volatile**, stationary deviations from the shared trend among these three variables produce movements in the consumption–aggregate wealth ratio and predict future asset returns..."

Solving out for expected consumption growth allows the estimation of the intertemporal elasticity of substitution and provides an immediate interpretation of the correlation between excess-consumption and long-horizon returns on the market portfolio. Empirical estimation of (7)is a natural step to take at this stage. We shall devote the next section to this issue.

[4]Note that (6)determines the risk premium adjusted for the Jensen inequality term in terms of a weighted average of the Capital Asset Pricing Model and the Consumption Capital Asset Pricing Model.

3 Empirics: Cointegration and the estimation of deep parameters.

Our empirical exercise will be based on US data, in fact we consider an extended version of the Lettau and Ludvigson original data-set which considers over the period 1952:4-2003:2, quarterly observations for the following series: c_t, (log of) real consumption of non-durable and services, a_t,(log of) real financial wealth, y_t (log of) real labour income,$r_{m,t}$ quarterly returns on the S&P composite index.[5]

3.1 Identification Strategy

The objective of our investigation is the estimation of the long-run consumption function (7) .The estimation of this structural relation will allow to see how precisely the coefficient of intertemporal substitution ψ can be estimated, and to asses how tight is the relation between long-horizon returns and excess consumption.

The possibility of identifying ψ is related to the solution of two problems: finding a proxy for the log of consumption- wealth ratio and finding an instrument for expected long horizon portfolio returns.

The first problem has already been solved by LL. LL approximation starts from the consideration that aggregate wealth is the sum of asset holdings and human capital, the log of aggregate wealth may then be approximated as

$$w_t = \omega a_t + (1 - \omega) h_t$$

where ω equals the average share of asset holdings in total wealth, a_t is the log of asset holdings and h_t is the log of human capital. Using the log of labour income y_t as a proxy for human capital h_t, and the returns on the S&P composite index as a proxy for returns on total wealth we have:

[5] The first three series are taken directly from the authors' websites: http://www.ny.frb.org/rmaghome/economist/lettau/lettau.html and http://www.ny.frb.org/rmaghome/economist/ludvigson/ludvigson.html. A detailed description on the construction of these series is provided in the appendix to Lettau and Ludvigson(2001).

The S&P composite index has been taken from Robert Shiller's webpage.

$$c_t - \omega a_t + (1-\omega)\, y_t = (1-\psi)\, E_t \left[\sum_{j=1}^{\infty} \rho^j r_{m,t+j} \right] + \frac{\rho\,(k-\mu_m)}{1-\rho} \tag{8}$$

As consumption based models normally use expenditures on non-durables and services as a measure of consumption, LL before taking their model to the data, assume that log of total consumption is proportional to log of consumers expenditure on durables and services:

$$c_t = \lambda c_{n,t}.$$

By using this assumption in(8), we have

$$c_{n,t} - \frac{\omega}{\lambda} a_t + \frac{(1-\omega)}{\lambda} y_t = \frac{(1-\psi)}{\lambda} E_t \left[\sum_{j=1}^{\infty} \rho^j r_{m,t+j} \right] + \frac{\rho\,(k-\mu_m)}{1-\rho} \tag{9}$$

At this stage we adopt the methodology proposed by LL and use a cointegrating relation between the log of real consumption of non durables and services the log of real financial wealth and the log of real labour income as a proxy for the log of the consumption to wealth ratio:

$$\begin{aligned} c_t - w_t &\sim c_{n,t} - \hat{\beta}_a a_t - \hat{\beta}_y y_t = cay_t \\ \hat{\beta}_a &= 0.31,\ \hat{\beta}_y = 0.69 \end{aligned}$$

Note that the parameters of the cointegrating relation will be $[1, -(1/\lambda)\omega, -(1/\lambda)(1-\omega)]$, so we can identify $\lambda = \frac{1}{\hat{\beta}_a + \hat{\beta}_y}$.

To solve the second problem and find an instrument for long-run expected returns we need to pin down the parameter ρ, which is the complement to one of the mean consumption to total wealth ratio, and provide an estimate for future expected returns on the market portfolio. We calibrate $\rho = 0.94$, this number is obtained as the complement to one of the average consumption to financial wealth ratio (0.18) multiplied by one-third. We multiply by one third the average consumption-financial wealth ratio as the cointegration results that we use suggest that the share of financial wealth in total wealth is about one-third. To derive a proxy for the long-run expected returns we consider the following VAR:

$$\mathbf{X}_t = \mathbf{A}_t(\mathbf{L})\mathbf{X}_{t-1} + \mathbf{u}_t \tag{10}$$
$$\mathbf{X}_t = \begin{bmatrix} r_{m,t} \\ cay_t \\ \Delta c_t \\ \Delta a_t \end{bmatrix}.$$

(10)is constructed by considering the stationary VAR representation of a cointegrated system proposed by Campbell and Shiller(1987) and formally derived in Mellander et al.(1993). We consider the VAR adopted by LL and augmented it by another stationary variable, the quarterly return on the S&P composite index. This is the empirical counterpart of the market portfolio. As we focus on modelling expected returns in real-time, after initialization, at each point in time we re-estimate (10)and project it forward for a long-horizon (we consider forty quarters as $0.94^{40} = 0.01$). This allows us to construct our expected long-run return $\hat{S}^*_{t,T}$:

$$\hat{S}^*_{t,T} = \sum_{j=1}^{40} \rho^j E[\Delta r_{m,t+j} \mid \Omega_t] \tag{11}$$

where Ω_t is the information set included in our VAR. Note that, by recursively estimating the system, we allow for time variation in the parameters determining the short-run dynamics of our system. However we keep the cointegrating parameters constant. To sustain this choice we have estimated the cointegrating parameters on our initialization sample and tested if we could restrict the cointegrating coefficients to the value adopted by LL. The null that the cointegrating vector between $c_{n,t}, a_t$ and y_t is $\begin{bmatrix} 1 & 0.31 & 0.69 \end{bmatrix}$ cannot be rejected both on our initialization sample 1952:4 1980:4 and on our full-sample 1952:4-2003:2[6]. Having a measure for expected long-run returns allows GMM estimation of the following model:

[6]The cointegrating vector is originally estimated by LL over the sample 1950:1 1998:4.

$$cay_t = \frac{(1-\psi)}{\lambda}S_t + k + u_t \tag{12}$$

$$S_t = E_t\left[\sum_{j=1}^{40}\rho^j r_{m,t+j}\right] \tag{13}$$

where S_t is instrumented with $\hat{S}^*_{t,T}$. Estimated parameters will then be used to assess the predictive power of $.cay_t$ for future expected and realized long-run returns.

3.2 Empirical Results

Our empirical results are reported in Figures 1-2 and Table 1-2. The first step is the estimation of the cointegrating system. Table 1 shows that the long-run coefficients in the LL cointegrating relation are stable over time, so we estimate the stationary representation of a Vector Error Correction Model by imposing the cointegrating relations originally estimated by LL. We report initial estimates of parameters in our cointegrating system in Table 1, while Figure 1 illustrates the results of recursive estimation of all statistically significant coefficients over the initialization sample The results in Table 1 confirms the interesting properties of the data on consumption, wealth, labor income and stock market returns explored by LL. The asset growth equation shows that cay_t predicts asset growth, implying that deviations in asset wealth from its shared trend with labor income and consumption uncover an important transitory variation in asset holdings. The equation for stock returns confirms that cay_t predicts asset growth because the estimated trend deviation forecasts asset returns. Consumption growth is somewhat predictable by its own lags and by lags of stock market returns consistently with the fact that transitory variation in the (log) levels of a series requires forecastability of the growth rates. As predicted by the Life-Cycle Permanent Income Theory the Error Correction term does not enter at a statistically significant level in the equations for consumption. When log consumption deviates from its habitual ratio with log labor income and log assets, it is asset wealth, rather than consumption or labor income, that adjusts until the equilibrating relationship is restored.

Recursive estimation of significant coefficients reveals the presence of some short-run parameters instability, that does not alter the main result of estimation on the initial sample. However, the declining pattern of the effect of cay_t on stock market returns and fluctuations in assets, paired with the increasing estimate of persistence in cay_t gives some support to the view that the time-variation of the ratio of total to non-durable consumption and of the relative price of durable to non-durable consumption goods could have some importance as omitted information (see Palumbo, Rudd and Whelan(2002), Fernandez-Corugedo, Price and Blake(2003)) [7] We shall consider this issue more closely in the next section.

Recursive estimation of the cointegrated system allows us to project forward at each point in time, from 1981:1 onwards, long-run stock market returns, and construct $\hat{S}^*_{t,T}$,a proxy in real-time for long-run expected returns. $\hat{S}^*_{t,T}$ is a natural instrument to estimate the forward-looking consumption function (7)and the elasticity of intertemporal substitution ψ.

We report the results from GMM estimation in Table 2. ψ is estimated rather precisely at 0.76. On the basis of this results we can clearly explain the positive relation between excess-consumption and long-run expected returns originally found by LL. The analysis of the reduced form implicit in our GMM estimation reveals that our cointegrating system generates an estimate for long-run expected returns, that have some power to predict realized returns, although eigthy-five per cent of the variance of long-run returns cannot be explained by our projections. As a consequence it seems inappropriate to use ex-post realized returns as a proxy for ex-ante expected returns when testing implication of the theory on long-run expected returns.

It is now interesting to use the estimated coefficient in our forward looking consumption function to assess what is the predictive power for long-run expected returns and long-run ex-post observed returns. We obtain projections of the relevant variables on cay_t and analyze graphically in Figure 2 the relation between ex-post long-run realized returns and ex-ante long-run expected returns. The figure clearly shows that cay_t does much better in predicting long-run returns than the VAR based projections, this evidence confirms the forward-looking nature of this variable.

[7] Note, however, that the recursive application of the Johansen procedure always rejects the null hypothesis of at most zero cointegrating relation between consumption of non durables and services, labour income, and financial assets. Hence, our construction of cay_t is robust to these signs of instability in the short-run dynamics of the system.

4 Non-durables

In the previous section we have followed the common approach and analyzed non-durable consumption in our structural model. We reported some evidence of instability of parameters determining long-run equilibrium consumption. In this section we shall investigate instability further and we shall assess if the explicit consideration of non durable expenditure makes our results sharper.

In our derivation of the equilibrium relation for consumption we have followed the assumption that the log of total real consumption has been constantly proportional to the log of real consumption of non-durables and services. We plot in Figure 3 the ratio of these two variables. The empirical evidence does not favour the hypothesis of constancy for the parameter λ in the relation $c_t = \lambda c_{n,t}$, .and the estimate of λ derived directly form the data differs slightly from that implied by the coefficients in the cointegrating vector, although the restriction that $\hat{\beta}_a + \hat{\beta}_y = \frac{\bar{c}_t^n}{\bar{c}_t} = \frac{1}{1.04}$ is not rejected in our full sample estimation of the cointegrating vector.. Time variation in λ does have some implication for our structural estimate of the elasticity of intertemporal substitution ψ. In fact, when we estimate our deep parameter of interest using GMM on the solved out forward looking consumption function we keep λ constant at the value $1/0.9$ estimated from the cointegrating relation, therefore time variation in λ would cause time variation in our estimate of the EIS. Figure 4, that reports recursive estimates of ψ, shows that this is indeed the case.

Following the idea put forward by Fernandez-Corugedo et al.(2003) we consider the possibility of capturing the time-variation in λ by the fluctuations in the relative price of durables to non-durables. The graphical evidence of Figure 5 shows that indeed this variable has the capability of explaining the increase in the expenditure on non-durables that caused the increase in λ over the last part of our sample.

So we re-do our empirical exercise assuming a long-run linear relationship between durable consumption, non-durable consumption and the relative price:

$$c_{d,t} = \phi_1 c_{n,t} - \phi_2 p_t^d$$

and by substituting the original assumption of proportionality between $c_{n,t}$ and c_t with the following:

$$\begin{aligned} c_t &= c_{n,t} + c_{d,t} \\ c_{d,t} &= \phi_1 c_{n,t} - \phi_2 p_t^d \end{aligned}$$

Our revised structural model is :

$$c_{n,t} - \frac{\omega}{1+\phi_1} a_t - \frac{(1-\omega)}{1+\phi_1} y_t - \frac{\phi_2}{\phi_1} p_t^d = \frac{(1-\psi)}{1+\phi_1} E_t \left[\sum_{j=1}^{\infty} \rho^j r_{m,t+j} \right] + \frac{\rho (k - \mu_m)}{1-\rho} \tag{14}$$

As before we proceed to cointegration analysis and derive a proxy fir the left-hand side, then we specify a CVAR to project expected returns $r_{m,t+j}$ and finally we implement GMM estimation of the elasticity of intertemporal substitution ψ.

We implement cointegration analysis by augmenting by one variable, p_t^d, the VAR originally considered by LL. The results are reported in Table 3 and Figure 6. The evidence shows clearly that the inclusion of p_t^d in the cointegrating vector makes all parameters much more stable over time. Importantly, the relative price seems to be particularly relevant in the last part of the sample, exactly when the ratio λ has been fluctuating on a clear upward trend.

The results of estimation of the parameters in the cointegrating VAR used to project returns are reported in Table 3 and Figure 7. Almost all results of the estimation of the restricted model are confirmed, the most notable exception is that in the new specification consumption reacts directly to the disequilibrium, while in the original LL specification the loading of deviations of consumption to its long-run equilibrium value in the consumption function was estimated at zero.

Table 4 and Figure 8 reports the results of estimation of the structural model, where $\frac{1}{1+\phi_1}$ has been estimated from the sum of coefficients on wealth and labour income in the cointegrating relation and set to .98. Our main results is that the estimate of EIS does not differ substantially from that obtained using the LL specification but it is much more stable in the recursive estimation.

Finally we compare ex-post long run returns with those predicted by the VAR and by long-run consumption.

The results reported in Figure 9 confirms the finding that deviations from long-run equilibrium consumption are a much better predictor of long-run stock markets returns than VAR projections, moreover the extended model generates a much tighter fit of long-run returns than the original model.

5 Conclusions

We started from the recent evidence that the well-known asset pricing puzzles generated by the difficulties of matching high frequency consumption fluctuations with asset prices and a sensible parameterization for (the representative) consumer's preferences seems to find some interesting solutions when low-frequency fluctuations in consumption are considered. Low frequency fluctuations in consumption are considered along two different dimensions: long-run consumption growth and deviations of consumption from its long-run equilibrium path. Technically speaking the first strand of the literature concentrates on multiperiod differences in log consumption, the second concentrates on the cointegrating relation for consumption. Interestingly, the first strand of the literature concentrates only on the (multiperiod) Euler equation for the consumer optimization problem, while the second strand of the literature concentrates exclusively on the (linearized) intertemporal budget constraint. In this paper, we have used a recursive Epstein-Zin utility function and the linearized intertemporal budget constraint to derive an explicit long-run consumption function. The forward looking consumption function constitutes a tight relation between long-run stock market returns and a cointegrating relation linking consumption to wealth. Such a relation is determined by the elasticity of intertemporal substitution. The empirical estimation of the forward-looking consumption function delivers a precise estimate of the coefficient of intertemporal substitution and a shows that deviation of consumption from its long-run trend has indeed some predictive power for long-run expected stock market returns. The explicit inclusion of non-durables consumption in the model does not deliver different estimates of EIS over the full-sample but makes the empirical estimates much more stable over time. Our empirical investigation also shows that there is a sizeable difference between ex-ante expected long-run returns and ex-post realized returns. Hence, in testing theoretical predictions on the relations between fluctuations in macroeconomic variables and long-run fluctuations in asset prices, it is important to avoid using ex-post realized returns as a proxy for ex-ante expected returns. Our main conclusion is that indeed there is empirical evidence in support of the observation that consumption is striking back.

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Table 1: Estimates from of a Cointegrated System

This panel contains the results of the application of the Johansen (1995) procedure allowing an intercept in the cointegrating vector and in the VAR. The panel below reports the estimate of the cointegrating parameters and the result of the cointegration test over different samples.

Panel 1: The Cointegrating Relation			
Sample	Cointegrating Parameters		
	c_t	a_t	y_t
1952:4-1980:4	1.000	0.32	0.57
		(0.071)	(0.053)
1952:4-1998:3	1.000	0.31	0.59
		(0.031)	(0.03)
1952:4-2003:2	1.000	0.26	0.62
		(0.029)	(0.03)

Cointegration Rank Test (Maximum Eigenvalue) for 1952:4 to 1980:4				
Hypotesized No. of CE(s)	Eigenvalue	Max - Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.17	20.84	22.30	0.078
At most 1	0.11	13.52	15.89	0.114

Cointegration Rank Test (Maximum Eigenvalue) for 1952:4 to 1998:3				
Hypotesized No. of CE(s)	Eigenvalue	Max - Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.15	31.09	22.30	0.002
At most 1	0.09	17.36	15.89	0.029

Cointegration Rank Test (Maximum Eigenvalue) for 1952:4 to 2003:2				
Hypotesized No. of CE(s)	Eigenvalue	Max - Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.16	34.86	22.30	0.0005
At most 1	0.07	14.56	15.89	0.0799

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level.

* denotes rejection of the hypothesis at the 0.05 level.

**MacKinnon-Haug-Michelis (1999) p-values

Panel 2: The Cointegrated system				
	Equation			
Dependent Variable	$r_{m,t}$	cay_t	Δc_t	Δa_t
$r_{m,t-i,i=1,2}$	-0.21	-0.02	**0.036**	-0.01
$(s.e.)$	(0.21)	(0.02)	**(0.017)**	(0.060)
$\Delta c_{t-i,i=1,2}$	-1.33	-0.17	**0.41**	0.33
$(s.e.)$	(1.58)	(0.16)	**(0.12)**	(0.40)
$\Delta a_{t-i,i=1,2}$	**1.71**	0.004	-0.08	0.20
$(s.e.)$	**(0.91)**	(0.09)	(0.105)	(0.23)
cay_{t-1}	**2.40**	**0.75**	-0.055	**0.55**
$(s.e.)$	**(0.60)**	**(0.06)**	(0.044)	**(0.15)**
$\bar{R}^2$	0.16	0.61	0.21	0.16

This table reports the sum of estimated coefficients from cointegrated sytem estimated by SURE of th column variable on the row-variable, standard errors for the sum are reported in parentheses. Significant coefficients at the five percent level are highlighted in bold face. cay_t is $c_{n,t} - 0.31a_t - 0.69y_t - 0.73$, where the estimates form the cointegrating coefficients are taken from Lettau and Ludvigson(2001) and they coincide with those delivered by the implementation of the VAR representation for our system on the sample 1952:4 1998:3. c_t is consumption, .a_t is asset wealth, y_t is labor income and r_t^m are quarterly returns from the S&P composite index. The coefficient reported in Table 1 are based on the estimation of the sytem on our initialization sample: 1952:4 1980:4.

Table 2: GMM Estimates of the coefficient of intertemporal substitution

This table reports the results of GMM estimation of the following model:

$$
\begin{aligned}
cay_t &= \frac{(1-\psi)}{\lambda} S_t + k + u_t \\
S_t &= E_t \left[\sum_{j=1}^{40} \rho^j r_{m,t+j} \right]
\end{aligned}
$$

where the GMM instruments where a constant and $\hat{S}^*_{t,T} = \sum_{j=1}^{40} \rho^j E[\Delta r_{m,t+j} \mid \Omega_t]$. $E[\Delta r_{m,t+j} \mid \Omega_t]$ are the recursive projection for stock market returns based the recursive estimation of the cointegrated system (10) .The second row of the Table reports the results of estimation of the implicit reduced form in our Choice of instruments: $S_t = \hat{\beta}_0 + \hat{\beta}_1 \hat{S}^*_{t,T} + \hat{\epsilon}_t$

Sample 1981:1 1996:3			
	Structural Model		
Parameters	ψ	k	
	0.85	-0.057	
s.e.	(0.03)	(0.013)	

	Reduced From		
Parameters	$\hat{\beta}_0$	$\hat{\beta}_1$	$\bar{R}^2$
	0.29	0.42	0.14
	(0.04)	(0.12)	

Table 3: Estimates from of a Cointegrated System including the relative price of durables to non-durables

This panel contains the results of the application of the Johansen (1995) procedure allowing an intercept in the cointegrating vector and in the VAR. The tables below reports the estimate of the cointegrating parameters and the result of the cointegration test over different samples.

Panel 1: The Cointegrating Relation				
Sample	c_t	a_t	y_t	pd_t
1952:4-1980:4	1.000	-0.32	-0.60	-0.046
		(0.098)	(0.108)	(0.097)
1952:4-1998:3	1.000	-0.35	-0.63	-0.13
		(0.038)	(0.042)	(0.06)
1952:4-2003:2	1.000	-0.35	-0.63	-0.13
		(0.038)	(0.034)	(0.06)

Cointegration Rank Test (Maximum Eigenvalue) for 1952:4 to 1980:4				
Hypotesized No. of CE(s)	Eigenvalue	Max - Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.22	27.69	28.59	0.064
At most 1	0.14	16.70	22.30	0.25

Cointegration Rank Test (Maximum Eigenvalue) for 1952:4 to 1998:3				
Hypotesized No. of CE(s)	Eigenvalue	Max - Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.21	28.58	28.59	0.0005
At most 1	0.099	19.36	22.30	0.122

Cointegration Rank Test (Maximum Eigenvalue) for 1952:4 to 2003:2				
Hypotesized No. of CE(s)	Eigenvalue	Max - Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.205	46.61	28.59	0.0001
At most 1	0.084	17.85	22.30	0.186

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level.

* denotes rejection of the hypothesis at the 0.05 level.

**MacKinnon-Haug-Michelis (1999) p-values

Panel 2: The Cointegrated system					
	Equation				
Dependent variable	$r_{m,t}$	cay_t^{pd}	Δc_t	Δa_t	Δpd_t
$r_{m,t-i,i=1,2}$	-0.32	-0.02	**0.04**	-0.03	**0.06**
$(s.e.)$	(0.22)	(0.02)	(0.016)	(0.06)	(0.03)
$\Delta c_{t-i,i=1,2}$	-1.02	-0.24	**0.37**	0.33	0.02
$(s.e.)$	(1.51)	(0.19)	(0.11)	(0.40)	(0.20)
$\Delta a_{t-i,i=1,2}$	1.76	0.03	-0.12	0.13	-0.18
$(s.e.)$	(0.93)	(0.12)	(0.07)	(0.24)	(0.12)
$\Delta pd_{t-i,i=1,2}$	**2.04**	**-0.21**	0.06	**0.63**	**0.33**
$(s.e.)$	(0.81)	(0.10)	(0.06)	(0.22)	(0.10)
cay_{t-1}^{pd}	**1.53**	**0.84**	**-0.07**	**0.25**	0.10
$(s.e.)$	(0.42)	(0.05)	(0.03)	(0.11)	(0.06)
$\bar{R}^2$	0.26	0.75	0.23	0.20	0.22

This table reports the sum of estimated coefficients from cointegrated sytem estimated by SURE of th column variable on the row-variable, standard errors for the sum are reported in parentheses. Significant coefficients at the five percent level are highlighted in bold face. cay_t^{pd} is $c_{n,t} - 0.35a_t - 0.63y_t - 0.13pd_t + 0.36$, where c_t is consumption, $.a_t$ is asset wealth, y_t is labor income, pd_t is the relative price of durables to non-durables and r_t^m are quarterly returns from the S&P composite index. The coefficient reported in Table 3 are based on the estimation of the sytem on our initialization sample: 1952:4 1980:4.

Table 4: GMM Estimates of the coefficient of intertemporal substitution including the relative price of durables

This table reports the results of GMM estimation of the following model:

$$
\begin{aligned}
cay_t^{pd} &= \frac{(1-\psi)}{1+\phi_1} S_t + k + u_t \\
S_t &= E_t\left[\sum_{j=1}^{40} \rho^j r_{m,t+j}\right]
\end{aligned}
$$

where the GMM instruments where a constant and $\hat{S}^*_{t,T} = \sum_{j=1}^{40} \rho^j E[\Delta r_{m,t+j} \mid \Omega_t]$. $E[\Delta r_{m,t+j} \mid \Omega_t]$ are the recursive projection for stock market returns based the recursive estimation of the cointegrated system (10) .The second row of the Table reports the results of estimation of the implicit reduced form in our Choice of instruments: $S_t = \hat{\beta}_0 + \hat{\beta}_1 \hat{S}^*_{t,T} + \hat{\epsilon}_t$

Sample 1981:1 1996:3			
	Structural Model		
Parameters	ψ	k	
	0.84	-0.061	
s.e.	(0.03)	(0.013)	
	Reduced From		
Parameters	$\hat{\beta}_0$	$\hat{\beta}_1$	$\bar{R}^2$
	0.25	0.75	0.43
	(0.03)	(0.11)	

Figure 1: Recursive estimates of significant coefficients from the Cointegrated System

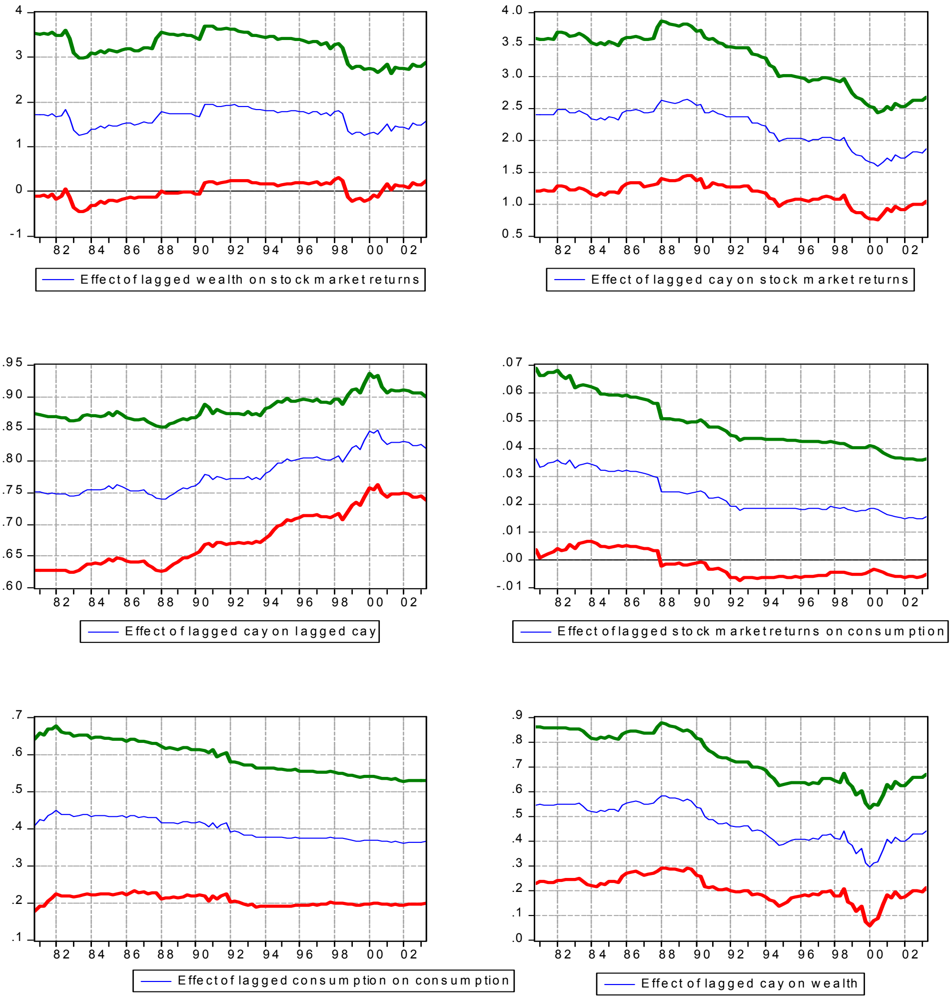
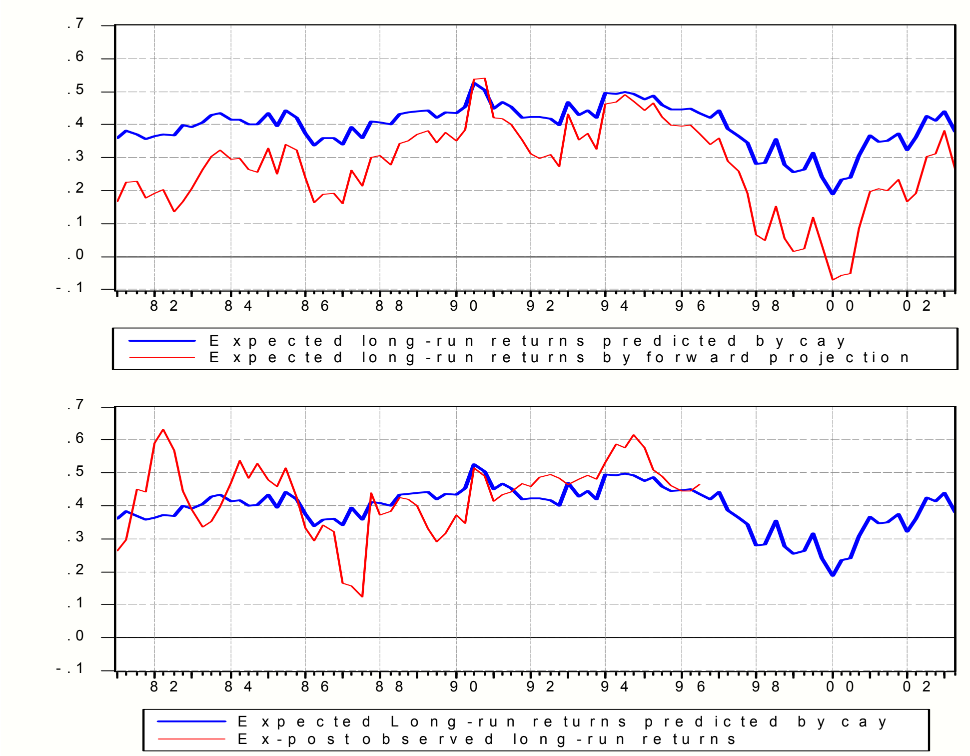


Figure 2: Ex-ante and ex-post long-run returns



Expected long-run returns by forward projection are constructed as $\hat{S}^*_{t,T} = \sum_{j=1}^{40} \rho^j E[\Delta r_{m,t+j} \mid \Omega_t]$. $E[\Delta r_{m,t+j} \mid \Omega_t]$ are the projections for stock market returns based the recursive estimation of the cointegrated system. Expected long-run returns predicted by cay are obtained using the GMM estimated coeffcients to derive $\hat{S}^*_{t,T} = \frac{1}{\left(1-\hat{\psi}\right)} cay_t + \frac{\hat{k}}{\left(1-\hat{\psi}\right)}$.Ex-post observed long-run returns are constructed by setting $E_t\left[\sum_{j=1}^{40} \rho^j r_{m,t+j}\right] = \sum_{j=1}^{40} \rho^j r_{m,t+j}$ in the formula $S_t = E_t\left[\sum_{j=1}^{40} \rho^j r_{m,t+j}\right]$.

Figure 3: Ratio between total consumption and consumption of non-durable and services

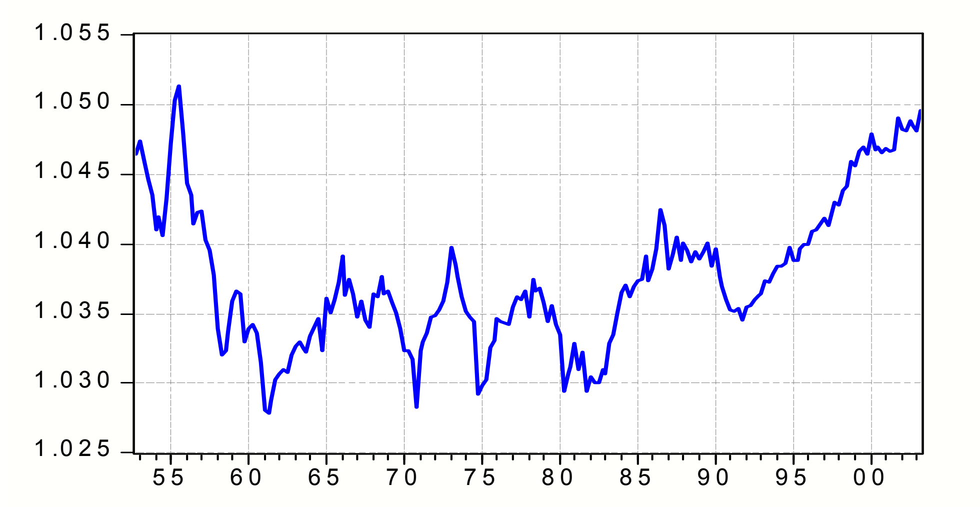


Figure 4: recursive estimate of CIS using excess consumption in the LL specification:

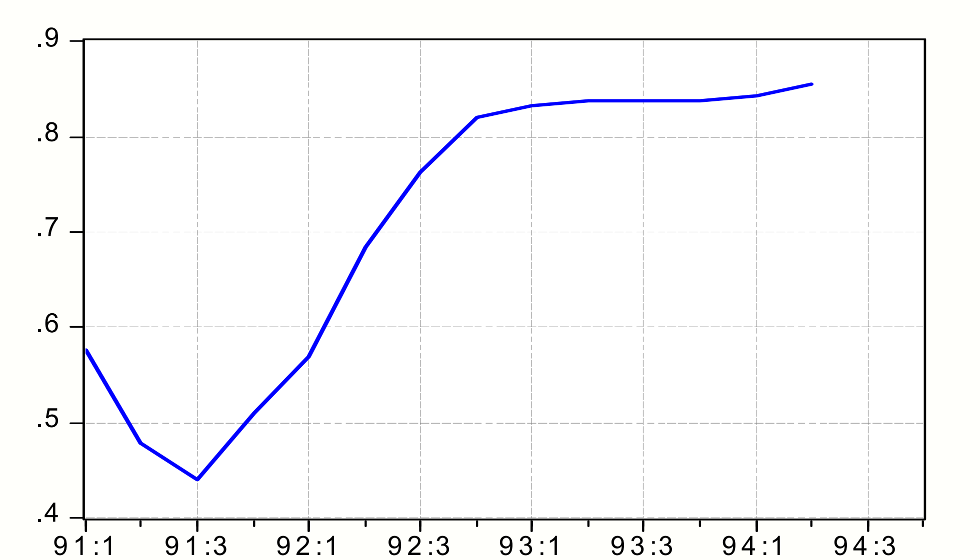


Figure 5: The relative price of durables to non durables goods

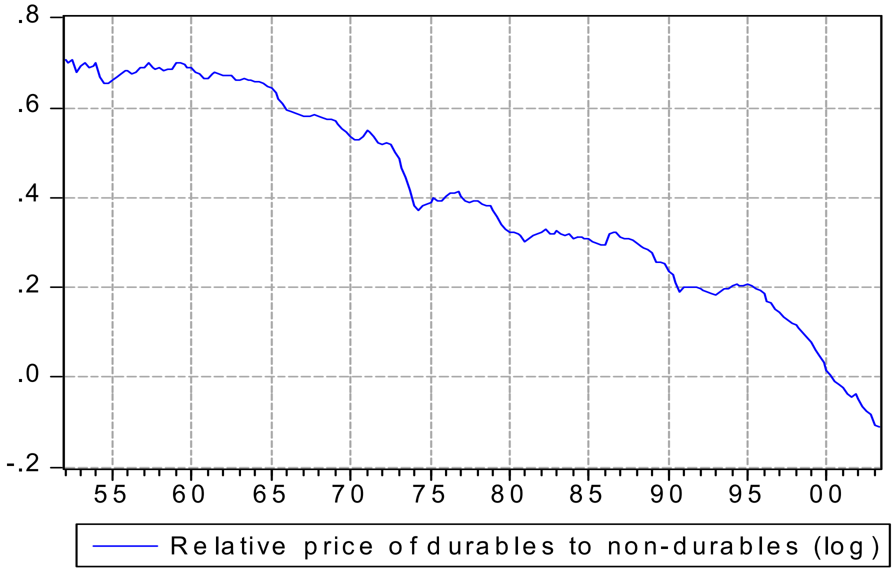


Figure 6.1: Recursive estimates of VEC coefficients

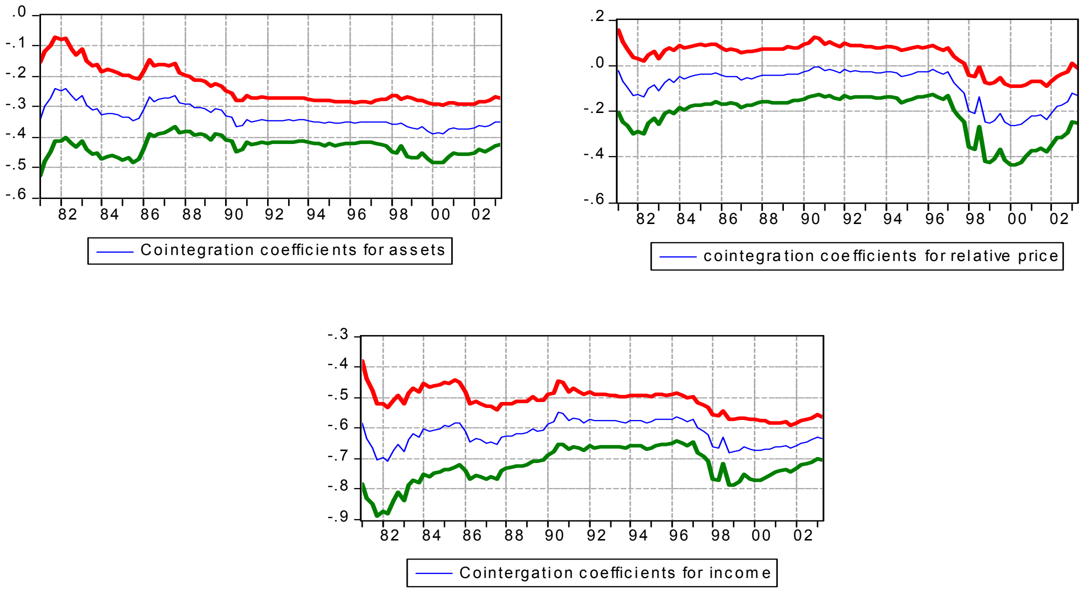


Figure 6.2: Recursive estimates of the VEC coefficients with and without the relative price of durables to non durables

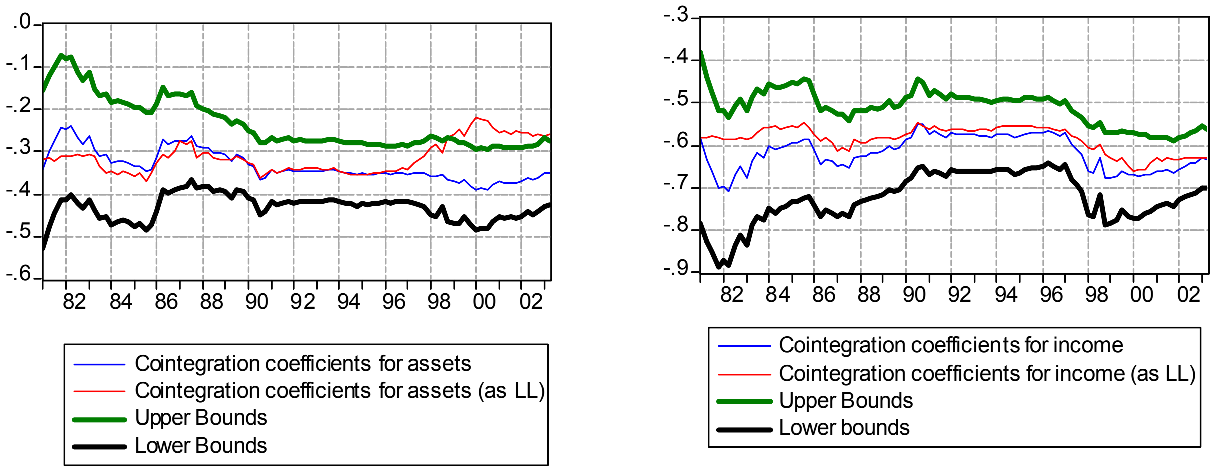


Figure 7: Recursive estimates of significant coefficients from the Cointegrated VAR including the relative price of durables

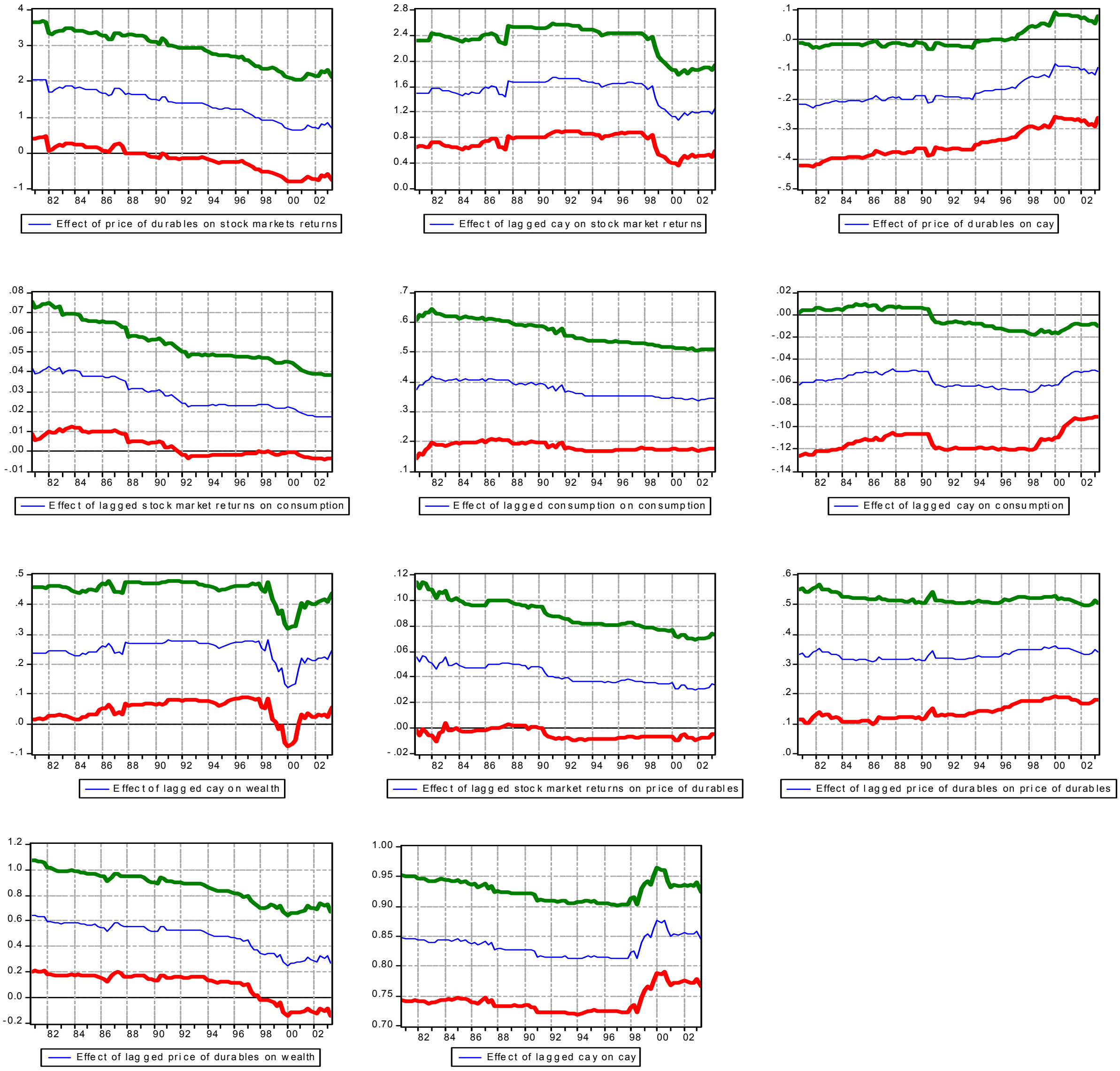


Figure 8: recursive estimate of EIS using excess consumption in the specification with the relative price of durables

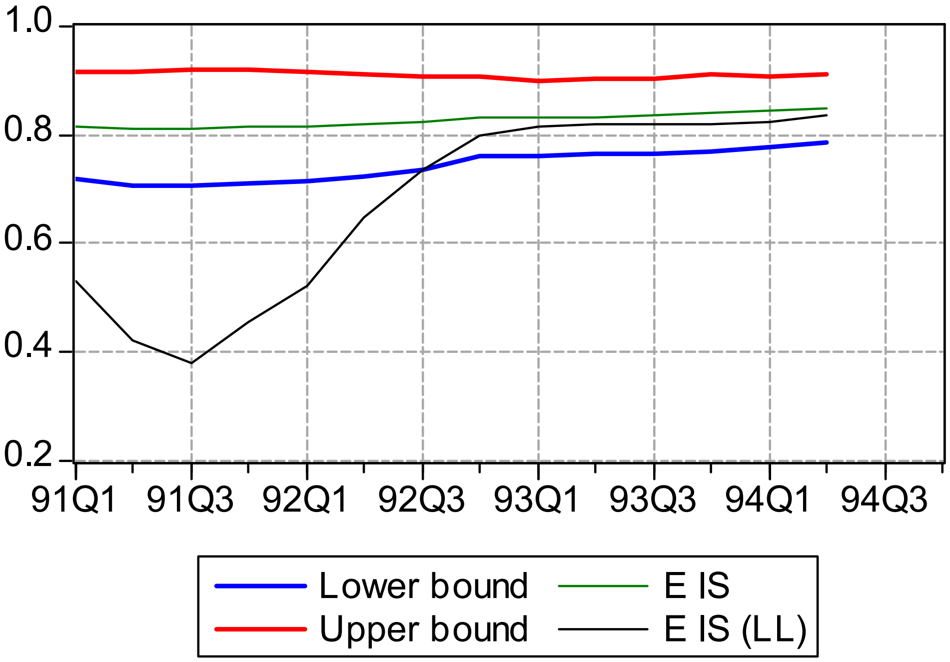
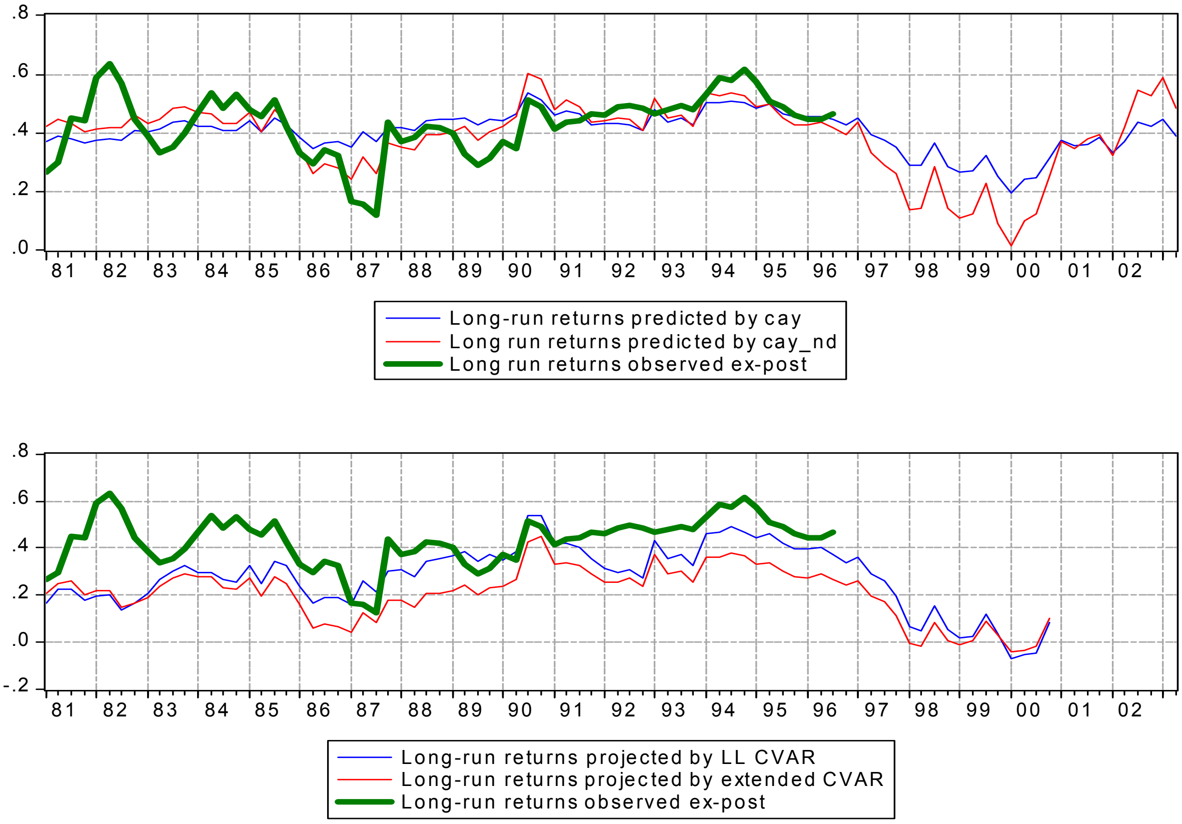


Figure 9: Ex-ante and ex-post long-run returns



Expected long-run returns by forward projection are constructed as $\hat{S}^{*}_{t,T} = \sum_{j=1}^{40} \rho^{j} E[\Delta r_{m,t+j} \mid \Omega_t]$. $E[\Delta r_{m,t+j} \mid \Omega_t]$ are the projections for stock market returns based the recursive estimation of two cointegrated systems: the one adopted by LL and the extended one including the relative price of non-durables.. Expected long-run returns predicted by cay_t^{pd} and cay_t are obtained using the GMM estimated coefficients.Ex-post observed long-run returns are constructed by setting $E_t\left[\sum_{j=1}^{40} \rho^j r_{m,t+j}\right] = \sum_{j=1}^{40} \rho^j r_{m,t+j}$ in the formula $S_t = E_t\left[\sum_{j=1}^{40} \rho^j r_{m,t+j}\right]$.