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WORKING PAPER SERIES

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Working Paper n. 318

March 2007

IGIER – Università Bocconi, Via Salasco 5, 20136 Milano –Italy
http://www.igier.uni-bocconi.it

Term Structure Forecasting: No-arbitrage Restrictions vs. Large Information set

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This version: March 2007

Abstract

This paper addresses the issue of forecasting the term structure. We provide a unified state-space modelling framework that encompasses different existing discrete-time yield curve models. within such framework we analyze the impact on forecasting performance of two crucial modelling choices, i.e. the imposition of no-arbitrage restrictions and the size of the information set used to extract factors. Using US yield curve data, we find that: a. macro factors are very useful in forecasting at medium/long forecasting horizon; b. financial factors are useful in short run forecasting; c. no-arbitrage models are effective in shrinking the dimensionality of the parameter space and, when supplemented with additional macro information, are very effective in forecasting; d. within no-arbitrage models, assuming time-varying risk price is more favorable than assuming constant risk price for medium horizon-maturity forecast when yield factors dominate the information set, and for short horizon and long maturity forecast when macro factors dominate the information set; e. however, given the complexity and the highly non-linear parameterization of no-arbitrage models, it is very difficult to exploit within this type of models the additional information offered by large macroeconomic datasets.

Keywords: Yield curve, term structure of interest rates, forecasting, large data set, factor models

JEL Classification: C33, C53, E43, E44

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1 Introduction

Yields of maturities longer than one period are risk-adjusted averages of expected future short-rates. Short term rates are monetary policy instruments, controlled by central banks. Forecasting the term structure requires forecasting risk as perceived by the market and forecasting future monetary policy rates. This paper asks what is the best model and the best information set to be used towards this objective.

The traditonal approach to modelling the yield curve in finance has followed the "parsimony principle": all the relevant information to price bonds at any given point in time is summarized by a small number of factors. Yield curve models fit different yields to maturity with a small number of factors (Litterman and Scheinkman, 1991). Such factors are obtained by using a variety of decomposition methods, the information set typically used is that of a number of observable yields.

One of the preferred modelling choices uses interpolation methods to extract latent factors from observed yields. The Nelson-Siegel (1987) approach is the most popular among market and central-bank practitioners. Nelson and Siegel three factor model explains most variances of yields at different maturity with a very good in-sample fit. Recently, Diebold-Li (2005) have successfully considered the out-of-sample forecasting performance of this model by assuming that the three factors follow AR (1) or VAR(1) processes.

An alternative strand of the literature concentrated on no-arbitrage latent factor models, in which a linear model is adopted for the latent factors and restrictions on their loadings are imposed to rule out arbitrage strategies on bonds of different maturities. No-arbitrage restrictions hence serve not only for reducing the dimension of the parameter space, but also contribute to the theoretical consistency of the model. Dai and Singleton (2000) and Piazzesi (2003) have surveyed the specification issues of affine term structure models in continuous time and discrete time respectively. Duffee (2002) showed the usefulness of essentially affine term structure model (A0(3)) in forecasting among a group of affine models. In A0(3), yields and risk prices are affine functions of three latent factors and there is no dependence between the conditional variances and the number of state variables. The recent popular discrete affine models (Ang, Piazzesi (2003), Ang, Dong and Piazzesi (2005)) also belong to this category.

Models mentioned above are traditionally based only on the information contained in the term structure. However, financial markets are not insulated from the rest of the economy. The feedback from the state of the economy to the short term interest rate is explicitly considered in the monetary policy reaction function introduced by Taylor (1993) and by now very widely adopted to explain the behaviour of central banks.

Several papers indicate that macroeconomic variables have strong effects on future movements of the yield curve (among others, Ang and Piazzesi (2003), Diebold, Rudebusch, and Aruoba (2005) and Rudebusch ad Wu (2003)). Ang and Piazzesi (2003) show that a mixed factor model (with three latent financial factors plus output and inflation) performs better than a yields-only model in terms of one step ahead forecast at quarterly frequency.

One question that may arise in this context is how to efficiently summarize the large amount of macroeconomic information used by Central Banks in forming policy for forecasting purposes. Factors models suited to deal with large cross-sections have therefore become increasingly popular in the profession. As shown in Stock and Watson (2002) and Forni, Hallin, Lippi, and Reichlin (2003), by decomposing large panels of time series in common and idiosyncratic components, the dimensionality can be greatly reduced and forecasting efficiency improved. Giannone, Reichlin and Sala (2004) show that a two dynamic factor model produces forecasting accuracy of the federal funds rate similar to that of the market.

Recently Moench (2005) has proposed a no-arbitrage macro factor-augmented VAR and finds better forecasts results at horizon from 6 to 12 months ahead, compared to a model with only financial factors.

In this paper we shall concentrate on understanding the relative importance of no-arbitrage restrictions versus large information set in forecasting the yield curve.

The paper is organized as follows.

In Section 2 we propose a unified state-space framework to evaluate the effects of incorporating factor information and/or no-arbitrage restrictions on the forecasting performance of empirical models of the yield curve. In Section 3 we apply our framework to the US yield curve and to a panel of 171 macroeconomic time-series and evaluate the forecasting performance of various models. The models will differ on how factors are extracted and on how additional macroeconomic information is incorporated . Section 4 is devoted to the discussion of our empirical results and Section 5 to conclusions.

2 The general state-space representation

We study the dynamics of the term structure in the following state-space framework. $y_{t,t+n}$ is the yield-to-maturity at time t of a bond maturing at time $t+n$. Yields with different maturities are collected in a vector $y_t =$

$[y_{t,t+1}\ , y_{t,t+2}\ , \ldots , y_{t,t+k}]'$. Equation (1) is the measurement equation, in which different yields $y_{t,t+n}$ are assumed to be determined by a set of state variables, collected in the vector X_t:

$$y_{t,t+n} = \frac{-1}{n}\left(A_n + B_n' X_t\right) + \varepsilon_{t,t+n} \qquad \varepsilon_t \sim i.i.d.N(0, \sigma^2 I) \tag{1}$$

$$X_t = \mu + \Phi X_{t-1} + v_t \qquad v_t \sim i.i.d.N(0, \Omega) \tag{2}$$

The variables in X_t can be either endogenous (that is, some of the elements of y_t are also included in X_t) or exogenous, observable or latent.

Equation (2) is the state equation in which the states X_t are assumed to follow a VAR(1) process.

The system composed by (1) and (2) is very general and can accommodate very different specification strategies. Equation (1) illustrates how the yield curve is fitted. This can be done either by pure interpolation methods or by using models that impose no-arbitrage restrictions. When no-arbitrage restrictions are imposed a more parsimonious parameterization emerges as cross-equations restrictions are derived from economic theory. In equation (2) different specifications of the information set can be adopted according to the choice of what variables to include in X_t. As we shall see below, some models will include only factors extracted from yield curve data, while others a combination of factors from the yield curve and factors from macroeconomic data. Additional specifications are possible, as different choices are available to measure the relevant factors.

We take the forecasting performance as the metric to evaluate alternative specifications.

We shall consider the following alternative specifications (see Table 1 for a summary):

1. Three factors extracted *à la* Nelson and Siegel (NS, henceforth), assumed to follow an unrestricted VAR (Diebold-Li (2005)). In this case we have:

$$B_n' = \left[-n\ , -\left(\frac{1 - e^{-\lambda n}}{\lambda}\right)\ , -\left(\frac{1 - e^{-\lambda n}}{\lambda} - n e^{-\lambda n}\right)\right] \text{ and } A_n = 0$$

We denote the three factors as $NS_t = [NS_{1,t}\ NS_{2,t}\ NS_{3,t}]'$ and define $X_t = NS_t$. Equation (1) takes the form:

$$y_{t,t+n} = NS_{1,t} + NS_{2,t}\left(\frac{1 - e^{-\lambda n}}{\lambda n}\right) + NS_{3,t}\left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right) + \varepsilon_{t,t+n} \tag{3}$$

The vector NS_t is assumed to follow an unrestricted VAR(1):

$$NS_t = \mu + \Phi NS_{t-1} + v_t \tag{4}$$

$NS_{1,t}$, $NS_{2,t}$, and $NS_{3,t}$ are estimated as parameters in a cross-section of yields, letting n, the maturity date, vary. In the time series dimension, $NS_{1,t}$, $NS_{2,t}$, and $NS_{3,t}$ have an immediate interpretation as latent factors. The loading on $NS_{1,t}$ is the only element in B_n' that does not decay to zero as n tends to infinity and can be interpreted as the long-term factor, the level of the term-structure. The loading on $NS_{2,t}$ is a monotone function that starts at 1 and decays to zero; it can be viewed as a short-term factor, the slope of the term structure. If we define $f_{1t} = NS_{1,t}+ NS_{2,t}$ we can interpret it as the risk-free rate or the monetary policy instrument. $NS_{3,t}$ is a medium term factor: its loading starts at zero, increases and then decays to zero, with the speed of decay determined by the parameter λ. This factor is usually interpreted as the curvature of the yield curve (more on this below).

The dynamics of the three factors is described by an unrestricted VAR. Forecasts for yields at any future date and at any maturity can be constructed by iterating the VAR forward and exploiting the relations between factors in equation (3) and the yields. This model is considered as the benchmark among the unrestricted models.

2. No-arbitrage VAR, in which long yields are risk-adjusted expectations of average future short-rates: the coefficients of the state-space model are restricted so as to rule out arbitrage opportunities (see Appendix 1 for details). In this case, defining the market price for risk associated with the state variables X_t as $\Lambda_t = \lambda_0 + \lambda_1 X_t$ and given the measurement equation of the short rate, $y_{t,t+1} = -\left(A_1 + B_1' X_t\right) + \varepsilon_{t,t+1}$, it is possible to show that no-arbitrage imposes the following structure on the coefficients of the measurement equation (for $n > 1$):

$$\begin{aligned} A_{n+1} &= A_n + B_n' \left(\mu - \Omega\lambda_0\right) + \tfrac{1}{2} B_n' \Omega B_n + A_1 \\ B_{n+1}' &= B_n' \left(\Phi - \Omega\lambda_1\right) + B_1' \end{aligned}$$

The restrictions imply that once the coefficients on the short rate equation (A_1, B_1') are fixed, all the other coefficients for longer maturity yields are determined by the parameters in the state equation and the risk pricing equation:

$$B_{n+1} = \left[\sum_{i=0}^{n} \left(\Phi' - \lambda_1' \Omega\right)^i\right] B_1$$

$$A_{n+1} = (n+1) A_1 + \sum_{i=0}^{n} B^{(i)}, \text{ where } B^{(i)} = B_i' \left(\mu - \Omega\lambda_0\right) + \tfrac{1}{2} B_i' \Omega B_i.$$

In this setup, following Chen and Scott (1993), factors are unobservable and are extracted by inverting the measurement equation by assuming that a number of yields equal to the number of factors is observed without error (see details in Appendix 2). The Chen and Scott factors are denoted as $CS_t = [CS_{1,t} \ CS_{2,t} \ CS_{3,t}]'$. We define $X_t = CS_t$.

In the class of no-arbitrage models, we estimate both a model with constant risk price ($\lambda_0 \neq 0$ and $\lambda_1 = 0$) and a model with time-varying risk prices ($\lambda_0 \neq 0$ and $\lambda_1 \neq 0$).

A second taxonomy considers small versus large information sets.

3. In the class of small information set models we consider the following specifications.

3a) In the unrestricted case, in addition to the NS_t factors, we also consider the unemployment rate (u_t) and the inflation rate (π_t) as additional states.

3b) In the restricted case, in addition to the CS_t factors, we also consider the unemployment rate (u_t) and the inflation rate (π_t) as additional states.

3c) We use the standard Taylor rule variables as state vector: $X_t = [FFR_t \quad u_t \quad \pi_t]'$. The Federal Funds Rate (FFR_t) in this case can be interpreted as a yield factor.

4. In the class of large information set models, we extract common factors from a large panel of macroeconomic variables ($N = 171$). We estimate factors by static principal components, as suggested by Stock and Watson (2002) and we call them $mf_t = [mf_{1,t} \quad mf_{2,t} \ldots mf_{k,t}]'$. We evaluate the forecasting performance of "large N" macroeconomic factors in the following specifications:

4a) The states are the macro factors: $X_t = mf_t$

4b) In the unrestricted case, the macro factors are added to the factors from the yield curve: $X_t = [NS_t' \quad mf_t']'$.

4c) In the restricted case, the macro factors are added to the factors from the yield curve: $X_t = [CS_t' \quad mf_t']'$.

4d) The macro factors are used as explanatory variables in a sort of "generalized" Taylor rule (see Bernake and Boivin, 2003): $X_t = [FFR_t \quad mf_t']'$, both in the unrestricted and restricted model.

Moench (2005) has showed that this class of models has good forecasting performance. We will introduce two to four ($k = 2, 3, 4$) macro factors.

3 Data and econometric methodology

3.1 Data and macro factor construction

Our basic data set consists of a set of zero-coupon equivalent US yields (1974:02-2001:12), provided by Brousseau, V. and B. Sahel (1999). We consider zero-coupon equivalent yields for US data measured at the following 11 maturities: 1-month, 2-month, 3-month, 6-month, 9-month, 1-year, 2-year, 3-year, 5-year, 7-year, and 10-year. The dynamics of the yield curve is shown in Figure 1.

The macro panel which contains 171 US macro monthly time series for the sample 1974:2-2002:12 is the same as used in Giannone, Reichlin and Sala (2004). In extracting common factors from the macro panel we have made the following methodological choices.

First, we transform the data so as to obtain stationarity. We take annual log-difference for the series that contain trends (production indices, price indices including asset prices, money stock) while series stationary by their nature (interest rates, capacity utilization, surveys, etc.) are considered in levels.

# of series	Categories	Transformation
21	IP indices	$\ln X_t - \ln X_{t-12}$
39	Labor market:	
	(employment, payroll, hrs worked, wages)	$\ln X_t - \ln X_{t-12}$
17	Sales, consumption spending	$\ln X_t - \ln X_{t-12}$
12	Inventory and orders	$\ln X_t - \ln X_{t-12}$
22	Financial markets, money and loans	$\ln X_t - \ln X_{t-12}$
25	Price indices	$\ln X_t - \ln X_{t-12}$
3	Import & export	$\ln X_t - \ln X_{t-12}$
9	Interest rates	X_t
23	Capacity utilisation, ISM mfg production and	
	inventory indices, surveys, etc.	X_t

Second, we estimate the factors by principal components (Stock and Watson (2002))

Third, we extract common factors from the whole panel in which also some of the yields used to describe the term structure. Our results are robust to excluding them.

Fourth, the first 4 common factors are used in our analysis. We rank the factors according to their explanatory power for the whole macro panel[1].

[1] This is different from Ng and Ludvigson (2006). They construct a composite factor by

As reported in Table 2, the first four factors explain up to 68% of the total variance in the transformed macro panel. The first factor highly correlates with output growth, the second represents inflation, the third is close to change in inventory, and the fourth is close to a factor of effective exchange rate.

When we use observed macro variables in the state equation, we choose the unemployment rate and the yearly CPI inflation. We choose these two series as they are not subject to revisions and are available in real-time. They are plotted in Figure 3.

3.2 Estimation specification

In the unrestricted model, the NS factors are modelled as a VAR(1).

In the restricted model under no-arbitrage, we assume that the factors have zero mean $\mu = 0$, and that the VAR matrix Φ is lower triangular, with diagonal variance covariance matrix Ω for the error term. This is the most general identified representation of the class of essential A0(3) models (Dai and Singleton (2000)). As explained above, we estimate two specifications. The first one assumes constant prices of risk: $\lambda_0 \neq 0$ and $\lambda_1 = 0$. The second one assumes time-varying prices of risk, $\lambda_1 \neq 0$. As discussed in Dai and Singleton (2002), this specification is the combination of nonzero factor correlations through the matrix, Φ, and state-dependent market prices of risk, λ_1. In order to reduce the number of parameters to be estimated, we assume λ_1 to be diagonal. Also, since the price of risk is associated with shocks to states, v_t ($\sim$ i.i.d. N(0, Ω)), which are generally assumed to be independent (hence Ω diagonal), it is reasonable to assume independent pricing of such risks.

In addition, we normalise the latent factor loadings in the short rate equation, $(-B_1)$, to be a vector of ones, and we restrict $(-A_1)$ to be the historical mean of the short rate.

When the state vector is assumed to be composed by both CS_t factors and macro variables or factors, we still assume that the VAR matrix Φ is lower triangular, but we order the macro variables or factors before the CS_t factors, and order the unemployment rate (or the real factor) before inflation (or the nominal factor). In the restricted model with observable states (FFR with macro variables/factors or only macro factors), we follow a two step

combining several common factors according to their in sample significance in explaining the bond risk premia. We have tried to rank the factors according to their contribution to R-squares of yields, but we did not find clear evidence suggesting that such a strategy improves out-of-sample forecasting.

procedure. We first estimate the VAR for the states, then given $\left(\hat{\mu}, \hat{\Phi}, \hat{\Omega}\right)$, we estimate the prices of risk, λ_0 and λ_1. In this setting, we do not restrict Φ to be lower triangular. In the short rate equation, we normalise the loadings on the latent factors to be one, and set $-A_1 = \bar{r} + B_1'\bar{X}$.

3.3 Forecast

We analyze the properties of forecasts at different horizons. In our multi-period ahead forecast, we choose iterated forecast procedure, where multiple step ahead states are obtained by iterating the one-step model forward[2]:

$$\hat{X}_{t+h|t} = \sum_{i=0}^{h} \hat{\Phi}^i \hat{\mu} + \hat{\Phi}^h \hat{X}_t$$

Forecasts based on different specifications are computed as follows:

3.3.1 *Unrestricted models*

1. Diebold-Li (2005).

We obtain the Nelson and Siegel factors from equation (3). We fix λ, the parameter governing the speed of decay in the exponential function, at 0.077, as calibrated in Diebold, Rudebusch and Aruoba (2006)[3]. Figure 4 shows the NS factors as well as their empirical correpondants. The first NS factor closely represents the 10 year yield level. Empirically, the second NS factor corresponds to the spread between long and short yields: (10-year - 1-month). The third NS factor is close to: ($2\times$ 2-year $-$ (10-year $+$ 3-month)).

After having extracted the factors and estimated the unrestricted VAR(1), we obtain forecasts by iterated projections:

$$\hat{NS}_{t+h|t} = \sum_{i=0}^{h} \hat{\Phi}^i \hat{\mu} + \hat{\Phi}^h \hat{NS}_t$$

and by using the NS parameterization:

[2] We choose iterated forecast $\hat{X}_{t+h|t} = \sum_{i=0}^{h} \hat{\Phi}^i \hat{\mu} + \hat{\Phi}^h \hat{X}_t$, instead of direct forecast $\hat{X}_{t+h|t} = \hat{\mu}_h + \hat{\Phi}_h \hat{X}_t$. Though the direct forecast might have an advantage when the model is misspecified. We find from our unrestricted VAR forecast, that except the Taylor Rule VAR with two macro variables or two macro factors, iterated forecast does always better than direct forecast. Further, the no-arbitrage model is restricted such that only one period model can be estimated, hence only iterated forecast can be implemented.

[3] The factors extracted are insensitive to the choice of λ.

$$\hat{y}_{t+h|t} = \hat{NS}_{1,t+h|t} + \hat{NS}_{2,t+h|t}\left(\frac{1-e^{-\lambda n}}{\lambda n}\right) + \hat{NS}_{3,t+h|t}\left(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right)$$

2. Diebold-Li (2005) financial factors with macro variables/factors in unrestricted VAR form.

Here, the Nelson-Siegel factors are extracted as before. The factors together with macro variables/factors are modeled as a VAR(1). The state vector in this case becomes: $X_t = [NS_t \quad z_t]'$, where z_t contains the macro information.

We disregard the specific shape of the NS interpolants and project directly the yields on the states:

$$\begin{aligned} \hat{y}_{t+h|t} &= \hat{a} + \hat{b}\hat{X}_{t+h|t} \\ \hat{X}_{t+h|t} &= \sum_{i=0}^{h} \hat{\Phi}^i \hat{\mu} + \hat{\Phi}^h \hat{X}_t \end{aligned} \tag{5}$$

We find that this leads to better forecast than fixing the loadings on NS factors as specified by equation (3). An interpretation of this is that the good forecasting performance of the Diebold-Li model largely comes from the NS factor extraction in the first step and not from the restrictions imposed by the exponential functions.

3. Interest rate rule type VAR with the state equation in the unrestricted form.

In this setting, the yields are directly projected onto the states. The esimation of both measurement equation and state dynamics are implemented by OLS as in (5).

3.3.2 *Models with no-arbitrage restrictions*

The forecast of the no-arbitrage model is obtained by following the iterated procedure for h step ahead as in (5), where the parameters in the relevant forecasting model are subject to the no-arbitrage restrictions.

4 Empirical Results

To describe our empirical results we need to define a measure of forecasting performance, a sample for estimation and a forecasting horizon.

Our chosen measure of forecasting performance is the ratio of the root mean square forecast error (RMSFE) of each models to the RMSFE of a random walk forecast.

We set the sample size fixed at 212 periods. By moving the sample forward one observation at a time, we implement rolling estimation.

We consider a range of forecasting horizons (denoted by h): 1 month, 3 months, 6 months, 9 months, 12 months, 18 months, and 24 months.

For each forecasting horizon, we conduct our exercise for all dates in the period 1995:11 - 2001:12, for a total of 74 periods. For example, yield values realised at 1995:11 are compared with their one-month ahead forecast made at period 1995:10, three-month ahead forecast made at 1995:8, etc., up to 24-month ahead forecast made at 1993:11.

We show comparisons of forecasting results from different groups of models in Tables 3-6. Our empirical results are reported by indicating better forecasts with respect to the random walk by bold characters. The best model for each combination of horizon-maturity forecast is indicated with white numbers on a black background.

In the restricted estimation under no-arbitrage with latent factors, we extract yield-curve factors with the method proposed by Chen-Scott (1993)[4] and implement one-step joint estimation of both the state and the measurement equation. In the restricted estimation with fully observable factors, we follow two-step procedure to estimate state VAR by OLS and measurement equation by MLE subsequently.

We consider constant risk price ($\lambda_1 = 0$, second row), and time-varying risk price (λ_1 diagonal, third row).

Table 3 shows results from models in which the states are either three factors extracted from the yield curve (first column) or four mf factors extracted from the macro panel (second column).

Results in the first column indicate that when only factors extracted from the yield curve are considered, no-arbitrage models (second and third row) offer better forecasting performance with respect to the unrestricted Diebold

[4] See Appendix 2 for technincal details. In the restricted case with Chen-Scott (1993) method, the selection of yields priced without error is crucial. In general, different yields carry different information, so that forecast performance varies along the selection. In the one latent factor case, 2 year yield provides better anchor for forecasting the whole curve. In the two factor case, (1 month, 2 year) or (2 year, 7 year) combination forecasts better, but have different pattern. In the three factor case, it is important to keep 2 year in the set, then choosing short yield from (1 month, 3 month) and long yield from (7 year, 10 year) gives similar result. We show result of (3 month, 2 year, 10 year) as the set of yields measured without error. Comparisons are available upon request.

and Li model. It can also be noticed that especially for models with macro factors (second column), models with and without no-arbitrage performs very similarly.

When comparing models with yield curve factors to models with macro factors (first vs. second column), we notice a few features.

First, both approaches provide good forecasting performance but at different horizon-maturity combinations: yield curve factors deliver better forecasts at the short end of the curve for short forecasting horizons; macro factors deliver excellent forecasts (the best among the models we consider) for medium maturities and medium forecast horizons. The performance of macro factors is remarkable: they combine macroeconomic information without any explicit focus on the term structure but still they obtain a very good forecasting performance for the term structure.

It is also interesting to notice that macro factors in isolation do not improve upon the random walk forecast for very short forecasting horizon. We will comment more on this below.

When comparing models with and without no-arbitrage restrictions (first vs. second and third rows), we observe the following features:

First, in the yield factor models (first column), no-arbitrage restrictions help to improve te medium horizon-maturity forecast. However, the dynamics of the two types of yield factors in the restricted and unrestricted models is very similar. Figure 5 compares the Nelson-Siegel factors estimated from the unrestricted models and the CS latent factors extracted from the restricted models. The three NS factors and the three CS no-arbitrage factors are highly correlated.

Second, in macro factor models (second column), the gains from no-arbitrage restrictions for the medium horizon-maturity are at the price of short horizon and long maturity forecast.

Third, within the no-arbitrage models (second and third rows), models with time-varying risk price (third row) improves upon the models with constant risk price (second row), fore medium horizon-maturity forecast in yield factor model and short horizon and long maturity forecast in macro factor model.

Table 4 shows the results from models with four factors: three yield factors and one macro variable/factor. In the first row we report the results for models estimated without imposing no-arbitrage restrictions in which the state equation is an unrestricted VAR. The three NS factors are included in the state equation along with the unemployment rate (left column) or the first macro factor (right column). In the second and third rows we report the results by imposing the no-arbitrage restrictions. As in Table 3, we consider explicitly the cases of constant risk premia (second row) and of time-varying risk premia (third row).

Results show that the inclusion of the first macro factor improves upon the basic Diebold-Li specification at short-medium horizon for short-medium maturities, while the inclusion of unemployment in general does not: the information summarized in the first macro factor dominates upon the macro information contained in the single real macro variable we use.

The same is true for restricted models. The inclusion of unemployment among the states does not in general improve with respect to the 3 CS factors specification.

On the contrary, the inclusion of the first macro factor does improve the forecasting performance of the model, especially at short forecasting horizon and for medium maturities and improves even more when no-arbitrage restrictions are imposed (as one can see by moving downwards in the right column of Table 4). It is interesting to notice how the short term forecasting performance depends strongly on the inclusion of financial factors: if financial factors are not included, the 1-step ahead forecast is always worse than the random walk benchmark. If financial factors are included, the forecasting performance consistently improves with respect to the random walk.

When comparing models with or without no-arbitrage restrictions, we still find that no-arbitrage restrictions help to improve the medium horizon-maturity forecast. Within no-arbitrage models, time-varying risk price is preferred to constant risk price for short-to-medium horizon-maturity forecast.

Table 5 shows the results from richer models. The left column contains the forecasting performance of models estimated with 2 macro variables in addition to the three financial factors. The two macro variables, unemployment and inflation, are taken as representative of the real and the nominal side of the economy (see Giannone, Reichlin and Sala, 2004). The right column reports results from models in which macro factors are added to financial factors. In the first row we consider models with an unrestricted state equation. The second and third rows illustrate results obtained by imposing no-arbitrage restrictions with constant risk premia (second row) or time-varying risk-premia (third row).

The inclusion of inflation along with unemployment improves the forecasting performance (top-left quadrant) with respect to both the no-macro variables case and the unemployment-only case in unrestricted models. In no-arbitrage models with inflation among the states (second and third row of the left column), the forecasting performance slightly deteriorates with respect to previous cases, with one significant exception: models with time varying price of risk deliver very precise forecasts at long forecast horizon and for long maturities.

When macro factors are included (right column), the unrestricted model with 4 macro factors along with 3 NS yield factors obtains the largest number of black cells, especially for 2 sets of horizon- maturity: long forecasting horizons and short maturity and short forecasting horizons and long maturities.

When no-arbitrage is imposed the performance of the model with 4 macro factor deteriorates dramatically (results not displayed[5]): the interaction between a large number of parameters to be estimated and the imposition of non-linear restrictions does not allow to obtain good forecasts. When only 2 macro factors are considered together with 3 CS factors, the forecasting performance does not improve (and in some circumstances deteriorates) with respect to models without macro factors or with only 1 macro factor.

Within the no-arbitrage models, the advantage of time-varying risk price with respect to constant risk price is still evident for short-to-medium horizon-maturity forecast.

In Table 6 we consider generalized Taylor rules by comparing the performance of forecast based on unrestricted Taylor rules VAR with those based on no-arbitrage Taylor rules. These results show clearly that the exclusion of factors extracted from the term structure causes a sizeable worsening in the forecasting performance especially for short horizon and long maturity yields.

It is nevertheless clear (see especially Table 6-bis) that Taylor rules with macro factors do better than Taylor rules with observable macro variables.

As in the pure macro factor case (Table 3, second column), when macro factors dominate in the information set, the no-arbitrage restrictions do not bring much gains in the forecast in general.

However, as long as no-arbitrage restrictions are imposed, assuming time-varying risk price is preferrable to constant risk price for short horizon and long maturity forecast.

Let us come back to the finding that macro factors alone do not produce good forecasts at the 1-step ahead horizon. We interpret this result as not surprising. As macro factors capture comovements among variables and as comovement manifests itself especially at business cycle frequency, it is reasonable to expect that the forecasting power of macro factors becomes

[5] Results are available by request from the authors.

significant at longer horizon than 1 or 3 months ahead. In the very short run macroeconomic variables are driven by noise than signal and their use does not improve upon term structure information.

5 Conclusions

In this paper we evaluate term structure models from the perspective of their forecasting performance. We have used a common state-space representation to nest different classes of term structure models. We have then investigated the impact on forecasting performance of alternative choices in the modelling strategy. In particular we concentrated on the effect of the no-arbitrage restrictions and on the choice of the information set in constructing factor models for the term structure. Our aim was to decompose the specific contribution to forecast of these two aspects and to answer to the following questions: how do no-arbitrage restrictions and macro information affect the forecasting performance individually and jointly? What is the optimal forecasting strategy for different yields at different horizons?

Our results point to the following conclusions:

a. macro factors are very useful in forecasting at medium/long forecasting horizon;

b. financial factors are useful in short run forecasting;

c. no-arbitrage models are effective in shrinking the dimensionality of the parameter space and, when supplemented with additional macro information, are very effective in forecasting;

d. within no-arbitrage models, assuming time-varying risk price is more favorable than assuming constant risk price for medium horizon-maturity forecast when yield factors dominate the information set, and for short horizon and long maturity forecast when macro factors dominate the information set;

e. however, given the complexity and the highly non-linear parameterization of no-arbitrage models, it is very difficult to exploit within this type of models the additional information offered by large macroeconomic datasets.

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7 Appendix 1. No-Arbitrage Restrictions on Bond Pricing Parameters

1. **State variable dynamics.**

   Transition equation for X_t follows VAR(1):

$$X_t = \mu + \Phi X_{t-1} + v_t,$$

v_t is i.i.d. N(0, Ω).

2. **Short rate equation.**

$$r_t = \delta_0 + \delta_1' X_t$$

δ_0: a scalar.
δ_1: $K \times 1$ vector.

3. **Time-varying prices of risk** (associated with the sources of uncertainty v_t).

$$\Lambda_t = \lambda_0 + \lambda_1 X_t$$

Λ_t: $K \times 1$ vector.
λ_0: $K \times 1$ vector.
λ_1: $K \times K$ matrix.
If investors are risk-neutral, $\lambda_0 = 0$ and $\lambda_1 = 0$, hence $\Lambda_t = 0$, no risk adjustment. If $\lambda_0 \neq 0$ and $\lambda_1 = 0$, then price of risk is constant.

4. **Pricing kernel.**

   No arbitrage opportunity between bonds with different maturities implies that there is a discount factor m linking the price of yield of maturity n this month with the yield of maturity $n-1$ next month.

$$P_t^{(n)} = E_t\left[m_{t+1} P_{t+1}^{(n-1)}\right]$$

The stochastic discount factor is related to the short rate and risk perceived by the market,

$$m_{t+1} = \exp\left(-r_t - \tfrac{1}{2}\Lambda_t' \Omega \Lambda_t - \Lambda_t' v_{t+1}\right)$$

No-arbitrage recursive relation can be derived from the above equations as:

$$
\begin{aligned}
P_t^{(n)} &= E_t\left[m_{t+1}P_{t+1}^{(n-1)}\right] = E_t\left[m_{t+1}m_{t+2}P_{t+2}^{(n-2)}\right] \\
&= E_t\left[m_{t+1}m_{t+2}...m_{t+n}P_{t+n}^{(0)}\right] = E_t\left[m_{t+1}m_{t+2}...m_{t+n}\cdot 1\right] \\
&= E_t\left[\exp\left(-\sum_{i=0}^{n-1}\left(r_{t+i}+\tfrac{1}{2}\Lambda'_{t+i}\Omega\Lambda_{t+i}+\Lambda'_{t+i}v_{t+1+i}\right)\right)\right] \\
&= E_t\left[\exp\left(A_n+B'_nX_t\right)\right] = E_t\left[\exp\left(-ny_{t,n}\right)\right] \\
&= E_t^Q\left[\exp\left(-\sum_{i=0}^{n-1}r_{t+i}\right)\right]
\end{aligned}
$$

E_t^Q denotes the expectation under the risk-neutral probability measure, under which the dynamics of the state vector X_t are characterised by the risk-neutral vector of constants and autoregressive matrix:

$$
\begin{gathered}
\mu^Q = \mu - \Omega\lambda_0 \\
\Phi^Q = \Phi - \Omega\lambda_1
\end{gathered}
$$

Affine functions of the state variables for yields are:

$$
\begin{gathered}
p_{t,t+n} \equiv \ln P_t^{(n)} = A_n + B'_nX_t \\
y_{t,t+n} = a_n + b'_nX_t = \tfrac{-1}{n}\left(A_n + B'_nX_t\right)
\end{gathered}
$$

where the coefficients and follow the difference equations:

$$
\begin{gathered}
A_{n+1} = A_n + B'_n\left(\mu - \Omega\lambda_0\right) + \tfrac{1}{2}B'_n\Omega B_n + A_1 \\
B'_{n+1} = B'_n\left(\Phi - \Omega\lambda_1\right) + B'_1
\end{gathered}
$$

with $a_1 = \delta_0 = -A_1$ and $b_1 = \delta_1 = -B_1$.
These can be derived from the pricing kernel equation.

$$
\begin{aligned}
P_t^{(n+1)} &= E_t\left[m_{t+1}P_{t+1}^{(n)}\right] \\
&= E_t\left[\exp\left\{-r_t-\tfrac{1}{2}\Lambda'_t\Omega\Lambda_t-\Lambda'_tv_{t+1}\right\}\exp\left\{A_n+B'_nX_{t+1}\right\}\right] \\
&= \exp\left\{-r_t-\tfrac{1}{2}\Lambda'_t\Omega\Lambda_t+A_n\right\}E_t\left[\exp\left\{-\Lambda'_tv_{t+1}+B'_nX_{t+1}\right\}\right] \\
&= \exp\left\{-\delta_0-\delta'_1X_t-\tfrac{1}{2}\Lambda'_t\Omega\Lambda_t+A_n\right\} \\
&\quad\cdot E_t\left[\exp\left\{-\Lambda'_tv_{t+1}+B'_n\left(\mu+\Phi X_t+v_{t+1}\right)\right\}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \exp\left\{-\delta_0 - \delta_1' X_t - \tfrac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'\left(\mu + \Phi X_t\right)\right\} \\
&\quad \cdot E_t\left[\exp\left\{-\Lambda_t' v_{t+1} + B_n' v_{t+1}\right\}\right] \\
&= \exp\left\{-\delta_0 + A_n + B_n'\mu + \left(B_n'\Phi - \delta_1'\right)X_t - \tfrac{1}{2}\Lambda_t'\Omega\Lambda_t\right\} \\
&\quad \cdot E_t\left[\exp\left\{\left(-\Lambda_t' + B_n'\right)v_{t+1}\right\}\right] \\
&= \exp\left\{-\delta_0 + A_n + B_n'\mu + \left(B_n'\Phi - \delta_1'\right)X_t - \tfrac{1}{2}\Lambda_t'\Omega\Lambda_t\right\} \\
&\quad \cdot \exp\left\{E_t\left[\left(-\Lambda_t' + B_n'\right)v_{t+1}\right] + \tfrac{1}{2}var\left[\left(-\Lambda_t' + B_n'\right)v_{t+1}\right]\right\} \\
&= \exp\left\{-\delta_0 + A_n + B_n'\mu + \left(B_n'\Phi - \delta_1'\right)X_t - \tfrac{1}{2}\Lambda_t'\Omega\Lambda_t\right\} \\
&\quad \exp\left\{\tfrac{1}{2}var\left[\left(-\Lambda_t' + B_n'\right)v_{t+1}\right]\right\} \\
&= \exp\left\{-\delta_0 + A_n + B_n'\mu + \left(B_n'\Phi - \delta_1'\right)X_t - \tfrac{1}{2}\Lambda_t'\Omega\Lambda_t\right\} \\
&\quad \cdot \exp\left\{\tfrac{1}{2}E_t\left[\left(-\Lambda_t' + B_n'\right)v_{t+1}v_{t+1}'\left(-\Lambda_t + B_n\right)\right]\right\} \\
&= \exp\left\{-\delta_0 + A_n + B_n'\mu + \left(B_n'\Phi - \delta_1'\right)X_t - \tfrac{1}{2}\Lambda_t'\Omega\Lambda_t\right\} \\
&\quad \cdot \exp\left\{\tfrac{1}{2}\left[\Lambda_t'\Omega\Lambda_t - 2B_n'\Omega\Lambda_t + B_n'\Omega B_n\right]\right\} \\
&= \exp\left\{-\delta_0 + A_n + B_n'\mu + \left(B_n'\Phi - \delta_1'\right)X_t - B_n'\Omega\Lambda_t + \tfrac{1}{2}B_n'\Omega B_n\right\} \\
&= \exp\left\{-\delta_0 + A_n + B_n'\mu + \left(B_n'\Phi - \delta_1'\right)X_t - B_n'\Omega\Lambda_t + \tfrac{1}{2}B_n'\Omega B_n\right\} \\
&= \exp\left\{-\delta_0 + A_n + B_n'\mu + \left(B_n'\Phi - \delta_1'\right)X_t - B_n'\Omega\left(\lambda_0 + \lambda_1 X_t\right) + \tfrac{1}{2}B_n'\Omega B_n\right\} \\
&= \exp\left\{-\delta_0 + A_n + B_n'\left(\mu - \Omega\lambda_0\right) + \tfrac{1}{2}B_n'\Omega B_n + \left(B_n'\Phi - B_n'\Omega\lambda_1 - \delta_1'\right)X_t\right\} \\
&= \exp\left\{\left[A_1 + A_n + B_n'\left(\mu - \Omega\lambda_0\right) + \tfrac{1}{2}B_n'\Omega B_n\right] + \left[B_n'\Phi - B_n'\Omega\lambda_1 + B_1'\right]X_t\right\}
\end{aligned}
$$

5. **An alternative presentation for the no-arbitrage coefficients.**

In order to understand intuitively how these restrictions are imposed directly on the coefficients in the yield equation, we can write them in the following affined form.

Given that

$$
\begin{gathered}
p_{t,t+n} = A_n + B_n' X_t \\
y_{t,t+n} = a_n + b_n' X_t = \tfrac{-1}{n}\left(A_n + B_n' X_t\right)
\end{gathered}
$$

we can derive

$$
\begin{gathered}
b_{n+1} = \tfrac{1}{(n+1)}\left[\sum_{i=0}^{n}\left(\Phi' - \lambda_1'\Omega\right)^i\right] b_1 \\
a_{n+1} = a_1 - \tfrac{1}{(n+1)}\sum_{i=1}^{n} B^{(i)}
\end{gathered}
$$

where $B^{(i)} = B_i'\left(\mu - \Omega\lambda_0\right) + \frac{1}{2}B_i'\Omega B_i$.

8 Appendix 2. The likelihood function with Chen-Scott (1993) method

(The likelihood function representation follows closely to Ang, Piazzesi (2003).)

In order to be able to extract factors under no-arbitrage restrcitions, we employ the method by Chen and Scott (1993). Assume that there are K factors in the state equation and that among them, K_2 factors are unobserved. When the number of yields N exceeds number of unobserved factors, K_2, following Chen and Scott (1993), we assume that K_2 yields, y_t^{NE}, are observed without measurement errors, and that $N - K_2$ yields, y_t^E, are measured with error u_t^m. The state vector contains both observed variables X_t^o and latent factors X_t^u, thus $X_t = [X_t^o; \quad X_t^u]$

The measurement equation can be written as following:

$$y_t = a + b^o X_t^o + b^u X_t^u + b^m u_t^m$$

where $y_t = \begin{bmatrix} y_t^{NE} \\ y_t^E \end{bmatrix}$, $a = \begin{bmatrix} a^{NE} \\ a^E \end{bmatrix}$, $b^o = \begin{bmatrix} b^{NE,o} \\ b^{E,o} \end{bmatrix}$, $b^u = \begin{bmatrix} b^{NE,u} \\ b^{E,u} \end{bmatrix}$, and $b^m = \begin{bmatrix} \mathbf{0}_{(K_2 \times (N-K_2))} \\ b^{E,m} \end{bmatrix}$

For a given parameter vector $\theta = (\mu, \Phi, \Omega, \delta_0, \delta_1, \lambda_0, \lambda_1)$, the unobserved factors X_t^u will be solved from the yields and the observed variables X_t^o as: $X_t^u = \left(b^{NE,u}\right)^{-1} \left[Y_t^{NE} - a^{NE} - b^{NE,o} X_t^o\right]$.

Denoting the normal density functions of the state variables X_t^u and the error u_t^m as f_X and f_{u^m} respectively, the joint likelihood $\pounds(\theta)$ of the observed data on zero coupon yields Y_t and the observable factors X_t^o is given by:

$$\begin{aligned}
\pounds(\theta) &= \prod_{t=2}^{T} f\left(y_t, X_t^o | y_{t-1}, X_{t-1}^o\right) \\
\log(\pounds(\theta)) &= \sum_{t=2}^{T} \log|\det(J^{-1})| + \log f_X\left(X_t^o, X_t^u | X_{t-1}^o, X_{t-1}^u\right) + \log f_{u^m}(u_t^m) \\
&= -(T-1)\log|\det(J)| - \tfrac{(T-1)}{2}\log(\det(\Omega)) \\
&\quad -\tfrac{1}{2}\sum_{t=2}^{T}\left(X_t - \mu - \Phi X_{t-1}\right)' \Omega^{-1} \left(X_t - \mu - \Phi X_{t-1}\right) \\
&\quad -\tfrac{(T-1)}{2}\log\sum_{i=1}^{N-K_2}\sigma_i^2 - \tfrac{1}{2}\sum_{t=2}^{T}\sum_{i=1}^{N-K_2}\frac{\left(u_{t,i}^m\right)^2}{\sigma_i^2}
\end{aligned}$$

(The constant terms like $\frac{(T-1)}{2}\log(2\pi)$ are ignored.)

The Jacobian term is: $J = \begin{pmatrix} I_{K-K_2} & 0_{(K-K_2)\times K_2} & 0_{(K-K_2)\times(N-K_2)} \\ B^o & B^u & B^m \end{pmatrix}$.

Figure 1. Yield data

(1974:02-2001:12)

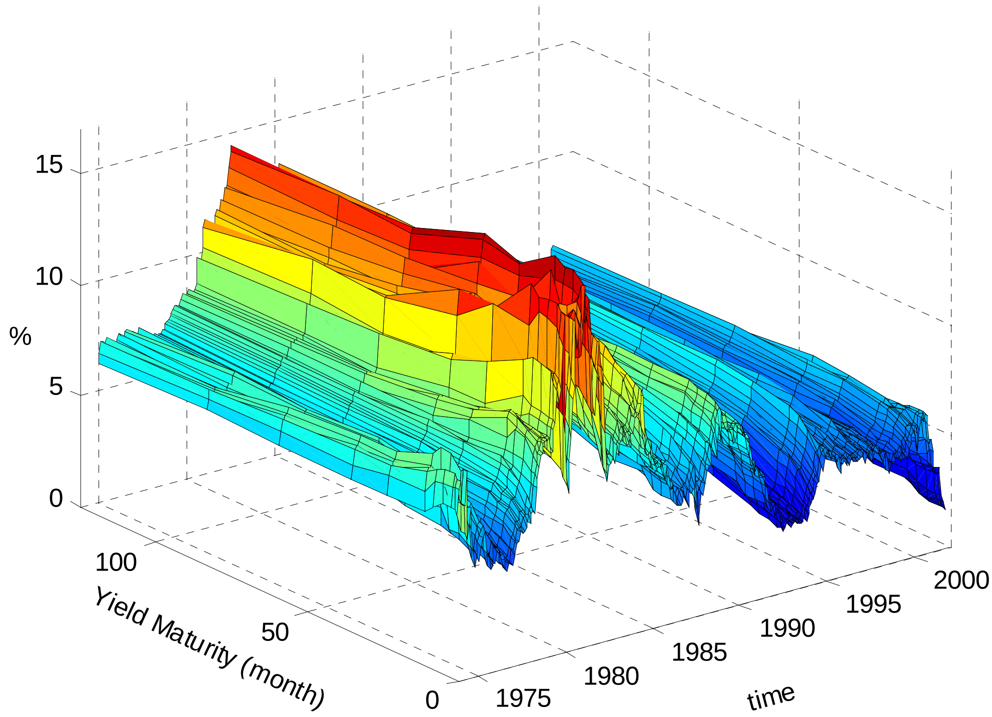


Figure 2. Macro variables

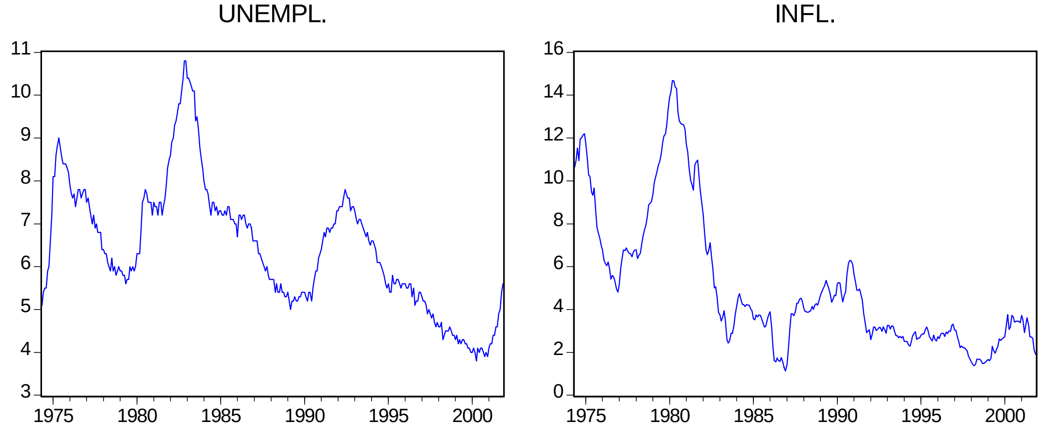


Figure 3. First four macro factors

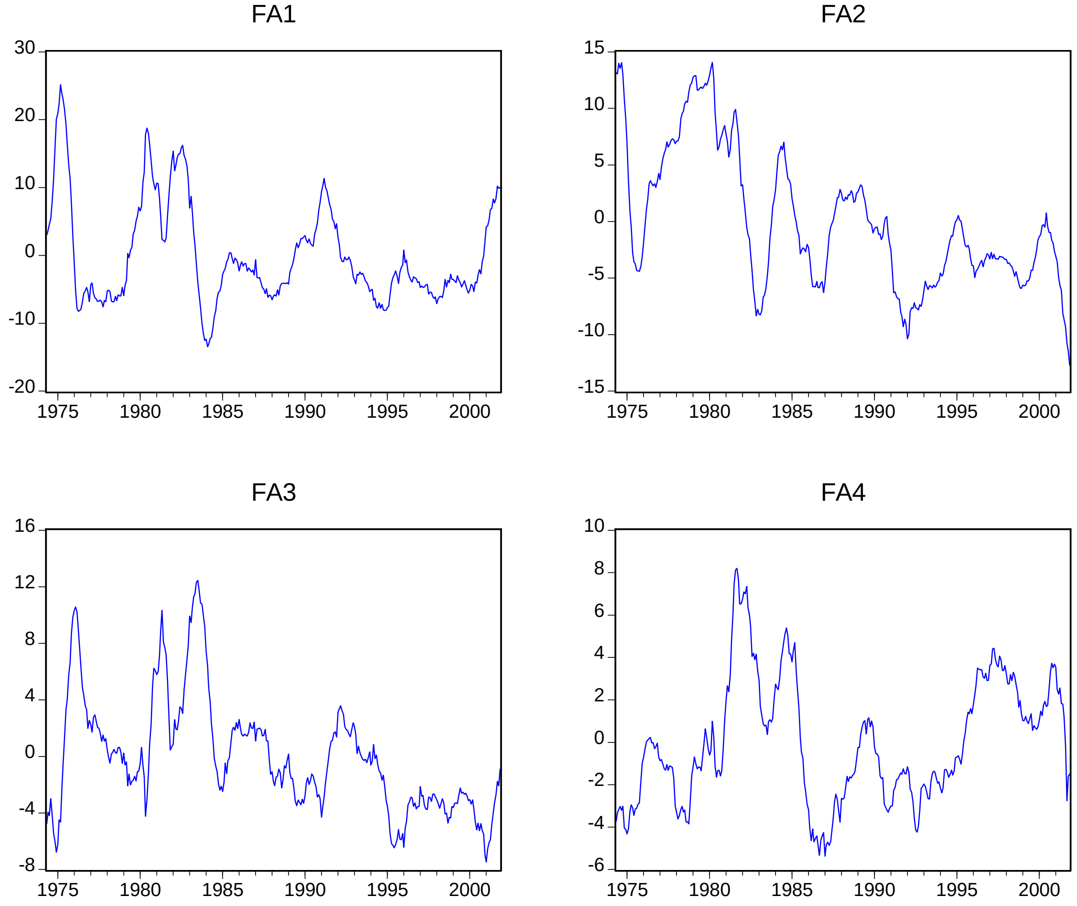
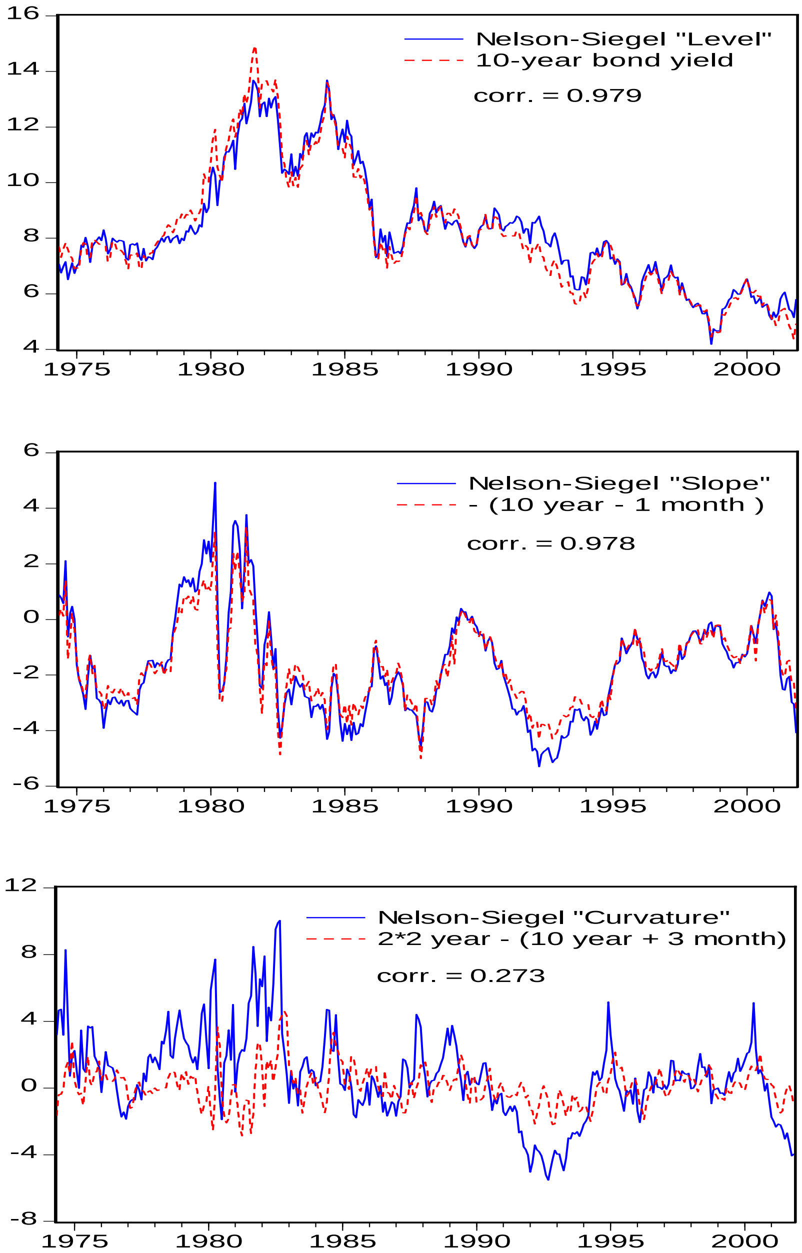
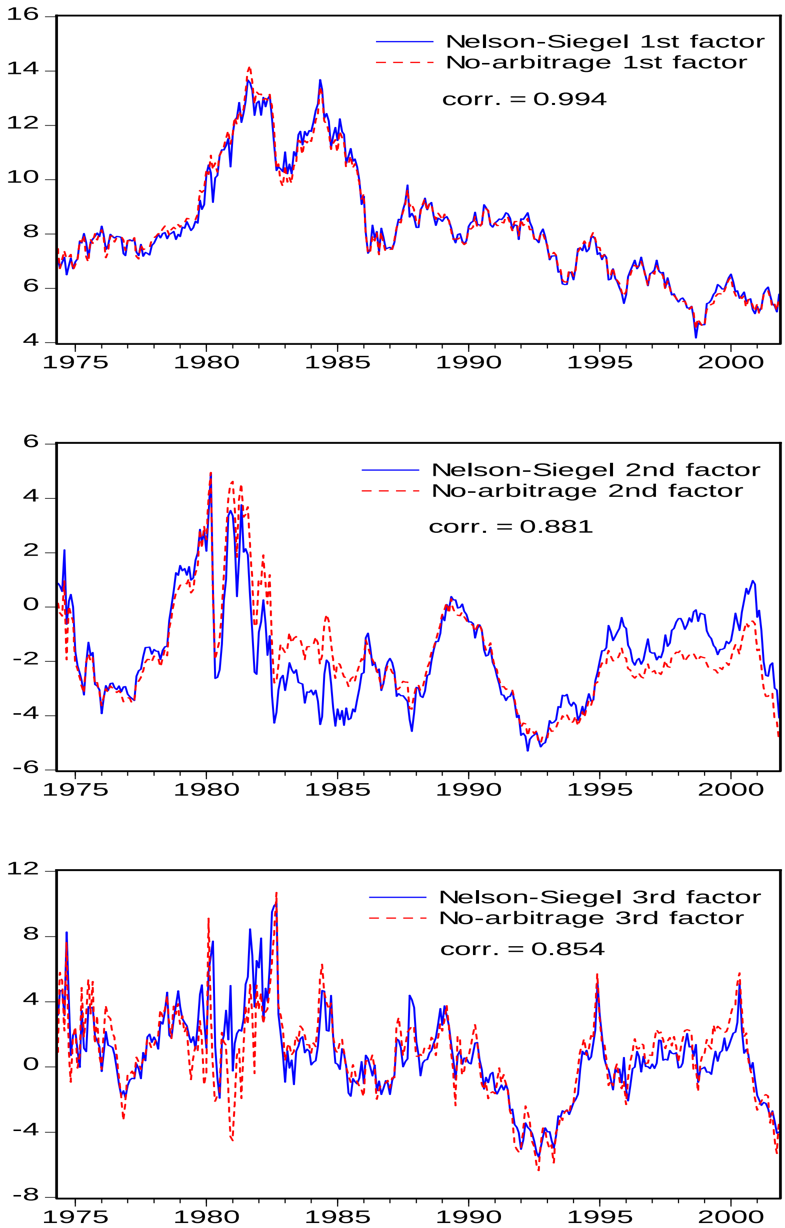


Figure 4. Nelson-Siegel Factors and Yields



Note: This figure shows Nelson-Siegel factors and corresponding empirical proxies directly obtained by yields.

Figure 5. Nelson-Siegel Factors and No-arbitrage Factors



Note: No-arbitrage factors shown here are extracted under time-varying risk price ($\lambda_1 \neq 0$). They are rescaled to make the comparison. No-arbitrage factors extracted under non time-varying risk price ($\lambda_1 = 0$) are very similar. The correlations between the corresponding factors are all 0.999.

Tabel 1. Modeling Framework

State variables		Unrestricted model		Restricted Model (No-arbitrage)		
# Financial factors	# Macro Variables /factors	Small information set: {Yield factors, unempl., inflation}	Large information set: {Macro factors}	Small information set: {Yield curve, output, inflation}	Large information set: {Macro factors}	Reported in table
3	0	3 Nelson Siegel (NS) factors (Diebold-Li, 2003)		3 latent factors by Chen-Scott (CS) (1993) method		Table 3
	1	unempl. + 3 NS	mf1 + 3 NS	unempl. + 3 CS	mf1 + 3 CS	Table 4
	2-4	unempl., inflation + 3 NS	mf1, mf2,mf3,mf4 + 3 NS	unempl., inflation + 3 CS	mf1, mf2 + 3 CS	Table 5*
2	1	unempl. + 2 NS	mf1 + 2 NS	unempl. + 2 CS	mf1 + 2 CS	
	2	unempl., inflation + 2 NS	mf1, mf2 + 2 NS	unempl., inflation + 2 CS	mf1, mf2 + 2 CS	
	3-4		mf1, mf2, mf3 + 2 NS		mf1, mf2, mf3 + 2 CS	
1 (Generalised Taylor Rule)	2	unempl., inflation + FFR	mf1, mf2 + FFR	unempl., inflation + FFR	mf1, mf2 + FFR	Table 6
	3		mf1, mf2, mf3 + FFR		mf1, mf2, mf3 + FFR	
	4		mf1, mf2, mf3, mf4 + FFR		mf1, mf2, mf3, mf4 + FFR	
0	4		mf1, mf2, mf3, mf4		mf1, mf2, mf3, mf4	Table 3

NS: Nelson-Siegel factors used in Diebold-Li model; **CS**: Latent factor extracted by using Chen-Scott method; **mf**: macro common factors.
We report forecast comparison for highlighted models in Table 3 to Table 6.
Table 5*: Table 5 shows small information set or large information set augmented with 3 yield factors. In the unrestricted large N case, we show the result of 3NS with 4 mf; in the restricted large N case, we show 3 NS with 2 mf, because as the number of mf increases to 3 or 4, so does the parameter uncertainty, hence the results deteriorate very much.

Tabel 2. Factor loadings

Factors are extracted from a panel with 171 macro variables (1974:2-2002:12). We report in the following the first four factors with the eight variables with which they are most highly correlated. The R^2 indicates the total explanatory power of the single factor on the corresponding variable. The first four factors together explain 68.53% of the total variation in the macro panel. The number of series is their original order in the panel used in Giannone, Reichlin and Sala (2004).

Factor 1	**Total variance explained: 32.56%**	R^2
11	Index of IP: Mfg	0.910
17	Index of IP: Non-energy, total	0.906
33	No. Of unemployed in the civ. labor force (CLF)	0.889
1	Index of IP: Total	0.887
12	Index of IP: Mfg, durables	0.884
20	Index of IP: Non-energy excl CCS	0.883
9	Index of IP: Materials, nonenergy, durables	0.878
8	Index of IP: Materials	0.869
Factor 2	**Total variance explained: 21.17%**	
168	Philadelphia Fed Business Outlook: Prices received	0.752
167	Philadelphia Fed Business Outlook: Prices paid	0.745
93	Mfg new orders: nondurable (in mil of current $)	0.691
95	Mfg unfilled orders: all mfg industries (in mil of current $)	0.645
139	CPI: commodities	0.639
144	CPI: all items less medical care	0.636
150	PCE prices: nondurable	0.635
133	CPI: all items (urban)	0.624
Factor 3	**Total variance explained: 9.83%**	
122	Loans and Securities @ all comm banks: Securities, U.S. govt (in mil of current $)	0.593
27	Capacity Utilization: Utilities	0.448
83	Inventories: Mfg and Trade: Mfg, durables (mil of chained 96$)	0.446
121	Loans and Securities @ all comm banks: Securities, total (in mil of current $)	0.439
114	M1 (in bil of current $)	0.397
82	Inventories: Mfg and Trade: Mfg (mil of chained 96$)	0.395
36	Mean duration of unemployment	0.352
81	Inventories: Mfg and Trade: Total (mil of chained 96$)	0.341
Factor 4	**Total variance explained: 4.97%**	
99	Nominal effective exchange rate	0.607
100	Spot Euro/US	0.561
101	Spot SZ/US	0.506
59	Employment on nonag payrolls: Government	0.399
102	Spot Japan/US	0.327
117	Monetary base, adjusted for reserve requirement (rr) change (bil of $)	0.309
103	Spot UK/US	0.272
118	Depository institutions reserve: Total (adj for rr changes)	0.266

Table 3. Models in which factors are not mixed

RMSFE ratio with respect to Random Walk *h* months ahead

Unrestricted

Three Yield Factor models

Diebold-Li, VAR(1) states: {NS1 NS2 NS3}

h	1	3	6	9	12	18	24
m01	0.86	0.85	0.89	0.92	0.99	1.19	1.23
m02	0.91	0.85	0.89	0.92	0.97	1.16	1.21
m03	0.98	0.88	0.89	0.92	0.95	1.11	1.18
m06	1.02	0.94	0.94	0.94	0.94	1.06	1.19
m09	1.03	0.96	0.96	0.97	0.95	1.06	1.21
y01	0.79	0.91	0.94	0.94	0.92	1.03	1.18
y02	1.15	1.03	1.01	1.00	0.95	1.01	1.25
y03	0.79	0.93	0.96	0.96	0.92	1.02	1.32
y05	1.01	0.99	1.02	1.06	1.03	1.15	1.55
y07	1.00	0.98	1.01	1.06	1.05	1.19	1.60
y10	0.99	0.99	1.07	1.15	1.16	1.32	1.73

Four Macro Factor models

VAR(1) states: {mf1 mf2 mf3 mf4}

h	1	3	6	9	12	18	24
m01	1.83	1.06	0.82	0.74	0.79	0.97	1.06
m02	2.16	1.09	0.80	0.73	0.77	0.95	1.07
m03	2.42	1.10	0.81	0.74	0.77	0.94	1.07
m06	2.21	1.03	0.79	0.74	0.76	0.89	1.08
m09	1.93	0.94	0.77	0.74	0.75	0.87	1.07
y01	1.82	0.87	0.75	0.73	0.75	0.88	1.06
y02	1.69	0.93	0.78	0.74	0.73	0.79	1.01
y03	1.89	0.97	0.80	0.74	0.72	0.78	1.01
y05	2.42	1.33	1.00	0.87	0.79	0.82	1.07
y07	2.85	1.59	1.19	1.01	0.92	0.93	1.18
y10	3.55	1.98	1.51	1.27	1.12	1.10	1.33

Restricted

Three Yield Factor models

A0(3), ne =(m03, y02, y10)
VAR(1) states: {CS1 CS2 CS3}

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$

h	1	3	6	9	12	18	24
m01	0.77	0.71	0.72	0.76	0.93	1.25	1.28
m02	0.80	0.72	0.74	0.77	0.93	1.24	1.28
m03	0.84	0.75	0.77	0.80	0.93	1.21	1.26
m06	0.93	0.82	0.82	0.83	0.92	1.13	1.20
m09	0.99	0.84	0.85	0.86	0.92	1.10	1.15
y01	0.65	0.80	0.87	0.86	0.92	1.09	1.13
y02	0.98	0.97	0.97	0.95	0.95	1.01	1.02
y03	0.83	0.93	0.97	0.93	0.92	0.99	1.01
y05	1.10	1.03	1.02	0.99	0.96	0.97	0.98
y07	1.18	1.06	1.02	0.99	0.97	0.97	0.97
y10	1.00	1.00	0.99	0.98	0.97	0.99	1.03

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ **diagonal**

h	1	3	6	9	12	18	24
m01	0.79	0.73	0.72	0.71	0.83	1.11	1.18
m02	0.82	0.74	0.73	0.72	0.84	1.10	1.18
m03	0.85	0.76	0.75	0.75	0.84	1.08	1.17
m06	0.95	0.84	0.82	0.80	0.85	1.06	1.20
m09	1.02	0.88	0.87	0.84	0.87	1.06	1.21
y01	0.67	0.82	0.87	0.82	0.86	1.05	1.18
y02	0.97	0.97	0.96	0.91	0.91	1.02	1.19
y03	0.74	0.89	0.92	0.87	0.86	1.00	1.20
y05	1.00	0.97	0.96	0.94	0.94	1.06	1.32
y07	1.07	0.99	0.96	0.94	0.97	1.10	1.38
y10	0.99	1.00	1.03	1.05	1.09	1.25	1.55

Four Macro Factor models

VAR(1) states: {mf1 mf2 mf3 mf4}

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$

h	1	3	6	9	12	18	24
m01	2.50	1.40	1.01	0.88	0.91	1.10	1.16
m02	2.97	1.45	1.01	0.88	0.89	1.07	1.15
m03	3.34	1.48	1.01	0.88	0.87	1.03	1.13
m06	2.73	1.22	0.88	0.79	0.78	0.89	1.06
m09	2.05	0.98	0.77	0.71	0.71	0.83	1.03
y01	1.55	0.77	0.68	0.65	0.67	0.79	1.01
y02	1.47	0.84	0.69	0.66	0.67	0.81	1.12
y03	2.26	1.16	0.84	0.74	0.71	0.85	1.18
y05	4.32	2.21	1.54	1.31	1.18	1.28	1.73
y07	5.21	2.78	2.01	1.75	1.60	1.70	2.25
y10	7.03	3.82	2.86	2.47	2.18	2.21	2.75

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ **diagonal**

h	1	3	6	9	12	18	24
m01	2.12	1.23	0.95	0.84	0.88	1.05	1.12
m02	2.38	1.19	0.87	0.78	0.81	0.97	1.08
m03	2.64	1.20	0.88	0.79	0.80	0.95	1.08
m06	2.16	0.99	0.77	0.72	0.73	0.86	1.07
m09	1.78	0.88	0.74	0.70	0.71	0.83	1.06
y01	1.54	0.79	0.71	0.68	0.70	0.82	1.04
y02	1.68	0.95	0.82	0.77	0.75	0.81	1.05
y03	1.75	0.96	0.83	0.77	0.73	0.80	1.03
y05	2.41	1.34	1.06	0.93	0.86	0.90	1.16
y07	2.93	1.62	1.24	1.07	0.99	1.01	1.29
y10	4.38	2.40	1.81	1.52	1.36	1.31	1.62

Notes:
1) "ne" stands for the set of yields assumed priced without errors.

2) Illustration on the display of RMSFE ratio:

< 0.80	0.80 – 0.90	0.90 – 1.00	> 1.00

3) Best RMSFE ratio across all models: e.g. 0.69

Table 4. Models with mixed factors: 1 macro and 3 yield factors

RMSFE ratio with respect to Random Walk *h* months ahead

Unrestricted

Small N — VAR(1) states: {unempl NS1 NS2 NS3}

h	1	3	6	9	12	18	24
m01	0.84	0.84	0.90	0.91	0.97	1.25	1.36
m02	0.94	0.91	0.95	0.95	1.00	1.28	1.41
m03	1.00	0.95	0.97	0.97	1.01	1.25	1.41
m06	1.05	1.01	1.01	0.98	0.98	1.18	1.40
m09	1.08	1.03	1.03	1.00	0.98	1.17	1.39
y01	0.76	0.96	1.02	0.99	0.98	1.17	1.40
y02	1.20	1.08	1.03	1.00	0.94	1.02	1.28
y03	0.81	0.96	0.99	0.97	0.92	1.02	1.32
y05	1.04	0.99	0.98	0.99	0.94	1.01	1.35
y07	1.00	0.98	0.99	1.01	0.98	1.08	1.43
y10	1.03	1.00	1.05	1.09	1.08	1.19	1.55

Large N — VAR(1) states: {mf1 NS1 NS2 NS3}

h	1	3	6	9	12	18	24
m01	0.82	0.75	0.79	0.84	0.95	1.22	1.29
m02	0.91	0.80	0.83	0.89	0.97	1.23	1.33
m03	0.94	0.81	0.84	0.90	0.98	1.21	1.33
m06	0.91	0.83	0.86	0.92	0.96	1.16	1.34
m09	0.91	0.84	0.88	0.94	0.96	1.14	1.35
y01	0.70	0.81	0.88	0.94	0.97	1.15	1.37
y02	1.13	0.96	0.94	0.98	0.96	1.06	1.33
y03	0.81	0.90	0.94	0.98	0.96	1.09	1.42
y05	1.03	0.98	0.99	1.05	1.04	1.17	1.57
y07	1.00	0.99	1.02	1.09	1.10	1.25	1.68
y10	1.04	1.03	1.12	1.21	1.23	1.40	1.84

Restricted

Small N — VAR(1) states: {unempl CS1 CS2 CS3}, ne =(m03, y02, y10)

Large N — VAR(1) states: {mf1 CS1 CS2 CS3}, ne =(m03, y02, y10)

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$ — Small N

h	1	3	6	9	12	18	24
m01	0.90	0.82	0.87	0.90	1.07	1.49	1.66
m02	0.82	0.78	0.83	0.87	1.03	1.42	1.58
m03	0.87	0.82	0.86	0.90	1.03	1.38	1.53
m06	0.95	0.87	0.89	0.91	1.00	1.28	1.44
m09	0.96	0.87	0.90	0.92	0.99	1.23	1.37
y01	0.66	0.85	0.93	0.93	1.00	1.23	1.36
y02	0.99	0.99	1.01	1.00	1.01	1.11	1.22
y03	0.78	0.95	1.00	0.99	0.98	1.09	1.21
y05	1.01	1.02	1.04	1.04	1.02	1.08	1.18
y07	1.10	1.07	1.06	1.06	1.04	1.08	1.17
y10	1.01	1.02	1.03	1.05	1.04	1.08	1.16

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$ — Large N

h	1	3	6	9	12	18	24
m01	0.77	0.69	0.72	0.76	0.93	1.26	1.32
m02	0.79	0.71	0.73	0.77	0.93	1.25	1.30
m03	0.83	0.75	0.77	0.80	0.93	1.22	1.27
m06	0.98	0.82	0.82	0.83	0.92	1.14	1.21
m09	1.07	0.86	0.86	0.86	0.93	1.10	1.15
y01	0.69	0.81	0.87	0.85	0.92	1.09	1.12
y02	0.98	0.98	0.97	0.95	0.95	1.00	1.02
y03	0.88	0.95	0.97	0.93	0.91	0.98	1.01
y05	1.05	1.01	1.00	0.98	0.95	0.97	0.99
y07	1.07	1.03	1.00	0.98	0.96	0.98	1.01
y10	1.00	1.00	0.99	0.98	0.98	1.00	1.03

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ diagonal — Small N

h	1	3	6	9	12	18	24
m01	0.82	0.76	0.77	0.76	0.87	1.23	1.37
m02	0.83	0.76	0.76	0.76	0.86	1.20	1.34
m03	0.88	0.80	0.80	0.78	0.86	1.17	1.32
m06	0.95	0.85	0.83	0.80	0.85	1.12	1.30
m09	0.97	0.86	0.85	0.82	0.86	1.10	1.27
y01	0.67	0.83	0.87	0.83	0.86	1.09	1.25
y02	0.98	0.96	0.94	0.90	0.90	1.04	1.19
y03	0.82	0.92	0.93	0.88	0.87	1.03	1.19
y05	1.02	0.98	0.96	0.95	0.95	1.06	1.23
y07	1.10	1.01	0.97	0.97	0.99	1.10	1.26
y10	0.99	0.99	1.00	1.03	1.07	1.19	1.39

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ diagonal — Large N

h	1	3	6	9	12	18	24
m01	0.78	0.67	0.64	0.71	0.93	1.32	1.38
m02	0.78	0.67	0.66	0.75	0.95	1.33	1.42
m03	0.81	0.68	0.69	0.79	0.96	1.33	1.41
m06	0.96	0.77	0.77	0.86	0.99	1.30	1.45
m09	1.03	0.81	0.82	0.90	1.01	1.30	1.44
y01	0.66	0.74	0.82	0.89	1.00	1.28	1.41
y02	0.95	0.92	0.95	0.99	1.04	1.21	1.35
y03	0.68	0.84	0.93	0.97	1.01	1.20	1.37
y05	0.97	0.98	1.03	1.08	1.12	1.28	1.51
y07	1.02	1.00	1.04	1.09	1.14	1.30	1.54
y10	1.00	1.03	1.11	1.18	1.23	1.39	1.64

Notes:
1) "ne" stands for the set of yields assumed priced without errors.
2) Illustration on the display of RMSFE ratio:

< 0.80	0.80 – 0.90	0.90 – 1.00	> 1.00

3) Best RMSFE ratio across all models: e.g. 0.69

Table 5. Models with mixed factors: *n(>1)* macro and 3 yield factors

RMSFE ratio with respect to Random Walk *h* months ahead

Unrestricted

Small N

VAR(1) states: {unempl infl NS1 NS2 NS3}

h	1	3	6	9	12	18	24
m01	0.84	0.82	0.83	0.79	0.83	1.04	1.21
m02	0.92	0.88	0.86	0.82	0.85	1.07	1.26
m03	0.98	0.93	0.89	0.84	0.85	1.05	1.26
m06	1.06	1.00	0.94	0.86	0.84	1.00	1.26
m09	1.08	1.02	0.96	0.88	0.85	0.99	1.25
y01	0.77	0.97	0.96	0.87	0.84	0.98	1.24
y02	1.19	1.07	0.99	0.89	0.83	0.89	1.15
y03	0.81	0.97	0.95	0.87	0.80	0.88	1.17
y05	1.03	0.98	0.94	0.88	0.82	0.89	1.20
y07	1.00	0.97	0.94	0.89	0.85	0.94	1.28
y10	1.00	0.95	0.96	0.94	0.91	1.03	1.40

Large N

VAR(1) states: {mf1 mf2 mf3 mf4 NS1 NS2 NS3}

h	1	3	6	9	12	18	24
m01	0.83	0.73	0.70	0.72	0.78	0.82	0.85
m02	0.90	0.75	0.71	0.73	0.79	0.82	0.88
m03	0.92	0.77	0.73	0.74	0.79	0.81	0.89
m06	0.89	0.79	0.77	0.77	0.78	0.78	0.93
m09	0.92	0.82	0.80	0.79	0.78	0.78	0.94
y01	0.75	0.80	0.82	0.80	0.79	0.78	0.93
y02	1.14	0.96	0.89	0.84	0.78	0.74	0.96
y03	0.83	0.89	0.88	0.82	0.77	0.75	0.98
y05	1.00	0.94	0.88	0.83	0.76	0.77	1.05
y07	0.99	0.96	0.92	0.88	0.82	0.83	1.12
y10	1.02	0.99	1.00	0.97	0.91	0.95	1.24

Restricted

Small N

VAR(1) states: {unempl infl CS1 CS2 CS3}
ne =(m03, y02, y10)

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$

h	1	3	6	9	12	18	24
m01	0.83	0.79	0.88	0.93	1.10	1.54	1.70
m02	0.83	0.79	0.86	0.92	1.07	1.48	1.64
m03	0.87	0.83	0.89	0.94	1.06	1.43	1.58
m06	0.97	0.86	0.90	0.93	1.01	1.29	1.44
m09	1.02	0.87	0.91	0.94	1.00	1.23	1.34
y01	0.66	0.84	0.94	0.95	1.01	1.23	1.32
y02	1.00	1.01	1.03	1.02	1.02	1.09	1.15
y03	0.88	0.99	1.04	1.01	0.99	1.08	1.13
y05	1.04	1.05	1.06	1.06	1.03	1.04	1.08
y07	1.12	1.08	1.08	1.08	1.04	1.05	1.08
y10	1.01	1.03	1.05	1.06	1.04	1.04	1.08

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ **diagonal**

h	1	3	6	9	12	18	24
m01	0.79	0.72	0.72	0.70	0.80	0.97	1.02
m02	0.83	0.74	0.73	0.71	0.79	0.96	1.01
m03	0.88	0.77	0.76	0.74	0.80	0.94	1.00
m06	0.95	0.81	0.80	0.76	0.78	0.88	0.94
m09	0.99	0.83	0.82	0.78	0.78	0.84	0.89
y01	0.69	0.79	0.84	0.79	0.79	0.85	0.90
y02	0.98	0.97	0.94	0.87	0.82	0.80	0.85
y03	0.81	0.96	0.97	0.89	0.82	0.84	0.89
y05	1.10	1.07	1.03	0.95	0.87	0.85	0.93
y07	1.15	1.11	1.05	0.97	0.90	0.89	0.99
y10	1.01	1.02	1.00	0.95	0.89	0.88	0.98

Large N

VAR(1) states: {mf1 mf2 CS1 CS2 CS3}
ne =(m03, y02, y10)

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$

h	1	3	6	9	12	18	24
m01	0.78	0.71	0.73	0.77	0.92	1.25	1.31
m02	0.80	0.73	0.74	0.78	0.92	1.22	1.29
m03	0.85	0.76	0.77	0.80	0.92	1.19	1.26
m06	1.04	0.85	0.82	0.82	0.90	1.12	1.19
m09	1.13	0.88	0.86	0.85	0.91	1.09	1.14
y01	0.71	0.81	0.86	0.84	0.90	1.07	1.12
y02	0.98	0.96	0.95	0.93	0.93	0.99	1.03
y03	0.75	0.90	0.93	0.90	0.89	0.97	1.01
y05	1.01	0.98	0.98	0.97	0.95	0.98	1.01
y07	1.17	1.05	1.00	0.98	0.96	0.97	0.99
y10	1.00	0.99	0.99	0.98	0.97	1.00	1.03

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ **diagonal**

h	1	3	6	9	12	18	24
m01	0.77	0.66	0.67	0.68	0.83	1.14	1.22
m02	0.78	0.68	0.70	0.71	0.85	1.16	1.26
m03	0.83	0.71	0.74	0.74	0.86	1.15	1.27
m06	1.06	0.82	0.83	0.81	0.88	1.14	1.34
m09	1.13	0.85	0.87	0.84	0.90	1.14	1.37
y01	0.76	0.76	0.85	0.81	0.88	1.12	1.33
y02	0.96	0.94	0.96	0.90	0.92	1.07	1.36
y03	0.68	0.87	0.94	0.89	0.91	1.09	1.44
y05	1.00	1.03	1.05	1.03	1.05	1.23	1.68
y07	1.06	1.02	1.02	1.01	1.05	1.24	1.69
y10	1.02	1.07	1.12	1.15	1.20	1.41	1.86

Notes:
1) "ne" stands for the set of yields assumed priced without errors.

2) Illustration on the display of RMSFE ratio:

< 0.80	0.80 – 0.90	0.90 – 1.00	> 1.00

3) Best RMSFE ratio across all models: e.g. 0.69

Table 6. Generalised Taylor Rule: FFR and macro variables/factors

RMSFE ratio with respect to Random Walk *h* months ahead

Unrestricted

Small N

VAR(1) states: {FFR unempl infl}

h	1	3	6	9	12	18	24
m01	1.18	1.01	1.03	1.02	1.02	1.21	1.48
m02	1.47	1.16	1.11	1.08	1.06	1.26	1.55
m03	1.77	1.27	1.17	1.12	1.06	1.25	1.55
m06	2.13	1.43	1.24	1.15	1.06	1.19	1.56
m09	2.31	1.48	1.29	1.21	1.08	1.19	1.58
y01	2.27	1.52	1.32	1.22	1.11	1.21	1.61
y02	2.65	1.60	1.35	1.25	1.10	1.11	1.53
y03	2.68	1.64	1.36	1.26	1.11	1.15	1.61
y05	2.82	1.68	1.38	1.29	1.15	1.18	1.65
y07	2.95	1.76	1.46	1.37	1.25	1.29	1.77
y10	3.20	1.94	1.64	1.53	1.40	1.45	1.92

Large N

VAR(1) states: {FFR mf1 mf2}

h	1	3	6	9	12	18	24
m01	1.02	**0.78**	**0.86**	**0.97**	1.13	1.53	1.80
m02	1.23	**0.89**	**0.93**	1.03	1.16	1.57	1.86
m03	1.42	**0.98**	**0.99**	1.08	1.19	1.58	1.89
m06	1.77	1.16	1.12	1.17	1.23	1.56	1.97
m09	2.07	1.29	1.22	1.27	1.29	1.59	2.03
y01	2.29	1.42	1.30	1.33	1.36	1.67	2.12
y02	3.25	1.86	1.59	1.56	1.50	1.63	2.14
y03	3.91	2.21	1.81	1.72	1.62	1.77	2.35
y05	5.00	2.84	2.25	2.10	1.93	1.98	2.60
y07	5.51	3.16	2.52	2.32	2.15	2.18	2.79
y10	6.45	3.71	2.98	2.70	2.46	2.43	2.98

Restricted

Small N

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$

h	1	3	6	9	12	18	24
m01	2.16	1.30	1.08	**0.99**	**0.96**	**0.99**	1.04
m02	2.58	1.39	1.10	1.01	**0.95**	**0.98**	1.05
m03	2.95	1.47	1.13	1.03	**0.95**	**0.97**	1.06
m06	2.55	1.42	1.16	1.05	**0.95**	**0.98**	1.20
m09	2.33	1.40	1.20	1.11	**0.99**	1.05	1.35
y01	2.28	1.46	1.24	1.14	1.03	1.11	1.45
y02	3.61	2.07	1.68	1.54	1.36	1.43	2.00
y03	4.87	2.70	2.09	1.86	1.64	1.75	2.45
y05	7.15	4.00	3.06	2.72	2.38	2.42	3.28
y07	8.52	4.79	3.66	3.21	2.82	2.79	3.60
y10	10.73	6.01	4.61	3.95	3.39	3.20	3.85

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ **diagonal**

h	1	3	6	9	12	18	24
m01	2.14	1.43	1.22	1.09	**1.00**	**0.98**	1.04
m02	2.69	1.58	1.27	1.11	**0.97**	**0.85**	**0.92**
m03	2.92	1.56	1.23	1.08	**0.95**	**0.89**	**0.96**
m06	2.92	1.59	1.25	1.08	**0.92**	**0.84**	1.01
m09	2.75	1.54	1.25	1.11	**0.94**	**0.89**	1.10
y01	2.79	1.59	1.25	1.10	**0.93**	**0.86**	1.09
y02	2.93	1.65	1.34	1.19	**0.99**	**0.91**	1.24
y03	3.10	1.77	1.39	1.24	1.03	**0.96**	1.34
y05	3.78	2.16	1.70	1.52	1.29	1.22	1.71
y07	4.24	2.42	1.90	1.69	1.47	1.41	1.95
y10	5.25	2.97	2.32	2.03	1.75	1.68	2.25

Large N

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$

h	1	3	6	9	12	18	24
m01	2.76	1.64	1.28	1.16	1.24	1.61	1.87
m02	3.20	1.66	1.22	1.12	1.17	1.51	1.77
m03	3.55	1.66	1.18	1.08	1.12	1.42	1.66
m06	3.09	1.47	1.10	1.03	1.06	1.28	1.57
m09	2.47	1.27	1.04	1.03	1.05	1.26	1.55
y01	2.02	1.11	**1.00**	1.01	1.04	1.24	1.52
y02	2.25	1.47	1.33	1.33	1.29	1.39	1.80
y03	4.11	2.33	1.88	1.72	1.58	1.67	2.15
y05	6.66	3.71	2.79	2.44	2.12	2.05	2.59
y07	6.84	3.79	2.83	2.41	2.06	1.93	2.36
y10	7.22	4.02	3.04	2.55	2.15	1.95	2.28

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ **diagonal**

h	1	3	6	9	12	18	24
m01	1.00	**0.75**	**0.85**	**0.97**	1.13	1.54	1.80
m02	1.14	**0.81**	**0.89**	1.00	1.13	1.53	1.80
m03	1.32	**0.90**	**0.94**	1.04	1.14	1.50	1.78
m06	1.81	1.16	1.12	1.17	1.21	1.51	1.89
m09	2.33	1.38	1.27	1.30	1.31	1.59	2.01
y01	2.53	1.51	1.35	1.36	1.36	1.64	2.07
y02	3.62	2.03	1.72	1.67	1.58	1.72	2.27
y03	4.04	2.26	1.84	1.75	1.64	1.78	2.36
y05	4.89	2.79	2.24	2.10	1.93	1.99	2.60
y07	5.34	3.08	2.47	2.30	2.13	2.16	2.76
y10	6.75	3.89	3.13	2.84	2.57	2.54	3.11

Notes:

1) Illustration on the display of RMSFE ratio:

< 0.80	0.80 – 0.90	0.90 – 1.00	> 1.00

2) Best RMSFE ratio across all models: e.g. **0.69**

Table 6. Generalised Taylor Rule: FFR and macro variables/factors (continued)

RMSFE ratio with respect to Random Walk h months ahead

Unrestricted

Large N — VAR(1) states: {FFR mf1 mf2 mf3}

h	1	3	6	9	12	18	24
m01	1.06	0.82	0.84	0.83	0.86	0.96	1.03
m02	1.18	0.85	0.83	0.82	0.84	0.94	1.02
m03	1.27	0.88	0.85	0.83	0.83	0.93	1.02
m06	1.36	0.91	0.85	0.81	0.80	0.86	1.00
m09	1.42	0.90	0.84	0.81	0.78	0.83	0.98
y01	1.47	0.89	0.84	0.80	0.79	0.83	0.96
y02	1.97	1.06	0.86	0.80	0.76	0.74	0.92
y03	2.26	1.16	0.89	0.79	0.74	0.75	0.96
y05	2.79	1.48	1.07	0.92	0.83	0.85	1.14
y07	3.14	1.70	1.24	1.05	0.96	0.98	1.29
y10	3.73	2.04	1.53	1.30	1.16	1.18	1.48

Large N — VAR(1) states: {FFR mf1 mf2 mf3 mf4}

h	1	3	6	9	12	18	24
m01	1.04	0.81	0.83	0.82	0.86	0.96	1.04
m02	1.15	0.83	0.82	0.81	0.84	0.94	1.05
m03	1.23	0.85	0.83	0.82	0.83	0.93	1.05
m06	1.32	0.87	0.82	0.80	0.80	0.88	1.08
m09	1.39	0.86	0.81	0.80	0.79	0.86	1.07
y01	1.43	0.85	0.80	0.79	0.79	0.87	1.06
y02	1.87	1.01	0.82	0.77	0.75	0.78	1.02
y03	2.15	1.10	0.84	0.75	0.72	0.77	1.03
y05	2.57	1.39	1.01	0.85	0.77	0.81	1.09
y07	2.90	1.60	1.17	0.98	0.89	0.92	1.19
y10	3.50	1.95	1.47	1.23	1.09	1.09	1.34

Restricted

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$ (VAR(1) states: {FFR mf1 mf2 mf3})

h	1	3	6	9	12	18	24
m01	1.46	1.02	0.95	0.93	0.96	1.09	1.10
m02	1.69	1.07	0.96	0.92	0.93	1.05	1.08
m03	1.87	1.11	0.97	0.92	0.91	1.01	1.05
m06	1.64	1.02	0.89	0.84	0.81	0.86	0.98
m09	1.46	0.91	0.83	0.78	0.75	0.80	0.95
y01	1.30	0.83	0.77	0.73	0.71	0.77	0.93
y02	1.68	0.92	0.74	0.70	0.69	0.75	1.02
y03	2.23	1.20	0.89	0.79	0.75	0.82	1.10
y05	4.33	2.31	1.62	1.36	1.19	1.18	1.53
y07	5.07	2.74	1.96	1.65	1.45	1.42	1.79
y10	6.76	3.69	2.72	2.27	1.94	1.84	2.19

Constant risk price: $\lambda_0 \neq 0, \lambda_1 = 0$ (VAR(1) states: {FFR mf1 mf2 mf3 mf4})

h	1	3	6	9	12	18	24
m01	1.50	1.01	0.93	0.90	0.95	1.11	1.16
m02	1.68	1.03	0.91	0.88	0.91	1.05	1.13
m03	1.84	1.05	0.90	0.87	0.88	1.01	1.09
m06	1.63	0.96	0.84	0.80	0.80	0.87	1.04
m09	1.50	0.89	0.80	0.77	0.75	0.82	1.03
y01	1.44	0.85	0.78	0.73	0.72	0.79	1.01
y02	1.81	0.96	0.74	0.69	0.68	0.77	1.10
y03	2.54	1.31	0.94	0.81	0.76	0.89	1.26
y05	4.42	2.29	1.58	1.34	1.19	1.28	1.77
y07	5.03	2.68	1.91	1.65	1.49	1.59	2.13
y10	7.21	3.91	2.90	2.48	2.17	2.17	2.70

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ diagonal (VAR(1) states: {FFR mf1 mf2 mf3})

h	1	3	6	9	12	18	24
m01	1.14	0.87	0.87	0.85	0.87	0.97	1.03
m02	1.26	0.90	0.87	0.84	0.85	0.95	1.02
m03	1.38	0.93	0.88	0.85	0.84	0.93	1.01
m06	1.38	0.90	0.84	0.81	0.78	0.84	0.99
m09	1.47	0.88	0.82	0.79	0.76	0.81	0.97
y01	1.46	0.85	0.79	0.76	0.75	0.80	0.94
y02	1.98	1.04	0.84	0.80	0.76	0.75	0.95
y03	2.09	1.09	0.85	0.78	0.74	0.74	0.95
y05	3.03	1.60	1.14	0.98	0.88	0.86	1.13
y07	3.55	1.89	1.36	1.14	1.01	1.00	1.30
y10	4.64	2.52	1.86	1.54	1.34	1.31	1.62

Time-varying risk price: $\lambda_0 \neq 0, \lambda_1$ diagonal (VAR(1) states: {FFR mf1 mf2 mf3 mf4})

h	1	3	6	9	12	18	24
m01	1.11	0.85	0.86	0.86	0.89	0.99	1.06
m02	1.22	0.86	0.83	0.83	0.86	0.97	1.07
m03	1.34	0.88	0.84	0.83	0.84	0.94	1.06
m06	1.41	0.88	0.80	0.78	0.78	0.87	1.07
m09	1.52	0.89	0.79	0.77	0.76	0.84	1.06
y01	1.54	0.88	0.78	0.75	0.75	0.84	1.04
y02	1.92	1.01	0.80	0.76	0.74	0.79	1.05
y03	1.99	1.02	0.80	0.73	0.70	0.77	1.03
y05	2.70	1.44	1.04	0.90	0.82	0.87	1.17
y07	3.12	1.70	1.23	1.05	0.96	0.99	1.29
y10	4.28	2.36	1.78	1.50	1.33	1.32	1.61

Notes:

1) Illustration on the display of RMSFE ratio: < 0.80 | 0.80 – 0.90 | 0.90 – 1.00 | > 1.00

2) Best RMSFE ratio across all models: e.g. 0.69