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The option to wait in collective decisions*

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Abstract

We consider a model in which voters over time receive more information about their preferences concerning an irreversible social decision. Voters can either implement the project in the first period, or they can postpone the decision to the second period. We analyze the effects of different majority rules. Individual first period voting behavior may become "less conservative" under supermajority rules, and it is even possible that a project is implemented in the first period under a supermajority rule that would not be implemented under simple majority rule.

We characterize the optimal majority rule, which is a supermajority rule. In contrast to individual investment problems, society may be better off if the option to postpone the decision did not exist. These results are qualitatively robust to natural generalizations of our model.

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Keywords: Supermajority rules, information, investment, option value

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1 Introduction

In most political economy models, individuals know their preferences over candidates or social actions. In another branch of the literature, individuals know their fundamental preferences, but which action is best suited to implement them depends on an unknown state of the world. The main objective of this type of models is to analyze how individuals can aggregate dispersed information through strategic voting.[1]

In the present paper we focus on a third case that has received little attention so far: Collective decisions under uncertainty when individuals learn about their own preferences over time. In our model, individuals get additional information over time about their heterogeneous preferences regarding an investment project, and have to choose whether to implement it immediately, or delay the decision. In the latter case, they can either implement it after receiving additional information, or pass on it completely. While investment problems under uncertainty have been analyzed extensively for single decision makers,[2] the new feature of our paper is to investigate the interplay of *individual* learning and *social* decisions.[3] Our main focus is twofold: Firstly, we examine the effect of the majority rule on individual voting behavior and social decisions in this framework. From an ex-ante point of view, we show that a supermajority rule dominates simple majority rule with respect to social welfare.[4] Secondly, we analyze the social value of the opportunity to postpone the decision, and show that it can be either positive or negative; this contrasts with unilateral investment projects where the value of waiting is always positive.

Specifically, we consider the following dynamic social investment problem. In the first period, each voter knows whether he would be a winner or a loser in the first period, but his second period type is random. If the project is implemented in the first period, it is irreversible and payoff to voters accrue in both periods according to their type realizations. Alternatively, if the project is not implemented in the first period, the voters learn their respective second period types, and vote on whether to implement the project for the second period. We parameterize projects according to the relative size of the gain of winners to the loss of losers. A "good" project is one where this ratio is large (i.e., the ex-ante expected average payoff is positive), and vice versa.

[1] See, e.g., Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996), Feddersen and Pesendorfer (1998).

[2] See Dixit and Pindyck (1994) for a review of this literature.

[3] A short note on our terminology: In our model, voters (passively) "learn" as new information arrives over time. We do not model this learning process as an active information acquisition process in which, say, voters decide how much time to spend on learning.

[4] By a supermajority rule, we mean a voting rule that specifies that the status quo is only to be changed if a certain proportion of the electorate (greater than the 50%, the "simple majority") votes in favor of change.

A possible advantage of delaying investment in the first period is that agents learn more about their payoffs in the next period: There is an "option value of waiting". We analyze how the type of majority rule influences this option value of waiting, and thus, the voting behavior of individuals and the first period implementation decision. The expected second period payoff for a voter, if the project is delayed in the first period, may go in either direction as the majority rule changes: A higher majority rule may increase the risk that a "good" project with a positive expected value (i.e., one in which the winners gain more than losers lose) is not implemented in the second period, thus diminishing the option value of waiting and inducing voters to implement the project already in the first period. In contrast, a higher majority rule decreases the risk that a "bad" project is implemented in period 2, thus increasing the option value of waiting.

A higher option value of waiting makes voters more reluctant to implement the project already in the first period. Thus, a higher majority rule makes each voter more willing to agree to good projects, even if he is a loser today, and less willing to agree to bad projects, even if he is a winner today. There is also a second, direct, effect of a higher majority rule: More voters have to agree, making first-period implementation less likely. For bad projects, both effects go in the same direction, making implementation less likely for higher majority rules. In contrast, for good projects, the first effect may outweigh the second one, leading to more projects being implemented in period 1 under a higher majority rule.

On the normative side, we focus on an ex-ante point of view, that is, taking expectation over both voter type realizations and project types. We show that, relative to a situation where all decisions have to be made in the first period, the option to wait (weakly) increases the optimal majority rule in large electorates. Intuitively, higher majority rules have the advantage that, for socially bad projects, voters become more conservative and thus fewer of these projects are implemented, while for good projects, voters become more willing to implement in the first period. Moreover, since the best projects are already implemented in period 1, those projects that are reconsidered in period 2 form a negative selection from the set of all projects, and a higher majority rule is socially beneficial for these cases as well.[5]

We also characterize the ex-ante optimal supermajority rule explicitly under the additional assumption that project types are uniformly distributed at the constitutional stage. For any number of voters, the optimal supermajority rule in this case is between

[5]Even at the interim stage (i.e., in the first period when voters know the project type and their own first-period type), simple majority rule may be Pareto inefficient for some bad projects. This is the case if, under simple majority rule, there is a majority of voters who approve immediate implementation; but even those voters would prefer to postpone implementation, if the majority rule is changed to unanimity rule. In contrast, transition from unanimity rule to simple majority rule cannot yield an ex-post Pareto improvement.

$7/11 \approx 63.6\%$ and the smallest implementable rule greater or equal to $2/3$. (For example, with 5 voters, the smallest rule greater than a $2/3$ rule is 4 out of 5, or 80%. As the number of voters increases, this upper limit of the interval of possibly optimal majority rules converges to $2/3$). Many organizations use indeed supermajority rules close to these numbers.

It is also interesting to analyze the social ex-ante value of the option to wait. In unilateral investment problems, this option value is always nonnegative, and often positive: A firm's expected discounted profit from an investment opportunity is strictly larger than what it could get if it were forced to either invest immediately, or forgo the investment completely. In contrast, a *society* may be better off if it is forced to invest either immediately or not at all, rather than having the option of postponing this decision. Indeed, we show that, from an ex-ante point of view (and with uniformly distributed project costs), this is the case even if society chooses the *optimal* majority rule for the case when waiting is possible.

Our results shed light on an important question in the endogenous determination of institutions: Why do some organizations choose supermajority rules, and which features of decision problems influence this choice? Majority rules within organizations vary considerably, from simple majority rule to unanimity rule. Often, the choice of the majority rule that is to govern future decision making is a contentious issue itself, such as in the recent EU summit, which eventually adopted a supermajority rule. Most countries use supermajority rules for a change of the constitution, and, often implicitly, for "normal" legislation.[6] This paper contributes to the literature on the relative advantages of different majority rules (discussed in more detail in Section 6), by providing a new rationale for supermajority rules.

Several previous papers have analyzed supermajority rules from an economic point of view. Buchanan and Tullock (1962) argue that, under a simple majority rule, a majority of people may implement socially bad projects because they can externalize a part of the associated cost to the losing minority, while under unanimity rule, only Pareto improving projects are implemented. However, Guttman (1998) shows that unanimity rule leads to a rejection of many projects that are not Pareto improvements, but nevertheless worthwhile from a reasonable social point of view. Assuming that the social goal is to minimize the sum of both types of mistakes, he shows that, in a symmetric setting, simple majority rule is optimal. Our model is constructed symmetrically, so that simple majority rule is optimal if voters have to make a once-and-for-all

[6]For example, in parliamentary systems with a strong committee organization, a legislative proposal usually needs the support of *both* the respective committee and the house. In parliamentary systems with two chambers, certain legislative proposals need the support of both chambers. Tullock (1998), p. 216, estimates that legislative rules in the US for changing the status quo are "roughly equivalent to requiring a 60% majority in a single house elected by proportional representation."

decision about the project in the first period. However, with the option to postpone a decision to the second period, we show that a supermajority rule is optimal.[7]

In terms of the dynamic setup, the paper most related to ours is Glazer (1989), where voters in period 1 choose between i) implementing an irreversible long run project that delivers benefits both in period one and in period two, ii) implementing a short run project, whose costs and benefits accrue in period one and iii) not investing. In cases (ii) or (iii), the electorate decides again in period 2 whether to implement a short run project for period 2. While each voter's second period benefit is the same as his first period benefit, there is exogenous uncertainty about the outcome of a possible election in period 2. Glazer shows that voters exhibit a bias towards implementing the long run project. Two effects drive this result. First, even if it were cheaper to implement two short term projects, a first period decisive voter with a positive net payoff may prefer to disempower the second period electorate by committing to a long-term project. Second, even if the first period decisive voter has a negative net payoff from the long-term project, he may prefer to implement it, if he is sufficiently afraid that a short-term project would be implemented in period 2 that would be even worse for him.

Apart from the fact that we have no first period short term project, our analysis differs from Glazer (1989) in two crucial points. First, we explicitly analyze a dynamic voting game in which each voter is uncertain about his second period preferences. This generates endogenous uncertainty about the second period voting outcome, and allows us to analyze how different majority rules affect this uncertainty and hence the first period voting behavior. The focus of our analysis is on the positive and normative implications of different majority rules in dynamic investment problems. Second, while Glazer's second effect is also present in our model for socially undesirable projects, there is not necessarily excessive commitment to early implementation in our model. Indeed, for socially beneficial projects, voters in our model are excessively conservative. Interestingly, in our model, supermajority rules mitigate *both* the tendency of voters to excessively commit to socially undesirable projects, and their reluctance to implement socially beneficial projects.

Our model is also related to a small literature in which voters learn about their preferences over time. Strulovici (2007) analyzes a model in which a society has to choose in continuous time between a risky and a secure project. Ex-ante, all individuals are identical; over time, some individuals discover that they are winners and then receive a payoff forever after. The arrival rate is unknown, and voters continuously update their beliefs as long as the risky action is played. In contrast to our model, information

[7] Other rationales for supermajority rules are discussed in Section 5 and include the problem of time inconsistency of optimal policies under simple majority rule (Gradstein (1999), Dal Bo (2006)), the possibility of electoral cycles under simple majority rule (Caplin and Nalebuff (1988)), and a strategic use of supermajority rules when preferences change deterministically (Messner and Polborn (2004)).

arrives only as long as the risky action is played, and the project is reversible. Voters decide under simple majority rule or unanimity rule when (and if at all) to stop experimentation with the uncertain action. Strulovici (2007) finds that society always stops experimentation too early compared with a utilitarian optimum, and that unanimity rule may lead to more or less experimentation than simple majority rule.

Compte and Jehiel (2008) analyze a collective search and bargaining process in which proposals arrive exogenously and over time. The tradeoff they study is that unanimity rule guarantees that only efficient projects are implemented, but it takes less time to reach an implementation decision under simple majority rule.

Another social learning paper in which new information arrives only as long as society is experimenting is the multiperiod model of Callander (2008). Citizens know how the status quo policy translates into outcomes, but the farther a policy is away from the status quo policy, the less certain are its consequences. Callander shows that an initial phase of experimentation and learning is eventually terminated if a policy achieves an outcome that is sufficiently close to the ideal outcome of the median voter.

Fernandez and Rodrik (1991) analyze a model of voting on reform projects that generate winners and losers. They show that a project that ex-post benefits the majority of the population need not be implemented, if the ex-ante expected benefit is negative for a majority of the population. If, instead, a majority of the population has positive ex-ante expected benefits, but ex-post, payoffs are negative for a majority, then a reform may be implemented initially, but would be reversed after payoff information becomes known. Thus, there is a bias in favor of the status quo. In contrast to Fernandez and Rodrik (1991), we analyze a setting in which reforms are not reversed, so that there is no status quo bias in our setup. Also, our focus is on comparing different majority rules and how they influence voting behavior and implementation decisions, while Fernandez and Rodrik (1991) only consider simple majority rule.

The paper proceeds as follows. In the next section, the model is presented. Our main results follow in Section 3. In Section 4, we analyze several extensions of the model. Previous literature is discussed in Section 5, and Section 6 concludes.

2 The model

2.1 Description

A group of N (odd) risk neutral individuals has to decide whether to undertake an investment project that creates costs and benefits (described in more detail below) for all members of society. The decision about the project has two stages. At the beginning of period 1, the group has to choose between implementing the project right away and postponing the decision to the beginning of period two. In the latter case, the group has to make the final decision on whether or not the project should be implemented at the beginning of period 2. In both periods, the decision is governed by a *voting rule* indexed by m. Specifically, the project is implemented if and only if at least m individuals approve. The majority requirement may range from simple majority to unanimity, i.e. $m \in \{(N+1)/2, \ldots, N\}$.

If the project is implemented in or before period t, then individual i receives a gross payoff of V_t^i in period t. We refer to V_t^i as i's *type* in period t, and assume that V_t^i is either 0 or 1, each with a probability of 1/2. In addition, the project generates a constant per-period flow of costs, $c \in (0,1)$, for each individual of the society, so that i's net payoff in period t is $V_t^i - c$.[8] Payoffs are measured relative to non-implementation, that is, in each period that the project is not implemented, each individual receives a net payoff of 0. For simplicity, we assume that individuals do not discount the future, so that they value future and current payoffs equally.

At the time of the election in period 1, individuals only know their own period 1 type, but not their period 2 type. In the period 2 election (if any), individuals know also their period 2 type. In elections, each individual votes for the option that would provide him with the higher expected utility: In period 2, voter i votes for the project if and only if $V_2^i = 1$. In period 1, voter i votes for project implementation if and only if he weakly prefers immediate implementation to the expected payoff from postponing (given that all voters behave in period 2 as described above). Formally, we use iterated elimination of weakly dominated strategies, a standard refinement in voting games.[9]

Eventually, we are interested in the endogenous determination of the voting rule. We envision that this choice occurs at an initial stage, before type realizations for the project are known. Thus, all voters are identical and agree to choose the majority rule that maximizes their ex-ante expected payoffs. Note that we can also interpret such a constitution normatively as the one that maximizes ex-ante utilitarian welfare.

Ex-ante payoffs, and thus the optimal majority rule, also depend on the cost parameter c. Typically, however, it is not feasible to construct a constitution where the applicable majority rule depends on the cost parameter of the project under consideration, since there would be verifiability problems, and such a rule would unavoidably lead to conflicts of interpretation. Thus, we focus on the majority rule that is optimal *in expectation*, when c is drawn from some distribution with density $f(c)$.

[8]Clearly, we could just specify the net payoff of each individual through one variable, but our approach allows us to use c in order to easily distinguish projects with a high expected average payoff (i.e., low c) from those with a low expected average payoff.

[9]This refinement, for example, eliminates (rather strange) Nash equilibria of the voting game in which everybody opposes investment, even if he would benefit from implementation.

2.2 Discussion of modeling choices

We now briefly discuss several modeling choices.

Cost parameter. While we call c the "cost" of the project, it can also be interpreted as a utility index without an explicit monetary cost interpretation. Different values of c can be used to capture any situation in which some voters gain and others lose from changing the status quo. The utility of the status quo is normalized to zero for every voter, and c is a parameter that measures the size of the gains of those people who are better off than in the status quo, relative to the losses of those who would prefer the status quo.

Available choices. We also restrict society to make a decision through voting and assume that project proposals cannot contain transfer payments between different voters. If, instead, types are observable and transfer payments are feasible, then, by the Coase theorem, any majority rule leads to implementation if and only if the project creates more benefits than costs. The assumption that transfer payments are not feasible is standard in most of the political economy literature and also appears to be quite realistic in many applications, for example because of informational constraints or legal provisions against vote buying. However, we do not model explicitly why this is the case.

The decision in the first period is restricted to the first-period implementation decision. For example, the first period electorate cannot choose to wait *and* commit the second period electorate to implement in the second period, or cannot choose to wait *and* forbid the second period electorate to consider implementation. They can also not choose to change the majority rule for the second period. We also do not allow society to choose different majority rules to apply for "first-period projects" and "second-period projects". Again, the reason is that it might be very difficult to describe ex-ante whether a investment proposal falls in the first or the second category. While there are cases in which a majority of the first period electorate would like to take such measures, the assumption that today's electorate cannot commit a future electorate is both standard in the literature, and quite realistic for most democratic institutions, as such attempts would be very controversial (at least ex-post).

Dynamic framework. The purpose of the model is to provide a simple framework for the analysis of intertemporal implementation decisions, the issue of learning and the effects of different majority rules. To keep the setup as simple as possible, our model has only two time periods. It is, in principle, not too difficult to extend this model to a setup with payoffs in infinitely many periods; however, a key assumption is that voters learn their preference for or against the project after some finite time (so that the relevant part of learning is concentrated in early periods).

For example, we could generalize our model as follows. Once a project is implemented, it generates an infinite stream of payoffs for each voter (depending on the voter's type, as in our model, and discounted using a discount factor of δ). In the first period, voters know only their first period type. In the second period, they learn whether they are a high or low type for the remaining periods (or, more generally, the frequency with which they will be high types in the future). Thus, voting behavior from the second period on will be type dependent and thus, implementation either occurs in one of the first two periods, or not at all. As in our model, backwards induction can then be used to determine first period voting behavior.

Note that our model corresponds to the case of $\delta = 1/2$ in the infinite period model, i.e., the payoff in the first period equals the present value of future payoffs (after all learning has occurred). For higher (lower) values of δ, the payoff in the period for which initial information is available is less (more) important than the future payoffs that are uncertain from today's point of view.

The principal reason for focusing on the simpler two period model is that the additional parameter (δ) complicates computations and proofs significantly without yielding qualitatively new insights.

Extensions. Finally, we assume that voters' second period valuations are independent of each other and of their first period types. We relax both of these assumptions in Section 4 and show that the qualitative results of the basic model are quite robust.

2.3 Application – Voting on hiring

There are many applications in which societies have to decide on investment projects with uncertain returns. Here, we just present an example that illustrates some of the main features and questions of the model. The reader who is more interested to proceed to the theoretical results can, without problems, skip the remainder of this section.

Late in the junior job market, an economics department can choose to offer an assistant professorship to Candidate A, who would accept such an offer. A decision to hire A is irreversible in the next years (i.e., the line is filled, and A can be fired only after considerable delay at tenure time). Payoffs accrue to department members, both now and in the future, depending on how good a researcher and colleague Professor A turns out to be. From today's perspective, this is a random event, and also a question in which individual tastes of existing department members may differ, so department members may disagree even ex-post whether hiring A was a good decision.

Alternatively, the department can choose to leave the position unfilled and wait till the following year, when there is a new draw of an available candidate, say, B. Again, the department can decide whether to hire B or not. (In principle, the department's hiring problem is an infinite period problem, assuming that their dean would always renew searches for lines that were not filled in the previous year. In the interest of tractability, we simplify the infinite period world to a two-period one in our model.)

Individuals' payoffs in our model and in this application differ somewhat: In our model, an individual's second period payoff is the same, whether the project was implemented in the first period or only in the second period. In contrast, in this application, an individual voter's realized second period payoff from A (if he was hired in the first period) may very well differ from that individual's payoff from B (in the same period). However, all that matters for individual voters when they decide how to vote in the first period is the *expectation* of their second period payoff, so the difference between realized payoffs in the model and in the application is immaterial for our main results.

A central result of our model is that supermajority rules outperform simple majority rule with respect to voters' ex-ante expected utility. While we have only anecdotal evidence, supermajority rules appear also prevalent in groups that decide on hiring and/or promotion through voting.[10] We also show that – in contrast to individual investment problems – the ability to postpone the decision can hurt a society. Thus, groups have an incentive to construct rigid rules in an attempt to commit against reconsideration, if possible. For example, consider a tenure decision for a marginal (i.e., neither awful nor great) candidate. If there is uncertainty about the quality of the candidate's unpublished work, it would appear wise to postpone the decision on whether to grant tenure by an additional year or two. However, university regulations usually preclude such a course of action and force an immediate decision. While such a rigid rule would often lower and never increase the utility of a *single* decision maker, our model shows that it may be strictly welfare increasing in a group decision problem.

3 Results

3.1 The benchmark case: No option to wait

We first analyze the benchmark case in which the electorate has to take the decision about the project once and for all in period 1. That is, a first period rejection of the project is final. Voter i's expected total payoff from immediate implementation is

$$U_I^i(V_1^i, c) = V_1^i + E(V_2^i) - 2c = V_1^i + \frac{1}{2} - 2c. \tag{1}$$

Each voter approves the project if and only if its net present value is nonnegative.[11] Thus, a voter with first period type $V_1^i = 1$ (a *high type*) votes in favor if and only if

[10] This is certainly true, as a practical matter, for promotion votes in universities. A candidate who receives a bare majority of favorable votes in his own department usually is in severe problems at the college or university level.

[11] As a tie-breaking assumption, we assume that voters who are indifferent always approve the project. No results of our model qualitatively depend on this assumption.

$1 + E(V_2^i) - 2c = 3/2 - 2c \geq 0$, hence if $c \leq 3/4$. Similarly, a *low type* voter ($V_1^i = 0$) casts a favorable ballot if and only if $E(V_2^i) - 2c = 1/2 - 2c \geq 0$, or $c \leq 1/4$.

Thus, projects with adjustment cost $c \leq 1/4$ are unanimously approved, and those with $c \geq 3/4$ are unanimously rejected. The realization of first period types and the majority rule matter only for projects with intermediate adjustment costs, i.e. $c \in (1/4, 3/4]$. For these, the ex-ante probability of implementation (i.e., before nature draws voters' types) decreases in m.

To determine the majority rule that maximizes the ex-ante expected payoff of voters, denote a single voter i's ex-ante expected payoff under majority rule m with N voters when the adjustment cost is c by $\tilde{\pi}_i(c, m, N)$. Given our previous observations, it follows that if $c \leq 1/4$ then $\tilde{\pi}_i(m, N, c) = E(V_1^i) + E(V_2^i) - 2c = 1 - 2c$ for all m, because the project is unanimously approved, no matter which types are realized. If $c > 3/4$, then $\tilde{\pi}_i(m, N, c) = 0$ for all m, because the project is unanimously rejected, no matter which types are realized.

If, instead, $c \in (1/4, 3/4]$ then the realization of types matters. It is useful to define $p(m, N) = 2^{-(N-1)}\binom{N-1}{m-1}$ as the probability that there are exactly $m - 1$ high types among the other $N - 1$ voters. We can think of $p(m, N)$ as the probability of voter i being pivotal, if the majority rule is m. Also, let $q(m, N) = \sum_{j=m}^{N-1} \binom{N-1}{j} 2^{-(N-1)}$ be the probability that there are m or more high types among the other $N - 1$ voters. From the point of view of an individual voter i, $q(\cdot)$ is the probability that the project is implemented through the votes of the other voters, independent of voter i's preference on the project.

It is useful to distinguish these two different events (i.e., voter i being pivotal for implementation, and not being pivotal), because they give rise to different conditional expected implementation payoffs. When there are m or more high types among the other $N - 1$ voters, then voter i's expected payoff is simply the ex-ante expected implementation payoff, $E(V_1^i) + E(V_2^i) - 2c$. In contrast, if there are exactly $m - 1$ high types among the other $N - 1$ voters, then the project is implemented with probability $1/2$ (namely, if and only if i's type is $V_1^i = 1$), in which case voter i's implementation payoff over both periods is $1 + E(V_2^i) - 2c = 3/2 - 2c$. Thus, we have

$$\tilde{\pi}(m, N, c) = \begin{cases} 1 - 2c & \text{if } c \leq 1/4 \\ q(m, N)\,(1 - 2c) + \dfrac{p(m, N)}{2}\left(\dfrac{3}{2} - 2c\right) & \text{if } 1/4 < c \leq 3/4 \\ 0 & \text{if } c > 3/4 \end{cases} \tag{2}$$

where we have dropped the index i, since this payoff is identical for all individuals. To save on notation, we will also suppress the argument N in functions like $\tilde{\pi}$, p or q, when no confusion can arise (i.e., when we consider a situation in which N is fixed). We now show that $\tilde{\pi}(m, c)$ is a piecewise linear function of c that jumps downward at $c = 1/4$

and upward at $c = 3/4$.

Claim 1. *For any m, $\tilde{\pi}$ is a piecewise linear function of c with* $\lim_{c\downarrow 1/4} \tilde{\pi}(m,c) < \tilde{\pi}(m,1/4)$ *for all $m \leq N$, and* $\lim_{c\uparrow 3/4} \tilde{\pi}(m,c) < \tilde{\pi}(m,c)$ *for all $m < N$.*

Proof. We have $\lim_{c\downarrow 1/4} \tilde{\pi}(m,c) = [q(m)+p(m)]/2 < \tilde{\pi}(m,1/4) = 1/2$, and $\lim_{c\uparrow 3/4} \tilde{\pi}(m,c) = -q(m)/2$, while $\tilde{\pi}(m,3/4) = 0$. Note that for unanimity rule $q(N) = 0$, so that $\tilde{\pi}(N,\cdot)$ is discontinuous only at $c = 1/4$, but not at $c = 3/4$. □

Intuitively, at $c = 1/4$, high types strictly benefit from implementation, while low types are just indifferent. Hence, from an ex-ante perspective, voters strictly benefit if the project is implemented. Implementation always occurs for $c \leq 1/4$, while for $c > 1/4$, implementation depends on the realization of preference types and is thus not guaranteed. For example, under simple majority rule, the probability that a project with any $c \in (1/4, 3/4]$ is implemented is just 1/2. Hence, $\tilde{\pi}(m,c)$ drops at $c = 1/4$. Similarly, for $c = 3/4$, high types are just indifferent towards implementation, while low types strictly suffer. Thus, voters suffer from an ex-ante perspective if the project is implemented. Implementation never occurs for $c > 3/4$ (so that $\tilde{\pi}(m,c) = 0$ for all $c > 3/4$), while for $c \in (1/4, 3/4]$, implementation depends on the realization of preference types. Figure 1 illustrates Claim 1 and shows the ex-ante payoff for the case $N = 15$ and $m = 8$ (black curve) and $m = 9$ (the blue curve).

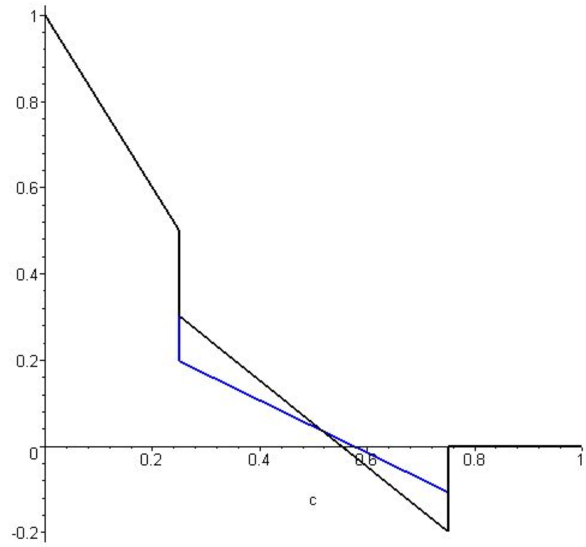


Figure 1: The function $\tilde{\pi}$ for $N = 15$, $m = 8$ (black) and $m = 9$ (blue)

We now analyze the optimal voting rule for different levels of c. First, observe that for $c \leq 1/4$ or $c > 3/4$ the voting outcome is unanimous, and thus payoffs do not depend on the majority rule, so that all majority rules perform equally well.

Second, for $c \in (1/4, 1/2]$, both the expected net benefit conditional on being pivotal, $1 + E(V_2) - 2c = 3/2 - 2c$ and the unconditional expected net benefit, $E(V_1) + E(V_2) - 2c = 1 - 2c$, are positive. Since the functions $q(\cdot)$ and $p(\cdot)$ are both decreasing in m, it follows that the unique optimal majority rule is simple majority.

Third, for $c \in (1/2, 3/4]$, voters expect to obtain a positive payoff only in case they are pivotal (i.e., $3/2 - 2c > 0 > 1 - 2c$). Thus, the optimal majority rule trades off the expected loss if a voter is not pivotal, and the expected gain if he is pivotal. For higher c, the pivot-benefits decrease while the non-pivot losses increase. To maintain optimality of the majority rule, the relative probability weight on benefits versus losses, $p(m)/q(m)$, must increase, and this ratio is increasing in m, as shown in Lemma 1.[12] Thus, intuitively, the optimal majority rule increases in c. In summary,

Proposition 1. *Suppose that society can either implement the investment project in period* 1*, or not at all.*

1. *If $c \leq 1/4$, or $c > 3/4$, all majority rules yield the same expected payoff $\tilde{\pi}(\cdot, c)$.*
2. *If $c \in (1/4, 1/2]$, then simple majority rule $m = (N+1)/2$ maximizes the expected payoff $\tilde{\pi}(\cdot, c)$.*
3. *Let $\lceil x \rceil$ denote the smallest integer greater or equal to x. For $c \in (1/2, 3/4)$ $\tilde{\pi}(\cdot, c)$ is single peaked in m, unless $(2c - 1/2)N$ is an integer; the majority rule that maximizes $\tilde{\pi}(\cdot, c)$ is given by $m^* = \left\lceil \left(2c - \frac{1}{2}\right) N \right\rceil$. In particular, for c close to $3/4$, the unique optimal majority rule is unanimity rule. If $(2c - 1/2)N$ is an integer, then $\tilde{\pi}(\cdot, N, c)$ is 'single-plateaued', where the plateau is given by the two points $m^* = (2c - 1/2)N$ and $m^{**} = (2c - 1/2)N + 1$.*

Proof. The first two statements are proved in the text above. For the third claim, note that $q(m) = q(m+1) + p(m+1)$, so that we have

$$\begin{aligned} \tilde{\pi}(m+1, c) - \tilde{\pi}(m, c) &= [q(m+1) - q(m)](1 - 2c) + [p(m+1) - p(m)]\left(\frac{3}{4} - c\right) \\ &= p(m+1)\left(c - \frac{1}{4}\right) - p(m)\left(\frac{3}{4} - c\right). \end{aligned} \tag{3}$$

Rearranging, (3) is positive if and only if

$$c\left[\frac{p(m+1)}{p(m)} + 1\right] > \frac{3}{4} + \frac{p(m+1)}{4p(m)}.$$

Using $p(m+1)/p(m) = (N - m)/m$, we can conclude that (3) is positive if and only if $m/N < 2c - 1/2$. Therefore, for $c \in [1/2, 3/4]$, the optimal majority rule is $m^* = \left\lceil \left(2c - \frac{1}{2}\right) N \right\rceil$, as claimed. Substituting shows that unanimity is the unique optimal majority rule for $c \in \left(\frac{3}{4} - \frac{1}{2N}, \frac{3}{4}\right]$. □

[12] This result is intuitive, because p/q goes to infinity for unanimity rule.

The optimal majority rule $m^*/N \approx \left(2c - \frac{1}{2}\right)$ has an intuitive interpretation as the one that maximizes utilitarian welfare: If there are $\hat{M}$ high types in the first period, then the per-capita expected utility from implementation is $\hat{m} - c + \frac{1}{2} - c$ (where $\hat{m} = \hat{M}/N$). A social planner would like to implement the project if and only if this expected utility is positive, and setting $m^* = \left\lceil \left(2c - \frac{1}{2}\right) N \right\rceil$ guarantees just that.

Now consider the problem of choosing an optimal majority rule when the constitution cannot condition the majority rule for a project on its adjustment cost c. From an ex-ante perspective, c is distributed according to some (arbitrary) distribution F. Of course, if F does not put any weight on the interval $(1/4, 3/4]$, then all majority rules perform equally well.

Proposition 2. *Suppose that society can either implement the investment project in period 1, or not at all. Furthermore, suppose that the constitution cannot condition the majority rule on c, and that c is drawn from a distribution with cumulative distribution function F that satisfies $F(3/4)-F(1/4) > 0$. Then $\tilde{\Pi}(\cdot, N)$ is generically single peaked with its peak at*[13]

$$m^* = \max\left\{\frac{N+1}{2}, \left\lceil \frac{(y-x)N}{2y} \right\rceil\right\},$$

where $x = \int_{1/4}^{3/4}(1-2c)dF(c)$ and $y = (F(3/4) - F(1/4))/2$. In particular, if F is symmetric around 1/2, then $x = 0$, so that simple majority rule is optimal.

Proof. See Appendix. □

Note that the condition $F(3/4) - F(1/4) > 0$ restricts attention to those distributions for which majority rules matter at all: Since all projects with $c \leq 1/4$ are unanimously accepted, and those with $c > 3/4$ are unanimously rejected, we know that, if $\text{Prob}(c \in (1/4, 3/4]) = 0$, then all majority rules yield the same surplus.

There are two important classes of distributions for which simple majority rule is optimal. First, as mentioned in the proposition , if F is symmetric around 1/2 (for example, if F is a uniform distribution on $[0, 1]$). The intuition for this result is as follows. If a voter is not pivotal for the implementation decision, then, by the symmetry of the distribution of c, expected gains and losses from implementation cancel out for any majority rule. However, if the individual is pivotal, then he receives a positive expected payoff. Thus, the best majority rule from an ex-ante perspective is the one that maximizes the probability of being pivotal, and that rule is simple majority rule.

Second, if the expected value of c, conditional on c being between 1/4 and 3/4, is lower or equal to 1/2, then $x \geq 0$, and thus $m^* = (N+1)/2$. In this case, the expected

[13] The term 'generically' refers to the fact that $\tilde{\Pi}$ may be single-plateaued instead of single-peaked if $(y-x)N/2y$ is an integer. In this case, both $(y-x)N/2y$ and $(y-x)N/2y+1$ are maximizers of $\tilde{\Pi}(\cdot, N)$.

payoff for a voter is positive even if he is not pivotal for implementation, and therefore simple majority rule (which maximizes the probability of implementation) is optimal.

3.2 Individual voting behavior and the option to wait

We are now ready to analyze the implications of the option to delay the decision on the implementation of the public project. We begin with the straightforward analysis of voting behavior in second period elections. Player i votes in favor of the project if and only if $V_2^i = 1$, and the project is implemented if and only if there are at least m players with a high second period type. Let $I_2(m)$ denote the event that the project is implemented in the second period, given that a majority of at least m votes is required; and let $P(I_2(m))$ denote the probability of this event.

Consider now the first period decision. If the project is not implemented in the first period, then player i can expect to obtain the payoff $\left(E[V_2^i|I_2(m)] - c\right) P(I_2(m))$. It is useful to write this expected continuation utility, the expected *value of waiting*, as

$$\begin{aligned} U_W(c,m) = p(m)E[\max\{V_2^i - c, 0\}] + q(m)(E(V_2^i) - c) = \\ \frac{p(m)}{2}(1-c) + q(m)\left(\frac{1}{2} - c\right), \end{aligned} \tag{4}$$

using the definitions of $p(\cdot)$ and $q(\cdot)$ from the last section. Since voter i's payoff from implementing the project immediately is $U_I^i(V_1^i, c) = V_1^i + 1/2 - 2c$, he will approve immediate implementation in period 1 if and only if

$$V_1^i + 1/2 - 2c \geq \frac{p(m)}{2}(1-c) + q(m)\left(\frac{1}{2} - c\right). \tag{5}$$

Note an important difference to the benchmark case without the option to wait: An individual voter's first period behavior as characterized by (5) depends on the majority rule m, because that rule determines the expected value of waiting.

If $c \leq 1/2$, then both terms on the right-hand side of (5) are positive. Thus, in this case, the option to wait induces voters to behave more conservatively than in situations where the decision may not be delayed. Moreover, since both $p(\cdot)$ and $q(\cdot)$ are decreasing functions of m, this tendency to behave more conservatively, is the stronger the lower the majority rule m. Thus, the cost threshold below which a low voter type is willing to approve a project shifts to the left as the option to delay the decision is introduced, and this shift is the stronger, the lower the majority rule.[14]

If $c > 1/2$, then the value of waiting is neither necessarily positive, nor is it necessarily decreasing in m. The reason for why the value of waiting can be negative in social decisions — in contrast to private decisions, where the value of waiting is always

[14] For $c \leq 1/2$, (5) implies that high types always favor implementation, so that their behavior does not change relative to the case that waiting is not possible.

positive — is that society sometimes implements projects that, from a social ex-ante point of view, are not beneficial. If the right-hand side of (5) is negative, then it is possible that a high type voter votes for immediate implementation of an investment project even though his expected overall payoff from this project is negative. The reason for this (seemingly strange) behavior is that the voter's payoff from immediate implementation is at least better than the expected payoff he would get if he forgoes immediate implementation and is then (perhaps) hit by implementation in the second period, when his type may be low. In this case, a higher majority rule may increase the value of waiting, as it increases the voters' protection in the next period against the implementation of a project that they oppose.

We now proceed to a more formal analysis of the value of waiting and its implications for individual voting behavior. Lemmas 1 and 2 are used repeatedly in the proofs of the following propositions, and are presented here in the text, because they are of independent interest and provide an intuition for the economic effects in our model.

Lemma 1 shows that a higher majority rule increases the probability of voter i being pivotal, relative to the probability that the project is implemented independent of voter i's preferences. This effect underlies a benefit of supermajority rules for projects with high c, because a voter always gets a nonnegative payoff if he is pivotal, but receives a negative expected payoff if the project is implemented independently of voter i's will.

Lemma 1. *The ratio* $\dfrac{p(m)}{q(m)}$ *is increasing in* m.

Proof. See Appendix. □

Lemma 2 shows that the value of waiting $U_W(c, m)$, defined in equation (4), is increasing in m if $m < Nc$ and decreasing in m if $m > cN$. Thus, for any $c \in [0, 1]$ the value of waiting is single-peaked in m.

Lemma 2. *If* $m < Nc$ *(*$m > Nc$*), then* $U_W(c, m) < U_W(c, m + 1)$ *(*$U_W(c, m) > U_W(c, m + 1)$*).*

Proof. See Appendix. □

Note that the condition $m > cN$ is satisfied for all majority rules if $c \leq 1/2$. Intuitively, an increase in m makes second period implementation of such projects (which are all socially valuable from an ex-ante perspective) less likely, so that the value of waiting decreases monotonically in m.

A similar argument holds if $m/N > c > 1/2$. Intuitively, $m/N > c$ means that, under majority rule m, the project is implemented only if the gross aggregate benefit (m) exceeds the total social cost (Nc). A further increase of the majority rule then implies that the project is not implemented in some situations where the project's average payoff is positive. Thus, from the perspective of period 1, the expected value of waiting decreases. Conversely, the value of waiting is increasing in m if $m < cN$, as the project is less often implemented when the average payoff of voters is negative.

For later reference, note that one of the implications of the single-peakedness of U_W in m is that, if for some c the value of waiting is negative for a given majority rule m, then the same must hold true for any majority rule $m' < m$ (i.e. $U_W(m, c) < 0$ implies $U_W(m', c) < 0$ for all $m' < m$).

The following Proposition 3 characterizes individual voting behavior in a situation where the decision on the project may be postponed. There are two cutoffs $\underline{c}$ and $\bar{c}$ such that first-period low types vote for implementation if $c \leq \underline{c}$ and first-period high types vote for implementation if $c \leq \bar{c}$. Thus, there are three different regimes: All projects with $c \leq \underline{c}$ are unanimously approved in the first period, all projects with $c > \bar{c}$ are unanimously rejected, and for projects with $c \in (\underline{c}, \bar{c}]$, implementation depends on whether there are at least m first-period high types.

Proposition 3 also characterizes the range in which the thresholds lie, and how they change with m. Intuitively, since $\underline{c} < 1/2$, the value of waiting is positive and decreases in m. A higher majority rule increases the willingness of low types to implement in the first period, as second period implementation becomes less likely. Thus, $\underline{c}$ increases in m. In contrast, $\bar{c} > 1/2$, and the value of waiting is non-monotonous in m in that region. For low majority rules, the value of waiting is negative for projects with costs close to $\bar{c}$, and increases with m. Thus, high types become more conservative when m increases, so that $\bar{c}$ decreases. In contrast, for high majority rules, the value of waiting is positive and decreases with a further increase in m, thus making high types less conservative, so that $\bar{c}$ increases in m for high levels of m. Thus, $\bar{c}$ is a U-shaped function of m.

Proposition 3. *For any majority rule m, there exist threshold values $\underline{c}(m)$ and $\bar{c}(m)$, with $\underline{c}(m) < \bar{c}(m)$, such that low types (high types) vote for first period implementation of a project if and only if $c \leq \underline{c}(m)$ ($c \leq \bar{c}(m)$).*

Moreover, $\underline{c}(m)$ is an increasing function of m and satisfies $1/12 \leq \underline{c}(m) < 1/4$. In contrast, $\bar{c}(\cdot)$ is U-shaped, assumes its minimum at $m = \lceil 3N/4 \rceil$ and satisfies $2/3 < \bar{c}(m) < 5/6$. Moreover, $\bar{c}((N+1)/2) > \bar{c}(N)$.

Proof. See Appendix. □

3.3 Ex ante payoffs under different majority rules

The key result of Proposition 3 is how the majority rule influences individual voting behavior. For low cost projects, individual voters behave more conservatively under lower majority rules. In contrast, for high cost projects, individual voters behave more conservatively under higher majority rules. We will now analyze the implications for voters' ex-ante payoffs.

We denote a player's ex-ante payoff, that is, his expected payoff given majority rule m and implementation cost c, but before the player's type is known, by $\pi(m, N, c)$. For $c \leq \underline{c}(m)$, all voters always approve the first-period implementation of the project, so that we have $\pi(m, c) = \tilde{\pi}(m, c) = 1 - 2c$ (where we again drop the variable N). If $c > \bar{c}(m)$, then all voters reject the project in the first period and so $\pi(m, c)$ in this case simply coincides with the value of waiting, $U_W(c, m) = q(m)(1/2 - c) + p(m)(1 - c)/2$. Finally, if $c \in (\underline{c}(m), \bar{c}(m)]$, then the project is approved in period 1 if and only if there are sufficiently many high type voters in period 1. In contrast to Section 3.1, after a first period rejection, which occurs with probability $[1 - q(m) - p(m)/2]$, the project may still be implemented in period 2, so that $\pi(m, c) = \tilde{\pi}(m, c) + [1 - q(m) - p(m)/2]U_W(c, m)$. Rearranging terms and dropping the arguments from the functions q and p, we thus have

$$\pi(m, c) = \begin{cases} 1 - 2c & \text{if } c \leq \underline{c}(m) \\ q(1 - 2c) + \frac{p}{2}\left(\frac{3}{2} - 2c\right) + \left(1 - q - \frac{p}{2}\right)\left[q\left(\frac{1}{2} - c\right) + \frac{p}{2}(1 - c)\right] & \text{if } c \in (\underline{c}(m), \bar{c}(m)] \\ q\left(\frac{1}{2} - c\right) + \frac{p}{2}(1 - c) & \text{if } c > \bar{c}(m). \end{cases} \tag{6}$$

It is easy to see that a result parallel to Claim 1 obtains: For any m, the ex-ante payoff $\pi(m, \cdot)$ is a piecewise linear function of c that exhibits a downward jump at $\underline{c}(m)$, and, unless $m = N$, an upward jump at $\bar{c}(m)$. Figure 2 depicts the ex-ante payoff for $N = 15$ and the cases $m = 8$ (black curve) and $m = 9$ (blue curve).

We now turn to an analysis of the optimal majority rule for a given level of c, which may be markedly different here from the benchmark case of Section 3.1.

There are several different effects. First, consider projects that are rejected by all voters in the first round under *any* majority rule (i.e., with $c > \bar{c}((N + 1)/2)$). In contrast to Section 3.1, these projects may now still be approved in the second period. Since those projects are projects with a negative expected net social benefit, it follows that higher majority rules outperform lower majority rules, since they decrease the probability that such projects will pass.

Second, the cost threshold below which high type voters are willing to implement projects in period 1 now depends on the majority rule. In a sense, under low majority rules, introducing the option to wait reinforces the electorate's tendency to behave too 'aggressively' in implementing projects with low expected net benefits. Specifically, take a setting where $\bar{c}(m + 1) < \bar{c}(m)$, and consider a project with a value of c that lies in between these two thresholds. Majority rule $m + 1$ guarantees that any project with a cost parameter in $(\bar{c}(m + 1), \bar{c}(m)]$ is at least not implemented in period 1. This is clearly beneficial if the project's expected net benefit is negative for both groups, i.e.

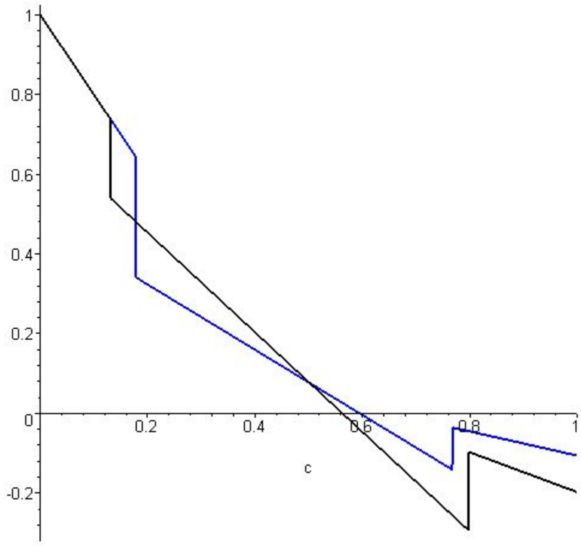


Figure 2: Ex-ante payoffs $\pi(8, 15, c)$ (black) and $\pi(9, 15, c)$ (blue)

if $\bar{c}(m+1) > 3/4$. Using results in the proof of Proposition 3, one can show that a sufficient condition for $\bar{c}(m+1) > 3/4$ is that $(m+1)/N \leq 2N/3$.

It might even be true that, *after* the first period types are realized, voters would unanimously agree to change the majority rule. Consider, for example, a society with simple majority rule, a project with c between 3/4 and $\bar{c}((N+1)/2)$, and a majority of high types. Without a change in the majority rule, the project is implemented by the support of all high types in the first period. However, *all* voters (including first period high types) would be better off if society switched to unanimity rule, thereby killing the project in the first period. Thus, a change from simple majority rule to unanimity rule leads to an ex-post Pareto improvement in this example.[15]

Another interesting effect arises in the case of projects with high expected net benefits. Since $\underline{c}(m) < \underline{c}(m+1)$, projects in the interval $(\underline{c}(m), \underline{c}(m+1))$ are approved in the first period for sure under majority rule $m+1$, while they may be rejected under majority rule m. As $\underline{c}(m+1) < 1/4$, these projects are characterized by high expected social benefits. Thus, from an ex-ante perspective, voters are better off if such projects are more likely to pass. Again, this effect favors higher majority rules.

[15]The fact that high types may choose to implement a project with a negative expected return even for themselves is an example of what Bai and Lagunoff (2007) call the *Faustian tradeoff* in politics, where today's policy is determined by a desire to influence either the identity or the set of feasible choices of a future policy maker. By implementing immediately, today's pivotal voters make sure that tomorrow's "leaders" have no power to make a decision. While implementation out of a concern that tomorrow's leaders could implement the project anyway is reminiscent of the act of committing suicide out of a fear of death, it is nevertheless rational.

We summarize these observations in the following proposition.

Proposition 4. *1. For $c \leq \underline{c}((N+1)/2)$, all majority rules always lead to first period implementation, and thus, the expected ex-ante payoff is the same under any majority rule.*

2. For $c \in (\underline{c}((N+1)/2), \underline{c}(N))$, unanimity is an optimal majority rule. In particular, it strictly dominates simple majority rule.

3. Unanimity rule also dominates simple majority for $c > 3/4$.

While Proposition 4 emphasizes the positive effects of supermajority rules, there are also countervailing effects that increase the benefit of low majority rules. Because all thresholds $\underline{c}$ decrease below $c = 1/4$, the behavior of the electorate becomes more conservative under any majority rule. Of course, for $c \in (\underline{c}(N), 1/4)$ the negative welfare consequences of this effect are less severe with a lower majority rule. Moreover, for projects with $c \in (1/4, 1/2)$ the fact that the project may still be implemented in period two, even if it is rejected in period one, reinforces the relative advantage of low majority rules. Clearly, to this stronger relative advantage corresponds a larger comparative disadvantage for projects with negative expected net benefits.

3.4 Average ex-ante payoffs

The option to wait produces effects both in favor and against low majority rules, so it is natural to ask which of these effects dominates. In what follows we assume again that c is drawn from some distribution F and denote the average ex-ante payoff of an individual voter under majority rule m by $\Pi(m, N)$. That is, $\Pi(m, N) = \int_0^1 \pi(m, N, c) dF(c)$.

Our first result obtains in the case of a large electorate and shows that the optimal majority rule with the option to wait is weakly larger than the optimal majority rule in the benchmark case. This result holds for a very large class of distributions of c.

Proposition 5. *Let s denote a proportional majority rule, i.e. $s = m/N$. Let S^* denote the set of ex-ante optimal proportional majority rule when voters have the option to wait, c is distributed according to F and N goes to infinity. That is,*

$$S^* = \{s^* | \lim_{N\to\infty} \Pi(\lceil s^* N \rceil, N) \geq \lim_{N\to\infty} \Pi(\lceil sN \rceil, N) \text{ for all } s \in [1/2, 1]\}$$

Similarly, let

$$\tilde{S}^* = \{\tilde{s}^* | \lim_{N\to\infty} \tilde{\Pi}(\lceil \tilde{s}^* N \rceil, N) \geq \lim_{N\to\infty} \tilde{\Pi}(\lceil sN \rceil, N) \text{ for all } s \in [1/2, 1]\}$$

be the set of optimal proportional majority rules in the limit when there is no option to wait. Then $\inf S^* \geq \sup \tilde{S}^*$ for all distributions F with $1 - F(3/4) + F(1/4) - F(1/6) > 0$ (i.e., whenever $\text{Prob}(c \in (1/6, 1/4] \cup (3/4, 1]) > 0$).

Proof. See Appendix. □

What are the intuitive benefits of increasing the majority rule when society has the option to wait? Consider a distribution that is symmetric around $c = 1/2$, so that (by Proposition 2) simple majority rule is optimal in the benchmark case. Now introduce the option to wait. For projects with $c = 1/4$, low types now strictly prefer to wait, as the expected value of waiting is positive. Specifically, in period 2, all projects are implemented with probability 1/2, and so, for given c, the value of waiting is $U_W = \frac{1}{2}\left(\frac{1}{2} - c\right)$. Thus, in period 1, low types agree to implement if and only if $\frac{1}{2} - 2c \geq \frac{1}{2}\left(\frac{1}{2} - c\right)$, hence whenever $c \leq 1/6$. Similarly, first period high types vote for immediate implementation whenever $c \leq 5/6$. Thus, if $c \leq 1/6$, the project is implemented in the first period. Projects with $c \in (1/6, 5/6]$ are implemented with probability 1/2 in the first period, and projects with $c > 5/6$ are not implemented in period 1. In period 2, those projects rejected in the first period are reconsidered, and each has a probability of 1/2 of gaining sufficient support for implementation.

Now consider the effect of a supermajority rule that requires approval of sN voters, where $s > 1/2 + \varepsilon$, with $\varepsilon > 0$. Since the proportion of high types among all voters is almost certainly close to 1/2, the project will not be implemented in period 2 under any supermajority rule. Thus, the value of waiting is zero. Furthermore, a project is implemented in period 1 if and only if low types agree, that is, if $\frac{1}{2} - 2c \geq 0$. Hence, under a supermajority rule, all projects with $c \leq 1/4$ are implemented in the first period, and no projects with $c > 1/4$ are ever implemented.

Relative to simple majority rule, a supermajority rule therefore has three effects: First, it makes first-period low types more willing to implement low-c projects. Second, first-period high types become more reluctant to implement high-c projects. Third, it hampers implementation of projects in the second period. All three effects are socially desirable. This is quite obvious for the first two effects. For the third one, observe that those projects that are considered for implementation in the second period are an adverse selection from the set of all projects, because the best projects (with the lowest c) have already been implemented by unanimous consent in period 1. Since the initial distribution of c was symmetric around 1/2, the average ex-ante payoff from a project that is still available for implementation in period 2 is negative, so that a supermajority rule is strictly better in that period.

Since Proposition 5 holds for a large class of cost distributions, it cannot characterize the optimal majority rule with the option to wait precisely. Thus, while we show that the optimal majority rule cannot decrease (relative to the case without waiting), it remains unclear whether and by how much the optimal majority rule increases, and how this depends on N. In particular, as we show in the proof of Proposition 5, *any* supermajority rule yields the same ex-ante expected surplus in the limit, and thus the limit case cannot inform us on whether (in small or medium-sized electorates), the optimal majority rule is close to simple majority or unanimity rule.

The following Proposition 6 is therefore an important complement to Proposition 5: It pertains to a specific cost distribution, but shows that, in this case, the option to wait leads to a substantial increase in the optimal majority rule for any N. Remember that, under the assumption that c is uniformly distributed on $[0, 1]$, simple majority rule is optimal without the option to wait.

Proposition 6. *Suppose that* c *is ex-ante uniformly distributed on* $[0, 1]$*, so that* $\Pi(m) = \int_0^1 \pi(m, c)dc$.

i) $\Pi(m + 1) - \Pi(m) < 0$ *for all* $m \geq 2N/3$ *and*

ii) $\Pi(m + 1) - \Pi(m) > 0$ *for all* $(N + 1)/2 \leq m < 7N/11$.

Moreover, $\Pi((N + 1)/2) < \Pi(N)$.

Proof. See Appendix. □

While Proposition 6 does not determine the optimal majority rule exactly, it is clear from (i) that the optimal majority rule is at most $\lceil 2N/3 \rceil$, i.e., the lowest majority rule that is higher than a two-thirds majority. From (ii), it follows that the optimal majority rule is a supermajority rule with $m/N \geq 7/11 \approx 0.636$. In particular, if the number of voters N is large, then the optimal majority rule as a percentage of the electorate lies either within or arbitrarily close to the interval $[7/11, 2/3]$.

Interestingly, when the option to wait is introduced, simple majority not only loses its status as the optimal majority rule, but it actually becomes the worst majority rule. It is dominated even by unanimity (which is the worst of all supermajority rules that have $m \geq \lceil 2N/3 \rceil$). Thus, loosely speaking, choosing a "too high" supermajority rule has a lower welfare cost than choosing a majority rule that is "too low".

While Proposition 6 holds for the uniform cost distribution, it is intuitive that the result is robust. For different cost distributions that are 'close' to a uniform distribution, the optimal majority rule would be close to the one characterized in Proposition 6, and thus a supermajority rule. For example, one can show that, for any density of the distribution that satisfies $1/4 \leq f(c) \leq 2$ for all c, a supermajority rule is ex-ante better than a simple majority rule.[16]

[16]The (rather tedious) proof of this claim is available from the authors upon request. Also note that, while there may be even weaker assumptions under which supermajority rules are optimal, there are some distributions for which the result does not hold. For example, if $c = 1/4$ with certainty, then simple majority rule is optimal for any N, with or without the option to wait.

3.5 Does the option to wait increase the welfare of voters?

In settings with a single decision maker, the option to postpone the investment decision always weakly increases the decision maker's expected profit. This is obvious, as the decision maker can still choose to go ahead and invest immediately, but sometimes he may strictly prefer to wait. While our setup here is similar, the answer to the title question is not obvious, as the option to wait influences a *game* between different voters, rather than the decision problem of a single decision maker.

Indeed, for some values of c, the option to wait hurts citizens from an ex-ante perspective. For example, projects with $c > 3/4$ are never implemented without the option to wait, as even the first period high types have a negative expected profit from their implementation. In contrast, with the option to wait and simple majority rule, each project that was rejected in period 1 has a 50 percent chance of being accepted in period 2; in most of the cases when the project is implemented in period 2, the percentage of winners is smaller than c, and so the project is socially undesirable. Moreover, with the option to wait, $\bar{c}(m) > 3/4$ for many majority rules, so that some projects that would definitely be rejected without the option to wait are actually implemented in period 1 with positive probability.

However, there are also project types for which the option of waiting increases expected social welfare. For instance, if $c \in (\underline{c}(m), 1/4]$, then a project may be rejected in period 1. Without the option to wait, such a rejection is final, while there is a second period chance for (on average beneficial) implementation with the option to wait. Thus, there exist some cost levels for which ex-ante welfare increases, and others where welfare decreases with the option to wait. Again, it is interesting to see which effect dominates from an ex-ante perspective.

The following Proposition 7 shows that, often, the option to wait harms voters in expectation. The first part of the proposition considers the limit case of $N \to \infty$; under some condition on the distribution (which is satisfied, for example, by any distribution that is symmetric around 1/2), the option to wait cannot strictly benefit voters. The second part again specializes to a uniform distribution of c and shows that, for a large range of low supermajority rules, the option to wait lowers ex-ante payoffs, and only under very high majority rules, the option to wait is guaranteed to have a positive social value in terms of average ex-ante payoffs. In particular, for $N > 5$, we show that the expected payoff without the option to wait and simple majority rule dominates the expected payoff with the option to wait and the optimal supermajority rule in this case.

Proposition 7. *1. Let s^* and $\tilde{s}^*$ be defined as in Proposition 5, and assume that F satisfies $\int_{1/4}^{3/4}(1-2c)dF(c) \leq 0$. Then, in a large electorate, the ex-ante expected utility is weakly lower with the option to wait: $\lim_{N\to\infty} \Pi(\lceil s^* N \rceil, N) \leq \lim_{N\to\infty} \tilde{\Pi}(\lceil \tilde{s}^* N \rceil, N)$.*

2. *Suppose that $F(c) = c$. If $N > 3$ and $m \leq \lfloor 3N/4 \rfloor$ then $\tilde{\Pi}(m) > \Pi(m)$. If, instead, $m \geq 13N/16$ then $\tilde{\Pi}(m) < \Pi(m)$.*

3. *Suppose that $F(c) = c$. For $N > 5$, the maximal average ex-ante payoff is strictly lower if voters have the option to wait than if they do not. That is,* $\max_m \tilde{\Pi}(m) > \max_m \Pi(m)$.

Proof. See Appendix. □

Parts 2 and 3 of Proposition 7 provide a generic and robust example that contrasts starkly with the value of waiting in individual decision problems, where an individual decision maker (facing the same distribution of c) would always strictly benefit from the option to wait.

For an intuition, consider a setting where N is large. Under simple majority rule without the option to wait, all projects with $c \leq 1/4$ are unanimously implemented, just as under the optimal supermajority rule. In addition, however, projects with $c \in (1/4, 3/4]$ are implemented under simple majority rule if and only if a majority of voters has a high type, and this is, on average, better (from an ex-ante perspective) than not implementing any of these projects.[17] Again, nothing in this argument relies on the assumption that the cost distribution is uniform, and the result that ex-ante expected utility is lower with the option to wait is thus quite robust.

4 Extensions

Like most political economy models, our model imposes some strong structural assumptions. In particular, we assume that there are only two different payoff types in each period, types are equally likely, and the second period type of a voter is independent of his first period type. We do this in order to generate tractability and comparability to the case without the option to wait.

In this section, we want to explore the robustness of the model when we loosen some of our assumptions. In Section 4.1, we analyze a setting where second period valuations of individuals are correlated with each other, so that voters are more likely to agree with the majority of other voters ex-post. In Section 4.2, the first- and second-period valuations of each individual are correlated, that is, first period high types are more likely to be second period high types than first period low types.

[17] Clearly, this argument requires that N is large, but finite, because when we take N to infinity, then $\lim \Pi(m, N) = \lim \tilde{\Pi}(m, N)$.

4.1 Systematic second-period risk

In our basic model, each voter's probability of being a high type in period 2 is 1/2. We now assume instead that, at the beginning of the second period, nature draws a parameter μ from a uniform distribution on $[0,1]$; then, each voter is assigned a high type with probability μ (and, correspondingly, a low type with probability $1-\mu$).

Note that this variation of the model would not at all affect the expected utility or first period actions of a single decision maker, as, from the perspective of the first period, the expected probability of being a high type in period 2 remains at 1/2 in this scenario. Consequently, both the implementation payoff and the value of waiting remain unchanged.

However, the model variation introduces correlation between the types of different voters in the second period: The probability of voter i being a high type in the second period, conditional on voter $j \neq i$ being a high type in that period, is $\text{Prob}(V_2^i = 1|V_2^j = 1) = \frac{\text{Prob}(V_2^i=1 \cap V_2^j=1)}{\text{Prob}(V_2^j=1)} = \frac{\int_0^1 \mu^2 d\mu}{\int_0^1 \mu d\mu} = 2/3$, while this probability is equal to 1/2 in the basic model. Effectively, while voters still do not know in the first period whether they will like the project in the second period, they are now more likely than in the basic model to agree with the majority of the other voters about the desirability of the project ex-post: Say, if μ turns out to be high, then it is likely that a particular voter i is a high type, and also likely that the majority of other voters agrees.

For example, consider the job market example from Section 2.3. Suppose that, whether a particular faculty member "likes" next year's candidate (i.e., receives a positive net payoff from the candidate being hired) depends stochastically on that candidate's "quality" (i.e., μ). A high quality candidate is more likely to be liked by each existing faculty member than a low quality candidate, so there is correlation between the opinions of different voters. However, if the department rejects this year's candidate, its voters do not know the quality of the (feasible) candidates next year, so next year's μ is a random variable from today's perspective.

What is the value of waiting in this setup, for a project with a given value of c? Just as in the basic model, we can condition on whether voter i is or is not pivotal for second period implementation. Writing $p_2(m, N, \mu)$ and $q_2(m, N, \mu)$ for the obvious generalizations of the functions $p(m, N)$ and $q(m, N)$ from the basic model, we have

$$U_W(c, m) = E[\max\{V_2^i - c, 0\}]p_2(m, \mu) + (E(V_2^i) - c)q_2(m, \mu) =$$
$$\int_0^1 \left[\binom{N-1}{m-1} \mu^{m-1}(1-\mu)^{N-m} \cdot \mu(1-c) + \sum_{k=m}^{N-1} \binom{N-1}{k} \mu^k (1-\mu)^{N-1-k} \cdot (\mu - c) \right] d\mu. \tag{7}$$

The first term in (7) refers to the case that individual i is pivotal, which happens with probability $\binom{N-1}{m-1}\mu^{m-1}(1-\mu)^{N-m}$; then, with probability μ, individual i is a high type and votes for implementation, in which case his payoff is $1 - c$. The second term refers to the case that there are at least m high types among the other voters; in this case, voter i's expected type is μ, so that his expected implementation payoff is $(\mu - c)$.

Just like in the basic model, intersecting $U_W(c, m)$ with $U_I(0, c)$ and with $U_I(1, c)$ yields the values for $\underline{c}(m)$ and $\bar{c}(m)$, respectively. All projects with $c \leq \underline{c}(m)$ are unanimously approved in the first period; all projects with $c > \bar{c}(m)$ are unanimously rejected in the first period; and for projects with $c \in (\underline{c}(m), \bar{c}(m)]$, voting is type dependent and the voting outcome in period 1 depends on the realization of voter types.

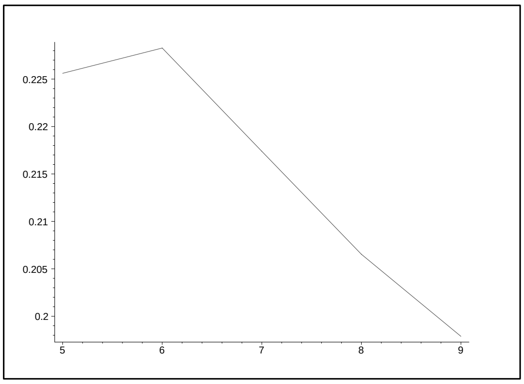
Observe that the function $U_I(V_1, c)$ is linear in V_1, so $E_{V_1} U_I(V_1, c) = U_I(EV_1, c)$. Therefore, we can write the ex-ante expected utility, given majority rule m, as

$$\Pi(m) = \int_0^{\underline{c}(m)} U_I(0.5, c)dc + \int_{\bar{c}(m)}^1 U_W(c, m)dc +$$
$$\int_{\underline{c}(m)}^{\bar{c}(m)} \left[\sum_{k=m}^N \binom{N}{k} \left(\frac{1}{2}\right)^N U_I\left(\frac{k}{N}, c\right) + \left(1 - \sum_{k=m}^N \binom{N}{k} \left(\frac{1}{2}\right)^N\right) U_W(c, m) \right] dc \tag{8}$$

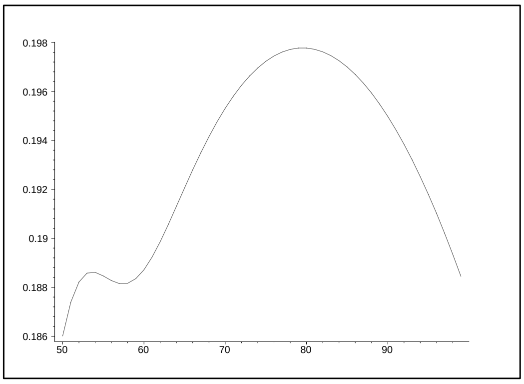
The first of these terms corresponds to those projects that have $c \leq \underline{c}(m)$ and are all implemented in period 1. All projects with $c > \bar{c}(m)$ are rejected in period 1, and each voter obtains the value of waiting. The third integral corresponds to projects with a cost between $\underline{c}(m)$ and $\bar{c}(m)$; if there are $k \geq m$ high types in period 1, these projects are implemented immediately and generate a per-capita utility of $U_I(k, c)$; otherwise, if $k < m$, the project is delayed and each voter obtains the value of waiting.

Figure 3 shows the function $\Pi(m, N)$ for $N = 9$, $N = 99$ and $N = 199$. For $N = 9$ in part (a), the optimal majority rule is $m = 6$, that is, a two-thirds majority rule, just as in the basic model. For $N = 99$ in part (b) and $N = 199$ in (c), the optimal m is approximately equal to $0.8N$, respectively. Thus, the optimal majority rule in these cases is considerably larger than in the basic model, where the optimal majority rule is approximately between 7/11 and 2/3 of the electorate.

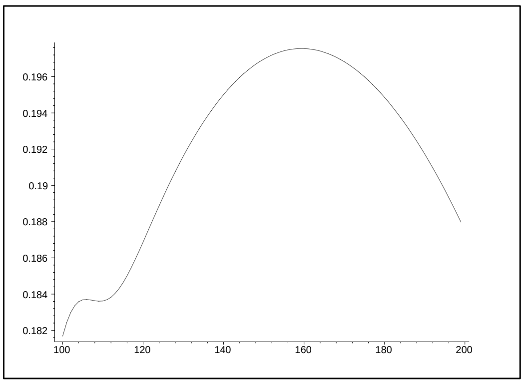
Intuitively, the reason for why the optimal majority rule increases when voters' second period payoffs are correlated is as follows. Remember that the optimal majority rule in the basic model is (about a 2/3) supermajority rule. If N is at least moderately large, then it is relatively unlikely that a project would be implemented in the second period. For this reason, low types are willing to agree to first period implementation of projects with a low c: They don't have much to lose in the first period, and potentially a lot to gain in the second period if their second period type is high, but are likely to receive this payoff only if the project is already implemented in the first period. As a consequence, projects with $c \leq \underline{c} \approx 1/4$ are implemented unanimously in period 1, and they make up the largest part of all projects that are implemented at all.



(a) $N = 9$



(b) $N = 99$



(c) $N = 199$

Figure 3: Ex-ante expected utility for different majority rules

Consider what happens when we leave the majority rule unchanged, but second period types are high with probability μ, where μ is itself a random variable. In this case, it is now much more likely that sufficiently many voters agree in the second period to implement the project. Thus, the value of waiting increases, and first period low types become less inclined to implement a project already in the first period (so $\underline{c}$ decreases). From an ex-ante perspective, the additional projects that are now rejected (i.e., projects with costs between the old and the new level of $\underline{c}$) are socially very valuable; since their c is less than $1/4$, from a per-capita perspective, they should be implemented based on their first period payoffs for high types alone (the per-capita surplus of these projects is positive even if all voters turn out to be low types in the second period). Thus, the decrease of $\underline{c}$ is very inefficient. Increasing the majority rule reduces the value of waiting, and thus, more projects are accepted (unanimously) in the first period. This effect outweighs the cost that, with a higher majority rule, slightly more (on average efficient) projects are rejected in the second period.

To gain more insight, consider the case of a very large society ($N \to \infty$). Let $s = m/N$ denote the proportional majority rule. In the second period, the project is implemented if and only if $\mu > s$. Thus, defining $\mathcal{U}_W(c,s) = \lim_{N\to\infty} U_W(c, \lceil sN \rceil, N)$, the value of waiting is

$$\mathcal{U}_W(c,s) = \text{Prob}(\mu \geq s)\left[E(\mu|\mu \geq s) - c\right] = (1-s)\left(\frac{1+s}{2} - c\right). \tag{9}$$

It is easy to check that $\mathcal{U}_W(c,s)$ is maximal for $s = c$. Intuitively, in the second period a utilitarian social planner would accept a project if and only if the realized percentage of winners μ is greater than c.

When are first period low types just indifferent between implementing the project and waiting? Solving $U_I(0,c) = \mathcal{U}_W(c,s)$ yields

$$\underline{c} = \frac{s^2}{2(1+s)} \tag{10}$$

Similarly, high types are indifferent between implementing in period 1 and waiting if

$$\bar{c} = \frac{1+\frac{s^2}{2}}{1+s} \tag{11}$$

Under simple majority rule, projects with costs between $\underline{c}$ and $\bar{c}$ are implemented with probability $1/2$ in the first period. From (10), we have $\underline{c} = 1/12$, and from (11), we have $\bar{c} = 3/4$ under simple majority rule. Thus, expected ex-ante utility under simple majority rule is

$$\begin{aligned}\lim_{N\to\infty} \Pi\left(\frac{N}{2}, N\right) = &\int_0^{1/12} (1-2c)dc + \int_{1/12}^{3/4} \left[\frac{1}{2}(1-2c) + \frac{1}{4}\left(\frac{3}{4} - c\right)\right] dc \\ &+ \int_{3/4}^{1} \frac{1}{2}\left(\frac{3}{4} - c\right) dc = \frac{11}{64} = 0.171875.\end{aligned} \tag{12}$$

Consider now a supermajority percentage $s \equiv m/N \geq 0.5 + \varepsilon$ (for some $\varepsilon > 0$). Since the percentage of high types in the first period is almost certainly within ε of 0.5, if $c > \underline{c}(m, N)$, there are almost never enough high types to implement the project in period 1. Thus, expected ex-ante utility under supermajority rule s is

$$\begin{aligned}\lim_{N\to\infty} \Pi(\lceil sN \rceil, N) &= \int_0^{\underline{c}} (1-2c)dc + \int_{\underline{c}}^1 \left(\frac{1+s}{2} - c\right)(1-s)dc \\ &= (1-\underline{c})\left[\underline{c} + \frac{1-s^2}{2} - \frac{1-s}{2}(1+\underline{c})\right] = \frac{s(2-s)(2+2s-s^2)}{8(1+s)}\end{aligned} \tag{13}$$

where the last line follows from substituting (10). Differentiating (13) with respect to s yields

$$\frac{3s^4 - 4s^3 - 10s^2 + 4s + 4}{8(1+s)^2}$$

Setting this equal to zero and solving yields that the optimal supermajority rule is approximately $s^* = 0.7985$.[18] Substituting the optimal value into Π yields an expected utility of about 0.1973, which is larger than the ex-ante utility under simple majority rule. Therefore, a supermajority rule of approximately 80% is optimal in the limit, which corresponds very well to the maximum in the graphs of Figure 3.

4.2 Intertemporal correlation

We now consider the case that each voter's first and second period type are positively correlated. Specifically, we assume that a voter's second period type coincides with his first period type with probability $r \in [0.5, 1]$, i.e. $Prob(V_2^i = 1 | V_1^i = 1) = Prob(V_2^i = 0 | V_1^i = 0) = r$. We continue to assume that each player has an equal chance of being a high or a low type in the first period.

Benchmark: No option to wait. As in the basic model, we start by considering the case of a one-off election in period 1. In our new setting, the net present value of a project with cost parameter c for voter i, is

$$U_I(V_1^i, c, r) = V_1^i + E[V_2^i | V_1^i] - 2c = \begin{cases} 1 + r - 2c & \text{if } V_1^i = 1 \\ 1 - r - 2c & \text{if } V_1^i = 0. \end{cases} \tag{14}$$

For example, a first period high type gets an immediate payoff of 1, and is a second period high type with probability r. A first period low type gets zero in the first period, and is a second period high type with probability $1 - r$.

If the first period decision is final, high types vote in favor of projects with $c \leq (1 + r)/2$, while low types only vote in favor of projects with $c \leq (1 - r)/2$. The

[18] The second order condition is satisfied at s^*.

stronger is the correlation across periods (i.e., the higher is r), the more extreme are these cost thresholds, because low types have only a very slight hope that they will profit from the project in period 2, while high types are very confident that they will remain high types. Essentially, the higher is r, the more the social decision problem resembles a situation with known benefits.

The ex-ante expected payoff of a project with cost c, given r and majority rule m, is

$$\tilde{\pi}(m,c,r) = \begin{cases} 1-2c & \text{if } c \leq \frac{1-r}{2} \\ q(m)(1-2c) + \frac{p(m)}{2}(1+r-2c) & \text{if } \frac{1-r}{2} < c \leq \frac{1+r}{2} \\ 0 & \text{if } c > \frac{1+r}{2} \end{cases} \tag{15}$$

Just as in the basic model, the ex ante equilibrium payoff is a piecewise linear function of c that exhibits a downward jump at $(1-r)/2$ and an upward jump at $(1+r)/2$.

Following the same arguments as in the proof of Claim 1, we can show that the optimal majority rule for a given c is

$$m^* = \lceil N(2c-(1-r))/2r \rceil,$$

which is a decreasing function of r. Thus, correlation strengthens the case for low majority requirements, and it is therefore intuitively clear that the result of Proposition 2 also holds here: If the project decision cannot be postponed and costs are drawn from some distribution that is symmetric around $1/2$, then simple majority yields the highest average ex-ante payoffs.

The option to wait and intertemporal correlation. We now turn to the case that society can implement the project in period 2, if it was turned down in period 1. In order to characterize first period voting behavior, we need to find the value of waiting. In contrast to the basic model, it matters here whether individuals can observe the payoff types of other voters. The number of first period high types influences the distribution of the number of second period high types, and thus the probability of implementation in period 2. Thus, if voters can observe the first period types of other voters, they will condition their behavior on it. If, instead, types are only privately observed, then each voter has to take into account the first period type distribution conditional on the event that his vote is decisive in the first period election.

In what follows, we assume that payoff types are publicly observed. We make this assumption for two reasons. First, this assumption is probably reasonable for applications with small electorates. Second, a model with publicly observed types is more tractable than a model with privately observed payoff types. In particular, with publicly observable payoff types, iterated elimination of weakly dominated strategies still delivers a unique strategy profile (up to tie breaking in situations where individuals are indifferent between their two first period actions, independently of other voters' behavior). The same is not true for a model with privately observed types, where the voting game may exhibit multiple (sequential) equilibria in iteratively weakly undominated strategies. A sufficiently detailed exposition of such a game would considerably increase the length of the paper. Moreover, in qualitative terms, the interesting results do not change substantially from the case considered here.[19]

We start our analysis by defining voter i's value of waiting, which now depends not only on the cost parameter c and the majority rule m, but also on the intertemporal correlation parameter r, voter i's first period type and the number of high types among other voters, h, which determines the distribution of the number of second period high types and hence the probability of second period implementation. Formally, we have

$$\begin{aligned} U_W(V_1^i, c, m, h, r) &= E[\max\{V_2^i - c, 0\}|V_1^i] p_2(m,h,r) + (E[V_2^i|V_1^i] - c) q_2(m,h,r) \\ &= \begin{cases} (1-c)(1-r)p_2(m,h,r) + (1-r-c)q_2(m,h,r) & \text{if } V_1^i = 1 \\ (1-c)rp_2(m,h,r) + (r-c)q_2(m,h,r) & \text{if } V_1^i = 0. \end{cases} \end{aligned} \tag{16}$$

The functions p_2 and q_2 are generalizations of the corresponding functions in the basic model and represent the probability of being pivotal in period two, and the probability that the project will pass in period 2 independently of voter i's will, respectively. In order to formally define p_2 and q_2, consider the transition function

$$t(\ell, k, r) = \sum_{j=0}^{k} \binom{\ell}{j} \binom{N-1-\ell}{k-j} (1-r)^{k+l-2j} r^{N-\ell-1-k+2j},$$

which describes the probability of moving from a first period type profile in which ℓ of the $N-1$ other players have high types to a second period profile in which k of them have high types. The functions p_2 and q_2 can now be written as

$$p_2(m,h,r) = t(h, m-1, r) \quad \text{and} \quad q_2(m,h,r) = \sum_{j=m}^{N-1} t(h,j,r).$$

Just as in the basic model, both (14) and (16) are linearly decreasing functions of c. Moreover, (14) decreases faster than (16). It is also obvious that, for c sufficiently close to 0, implementing the project immediately is strictly better than waiting, irrespective of the values of V_1^i, h, m and r. Similarly, if c is sufficiently close to 1, then delaying the project dominates investing immediately for all parameter values. Thus, for each triple (h,m,r), there are cost thresholds $\underline{c}(h,m,r)$ and $\bar{c}(h,m,r)$ at which low and high types switch from approval to rejection, respectively. Since

$$U_W(0, \frac{1-r}{2}, m, h, r) = \frac{(1-r)}{2}((1+r)p_2(m,h,r) + q_2(m,h,r)) > 0 = U_I(0, \frac{1-r}{2}, r),$$

[19] A formal analysis of the voting game with privately observed types is available from the authors upon request.

it follows that $\underline{c}(m,h,r) < (1-r)/2$ for all (m,h,r). Similarly, for high types we have

$$U_W(1,m,r,h,r) = (1-r)rp_2(m,l,r) < 1-r = U_I(1,r,r),$$

which implies that $\bar{c}(m,h,r) > r$ for all (m,h,r). Results that parallel Proposition 3 for the behavior of $\underline{c}$ and $\bar{c}$ can be obtained, but, in order to save some space, we refrain from presenting them explicitly.

Turning to the definition of the ex-ante expected payoff, it is convenient to calculate this in two steps: First, we integrate over a player's payoff type, given that h of other players are high types. Let $C^0_{m,h,r} = \{c|c \leq \underline{c}(m,h,r)\}$, $C^1_{m,h,r} = \{c|c \in (\underline{c}(m,h,r), \bar{c}(m,h,r)]\}$, and $C^2_{m,h,r} = \{c|c > \bar{c}(m,h,r)\}$. For any triple (c,m,r), voter i's expected payoff, conditional on there being h high types among the other voters, is

$$\pi_h(m,N,c,r) = \begin{cases} \dfrac{U_I(1,c,r)+U_I(0,c,r)}{2} & \text{if } c \in C^0_{m,h,r} \vee (c \in C^1_{m,h,r} \wedge h \geq m) \\ \dfrac{U_I(1,c,r)+U_W(0,m,c,h,r)}{2} & \text{if } c \in C^1_{m,h,r} \wedge h = m-1 \\ \dfrac{U_W(1,m,c,h,r)+U_W(0,m,c,h,r)}{2} & \text{if } c \in C^2_{m,h,r} \vee (c \in C^2_{m,h,r} \wedge h < m-1) \end{cases}$$

Second, we now take the expectation with respect to h, which gives

$$\pi(m,N,c,r) = \frac{1}{2^{N-1}} \sum_{h=0}^{N-1} \binom{N-1}{h} \pi_h(m,N,c,r).$$

As in the basic model, this function a piecewise linear function of c. Of course, $\pi(m,c,r)$ exhibits more than just two discontinuities, since both high and low type voters have multiple thresholds (one for each h) at which behavior switches. Figure 4 shows for the case that $r = 2/3$, the expected payoffs for $N = 15$ under the two majority rules $m = 8$ and $m = 9$.

One of the central results of the basic model was that, with the option to wait, simple majority is dominated by all supermajority rules. In comparison to the basic model, intertemporal correlation strengthens the case for simple majority rule. This is quite clear for $r = 1$, because the dynamic structure of our model then becomes irrelevant: If individuals' benefits are constant over time, then the set of voters who approve remains constant, and hence, a project is either implemented at once or never. This is exactly the same behavior as in the benchmark model without the option to wait, where we know that simple majority maximizes the ex-ante average payoff.

It is intuitive that, as we increase r from $1/2$ (i.e., the basic model) to $r = 1$ (i.e., perfect correlation), the optimal supermajority rule decreases. It is an interesting quantitative question to consider for which levels of r the optimal supermajority rule switches.

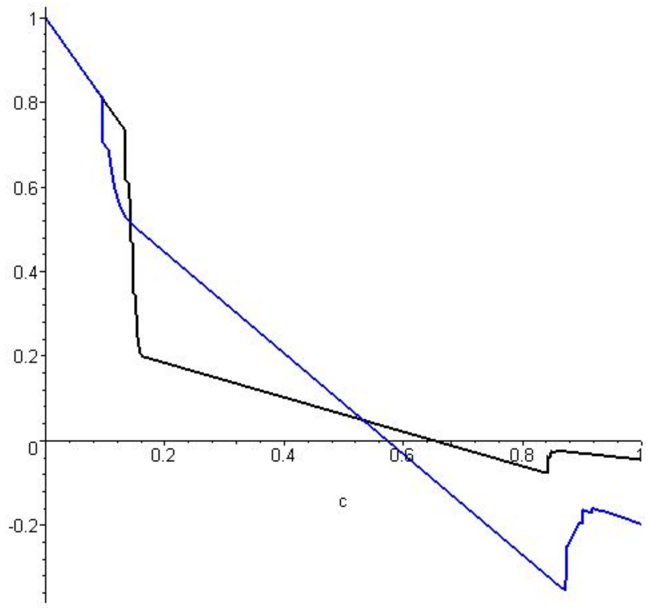


Figure 4: Ex-ante payoff π as function of c: $N = 15$, $r = 2/3$, $m = 8$ (blue) and $m = 9$ (black)

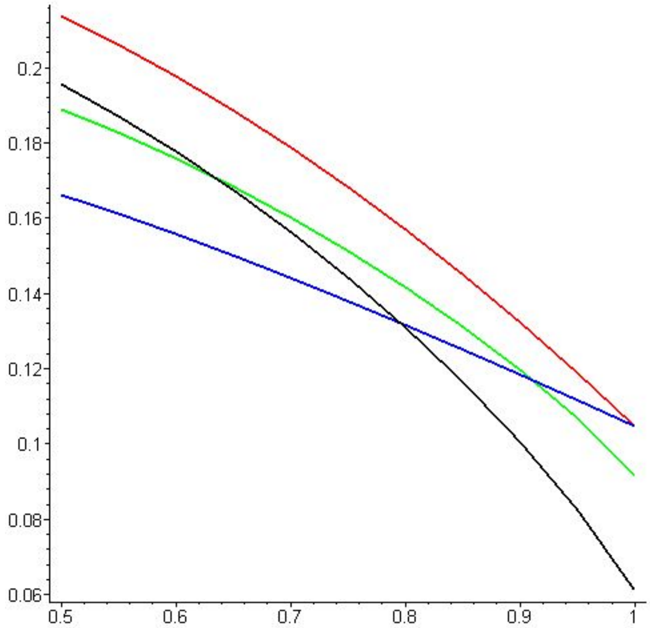


Figure 5: Π as function of r for $N = 15$, $m = 8$ (blue),$m = 9$ (green),$m = 10$ (black); and $\tilde{\Pi}$ for $m = 8$ (red)

Figure 5 shows, for $N = 15$, the average ex-ante payoff $\Pi(m, N, r) = \int_0^1 \pi(m, c, r)$ as a function of r for different majority rules. Consider first the three lower curves that intersect each other. The blue curve in Figure 5, the flattest of the three, represents the payoffs under simple majority, i.e. $m = 8$, while the green and the black one (the steepest one) show the payoffs under the supermajority rules $m = 9$ and $m = 10$, respectively. A two-thirds majority rule ($m = 10$) is the optimal majority rule in the basic model, and remains optimal up to approximately $r = 0.64$. For r between about 0.64 and 0.92, $m = 9$ (i.e. a 60% rule) is the optimal majority rule, and only for higher levels of correlation, simple majority rule is optimal. Thus, while intertemporal correlation decreases the optimal majority rule, our qualitative result concerning the optimality of supermajority rules is very robust even at high levels of correlation.[20]

The highest curve (red) shows the ex-ante expected payoff without the option of waiting. From our discussion in the basic model, we know already that, for $r = 0.5$ this payoff is higher than the payoff with the option to wait, even under the optimal supermajority rule. Figure 4 shows that this result continues to hold for $r > 1/2$, except at $r = 1$ where the difference between the payoffs with and without the option to wait becomes zero. This is intuitive, as with perfect correlation, a project is either implemented immediately or not at all, so payoffs are the same, whether or not second period implementation is, in principle, possible.

5 Previous literature on supermajority rules

Our results shed light on an important question in the endogenous determination of institutions: Why do some organizations choose supermajority rules, and which features of decision problems influence this choice? In this section, we relate our model to previous literature on this subject.

Majority rules within organizations vary considerably, from simple majority rule to unanimity rule. Often, the choice of the majority rule that is to govern future decision making is a contentious issue itself, such as in the recent EU summit, which, in the end, adopted some form of a supermajority rule. Supermajority rules are also used in most countries for a change of the constitution, and, often implicitly, for "normal" legislation. For example, in parliamentary systems with a strong committee organization, a legislative proposal usually needs the support of *both* the respective committee and the house. In parliamentary systems with two chambers, certain legislative proposals need the support of both chambers.[21]

[20] This result is further strengthened by the fact that $N = 15$ is relatively small in our example, so that even a simple majority rule requires the approval of $8/15 = 53.3\%$ of the population, and the smallest possible supermajority rule is already a 60% rule.

[21] Tullock (1998), p.216, estimates that legislative rules in the US for changing the status quo are

Several previous papers have analyzed arguments for supermajority rules from an economic point of view. Buchanan and Tullock (1962) argue for unanimity rule as the suitable rule governing social choices. Under a simple majority rule, a majority of people could be tempted to implement certain projects that are not socially desirable because they can "externalize" part of the cost associated with this project to the losing minority. Under unanimity rule, only Pareto improving projects are implemented. However, Guttman (1998) has argued that the unanimity rule leads to a rejection of many projects that are not Pareto improvements, but nevertheless worthwhile from a reasonable social point of view. Assuming that the social goal is to minimize the sum of both types of mistakes, he shows that simple majority rule is optimal. Our model is constructed in a way that simple majority rule would also be optimal if voters have to make a once-and-for-all decision about the project in the first period. However, with the option to postpone a decision to the second period, we show that (in the same symmetric setting), a supermajority rule is optimal.

Another rationale for supermajority rules is that this counteracts the problem of time inconsistency of optimal policies (see, e.g., Gradstein (1999) or Dal Bo (2006)). For example, a constitution that protects investment by inhibiting nationalization is valuable only if the constitution cannot be easily changed after investment has taken place. In our model, time inconsistency is not an issue.

As is well known, simple majority rule may lead to cycles in electoral preferences. A higher required majority reduces the possibility of cycles. Indeed, Caplin and Nalebuff (1988) show that a $(1-(n/(n+1))^n)$ supermajority rules out cycles, if voters have single peaked Euclidean preferences in an n–dimensional space. In our model, the decision is binary in each period, so cycles never arise in our model.

Aghion and Bolton (2003) analyze the optimal choice of the majority rules in a model where a polity has to decide simultaneously about public good provision and costly redistribution, and redistribution creates a deadweight loss. They assume that the constitutional rules are written before voters learn the costs and benefits of the public good. The optimal majority rule trades off the higher ex-post flexibility of low majority rules (which lead to more efficient public good provision) against the protection against excessive redistribution afforded by supermajority rules.

Eraslan and Merlo (2002) analyze an advantage of unanimity rule over all other majority rules in a model of bargaining with stochastic surplus. Under any majority rule requiring less than unanimity, the proposer and voters he selected into the minimum winning coalition have to fear that they might not be a part of a winning coalition in the future if no agreement is reached today. Therefore, agreement may be reached too

"roughly equivalent to requiring a 60% majority in a single house elected by proportional representation." See also Diermeier and Myerson (1999).

early from a social point of view (i.e., for a too small surplus).

Messner and Polborn (2004) analyze an overlapping generations model in which voters know that their preferences over reform projects will become more conservative over the remainder of their lives. The initial population decides on the majority rule to be used for later decisions. The median voter in the constitutional election prefers to implement a supermajority rule, which allows him to transfer power to his (more conservative) "average future self". In contrast to Messner and Polborn (2004), the electorate remains constant over time in our model, thus removing the incentive for the initial generation to use supermajority rules in order to transfer power from future voters to themselves. Also, the implementation decision on any reform project in Messner and Polborn (2004) is a simple one-time, up-or-down vote, while our focus here is on the timing of the implementation of reforms.

The implications of different majority rules have also been analyzed in settings where voters have congruent interests, but are only imperfectly informed about the consequences of the different alternatives. Inspired by Condorcet's famous Jury Theorem,[22] several authors have analyzed which majority rule is most efficient in aggregating the information that is dispersed in the electorate. Nitzan and Paroush (1985) find that the probability of a correct choice is maximized under simple majority rule. Feddersen and Pesendorfer (1998) analyze information aggregation with strategic voters and show that simple majority rule is optimal for information aggregation purposes, while unanimity rule is dominated by all other majority rules, if there are sufficiently many voters. Bond and Eraslan (2007) show that supermajority or unanimity rules may have an advantage for voters even in an information aggregation setting, if the proposal is made by an agenda setter whose interest is diametrically opposed to the voters. The intuition is that unanimity rule forces the agenda setter to make a more favorable proposal to voters, and this outweighs the disadvantage that, with positive probability, voters make mistakes under unanimity rule. The issue of information aggregation is not present in our model as all voters hold, at all times, the same information.

6 Discussion and conclusions

We analyze a model in which voters have to choose whether to implement a project immediately, or wait till the second period and reconsider the decision then. Our main focus is the interplay of *individual* learning and *social* decisions, and how this is influenced by the majority rule that governs the decision making process. We characterize

[22]Condorcet's Jury Theorem states that under simple majority rule, the probability with which a society facing a binary choice problem makes the correct choice, converges to one as the number of voters increases.

how the majority rule influences the value of waiting and individual voting behavior.

Our main conclusions were twofold: First, we show that, when society has the option to wait, the optimal majority rule increases relative to the case where postponing is not possible. For example, for a uniform project type distribution, the optimal majority rule changes from simple majority (without the option to wait) to a supermajority rule that is larger than $7/11 \approx 63.6$ percent, and, for a large society, also not larger than a two-thirds majority rule. Correlation between first and second period valuations reduces the size of the optimal majority rule. Perfect correlation re-establishes simple majority rule as the optimal rule; however, even if correlation is high, but not perfect, the optimal majority rule remains a supermajority rule.

Our second main result was that the option of waiting, which is always positive for individual decision problems, can be negative for our social decision problem. Indeed, we show that this is the case when the project cost is uniformly distributed from an ex-ante perspective, even if society adopts the optimal majority rule in the case that they have the option to wait.

Our model provides a fundamentally new rationale for societies choosing supermajority rules. Our principal effect in our model relies on voters' uncertainty over the consequences of project implementation, and the option value of waiting until new information is available. Thus, our model is most relevant for societies that frequently face decision problems with such characteristics.

For example, one can argue that the European Union fits this description quite well. The most important decisions that are made in the EU framework concern the admission of new members, transnational investment projects like the introduction of the Euro and the harmonization of industry regulations. One can argue that many of these projects are less "standard" (relative to the most important policy issues in the member states) and have uncertain payoff consequences for the member states. Interestingly, the European Council (the council of member state governments that makes the most significant decisions) uses a supermajority rule.

Also, most countries require supermajorities for changes of their constitution. Again, this area appears closer to the setting of this model than ordinary legislation issues: At the time when the constitution is written, future needs are difficult to foresee and potential winners and losers are unclear, and even once a proposal arises, the consequences of changes for the distribution of gains and losses are not necessarily clear.

In contrast, most ordinary legislation in national legislatures concerns social or economic issues where preferences are more stable and well-known. As we have seen in Section 4.2, the higher the correlation of voter types over time (and therefore, the smaller the opportunity of learning), the closer is the ex-ante optimal majority rule to simple majority rule.

One direction in which future research can expand on our model framework is as follows. In our model, individuals only choose how to vote. In some instances, individuals may also be able to adapt to the policy enacted and thereby influence the distribution of their payoff in the second period. This may be important, for example, in issues where the project is some sort of environmental regulation, say, increasing the private cost of some polluting activity. Adaptation (say, buying a smaller car, isolating one's home) may make compliance less costly over time, but the enacted policy (as well as the expectation of which regulation will be in force in the next period) will affect the optimal extent to which individuals adapt.

7 Appendix

Proof of Proposition 2. Observe that $\tilde{\Pi}(m) =$

$$
\begin{aligned}
& \int_0^{1/4}(1-2c)dF(c) + q(m)\int_{1/4}^{3/4}(1-2c)dF(c) + \frac{p(m)}{2}\int_{1/4}^{3/4}\left(\frac{3}{2}-2c\right)dF(c) \\
= & \int_0^{1/4}(1-2c)dF(c) + \left(q(m)+\frac{p(m)}{2}\right)\int_{1/4}^{3/4}(1-2c)dF(c) + \frac{p(m)}{2}\frac{F(3/4)-F(1/4)}{2} \\
= & \int_0^{1/4}(1-2c)dF(c) + x[q(m)+p(m)/2] + yp(m)/2
\end{aligned}
$$

We thus have

$$
\begin{aligned}
& \tilde{\Pi}(m+1) - \tilde{\Pi}(m) \\
= & \; x\left[q(m+1)+\frac{p(m+1)}{2}-q(m)-\frac{p(m)}{2}\right] + y\frac{p(m+1)-p(m)}{2} \\
= & \; -x[p(m+1)+p(m)]/2 + y[p(m+1)-p(m)]/2.
\end{aligned}
$$

It is straightforward to show that this increment is positive if and only if

$$
p(m+1)/p(m) = \frac{N-m}{m} \geq \frac{y+x}{y-x},
$$

or equivalently iff

$$
m \leq \frac{(y-x)N}{2y}.
$$

Hence the increment changes sign at most once. If $x \geq -y/N < 0$ then $(y-x)N/2y \leq (N+1)/2$ and thus in this case $\tilde{\Pi}(\cdot)$ is decreasing over the whole range of admissible majority rules. On the other hand, if $x < -y/N$, then $\tilde{\Pi}(\cdot)$ is increasing at low majority rules and decreasing at high majority rules. That is, unless $(y-x)N/2y$ happens to be an integer, $\tilde{\Pi}$, is single peaked in m, with its peak being $\lceil (y-x)N/2y \rceil$. If instead $(y-x)N/2y$ is an integer, then $\tilde{\Pi}$ is 'single-plateaued' where the plateau is given by the two points $m^* = (y-x)N/2y$ and $m^{**} = (y-x)N/2y+1$. $\square$

Lemma A1. *For all N and $(N+1)/2 \leq m \leq N$, $\dfrac{q(m,N)}{p(m,N)} \leq \dfrac{N-m}{2m-N}$.*

Proof. Fix N and m and let $s = m/N$. Observe that

$$
\begin{aligned}
\frac{q(m)}{p(m)} &= \sum_{\ell=m+1}^{N}\frac{p(\ell)}{p(m)} = \frac{\sum_{\ell=m}^{N-1}\binom{N-1}{\ell}}{\binom{N-1}{m-1}} = \sum_{\ell=m}^{N-1}\frac{(m-1)!(N-m)!}{\ell!(N-1-\ell)!} \\
&= \sum_{\ell=m}^{N-1}\frac{(N-\ell)\cdots(N-m)}{m\cdots\ell} = \sum_{\ell=m}^{N-1}\prod_{k=m}^{\ell}\frac{N-k}{k}. \qquad (17)
\end{aligned}
$$

Since $(N-k)/k \leq (N-m)/m = (1-s)/s$ for all $k \geq m$ it follows that the right hand side of (17) is smaller than

$$\sum_{\ell=m}^{N-1} \left(\frac{1-s}{s}\right)^{\ell-m+1} = \sum_{\ell=0}^{N-m-1} \left(\frac{1-s}{s}\right)^{\ell+1} \leq$$
$$\frac{1-s}{s} \sum_{\ell=0}^{\infty} \left(\frac{1-s}{s}\right)^{\ell} = \frac{1-s}{2s-1} = \frac{N-m}{2m-N}. \qquad \square$$

Proof of Lemma 1. Observe that $\frac{p(m+1)}{q(m+1)} \geq \frac{p(m)}{q(m)}$ if and only if

$$\frac{p(m)}{p(m+1)} \leq \frac{q(m)}{q(m+1)} = \frac{q(m+1)+p(m+1)}{q(m+1)} = \frac{p(m+1)}{q(m+1)} + 1. \tag{18}$$

Rearranging and using $p(m)/p(m+1) = m/(N-m)$, this becomes

$$\frac{N-m}{2m-N} \geq \frac{q(m+1)}{p(m+1)}. \tag{19}$$

By Lemma A1 we have that $\dfrac{q(m+1)}{p(m+1)} \leq \dfrac{N-(m+1)}{2(m+1)-N}$. Thus, since $\frac{N-m}{2m-N} > \frac{N-(m+1)}{2(m+1)-N}$, (19) is always satisfied, so that $\frac{p(m)}{q(m)}$ is increasing in m. $\square$

Proof of Lemma 2. Consider the difference $U_W(c, m+1) - U_W(c, m) =$

$$\begin{aligned} &\left(\frac{1}{2} - c\right)(q(m+1) - q(m)) + \frac{1-c}{2}(p(m+1) - p(m)) \\ =\; &\frac{1-c}{2}[p(m+1) - p(m)] - \left(\frac{1}{2} - c\right) p(m+1), \end{aligned} \tag{20}$$

where the second line results from substituting the identity $q(m) = p(m+1) + q(m+1)$. The expression in (20) is positive if and only if

$$c \geq \frac{p(m)}{p(m)+p(m+1)} = \frac{\binom{N-1}{m-1}}{\left[\binom{N-1}{m-1} + \binom{N-1}{m}\right]} = \frac{\binom{N-1}{m-1}}{\binom{N}{m}} = \frac{m}{N}. \qquad \square$$

Lemma A2. *$Np(\lceil 3N/4 \rceil, N) \leq 4$ for all N.*

Proof. Given that N is an odd number we have that either $3N+1$ is divisible by 4 (for $N = 5, 9, 13, \ldots$) or $3(N+1)/4$ is an integer (for $N = 3, 7, 11, \ldots$). In the first case we have that $\lceil 3N/4 \rceil = (3N+1)/4$, while in the latter case we have $\lceil 3N/4 \rceil = 3(N+1)/4$. Notice also that in either case we have $\lceil 3(N+4)/4 \rceil - \lceil 3N/4 \rceil = 3$.

Let $f(N) = Np(\lceil 3N/4 \rceil, N)$. In what follows we will show that $f(N+4) - f(N) < 0$. This is suffcient for proving our statement since it implies that $\max_N f(N) = \max\{f(3), f(5)\} = \max\{3\binom{2}{2}/2^2, 5\binom{4}{3}/2^4\} = 5/4 < 4$.

Letting $m = \lceil 3N/4 \rceil$ the increment $f(N+4) - f(N)$ is given by

$$\frac{N+4}{2^{N+3}}\binom{N+3}{m+2} - \frac{N}{2^{N-1}}\binom{N-1}{m-1} = \frac{N}{2^{N-1}}\binom{N-1}{m-1}\left[\frac{(N+4)(N+3)(N+2)(N+1)}{16(N+1-m)(m+2)(m+1)m} - 1\right].$$

Observe that $(N+j+2) < 4(m+j)$ for $j = 0, 1, 2$ and that $N+1 \leq 4(N+1-m)$. Thus the first term in the square brackets is a product of four numbers which are smaller than one, and thus the term in square brackets is negative. □

Proof of Proposition 3. We start by showing that the two thresholds lie within the claimed bounds. Consider first the threshold value for low payoff types, $\underline{c}(m)$. If $c = 1/4$, then the payoff of an immediate implementation of the project for low type voters is given by substituting in (1), $U_I(0, 1/4) = E[V_2] - 2 \cdot (1/4) = 1/2 - 1/2 = 0$. The payoff from waiting is $U_W(1/4, m) = (3/8)p(m) + (1/4)q(m)$, which is positive for all m. Therefore, low type voters always reject the project in period one if $c \geq 1/4$, i.e. $\underline{c}(m) < 1/4$.

At $c = 1/12$, we have $U_I(0, 1/12) = E[V_2] - 2 \cdot (1/12) = 1/3$, and $U_W(1/12, m) = (11/24)p(m) + (5/12)q(m, N) = (p(m)/2 + q(m))(5/12) + p(m)/4$. Given that both $p(m)/2+q(m)$ and $p(m)$ each cannot be larger than $1/2$, it follows that $U_W(1/12, m) < 5/24 + 1/8 = 1/3$. Hence, a low type voter strictly favors immediate implementation of a project with $c = 1/12$. Thus, $1/12 \leq \underline{c}(m) < 1/4$

As for the threshold $\bar{c}(m)$, notice that implementing the project right away for $c = 2/3$ and $c = 5/6$ yields to high type voters a payoff of $U_I(1, 2/3) = 3/2 - 2(2/3) = 1/6$, and $U_I(1, 5/6) = 3/2 - 2(5/6) = -1/6$, respectively. The corresponding expected payoffs from waiting are $U_W(2/3, m) = (1 - 2/3)p(m)/2 + (1/2 - 2/3)q(m) = (p(m) + q(m))/6 < 1/6$ and $U_W(5/6, m) = (1 - 5/6)p(m)/2 + (1/2 - 5/6)q(m) = p(m)/12 - q(m)/3 > -q(m)/3 > -1/6$. Thus, if $c \leq 2/3$, high types always favor immediate implementation, and if $c \geq 5/6$, they always reject immediate implementation. Thus, $\bar{c}(m) \in (2/3, 5/6)$.

Remember that the threshold $\bar{c}(m)$ solves the equation

$$\frac{3}{2} - 2c = \frac{1-c}{2}p(m) + \left(\frac{1}{2} - c\right) q(m).$$

Thus, we have

$$\bar{c}(m) = \frac{3 - q(m) - p(m)}{4 - 2q(m) - p(m)}.$$

Dropping the variables from the functions p and q and denoting their values at $m+1$ by p' and q' respectively we have

$$\bar{c}(m) \geq \bar{c}(m+1) \quad \Leftrightarrow \quad \frac{4-2q-p}{3-q-p} = 1 + \frac{1-q}{3-q-p} \leq 1 + \frac{1-q'}{3-q'-p'} = \frac{4-2q'-p'}{3-q'-p'}$$

which may be written equivalently as

$$\frac{3-q-p}{1-q} = 1 + \frac{2-p}{1-q} \geq 1 + \frac{2-p'}{1-q'} = \frac{3-q'-p'}{1-q'}.$$

Using $q' = q - p'$ and $p'/p = (N-m)/m$ we can express this condition as follows

$$\begin{aligned}(2-p)(1-q+p') - (2-p')(1-q) &= p\left(\frac{3p'}{p} - 1 + q - \frac{p'}{p}p + \frac{p'}{p}q\right) \\ &= p\left(\frac{3N-4m}{m} + q\frac{N}{m} - \frac{N-m}{m}p\right) \geq 0.\end{aligned}$$

Hence, it follows that $\bar{c}$ is decreasing at m if and only if

$$3N - 4m + Nq - (N-m)p \geq 0. \tag{21}$$

Now observe that, since $q > p'$ and $p'/p = (N-m)/m$, we have that

$$Nq - (N-m)p > Np' - (N-m)p = Np\frac{p'}{p} - (N-m)p = (N-m)p\left(\frac{N}{m} - 1\right) \geq 0.$$

Since $3N - 4m > 0$ for all $m < 3N/4$, we can therefore conclude that $\bar{c}$ must be decreasing at least up to $m = \lfloor 3N/4 \rfloor$.

Next we argue that if (21) is satisfied at $m < N$ then it must also hold at $m' = m+1$. To see this, evaluate (21) at m and at $m+1$, and observe that the difference is

$$\begin{aligned}3N - 4m + Nq - (N-m)p - \left[3N - 4(m+1) + Nq' - (N-m-1)p'\right] = \\ 4 + p'\left[2(N-m) - 1\right] > 0.\end{aligned}$$

It follows that $\bar{c}$ is increasing from $m = \lceil 3N/4 \rceil$ onwards if it is so at $m = \lceil 3N/4 \rceil$. Notice that at $m = \lceil 3N/4 \rceil$ we have $4m - 3N \geq 1$ and so we only have to show that $Nq(\lceil 3N/4 \rceil) - (N - \lceil 3N/4 \rceil)p(\lceil 3N/4 \rceil) < 1$.

From Lemma A1, we know that $p \geq q(2m-N)/(N-m)$, and thus

$$\begin{aligned}&Nq(\lceil 3N/4 \rceil, N) - (N - \lceil 3N/4 \rceil)p(\lceil 3N/4 \rceil, N) \\ &\leq 2(N - \lceil 3N/4 \rceil)q(\lceil 3N/4 \rceil, N) \leq \frac{N}{2}q(\lceil 3N/4 \rceil, N) \leq \frac{N}{4}p(\lceil 3N/4 \rceil, N) < 1.\end{aligned}$$

where Lemma A1 is used both for the first and the third inequality, and the last inequality follows from Lemma A2.

Finally, observe that since $q((N+1)/2) + p((N+1)/2)/2 = 1/2$, $p((N+1)/2) \leq 1/2$ and $q(N) = 0$ we have

$$\begin{aligned}\bar{c}((N+1)/2) &= \frac{5 - p((N+1)/2)}{6} \geq \frac{3}{4} \quad \text{and} \\ \bar{c}(N) &= \frac{3 - p(N)}{4 - p(N)} < \frac{3}{4}.\end{aligned}$$

□

Proof of Proposition 5. In the limit of $N \to \infty$, a voter's ex-ante expected utility for simple majority rule and without the option to wait is

$$\lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N) = \int_0^{1/4} (1-2c)dF(c) + \frac{1}{2}\int_{1/4}^{3/4} (1-2c)dF(c), \tag{22}$$

because all projects with $c \leq 1/4$ are implemented unanimously, and those with $c \in (1/4, 3/4]$ are implemented with probability $1/2$.

A voter's ex-ante expected utility for a proportional supermajority rule s, with $s > 1/2$, and without the option to wait, is

$$\lim_{N\to\infty} \tilde{\Pi}(\lceil sN \rceil, N) = \int_0^{1/4} (1-2c)dF(c) \tag{23}$$

because all projects with $c \leq 1/4$ are implemented unanimously, and those with $c > 1/4$ are (almost) never implemented, because they are only supported by high types, and the proportion of high types is almost surely less than s by the law of large numbers.

Similarly, when waiting is possible, a voter's ex-ante expected utility for a proportional supermajority rule s, with $s > 1/2$, is $\lim_{N\to\infty} \Pi(\lceil sN \rceil, N) = \lim_{N\to\infty} \tilde{\Pi}(\lceil sN \rceil, N)$. The reason is that, even when society can reconsider the decision in the second period, the proportion of high types in the second period is (almost) never sufficient for implementation.

Last, consider the ex-ante expected utility for simple majority rule and with the option to wait. In the second period, all projects are implemented with probability $1/2$, and so, for given c, the value of waiting is $U_{W,1/2} = \frac{1}{2}\left(\frac{1}{2} - c\right)$. Thus, in the first period, low types agree to implement if and only if $\frac{1}{2} - 2c \geq \frac{1}{2}\left(\frac{1}{2} - c\right)$, hence whenever $c \leq 1/6$; similarly, first period high types vote for immediate implementation whenever $c \leq 5/6$. Thus, if $c \leq 1/6$, the project is implemented in the first period. Projects with $c \in (1/6, 5/6]$ are implemented with probability $1/2$ in the first period, and projects with $c > 5/6$ are not implemented in the first period. In the second period, all projects that were not implemented in the first period are implemented with probability $1/2$. A voter's ex-ante expected utility under simple majority rule with the option to wait is thus

$$\begin{aligned}\lim_{N\to\infty} \Pi(\lceil N/2 \rceil, N) = &\int_0^{1/6} (1-2c)dF(c) + \frac{1}{2}\int_{1/6}^{5/6} (1-2c)dF \\ &+ \frac{1}{4}\int_{1/6}^{5/6} \left(\frac{1}{2} - c\right) dF(c) + \frac{1}{2}\int_{5/6}^{1} \left(\frac{1}{2} - c\right) dF(c)\end{aligned} \tag{24}$$

Suppose the claim in the proposition is false for some distribution F. Since expected utility under any supermajority rule, and with or without the option to wait, is equal to $\int_0^{1/4}(1-2c)dF(c)$, a contradiction to the claim can only arise if there exists $\tilde{s}^*(F) \in \tilde{S}^*$, with $\tilde{s}^*(F) > 1/2$, and $s^*(F) = 1/2 \in S^*(F)$. In this case,

$$\lim_{N\to\infty} \Pi(\lceil N/2 \rceil, N) \geq \lim_{N\to\infty} \Pi(\lceil \tilde{s}^*(F)N \rceil, N) = \lim_{N\to\infty} \tilde{\Pi}(\lceil \tilde{s}^*(F)N \rceil, N) \geq \lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N) \tag{25}$$

where the inequality signs follow from the optimality of $s^*(F)$ and $\tilde{s}^*(F)$, respectively, and the equality sign follows from $\lim_{N\to\infty} \tilde{\Pi}(\lceil sN \rceil, N) = \lim_{N\to\infty} \Pi(\lceil sN \rceil, N)$ for all $s > 1/2$. The last inequality in (25) implies that

$$\int_{1/4}^{3/4} (1-2c)dF(c) \leq 0. \tag{26}$$

Furthermore, note that $\lim_{N\to\infty} \Pi(\lceil N/2 \rceil, N) - \lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N) =$

$$-\frac{1}{4}\int_{1/6}^{1/4}(1-2c)dF(c)+\frac{1}{4}\int_{1/4}^{3/4}(1-2c)dF(c)+\frac{3}{4}\int_{3/4}^{5/6}(1-2c)dF(c)+\frac{1}{2}\int_{5/6}^{1}(1-2c)dF(c) \tag{27}$$

must be greater or equal to zero by (25). However, this inequality cannot hold, as all terms are nonpositive (the second term is nonpositive by (26)), and either the first, the third or the fourth term are strictly negative, by the assumption on F. This provides the desired contradiction.

Note that, while our assumption on F is not necessary for the Proposition to hold, some assumption is required. To see this, suppose that c has a one-point distribution with all mass on $c = 1/2$. In this case, without the option to wait, all majority rules yield the same ex-ante expected surplus of 0. Also, with the option to wait, all majority rules yield an ex-ante surplus of 0. Thus, for this example, $\sup \tilde{S}^* = 1 > \inf S^* = 1/2$. $\square$

Proof of Proposition 6. In the interest of a compact notation in what follows we write $m' = m + 1$, $q' = q(m')$ and $p' = p(m')$; the probabilities which refer to the majority rule m instead are simply denoted by p and q, respectively.

We have to show that the difference $\Pi(m') - \Pi(m)$ is negative whenever $m \geq 2N/3$. It is a matter of tedious but straightforward algebraic manipulations that this difference is equal to the ratio[23]

$$\frac{2pqp' + 8qp - 11p - 8pp' - q^2p - p(p')^2 - 16qp' + 21p' - 2q(p')^2 + 4(p')^2 + 3q^2p'}{4(4 - 2q + p')(4 - 2q - p)}. \tag{28}$$

The denominator of this expression is clearly positive. Thus the sign of the difference in average ex-ante payoffs coincides with the sign of the numerator of this expression. Denote this numerator by $d(p, q, p')$.

[23]The following expression is obtained by using $q' = q - p'$.

We first show that for any p and q, d is monotonically increasing in p'. We have

$$\begin{aligned}\frac{\partial d(p,q,p')}{\partial p'} &= 2pq - 8p - 2pp' - 16q + 21 - 4qp' + 8p' + 3q^2 \\ &\geq -8p - 2pp' - 16q + 21 - 4qp' \geq 13 - 2pp' - 4qp' > 0,\end{aligned}$$

where the second inequality sign in this expression follows from the fact that $q + p/2 \leq 1/2$.

Since $p' = (N-m)p/m$, we have that $p' \leq p/2$ for all $m \geq 2N/3$, and thus

$$d(p,q,p') \leq \max_{p' \leq p/2} d(p,q,p') = d(p,q,p/2) = p\left[\frac{pq}{2} - \frac{1}{2} - 3p + \frac{q^2}{2} - \frac{p^2}{4}\right] < \frac{p}{2}\left[pq - 1 + q^2\right].$$

Given that $p, q \leq 1/2$ we thus have that $d(p,q,p') < 0$ whenever $m \geq 2N/3$. This proves the first part.

Since $p' = (N-m)p/m = \frac{1-s}{s}p$ for $s = m/N$, we have $p' \geq ap$ for $m \leq sN$, where $a = (1-s)/s$. Monotonicity of d in p' therefore implies that

$$\begin{aligned}d(p,q,p') &\geq \min_{p' \geq ap} d(p,q,p') = d(p,q,ap) \\ &= \{[a(4-p) - 8]ap + [2pa(1-a) + (3a-1)q - 8(2a-1)]q + 21a - 11\}p.\end{aligned}$$

Now observe that the sign of $\partial d(p,q,ap)/\partial q$ coincides with the sign of $2pa(1-a) + (6a-2)q - 8(2a-1)$. Since

$$2pa(1-a) + (6a-2)q - 8(2a-1) < 8 - a(16 - 2(2q+p)) + 2q(1-a) \leq 8 - 14a$$

it follows that whenever $a \geq 8/14 = 4/7$ then $\min_{p' \geq ap} d(p,q,p')$ is decreasing in q. Using the fact that $q \leq (1-p)/2$ we thus have that for all $1 \geq a \geq 4/7$

$$\begin{aligned}d(p,q,p') &\geq \min_{q \leq (1-p)/2}\left\{\min_{p' \geq ap} d(p,q,p')\right\} = d(p,(1-p)/2,ap) \\ &= \frac{p}{4}(55a - 29 - 2ap - ap^2 - 14p + 12a^2p - p^2) =: D(p,a).\end{aligned}$$

$D(p,a)$ is clearly increasing in a. In the case of simple majority we have $a = (N-1)/(N+1)$, which is increasing in N. Thus under simple majority we have that $a \geq 2/3$ if $N \geq 5$ (for $N = 3$ all majority rules satisfy $m \geq 2N/3$). Since $D(p, 2/3) = p(23 - 30p - 5p^2)/12 > 0$ for all $p \in (0, 1/2)$ we can therefore conclude that at simple majority the increment of Π is positive.

The preceding observations allow us to restrict our attention in the remainder of the proof to supermajority rules $m > (N+1)/2$. Since for all such rules we have $1/2 \geq p(m-1)/2 + q(m-1)$ and $p(m) \geq p(m-1)/2$, the identity $q(m-1) = p(m) + q(m)$ implies $3p(m)/2 + q(m) \leq 1/2$ or equivalently $q \leq (1-3p)/2$. Exploiting this fact we can thus claim that if $m > (N+1)/2$ then we have for all $a \in (4/7, 1)$ that

$$\begin{aligned} d(p,q,p') &\geq \min_{q \leq (1-3p)/2} \left\{ \min_{p' \geq ap} d(p,q,p') \right\} = d(p,(1-3p)/2, ap) \\ &= \frac{p}{4}(50ap + 15p^2 a - 29 - 42p - 9p^2 + 8a^2p^2 + 55a + 12a^2p) =: \hat{D}(p,a). \end{aligned}$$

Since $\hat{D}(p,a)$ is strictly increasing in a it follows that for all $a \geq 4/7$ we have

$$d(p,q,p') \geq \hat{D}(p,4/7) = \frac{p(119 - 466p + 107p^2)}{196}.$$

It is straightforward to see that this expression is strictly positve for all $p \in (0, 1/4]$. Since $p(m)$ is decreasing in m for every N and also $p((N+3)/2)$ decreases with N, it follows that for $2N/3 > m > (N+1)/2$ we must have $p(m) \leq p(7,11) = 105/520 < 1/4$ (notice that only for $N \geq 11$ there are majority rules in the specified range). Hence, for all p which may arise for $2N/3 > m > (N+1)/2$ we know that $\hat{D}(p,a) > 0$, whenever $a \geq 4/7$. The condition $a = (1-s)/s \geq 4/7$ in turn is equivalent to $s = m/N \leq 7/11 \approx 0.636$. Thus we can conclude that $d(p,q,p') > 0$ for all $(N+1)/2 < m < 7N/11$. This proves statement ii).

Finally, using $p((N+1)/2)/2 + q((N+1)/2) = 1/2$ and $q(N) = 0$ it is straightforward to show that

$$\Pi(N) - \Pi((N+1)/2) = \frac{16 - 27p + 10p^2 + p^3}{48(4-p)},$$

which is strictly positive for all $p < 1$. □

Proof of Proposition 7. Suppose the first claim is false, i.e.

$$\lim_{N\to\infty} \Pi(\lceil s^* N \rceil, N) > \lim_{N\to\infty} \tilde{\Pi}(\lceil \tilde{s}^* N \rceil, N). \tag{29}$$

In the proof of Proposition 5, we have shown that $\lim_{N\to\infty} \Pi(\lceil sN \rceil, N) = \lim_{N\to\infty} \tilde{\Pi}(\lceil sN \rceil, N)$ for any $s > 1/2$, so that (29) cannot hold when s^* and $\tilde{s}^*$ are both greater than 1/2. Furthermore, by Proposition 5, we cannot have that $\tilde{s}^* > 1/2$ and $s^* = 1/2$. If $\tilde{s}^* = 1/2$ and $s^* > 1/2$, then

$$\lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N) \geq \lim_{N\to\infty} \tilde{\Pi}(\lceil s^* N \rceil, N) = \lim_{N\to\infty} \Pi(\lceil s^* N \rceil, N), \tag{30}$$

where the inequality follows from the optimality of $\tilde{s}^* = 1/2$. Last, if $\tilde{s}^* = 1/2$ and $s^* = 1/2$, then equation (27) shows that $\lim_{N\to\infty} \Pi(\lceil N/2 \rceil, N) < \lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N)$. Thus, (29) cannot hold, the desired contradiction.

For the proof of the second statement, we drop the arguments from the functions p and q (like in earlier proofs). Calculating the difference between $\Pi(m)$ and $\tilde{\Pi}(m)$ gives

$$\Pi(m) - \tilde{\Pi}(m) = \frac{2p + 3 + q^2 - 4q}{4(4 - 2q - p)} - \frac{3 + 2p}{16} = \frac{3p + 4q^2 - 10q + 4qp + 2p^2}{16(4 - 2q - p)}. \tag{31}$$

The denominator of this expression is clearly positive and thus the sign of the difference is determined by the numerator. Denote this numerator by $d(p, q)$.

We first show that $d(p, q)$ is negative for $m = (N + 1)/2$. Remember that, in this case, we have $p = 1 - 2q$ and thus

$$d = 3p + 4q^2 - 10q + 4qp + 2p^2 = 5 - 20q + 4q^2.$$

This expression is negative iff $q((N + 1)/2, N) \leq 5/2 - \sqrt{5} \approx 0.26$, which is satisfied for all $N > 5$.

Next consider any supermajority m such that $(N + 1)/2 < m \leq \lfloor 3N/4 \rfloor$. For any such majority rule we have that $m \leq N - 2$. Therefore, it follows that

$$q(m) \geq p(m + 1) + p(m + 2) = p(m + 1)\left(1 + \frac{N - m - 1}{m + 1}\right) = p(m)\frac{N - m}{m}\frac{N}{m + 1}.$$

Notice that the last term in this expression is decreasing in m. Thus we may write $p \leq (q\lfloor 3N/4 \rfloor \lceil 3N/4 \rceil)/((N - \lfloor 3N/4 \rfloor)N)$. Using Lemma A2, it can be shown that the right-hand side of this last inequality is smaller than $(12/5)q$.[24]

Next observe that the fact that m is a supermajority rule implies that $1/2 \geq p(m-1)/2 + p(m) + q(m) \geq 3p(m)/2 + q(m)$. Combining this observation with the preceding one, we obtain $p \leq \min\{(1 - 2q)/3, 12q/5\}$, or equivalently,

$$p \leq \begin{cases} 12q/5 & \text{if } q \leq 5/46 \\ (1 - 2q)/3 & \text{if } q > 5/46. \end{cases}$$

Notice that $d(12q/5, q) = -(14/5 - 628q/25)q < 0$ for all $q \leq 5/46$ and $d((1-2q)/3, q) = (11 - 104q + 20q^2)/9 < 0$ for all $5/46 < q \leq 1/2$. Since $d(p, q) \leq d(\min\{(1 - 2q)/3, 12q/5\}, q)$, this proves the first claim.

As for the second part of number 2, observe that a sufficient condition for the numerator of (31) to be positive is $3p > 10q$. From Lemma A1, we have $\frac{q}{p} \leq \frac{N-m}{2m-N}$ so that for all $m \geq \frac{13}{16}N$, the numerator of (31) is positive.

For number 3, note that Proposition 6 implies that the optimal majority rule with the option of waiting is lower or equal to $\lceil 2N/3 \rceil$, which is lower or equal to $\lfloor 3N/4 \rfloor$ for all $N > 5$. By the second statement of Proposition 7, for all such rules, the expected ex-ante payoff is higher without the option to wait. $\square$

[24] If $N = 7, 11, 15, \ldots$, then $12/5 - (\lfloor 3N/4 \rfloor \lceil 3N/4 \rceil)/((N - \lfloor 3N/4 \rfloor)N) = (N + (N-1)/2 + 8)/10N > 0$.

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