Institutional Members: CEPR, NBER and Università Bocconi

WORKING PAPER SERIES

Political Intergenerational Risk Sharing

Marcello D'Amato and Vincenzo Galasso

Working Paper n. 342

September 2008

IGIER – Università Bocconi, Via Guglielmo Röntgen 1, 20136 Milano –Italy
http://www.igier.unibocconi.it

Political Intergenerational Risk Sharing*

Marcello D'Amato
Università di Salerno, Csef.

Vincenzo Galasso
IGIER, Università Bocconi and CEPR

September 2008.

Abstract

In a stochastic two-period OLG model, featuring an aggregate shock to the economy, ex-ante optimality requires intergenerational risk sharing. We compare the level of time-consistent intergenerational risk sharing chosen by a social planner and by office seeking politicians. In the political setting, the transfer of resources across generations – a PAYG pension system – is determined as a Markov equilibrium of a probabilistic voting game. Negative shocks represented by low realized returns on the risky asset induce politicians to compensate the old through a PAYG system. Unless the young are crucial to win the election, this political system generates more intergenerational risk sharing than the (time consistent) social optimum. In particular, these transfers are more persistent and less responsive to the realization of the shock than optimal. This is because politicians anticipate their current transfers to the elderly to be compensated through offsetting transfers by future politicians, and thus have an incentive to overspend. Perhaps surprisingly, aging increases the socially optimal transfer but makes politicians less likely to overspend, by making it more costly for future politicians to compensate the current young.

Keywords: Pension Systems, Markov equilibria, social optimum.

JEL Classification: H55, D72.

*Marcello D'Amato, Università di Salerno, 84084 Fisciano (SA), damato@unisa.it. Vincenzo Galasso, Università Bocconi, via Roetgen 1, 20136, Milano, vincenzo.galasso@unibocconi.it. We thank Dirk Krueger, and participants at seminars in Bocconi and London for useful comments. Usual disclaimers apply.

1. Introduction

PAYG social security systems have been often advocated on the account that they are able to provide intergenerational risk sharing, which is not available in the private market. In fact, in the presence of uncertain wages and/or rate of returns, individuals would typically like to insure against a bad realization before they are even born[1]. If there exists a long term player, such as a government or a social planner, who can commit to carry out future policies on the behalf of yet-to-be-born generations, intergenerational risk sharing may arise. Since early contributions by Enders and Lapan (1982) and Merton (1983), PAYG social security systems have been acknowledged to be able to perform this task and a growing literature has developed on their optimal design.

Yet, life has recently become more difficult for social security systems. A parallel literature which provides evidence on the inefficiencies and the costs of these generous social security systems has in fact blossomed. The large drop in the employment rate among the middle aged and elderly workers (i.e., those aged 55+) experienced by most OECD countries since the late sixties, thanks to the existence of generous early retirement provisions (see Gruber and Wise, 1999), the rising labour cost, due to the large social security contributions, and the crowding out of capital induced by the reduction in private savings are only few of the by-products of these pension systems, which have been largely criticized. The upshot is that social security spending is inefficiently large. Moreover, population aging has added further emphasis on the cost of social security by questioning even the future financial sustainability of these systems. The reasons for this large spending may be political, as suggested by a recent political economy literature that underlines the importance of electoral constraints. Politicians who respond to electoral considerations in aging societies have an incentive to provide high pension spending (see Galasso, 2006).

Bohn (2003), and more recently Krueger and Kubler (2006), have taken an integrated look at the costs and benefits of PAYG schemes to suggest that the crowding out effect on the private savings may be so severe as to overweight the positive role of intergenerational risk sharing. Storeletten et al. (1999) considered the risk sharing properties of social security systems vis-à-vis idiosyncratic risks,

[1] Even in a dynastic environment in which parents care about the well being of their kids, and thus would like to purchase some insurance on their account, private markets may fail to work, to the extent that legal contracts signed by the parents cannot bind future generations (the kids).

such as wage fluctuations and mortality risk, and reached similar conclusions. Furthermore, including considerations on the labor market distortion induced by social security would even strengthen this argument. Yet, in a more general model, which – together with capital – features a long-lived asset (such as land), Gottardi and Kubler (2006) provide more optimistic results on the existence of a (ex-ante) Pareto improving social security system. So the jury is still out.

In this paper, we abstract from many distortionary aspects associated with social security and concentrate on the intergenerational risk sharing property of PAYG systems to examine whether – even in an environment with no additional distortions – electoral constraints may lead the politicians to choose "too much" social security spending. To assess whether politicians are to blame for excess spending in social security, we introduce a stochastic environment in which intergenerational risk sharing is (ex-ante) Pareto improving and derive the optimal level of social security as chosen by a time consistent social planner. The risk in our two-period stochastic overlapping generation economy comes from an aggregate shock to the stock market. Since young agents save for future consumption through a risky asset, this shock affects their net private wealth – and thus consumption, in old age. A social planner, who cares about the well-being of current and future generations, determines a time consistent policy, after the realization of the shock to the economy. The (ad interim) optimal linear risk sharing policy features a transfer from the young to the old, which depends negatively on the realization of the net private wealth of the elderly.

To model the incentives faced by a politician in determining the size and the design of the social security system, we consider a probabilistic voting environment in which politicians can only choose current policy but are unable to commit to future transfers. Lacking commitment to future policies, intergenerational risk sharing becomes harder to achieve, since young voters may not trust to receive back a transfer in their old age. To capture the intertemporal link between current and future policies in absence of commitment, we thus concentrate on Markov perfect equilibria of this probabilistic voting game, which require the equilibrium policy to depend only on the state of the economy. In this political system, we show that intergenerational risk sharing consisting of a transfer from the young to the old – as in a PAYG system – emerges in a negative state of the economy, and persists in future periods. Its size depends crucially on the political power of the elderly. In other words, if the stock market loss by the current generation is not negligible, and the electoral relevance of the old is sufficiently large, politicians will bail out the elderly. Politicians' incentives to intervene in case of a negative shock effectively create a quasi asset – the PAYG social security – whose returns are negatively correlated to the stock market returns. Moreover, this policy is persistent, since less disposable income for the current young generation leads to lower net private wealth in old age and thus to more future government intervention.

In this environment in which (ex-ante) intergenerational risk sharing is welfare enhancing, both the politicians and the social planner choose to resort to a state contingent PAYG social security system to ensure the individuals against the stock market risk. Yet, are office-seeking politicians more generous in setting up these systems – as the political economy literature would seem to suggest? Yes. Unless the young enjoy a strong political relevance in the election, the political system will induce more intergenerational risk sharing than the (time consistent) social optimum. Overspending stems from the fact that politicians exploit the expectations by current young voters, who anticipate that their current transfers will be compensated by offsetting transfers provided by future politicians. This strategic effect lowers the electoral cost to politicians. They hence have an incentive to overspend in social security to please the current elderly. In other words, politicians play strategically by "leaving" more risk sharing on the future generations than optimal. Furthermore, these transfers are persistent and less responsive to the realization of the shock than the optimal policy would require.

The effect of population aging of the behavior of the politicians are quite surprisingly, given the results in the current literature on the political economy of social security. In our environment, aging increases the socially optimal transfer chosen by the social planner; yet, it makes the politicians less likely to overspend. Aging in fact reduces the range of parameter values – defined as the relative political influence of the young and the social planner discount rate – over which the politicians choose more transfers than the social planner. This novel result depends on the fact that aging makes it more costly for future politicians to compensate the current young – and so reduces the room for strategic behavior by the current politicians.

Our analysis may also help to shed some light on the motivations behind the initial introduction of social security systems. In several countries, the initial design of social security system featured fully funded schemes, according to which the contributions by the workers were invested in financial assets – such as private share or nominal public bonds. At retirement, the accumulated assets were converted into an annuity. Stock market crashes, as in the US in 1929, and episodes of hyperinflation wiping the value of bond issued in nominal terms, as in France and Italy at the end of World War II, represented clear cases of negative shocks that called for government's intervention to transform the existing systems into PAYG schemes, as suggested in our analysis[2] (see also Perotti and Schwienbacher, 2008).

This paper is related to different strands of the literature. Several contributions have established important results on the Pareto optimal intergenerational risk sharing properties of social security both ex-ante, i.e., behind a veil of ignorance, and ad interim, that is, when the realization of one (or many) shock has occurred. The structure of our economy is closely related to Gordon and Varian (1988). Additional key contributions carried out in partial equilibrium framework include Allen and Gale (1997), Shiller (1999), Demange (2002), Matsen and Thogersen (2004) and Ball and Mankiw (2007). Instead Bohn (2003), Krueger and Kubler (2006), and Gottardi and Kubler (2006) consider also the general equilibrium effects arising from the introduction of a social security system, such as the crowing out of the private savings.

Our paper contributes also to the debate on the optimal social security design. Thogersen (2006) and De Menil et al. (2006) analyze the optimal mix of PAYG and partially funded scheme based on this risk sharing argument. Other contributions endorsing a double pillar, or mixed system, which integrates features of both schemes include Orszag and Stiglitz (2001), Shiller (1999), and Boldrin, Dolado, Jimeno, Peracchi (1999).

Yet, the closer literature is perhaps the one on the political support for intergenerational risk sharing policy, such as PAYG systems. As Orszag and Stiglitz (2001, see myth 9), we in fact recognize that, if a negative shock occurs, politicians may decide to "bail the elderly out". Rangel and Zeckhauser (2001) present several arguments suggesting that neither the market nor politicians are typically able to provide the optimal level of intergenerational risk sharing. Our contribution is however closer in spirit to Demange (2007), who characterizes the conditions for a PAYG social security system to be supported politically in absence of commitment on future policies. She finds that – besides the redistributive elements often embed in these pension systems – the political support depends on the degree of

[2] In the US, the 1935 Social Security Act established a system of Federal old-age benefits. There were two major provisions relating to the elderly: Title I: Grants to States for Old-Age Assistance, which supported state welfare programs for the needy aged, regardless of their previous contributions and Title II: Federal Old-Age Benefits, which provided monthly payments to workers who contributed to the system, upon reaching the age of 65. The former program represented an immediate response to the negative economic environment, while the latter scheme established the basis for the payment of future pension benefits (see more informations at www.ssa.org)

risk aversion of the decisive voter and on the availability of financial assets. Our results are in line with her sustainability conditions. In addition, we are able to characterize and to compare the risk sharing policy chosen by the social planner and by the politicians.

The paper is organized as follows: section 2 describes the model and the ex-ante Pareto improving role of intergenerational risk sharing. Section 3 and 4 analyze respectively the time consistent policy chosen by a social planner who cares about the current and future generations, and by office-seeking politicians in a probabilistic voting model. Section 5 compares these results and provides some comparative statics. Section 6 analyzes how the aging process affects the choice by the social planner and the politicians. Section 7 concludes. All the profs are in the appendix.

2. A Simple Stochastic Economy

We consider a two period overlapping generations model of a small open economy. Every period two generations are alive: young and old. Population grows at a constant rate n. Agents are endowed with one unit of labor in youth, which they supply inelastically to receive a wage, w. Agents evaluate old age consumption only, according to an increasing and concave utility function: $U\left(c_{t+1}^{t}\right)$, with $U'(.) > 0$, $U''(.) < 0$, where c_{t+1}^{t} represents consumption at time $t+1$, i.e., in old age, by an agent born at time t.

Output in the economy is given by

$$y_t = wL_t + R_t K_t \tag{2.1}$$

where K_t represents the stock of capital, i.e., the amount of savings, in the economy. Capital fully depreciates at every period. The return on capital is stochastic. Claims to capital represent the only (risky) asset in this economy, which pays a real return R_t distributed according to a cumulative function $G(R_t)$, with mean $E_{t-1}[R_t] = R$, variance $Var_{t-1}[R_t] = \sigma^2$ and no serial correlation $E(R_t R_{t+1}) = 0$. Limited liability applies in the stock market to the risky asset, which also features an upper bound $\bar{R}$ on its returns, $R_t \in [0, \bar{R}]\ \forall t$. The wage is deterministic and assumed to be unitary, $w = 1$.

The distribution of the stochastic returns represents a crucial element in a model that analyzes intertemporal risk sharing. Instead of recurring to a specific distribution function, we choose to consider distributions that obey to two criteria.

First, we assume that the average return from the risky asset is higher that the return from a PAYG social security system, $R > (1+n)$. Second, we assume that the distribution is rather spread out, so that the coefficient of variation, σ/R, is greater than one. These represent our first assumptions.

Assumption 1 $\sigma > R > (1+n)$.

Agents save their entire net endowment for old age consumption using the risky asset, so the budget constrain of an individual born at time t is

$$c_{t+1} = R_{t+1}(1 - T_t) + P_{t+1} \tag{2.2}$$

where $T_t\,(< 1)$ is the amount of taxes paid by the young to provide a transfer to the current old – as in a PAYG social security system, and $P_t = (1+n)\,T_t$ is the amount received by the old. It is also convenient to define the net private wealth of the elderly at time $t+1$ as $\varpi_{t+1} = R_{t+1}(1 - T_t)$.

In most of the paper, agents are assumed to have quadratic preferences, which will give raise to a mean variance representation:

$$U(c_{t+1}) = -\frac{1}{2}(c_{t+1} - \gamma)^2 \tag{2.3}$$

where the parameter γ plays a double role. In the deterministic formulation that will apply to the consumption of the elderly – once the shock has been realized, γ determines the marginal utility of consumption. In the expected utility formulation that will apply to the young, the parameter γ will be measuring the degree of risk aversion: a lower γ characterizes a more risk averse individual.

Hence, for this utility function to feature decreasing marginal utility and to be concave, we need to have that $\gamma > c_t\ \forall t$. A sufficient condition is that $\gamma > \bar{R} + (1+n)$. This amounts to require that the marginal utility of consumption is positive even when individuals pay no contributions in youth, obtain the largest possible return, $\bar{R}$, from their savings – consisting on their entire labor income – and receive the largest possible pension transfer, $P_{t+1} = (1+n)$, in old age[3]. We choose to impose an additional restriction on γ to guarantee that a policy

[3] In what follows, we will allow both the social planner and the politicians to choose a negative transfer – that is, a transfer from the elderly to the young – at least for some realization of the shock. If this occurs, the savings of the young may exceed their labor income. However, it is straightforward to show that the consumption level implied by the equilibrium policy chosen by the social planner and by the politicians (see sections 3 and 4) never exceeds $\overline{R}+(1+n)$. Hence, the sufficient condition at assumption 2 holds also for the following sections.

consisting of a positive contribution rate imposed on the young at time t, with no corresponding pension benefit at $t+1$, is associated with a negative expected marginal utility for the young. This occurs for $\gamma > S/R$ where $S = \sigma^2 + R^2$. We thus have our next two assumptions.

Assumption 2 $\gamma > \bar{R} + (1+n)$.

Assumption 3 $\gamma > S/R$.

Notice that in this economy, in absence of a social security system, consumption in old age is simply equal to the realization of the return on the risky asset: $c_{t+1} = R_{t+1}$. Hence, the expected consumption at time t is the mean of the distribution of returns $E_t[c_{t+1}] = E_t[R_{t+1}] = R$, and similarly for the variance $Var_t[c_{t+1}] = Var_t[R_{t+1}] = \sigma^2$. Clearly, the coefficient of variation for the consumption is $\sigma/R > 1$.

2.1. Ex-Ante Pareto Efficient Intergenerational Risk Sharing

In this simple economic setting, individuals have no mechanism to ensure against the risk of a negative aggregate stock market shock when old. In absence of intergenerational transfers, a low realization of the return on their savings affects their private wealth and hence their consumption. In this environment, agents may be willing to incur in a tax on their labor supply when young in order to receive a transfer from the next generation of individuals when old. As a simple benchmark to discuss ex-ante Pareto efficiency, consider the decision of a young generation that uses a constant intergenerational transfer scheme to maximize its expected utility. In other words, there exists a commitment device that allows the young to set up a PAYG pension system by contributing a share, α, of their labor income today in exchange for the same share of the young labor income in the next period. As largely discussed in the literature, in order for this arrangement to arise, some long term player – such as the government – who is able to commit to future policy needs to exist. In absence of this commitment device, agents would be unable to achieve intergenerational risk sharing. Once this commitment device is assumed to exist, the optimization problem becomes:

$$\begin{aligned} & \underset{\{\alpha\}}{Max} - \frac{1}{2} E_t \left(c_{t+1} - \gamma\right)^2 \qquad (2.4) \\ c_{t+1} &= R_{t+1}\left(1-\alpha\right) + \alpha\left(1+n\right) \end{aligned}$$

As made it clear by the following first order condition, this amounts to a simple portfolio decision problem in which the agents have to choose how to divide their saving between a safe asset that provides a return $(1+n)$, and a risky asset with a stochastic return R_{t+1}:

$$E_t\left(c_{t+1}-\gamma\right)\left(R_{t+1}-(1+n)\right)=0 \tag{2.5}$$

The share of savings allocated to the intergenerational risk sharing is

$$\alpha=\frac{\sigma^2+\left(R-(1+n)\right)\left(R-\gamma\right)}{\sigma^2+\left(R-(1+n)\right)}.$$

Simple algebra shows that depending on their degree of risk aversion, the young will want to have some intergenerational risk sharing. In particular, $\alpha>0$ for large enough degree of risk aversion, $\gamma<\frac{\sigma^2+R[R-(1+n)]}{R-(1+n)}=\frac{S-(1+n)R}{R-(1+n)}$, where $S=\sigma^2+R^2$. Since we are interested in an environment in which intergenerational risk sharing plays a role, this gives raise to our next assumption.

Assumption 4 $\gamma<\frac{S-(1+n)R}{R-(1+n)}$.

How does consumption in old age evolves in the presence of this intergenerational risk sharing transfer? Consumption in old age clearly depends on the realization of the return on the risky asset, as shown at eq. 2.4. The expected consumption at time t is thus equal to $E_t[c_{t+1}]=R-\alpha\left(R-(1+n)\right)$, which for $\alpha>0$ is lower than the expected consumption in absence of risk sharing, $E_t[c_{t+1}]<R$. However, also the variance is reduced, $Var_t[c_{t+1}]=(1-\alpha)^2\,\sigma^2<\sigma^2$. Finally, it is easy to see that the coefficient of variation for the consumption is lower with this risk sharing device than in absence of risk sharing.

To summarize, in this simple economic environment, even an uncontingent, constant policy can be ex-ante Pareto improving, if individuals are sufficiently risk averse[4]. This constant policy however rests on the assumption that current individuals can commit to future policies. In the remaining of the paper, we will allow for more complex policies, which relate the degree of risk sharing to the realization of the shock, but will drop the assumption of commitment. In choosing the intergenerational policies, the social planner and the office-seeking politicians will have to consider that future policies cannot be committed upon; quite on the opposite, they can be modified in the future, if it becomes convenient to do so.

[4]This is worth noticing that assumption 4 is always compatible with assumption 3 and is compatible with assumption 2 for distribution functions that have a sufficiently high variance, that is, if $\sigma^2>\left(R-(1+n)\right)\left(\overline{R}-R+(1+n)\right)$.

3. Intergenerational risk sharing: the Social Planner

In this section, we consider the intergenerational risk sharing decision for a social planner who cares for the well being of current and future generations, and has to choose a time consistent policy. In every period, after the realization of the shock has occurred, and hence the net private wealth of the elderly has became known, the social planner decides whether to transfer resources from the young to the elderly. We assume no commitment, so that the planner policy can be modified over time, if it is optimal to do so.

The social planner optimization problem at time t is thus

$$\underset{\{T_{t+i}\}_{i=0}^{\infty}}{Max}\; U(c_t) + \delta(1+n) E_t U(c_{t+1}) + \delta^2 (1+n)^2 E_t U(c_{t+2}) + ... \tag{3.1}$$

where δ represents the social planner discount rate, subject to the budget constraint and the utility function respectively at equations 2.3 and 2.2 in the previous section. Here, the planner decision variable is the policy, T_t, the state variable is $\varpi_t = R_t(1 - T_{t-1})$, which characterizes the net private wealth of the elderly at time t, and individual agents take no economic decisions. We can thus use the following recursive formulation:

$$V(\varpi_t) = \underset{\{T_t\}}{Max} \{U(\varpi_t, T_t) + \delta(1+n) E_t V(\varpi_{t+1})\}. \tag{3.2}$$

The first order condition of this optimization problem is:

$$-(\varpi_t + (1+n) T(\varpi_t) - \gamma) + \delta E_t (\varpi_{t+1} + (1+n) T(\varpi_{t+1}) - \gamma) R_{t+1} = 0. \tag{3.3}$$

The former term represents the marginal utility for the elderly of an increase in their consumption due to the intergenerational transfer, whereas the latter represents the reduction in marginal utility for the young from lower future consumption. To solve this optimization problem, we guess a linear time consistent policy, $T(\varpi_t) = A + B\varpi_t$, and verify that it satisfies eq. 3.3. Recall that $S = \sigma^2 + R^2$. The next proposition characterizes the optimal interior time-consistent linear policy function.

Proposition 3.1. *If $\delta \in \Lambda = \left(\frac{1}{R}, \frac{\gamma-(1+n)}{R\gamma - S}\right)$, there exists a time-consistent linear policy function $T^S(\varpi_t) = A + B\varpi_t$, that solves the social planner problem at eq.*

3.1, with $T^S(\varpi) < 1\ \forall \varpi$, and $T^S(\varpi) > 0$ for some ϖ, where

$$\begin{aligned} B &= -\frac{1}{\delta S} \\ A &= \frac{\delta\left[S - R\gamma\right] + \left(\gamma - (1+n)\right)}{\delta\left[S - R\left(1+n\right)\right]} \end{aligned}$$

This proposition suggests that if the social planner cares enough about the future generations[5], i.e., if $\delta \in \Lambda$, she will implement a linear interior intergenerational risk sharing mechanism, which provides to the elderly a transfer consisting of a constant share, A, which is reduced of a proportion B according to the realization of the state of the world, ϖ. Hence, in the worst case scenario, in which the elderly have zero private wealth, $\varpi = 0$ – for instance because of a very bad stock market shock, $R = 0$ – the social planner imposes a positive, large transfer on the young, $T^S(\varpi) = A \in (0, 1)$. Better realizations of the rate of return, and hence higher private wealth for the elderly, are associated with lower transfers from the young.

Clearly, a lower weight on the future generations, i.e., $\delta < 1/R$, may lead, in the occurrence of a particularly negative shock on the returns, to a 100% transfer of resources from the young to the elderly. These equilibria with full expropriation have the unpleasant feature of representing an absorbing state. In fact, the young generation on which a 100% tax rate has been imposed will reach old age with zero private wealth, $\varpi = 0$, which will in turn command a 100% tax rate on the young and so on. In the remaining of the paper, we will disregard these full expropriation equilibria and concentrate on interior equilibrium solution, thereby assuming that $\delta > 1/R$. On the contrary, extremely large weight on the future generations would lead to a transfer from the old to the young – that is, the opposite of a PAYG social security system – even in the worst case scenario, in which $\varpi = 0$. We will hence restrict out attention to the case in which $\delta < \frac{\gamma-(1+n)}{R\gamma - S}$. The next proposition further characterizes these interior-equilibrium risk sharing policy by presenting the results of some comparative statics.

Proposition 3.2. *For $\delta \in \Lambda$, an increase in (i) the discount factor, δ; or in (ii) average rate of return, R, reduces the fixed component, A, of the time-consistent linear policy function $T^S(\varpi_t)$, and makes the transfer less responsive to the shock. An increase in the variance of the shock, σ^2, increases the time-consistent linear policy function $T^S(\varpi_t)$.*

---

[5] It is easy to show that Λ is a non-empty set.

The intuition is straightforward. Given the realization of the shock, providing a transfer to the current elderly comes at the cost of lower expected utility for the young generations, because of the opportunity cost of using a PAYG system for risk sharing – recall that $R > (1+n)$. Hence, the higher the weight placed on these future generations, and the higher the average return of the risky asset – and hence the opportunity cost – the lower the fixed component of the transfer, which becomes also flatter. Higher variability of the returns clearly requires more risk sharing.

Let us analyze the behavior of the consumption in old age associated with the social planner policy. This is characterized by the next equation:

$$c_{t+1} = R_{t+1}\left(1 - T^S(\varpi_t)\right) + (1+n)\,T^S(\varpi_{t+1}).$$

The expected consumption at time t, given the realization of the state variable, ϖ_t, is thus equal to $E_t(c_{t+1} \mid \varpi_t) = R(1 - A - B\varpi_t)(1 + (1+n)B) + A(1+n)$, where A and B are defined at proposition 3.1, and the variance is $Var_t(c_{t+1} \mid \varpi_t) = (1 - A - B\varpi_t)^2(1 + (1+n)B)^2\sigma^2 < \sigma^2$. Finally, it is easy to see that the coefficient of variation for the consumption is lower with this risk sharing device than in absence of risk sharing, for ϖ_t such that $T^S(\varpi_t) = A + B\varpi_t \geq 0$.

We can now turn to the political arena.

4. Intergenerational risk sharing: the Politicians

In the political system, intergenerational risk sharing may arise because office-seeking politicians choose to transfer resources from young to the old (or viceversa). Politicians act after the stock market shock has occurred – and hence the return on the risky asset, R_t, is realized. They select the intergenerational policy characterized by the contribution T_t with the aim of winning the election.

To characterize the behavior of the politicians, we consider a probabilistic voting model (see Coughlin, 1992, and Persson and Tabellini, 2000). There exist two political candidates or parties, who compete in a majoritarian election. Each candidate determines her political platform, which is represented by the contribution, T, in order to maximize her probability of winning the election. The candidate who wins the election becomes the policy-maker, and implements the proposed policy. Elections take place every period, after the realization of the stochastic return on the portfolio of the current old. Hence, political candidates can condition their intergenerational risk sharing policy on the realized state of the world.

At every election, individual's voting decisions depend on the policy chosen by the political candidates – and on how this affects their utility, on the individual's political ideology towards the two candidates, and on a common popularity shock that may hit the candidates before the election. Politicians can use the intergenerational risk sharing policy to target the young and/or the old, in an attempt to increase their probability of winning the election, but they cannot affect the individuals' ideology or their own popularity. Within each age group, all individuals share the same economic preferences, thereby being equally affected by the candidates' platforms. Elderly care only about the current transfer; instead, the preferences of the current young – and thus their voting behavior – depend also on the expected future policy. If the young were myopic, they would only consider the direct effect of the current payroll tax on their savings and thereby on their future consumption. Young workers however realize that a current tax makes them more likely to be poorer tomorrow, and this may change the incentives for future politicians to intervene. But how will future politicians act after the realization of the return on their savings? Current young electors need to understand and evaluate how the decisions of the current politicians may affect the future politicians' policy choice. To make the results comparable with the time consistent policy determined by the social planner (see section 2), we choose to consider a Markov policy in which politicians cannot commit to future policy and the intergenerational risk sharing decision depends only on the state of the economy.

Besides the utility provided by the economic policy, individuals care also about the political ideology, with some people feeling ideologically closer to one candidate or another. The distribution of ideology within each group represents an important element of the candidate policy decision, as it determines the size of the swing voters, i.e., of the non-ideological voters who can be convinced to vote for a candidate if targeted with the appropriate policy. It is convenient to assume that the two groups, young and old, have a uniform distribution of ideology across agents.

In this environment, the two political candidates face the same optimization problem, and thus the political platforms converge, i.e. both candidates choose the same contribution, T (see Persson and Tabellini, 2000, and Hassler et al., 2003, in a framework with dynamic voting). Maximizing the probability of winning the election at time t is equivalent to maximizing the following expression, which may be interpreted as the welfare function of the policy-maker at time t:

$$W_t = \phi_o U(c_t) + (1+n)\phi_y E_t U(c_{t+1}) \tag{4.1}$$

where ϕ_o and ϕ_y represent the density of the uniform ideology distribution function in the two groups, old and young respectively. We normalize $\phi_o = 1$ and $\phi_y = \phi \geq 0$, where ϕ is the relative importance of non-ideological, or swing, voters among the young generation. This can be interpreted as a measure of how fiercely the young generations pursue their interests in the political arena.

Eq. 4.1 makes clear that political competition, as modelled in this probabilistic voting framework, entails a trade off between providing state contingent transfers (and utility) to current retirees and providing current negative, but expected positive transfers (and utility) to current workers. Hence, the votes of the young depend on the current politician policy as well as on its impact on tomorrow's policy. To model this intertemporal link, we focus on stationary Markov perfect equilibria, in which each politician's policy decision is contingent on the current state of the economy. At any time t, the state variable is the amount of old age consumption that can be financed out of private assets ϖ_t, which depends on the young' savings (or net income) and on the outcome of the stock market. Past policies hence directly contribute in defining the state of the economy. Clearly, each government anticipates that its current choice will affect the incentive faced by the future government and, therefore, the future design of the intergenerational risk sharing policy.

The optimization problem of a policy-maker at time t is thus

$$\underset{\{-\varpi \leq T(\varpi) \leq 1\}}{Max} U(c_t) + \phi (1+n) E_t U(c_{t+1}) \tag{4.2}$$

where the Markov strategy is $T_t = T(\varpi_t)$, the state variable is defined as $\varpi_t = R_t (1 - T_{t-1})$. Consumption can be written as $c_t = \varpi_t + (1+n) T(\varpi_t)$. Notice that $c_{t+1} = R_{t+1}(1 - T_t) + (1+n) T_{t+1}$ where T_{t+1} is the expected strategy played by future governments.

We can now formally define the linear Markov policy analyzed in this section.

Definition 4.1. *A policy $T^P(\varpi) = \theta + T'\varpi$, where θ and $T' = \frac{\partial T^P(\varpi_{t+1})}{\partial \varpi_{t+1}}$ are constant parameters, is a linear Markov perfect equilibrium of the intergenerational risk sharing game if it is a fixed point of the mapping from $T^e(.)$ to $T^P(.)$, where $T^e(.)$ is the expected policy function, $T^P(\varpi_t) \in \underset{T(\varpi_t)}{\arg\max} U(\varpi_t + (1+n) T(.)) + \phi(1+n) E_t U((1 - T(\varpi_t) R_{t+1} + (1+n) T^e(\varpi_{t+1}))$ and $T^P(\varpi_t) = T^e(\varpi_t)$.*

In what follows, we will characterize this equilibrium policy outcome for any well behaved utility function with $U' > 0$ and $U'' < 0$. We will return to the quadratic utility function later in this section.

The first order condition for the government problem is

$$U'(c_t) + \phi \frac{\partial E_t U(c_{t+1})}{\partial T^P(\varpi_t)} = 0 \qquad (4.3)$$

where – for $T_t^P > 0$ – the former term represents the marginal utility of increasing the consumption of the current old, while the latter defines the expected marginal disutility to the current young from imposing this risk sharing policy. This marginal cost can be decomposed as follows:

$$\frac{\partial E_t U(c_{t+1})}{\partial T^P(\varpi_t)} = E_t U'(c_{t+1}) [1 + (1+n) \frac{\partial T^P(\varpi_{t+1})}{\partial \varpi_{t+1}}] \frac{\partial \varpi_{t+1}}{\partial T^P(\varpi_t)}.$$

Notice that the impact of today's policy on tomorrow net private wealth is $\frac{\partial \varpi_{t+1}}{\partial T^P(\varpi_t)} = R_{t+1}$ and define $\frac{\partial T^P(\varpi_{t+1})}{\partial \varpi_{t+1}} = T'$. The first order condition of the maximization problem at eq. 4.2 becomes:

$$U'(c_t) - \phi[1 + (1+n)T'] E_t U'(c_{t+1}) R_{t+1} = 0. \qquad (4.4)$$

Thus, if an interior Markov equilibrium policy $T^P(\varpi_t)$ exists, it must satisfy $-1/(1+n) < T'(\varpi_t) \leq 0$. The above equation provides a first insight on this political intergenerational risk sharing. This policy is shaped by the political trade off between bailing out the current old from negative stock market shocks and imposing an expected cost on current young. In an interior Markov equilibrium, the intergenerational risk sharing agreement[6] features a transfer from the young to the old that is inversely related to the outcome of the stock market. It is important to notice that the political discretion by policy-makers in setting an intergenerational transfer policy creates a quasi-asset, whose returns are negatively correlated to stock market returns. Furthermore, by increasing T^P, the current government reduces, for any future realization of the stock market R_{t+1}, the level of private wealth, thereby requiring larger future intervention. This property creates a strategic effect that induces persistence in the shocks. In this model, a

[6] The structure of the problem faced by the politicians resambles the problem of optimal bequests strategies in altruistic economies where the current generation cares for the utilities of their immediate successors (see Phelps and Pollock 1973, Bernheim and Ray, 1989, and, more recently, Nowak, 2006, and references therein). The main difference is that in our political environment the weight on different generations depends on the relative size of non ideological voters whereas in the former class of models the relative weight between (state contingent) utility to ancestors and expected utility to descendants is dictated by altruism and other ethical considerations.

large current political intervention creates its own constituency for future large political interventions. Although this is commonly thought as the root of the persistence of inefficient policies (see Coate and Morris, 1999, and Conde-Ruiz and Galasso, 2003), in our context the tension between persistence and efficient allocation of risk is more subtle. The essence of intergenerational risk sharing is to spread current shocks on to future generations (see also Gordon and Varian, 1988), i.e., persistence is a crucial ingredient of an efficient policy. By transferring the burden of current negative shock to current workers, the politician triggers a reaction by all future governments who keep transferring such shock into the infinite future.

To obtain further insights on the intergenerational risk sharing policy chosen by the politicians, we continue our analysis using the quadratic utility function described at eq. 2.3. The first order condition of the maximization problem at eq. (4.2), which describes the stationary Markov policy chosen by the politician a time t, becomes:

$$-[\varpi_t+(1+n)\,T^P\left(\varpi_t\right)-\gamma]+\phi(1+(1+n)\,T')E_t[\varpi_{t+1}+(1+n)\,T^P(\varpi_{t+1})-\gamma]R_{t+1}=0. \tag{4.5}$$

subject to the linear policy $T^P(\varpi)=\theta+T'\varpi$. The next proposition characterizes the optimal linear policy function.

Proposition 4.2. *If* $\phi\in\Phi=\left(\frac{S/R}{S-R(1+n)},\frac{S(\gamma-(1+n))^2}{(S-R(1+n))\gamma(\gamma R-S)}\right)$, *there exists a linear Markov perfect policy function* $T^P\left(\varpi_t\right)=\theta+T'\varpi_t$, *with* $T^P\left(\varpi\right)<1\ \forall\varpi$, *and* $T^P\left(\varpi\right)>0\ for\ some\ \varpi$, *where*

$$T' = -\frac{1}{2\left(1+n\right)}\left(1-\sqrt{1-\frac{4\left(1+n\right)}{\phi S}}\right)$$

$$\theta = \frac{2\left(\gamma-\left(1+n\right)\right)-\phi\left(\gamma R-S\right)\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}{\phi\left[S-R\left(1+n\right)\right]\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}$$

This proposition characterizes the behavior of the sequence of politicians under a Markov perfect equilibrium when individual preferences have a mean-variance representation. As in the social planner case, the conditions on ϕ makes sure that the politicians care enough about the young generation to avoid equilibrium sequences featuring full expropriation of the young, in the occurrence of negative stock market shocks. Also in this case, in fact, full expropriation would become an absorbing state, since it would lead the young to have zero private wealth in old age, $\varpi = 0$, and thus trigger further full expropriation of future generations.

When the political relevance of the young is instead sufficiently large, politicians will introduce a linear intergenerational risk sharing scheme, featuring a positive constant component, θ, which is reduced by a share $T\prime$ according to the realization of the state of the world, ϖ. In the worst case scenario, $\varpi = 0$, the elderly obtain the largest transfer $T^P(\varpi) = \theta \in (0, 1)$. Higher levels of private wealth, ϖ, command lower transfers. Finally, if politicians care too much about the young, that is, ϕ is above the upper limit of Φ, they would fail to introduce a (PAYG) intergenerational risk sharing system, even when the worst case, $\varpi = 0$, occurs.

The next proposition provides some results on the comparative statics.

Proposition 4.3. *For $\phi \in \Phi$, the time-consistent linear policy function $T^P(\varpi) = \theta + T'\varpi$ increases if the average rate of return, R, decreases. For $\phi \in \Phi$ and $\phi < \frac{4S/R}{2S-R(1+n)}$, an increase in the political weight of the young, ϕ, reduces the time-consistent linear policy function $T^P(\varpi) = \theta + T'\varpi$ by decreasing its fixed component θ and by making it less responsive to the realization of the state variable, ϖ.*

The intuition is straightforward. A lower average return reduces the (opportunity) cost – recall that $R > (1 + n)$ – of using a PAYG system to provide risk sharing. Moreover, an increase in the political weight of the young, ϕ, increases the electoral cost of introducing risk sharing. The fixed component of the system thus shrinks, but the system becomes less responsive to the shocks. In other words, the young prefer less risk sharing with a flatter schedule.

Finally, we analyze the level of old age consumption associated with intergenerational risk sharing policy chosen by the politicians. This is characterized by the next equation:

$$c_{t+1} = R_{t+1}\left(1 - T^P\left(\varpi_t\right)\right) + (1+n)\, T^P\left(\varpi_{t+1}\right).$$

The expected consumption at time t, given the realization of the state variable, ϖ_t, is thus equal to $E_t\left(c_{t+1} \mid \varpi_t\right) = R\left(1 - \theta - T'\varpi_t\right)\left(1 + (1+n)\, T'\right) + \theta\,(1+n)$, where θ and $T\prime$ are defined at proposition 4.2, and the variance is $Var_t\left(c_{t+1} \mid \varpi_t\right) = \left(1 - \theta - T'\varpi_t\right)^2 \left(1 + (1+n)\, T'\right)^2 \sigma^2 < \sigma^2$. Again, the coefficient of variation for the consumption is lower with this risk sharing device than in absence of risk sharing, for for ϖ_t such that $T^S\left(\varpi_t\right) = \theta + T'\varpi_t \geq 0$.

5. How well do politicians do?

Both the politicians and the social planner have an incentive to provide intergenerational risk sharing in the stochastic environment introduced at section 2. Moreover, their linear time-consistent equilibrium policies share similar properties. As characterized at propositions 3.1 and 4.2, both policies consist of a constant component (A for the social planner and θ for the politicians), which is transferred to the elderly in the worst case scenario, i.e., for $\varpi = 0$, and of a share – respectively B and $T\prime$ – which reduces the maximum transfer in accordance to the realization of the state variable ϖ. Propositions 3.2 and 4.3 push these similarities even further, as they suggest that the steady state properties of the two policy functions are comparable.

We can now examine whether the politicians act exactly as the social planner. In other words, when is the interior linear Markov equilibrium policy chosen by the politician optimal? The next proposition provides the condition under which the interior linear Markov equilibrium policy $T^P(\varpi)$ coincides with the time consistent social optimum, $T^S(\varpi)$.

Proposition 5.1. *For $\delta \in \Lambda$ and $\phi \in \Phi$, if $\phi = f(\delta) = \delta \left[1 - \frac{1+n}{\delta S}\right]^{-1}$, the interior linear Markov equilibrium policy chosen by the politicians correspond to the time consistent social optimum, i.e., $T^P(\varpi) = T^S(\varpi)$ $\forall \varpi$. For $\phi < f(\delta)$, $T^P(\varpi) > T^S(\varpi)$ $\forall \varpi$, with $T\prime > B$ and $\theta > A$. For $\phi > f(\delta)$, $T^P(\varpi) < T^S(\varpi)$ $\forall \varpi$, with $T\prime < B$ and $\theta < A$.*

According to this proposition, the politicians involved in a Markov game among successive generations of players deliver the (time consistent) social optimum if they assign a political weight to the young which is larger than the weight assigned by the social planner to the future generations. In fact, for equal weights, i.e., for $\phi = \delta$, the politicians will provide more transfer than the social optimum. This transfer is characterized by a larger than optimal fixed component, $\theta > A$; and by a lower than optimal reduction associated to the state of nature, $T\prime > B$. In other words, the risk sharing policy is too generous, and not enough responsive to the state variable. These distortions come from the politicians' strategic behavior. In fact, in their decisions over the transfer policy, current politicians take into account that future politicians will compensate the current young in their old age for the tax that they contributed to the system in their youth. This stems from the fact that higher taxes on today environment lead to a lower private wealth in old age – that is, to a lower state variable in the following period – thereby triggering more transfers by the future politicians. The policy response of the future politicians thus reduces the current (electoral) cost of transferring resources to the elderly and leads to overspending, unless the young enjoy an unusually large political power.

These two intergenerational risk sharing policies have different implications for the consumption in old age. In both cases, old age consumption depends on the realization of the shock to the returns of the risky assets. However, for $\phi < f(\delta)$, that is, when the politicians are more generous than optimal in their risk sharing policy, they guarantee a higher than optimal expected consumption in old age, at the cost of having also a higher than optimal variance of consumption. By transferring too much resources to old age, and by failing to have these transfers depending more on the realization of the state variable, the politicians fail to provide the optimal risk sharing policy.

6. Aging

One of the most severe challenges for the social security systems in developed economies is brought by the demographic process of aging population. Aging decreases the returns from a PAYG system – thereby requiring either a reduction in the pension benefits or an increase in the contribution rates (or both), or other policy measures, such as a raise in the retirement age. Besides these economic effects, aging has also an important political effect. As suggested in a recent political economy literature, electoral constraints become more binding under aging, since elderly individuals, who are the more supportive of pension systems, are increasingly more relevant in the electorate.

How does aging affect the functioning of a PAYG pension system designed by a social planner to achieve intergenerational risk sharing? And how does aging affect the pension policy decisions of politicians who face electoral constraints? The next proposition summarizes the results for both the social planner and the politicians.

Proposition 6.1. *For $\delta \in \Lambda$ and $\phi \in \Phi$, population aging, i.e., a reduction in n, increases the time-consistent linear policy transfer chosen by the social planner, $T^S(\varpi_t)$. If $\phi < \frac{4S/R}{2S-R(1+n)}$, aging tilts the linear Markov equilibrium policy transfer chosen by the politicians, $T^P(\varpi_t)$, by reducing the fixed component, θ, and the responsiveness to the shocks. Moreover, population aging reduces the function $f(\delta) = \delta\left[1 - \frac{1+n}{\delta R}\right]^{-1}$.*

These results are quite surprising. In an intergenerational risk sharing framework, aging increases the cost of using the main risk sharing instrument – the PAYG pension system; but it increases the relative importance of the current generation of elderly both to the social planner and to the politicians. For the social planner, the latter effect dominates, and thus aging leads to higher transfers. Instead, if the political weight of the old is large enough – and therefore, the pension transfers are already very generous – the politicians display a more conservative behavior. Aging reduces the transfer level, θ, provided to the elderly in the worst case scenario, i.e., for $\varpi = 0$, but it also decreases the responsiveness to the shocks – that is, in good time the transfer level is reduced by a lower amount.

Aging thus affects the time-consistent (interior) linear intergenerational policy chosen by the social planner and by the politicians in opposite direction. As a result, aging has also an effect on the comparison between the social planner and the politician choice analyzed at Proposition 5.1. As shown in figure 2, aging moves downward the function $f(\delta) = \delta \left[1 - \frac{1+n}{\delta R}\right]^{-1}$, which characterizes the correspondence between the social planner and the politicians' decision. Politicians thus become less likely to "overspend" – that is, the range of parameters over which politicians provide more transfers than the social optimum is reduced. This, perhaps surprising, result stems from the fact that aging increases the cost of large current transfers in terms of tomorrow's adjustments. In other words, it becomes more costly for current politicians to provide too much risk sharing, since the PAYG system is less efficient and future politicians will hence be less keen on accommodating the previous policies by providing large transfers. Unlike most of the results in the current political economy literature, Proposition 6.1 thus suggests that aging will limit social security spending by reducing the room for the politicians' strategic behavior.

7. Concluding Remarks

The risk sharing properties of social security have long been recognized in the literature. In several stochastic environments, individuals would gain from having the opportunity to insure against aggregate shocks before they are born. Clearly, this is not possible. And once they are born – and the uncertainty is realized, there is no more room for risk sharing. Establishing a PAYG system thus requires the existence of a long term player, who can bind future, yet-to-be-born generations to carry out the risk sharing policy.

This paper shows that – even in absence of commitment – not only a social planner, who cares about current and future generations, but also office-seeking politicians choose to spread the risk across generations, by introducing a PAYG pension system. The gains from spreading aggregate risk across generations were already recognized in Gordon and Varian (1988). In an environment in which Arrow-Debreu securities are traded behind the veil of ignorance, Ball and Mankiw (2007) further characterized this statement. In their equilibrium, "all these generations- those who are old when the shock occurs and those who come later- suffer the same proportional loss in consumption from a bad shock", since "the generation who is hit by the shock buys insurance from future generations".

We show that social planner and politicians choose to adopt a state contingent social security system. The amount of resources transferred to the elderly by the working generation depends *negatively* on the elderly private wealth – and therefore on the realization of the aggregate shock to the returns of the risk asset. This state contingent social security thus constitutes a quasi asset, whose returns are negatively related to the market returns. This result is in line with Ball and Mankiw (2007) that propose an optimal intergenerational risk sharing plan with a negative correlation between the social security benefits and asset returns.

However, we also find that the intergenerational risk sharing schemes proposed by the social planner and by the politicians may differ. In particular, politicians are more likely to provide generous transfers that are less responsive to the aggregate shock. They hence introduce more persistent policies. In fact, politicians are willing to tax heavily current workers and to provide generous transfers to the current elderly, because they anticipate the response that future politicians will have to their current policy. Facing elderly individuals with low net wealth – due to the large contributions they paid in their youth – politicians will compensate them with generous pension transfers. This is consistent with Bohn (2003) findings that the current US social security system does not provide the optimal level of risk sharing, since it is too generous with the elderly and shifts most of the burden of risk sharing on to future generations. Moreover, since the initial introduction of the system, pension benefits have hardly been state contingent – that is, related to the elderly net wealth.

However, perhaps surprisingly, the aging process does not exacerbate the generous behavior of the politicians towards the elderly. In fact, due to population aging, politicians will indeed become less likely to "overspend". This is because aging reduces the return from social security, and hence increases the cost for the current politicians of leaving to future politicians more transfers to be paid (see Bohn, 2005, for an analysis of how future risks are beard by young and old generations under different pension arrangements).

The simple stochastic environment presented in this paper allows us to obtain analytical solutions and clear cut comparisons. As such, this paper represents a first step towards assessing the optimality of the politicians' risk sharing decisions. More analysis is however needed to explore other important aspects. For instance, other contributions have highlighted alternative risk sharing instruments that are not analyzed here. In particular, if a risk-free asset exists, individuals may – at least partially – reduce their portfolio risk by holding these risk-free assets. Indeed, as suggested by Allen and Gale (1997), the government (or social planner) may provide intergenerational risk sharing by using a buffer of these risk free assets. In the initial stage, social security would still be needed for risk sharing, meanwhile the risk-free assets are accumulated, but in later stage the buffer of risk-free assets would largely substitute the social security system. The existence of a risk-free asset would clearly allow to achieve a more complete intergenerational risk sharing, since it would not limit the amount of resource for risk sharing to the endowment of the young and the realized savings of the elderly. In other words, risk sharing would not be restricted to the resource of the two generations alive, but could be spread over the resources of more generations.

References

[1] Allen and Gale (1997), "Financial Markets, Intermediaries, and Intertemporal Smoothing", *Journal of Political Economy*, 105, 523-546.

[2] Ball, L. and N.G. Mankiw (2007): "Intergenerational risk sharing in the spirit of Arrow, Debreu, and Rawls, with applications to social security design", *Journal of Political Economics*, 115, 523-547.

[3] Bernheim, B.D., and D. Rey (1989) "Markov Perfect Equilibria in Altruistic Growth Economies with Production Uncertainty" *Journal of Economic Theory*, 47, 195-202.

[4] Bohn H. (1998), "Risk Sharing in a Stochastic Overlapping Generations Economy", *mimeo*, University of California, Santa Barbara.

[5] Bohn H. (2003), "Intergenerational Risk Sharing and Fiscal Policy", mimeo, Universiy of California, Santa Barbara.

[6] Bohn H. (2005), "Who Bears What Risk? An Intergenerational Perspective", Paper prepared for presentation at the 2005 Pension Research Council Symposium April 25-26, 2005 The Evolution of Risk and Reward Sharing in Retirement

[7] Boldrin, M., J.J. Dolado, J.F. Jimeno, F. Peracchi (1999) "The Future of Pension Systems in Europe. A Reappraisal," *Economic Policy (29).*

[8] Coate, S., and S. Morris, (1999). Policy persistence. *American Economic Review* 89, 1327–1336.

[9] Conde-Ruiz, J.I., and V. Galasso (2003), "Early Retirement", *Review of Economic Dynamics*, 6, 12–36

[10] Coughlin, P., (1992), "Probabilistic Voting Theory", Cambridge Press.

[11] Demange, G. (2002), "On optimality in intergenerational risk sharing", *Economic Theory*, 20, 1-27.

[12] Demange, G. (2007), "On sustainable Pay As You Go contribution rules", *mimeo.*

[13] De Menil, G., F. Murtin and E. Sheshinsky (2006): "Planning for the optimal mix of paygo tax and funded savings", *Journal of Pension Economics and Finance*, 5, 1-25.

[14] Enders, C. and H. Lapan, (1982) "Social Security Taxation and Intergenerational Risk Sharing", *International Economic Review* 23, 647-658.

[15] Galasso, V., (2006), The political future of social security in aging societies. The MIT Press.

[16] Gordon R. and H. Varian (1988), "Intergenerational Risk Sharing", *Journal of Public Economics*, 37, pp.185-202.

[17] Gottardi, P. and F. Kubler (2006), "Social Security and Risk Sharing", CESifo Working Paper Series No. 1705.

[18] Gruber, J., and D. Wise (1999). Social Security and Retirement Around theWorld. University of Chicago Press, Chicago.

[19] Hassler, J., J. V. Rodríguez Mora, K. Storesletten, and F. Zilibotti, (2003) "The Survival of the Welfare state" *American Economic Review*, 93(1), 87-112.

[20] Krueger, D. and F. Kubler, (2002), "Intergenerational Risk-Sharing via Social Security when Financial Markets Are Incomplete", *American Economic Review* 92, 407-410.

[21] Matsen E. and O Thogersen, (2004), "Designing Social Security- a portfolio choice approach", *European Economic Review*, 48, pp.883-904.

[22] Merton, R.C. (1983): "On the role of social security as a means for efficient risk sharing in an economy where human capital is not tradable", in: Z. Bodie and J.B. Shoven (eds.), Financial Aspects of the United States Pension System, The University of Chicago Press, 325-358.

[23] Nowak A. (2006), "On perfect equilibria in stochastic models of growth with intergenerational altruism", *Economic Theory*, 28 (1), pages 73-83.

[24] Orszag P. and J. Stiglitz (1999), "Rethinking Pension Reform: Ten Myths about Social Security Systems", in J. Stiglitz, ed., New Ideas about Old Age Security: Toward Sustainable Pension Systems in the 21st Century, World Bank.

[25] Perotti, E. and A. Schwienbacher (2008), "The Political Origin of Pension Funding", miemo University of Amsterdam.

[26] Persson, T., and G. Tabellini (2000), "Political Economics. Explaining Economic Policy", MIT Press.

[27] Phelps, E. S., and R. A. Pollak (1968), On Second-Best National Saving and Game-Equilibrium Growth, *Review of Economic Studies*, 35, 185-199.

[28] Rangel, A. and R. Zeckhauser (2001) "Can Market and Voting Institutions Generate Optimal Intergenerational Risk Sharing?", in: Campbell, John, and Martin Feldstein (eds.), Risk Aspects of Investment Based Social Security Reform, Chicago: University of Chicago Press, 113-141.

[29] Shiller, R.J. (1999), "Social security and institutions for intergenerational, intragenerational, and international risk-sharing", *Carnegie-Rochester Conference Series on Public Policy*, **50**, 165-204.

[30] Storesletten, K., C.I. Telmer and A. Yaron (1999), "The Risk Sharing Implications of Alternative Social Security Arrangements", *Carnegie-Rochester Conference Series on Public Policy*, 50, 213-259

[31] Thogersen, O. (2006) "Intergenerational Risk Sharing by Means of Pay-as-you-go Programs – an investigation of alternative mechanisms" CESIFO Working Paper No. 1759

Appendix

Proof of Proposition 3.1

Consider the optimization problem of the social planner at eq. 3.1. Its recursive formulation yields the following first order condition:

$$\frac{\partial U\left(c_t\right)}{\partial T_t}+\delta E_t\frac{\partial U\left(c_{t+1}\right)}{\partial \varpi_{t+1}}\frac{\partial \varpi_{t+1}}{\partial T_t}=0$$

where $\varpi_t = R_t\left(1-T_{t-1}\right)$ defines the state variable at time t. Using the utility function at eq. 2.3, the above expression can be written as eq. 3.3. To solve for interior equilibrium policies, we guess a linear solution: $T^{SP}(\varpi_t) = A + B\varpi_t$. Using simple algebra, from eq. 3.3 we obtain the following expression:

$$T^{SP}(\varpi_t)=-\frac{\varpi_t}{1+n+\delta S\left(1+B\left(1+n\right)\right)}+\frac{\gamma\left(1-\delta R\right)+\delta S\left(1+B\left(1+n\right)\right)-\delta\left(1+n\right)AR}{1+n+\delta S\left(1+B\left(1+n\right)\right)}.$$

Hence, we have

$$B = -\frac{1}{1+n+\delta S\left(1+B\left(1+n\right)\right)} \text{ and} \tag{7.1}$$

$$A = \frac{\gamma\left(1-\delta R\right)+\delta S\left(1+B\left(1+n\right)\right)-\delta\left(1+n\right)AR}{1+n+\delta S\left(1+B\left(1+n\right)\right)} \tag{7.2}$$

For $1+B\left(1+n\right)\neq 0$, we obtain

$$B = -\frac{1}{\delta S} \text{ and} \tag{7.3}$$

$$A = \frac{\gamma-\left(1+n\right)+\delta\left(S-\gamma R\right)}{\delta\left(S-\left(1+n\right)R\right)} \tag{7.4}$$

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, ϖ. That is, we require $T^{SP}(\varpi) < 1$ $\forall\varpi$. To guarantee that this condition holds, we need to impose $T^{SP}(\varpi) < 1$ for $\varpi = 0$, i.e., $A < 1$. Simple algebra yields $\delta > 1/R$. Additionally, we require risk sharing to take place, and thus a transfer from the young to the old to occur at least in some state. The more favorable case for this transfer to occur is for $\varpi = 0$. We hence need to have $A > 0$. Simple algebra shows that, since by assumption 3 $\gamma > S/R$, this occurs for $\delta < \frac{\gamma-(1+n)}{R\gamma-S}$. Q.E.D.

Proof of Proposition 3.2

For $\delta \in \Lambda$, recall that the time-consistent linear policy function is $T^S(\varpi_t) = A + B\varpi_t$ with A and B defined in proposition 3.1.

(i) consider a change in the discount factor, δ. Simple algebra shows that $\frac{\partial B}{\partial \delta} = \frac{1}{\delta^2 S} > 0$ and $\frac{\partial A}{\partial \delta} = -\frac{\gamma-(1+n)}{\delta^2(S-(1+n)R)} < 0$.

(ii) consider a change in the variance of the shock, σ^2. It is easy to see that $\frac{\partial B}{\partial \sigma^2} = \frac{1}{\delta S^2} > 0$ and $\frac{\partial A}{\partial \sigma^2} = \frac{(\delta R-1)[\gamma-(1+n)]}{\delta(S-(1+n)R)^2} > 0$. Hence, $\frac{\partial T^{SP}(\varpi)}{\partial \sigma^2} > 0$.

Finally, (iii) consider a change in the average rate of return, R. We have that $\frac{\partial B}{\partial R} = \frac{2R}{\delta S^2} > 0$ and $\frac{\partial A}{\partial R} = -\frac{(\gamma-(1+n))\left[\delta\left(\sigma^2-R^2\right)+2R-(1+n)\right]}{\delta(S-(1+n)R)^2} < 0$, since, by assumption 1, $\sigma^2 > R^2$. Q.E.D.

Proof of Proposition 4.2

Consider the first order condition at eq. 4.5, which describes the stationary Markov policy chosen by the politician a time t. Recall that the state variable is defined as $\varpi_t = R_t\left(1 - T^P_{t-1}\right)\ \forall t$, and that $T^P(\varpi_t) = \theta + T'\varpi_t$. Moreover, define $Q = 1 + (1+n)\,T'$. We need to obtain the value of the parameters T' and θ, which solve this FOC. Using simple algebra, from eq. 4.5 we obtain the following expression:

$$T^P(\varpi_t) = -\frac{\varpi_t}{1+n+\phi S Q^2} + \frac{\gamma + \phi S Q^2 - \phi Q R\left(\gamma - \theta\left(1+n\right)\right)}{1+n+\phi S Q^2}.$$

Hence, we have

$$T\prime = -\frac{1}{1+n+\phi S Q^2} \text{ and} \tag{7.5}$$

$$\theta = \frac{\gamma + \phi S Q^2 - \phi Q R\left(\gamma - \theta\left(1+n\right)\right)}{1+n+\phi S Q^2} \tag{7.6}$$

Since $Q = 1 + (1+n)\,T'$, we solve the expression at eq7.5 for $T\prime$ to find two solutions:

$$T_A\prime = -\frac{1}{2\left(1+n\right)}\left(1 + \sqrt{1 - \frac{4\left(1+n\right)}{\phi S}}\right) \text{ and} \tag{7.7}$$

$$T_B\prime = -\frac{1}{2\left(1+n\right)}\left(1 - \sqrt{1 - \frac{4\left(1+n\right)}{\phi S}}\right). \tag{7.8}$$

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, ϖ. That is, we require $T^P(\varpi) < 1 \; \forall \varpi$. To guarantee this condition for these two candidate solutions of $T\prime$, we need to impose the first order condition of the politicians (see eq. 4.5) evaluated at $\varpi = 0$ for $T = 1$ to be negative. Substituting $\varpi = 0$ and $T = 1$ in eq. 4.5, and imposing the expression to be negative yields the following inequality:

$$\phi(1 + (1 + n)\, T')R < 1 \tag{7.9}$$

Let's begin investigating the candidate solution $T\prime = T_A\prime$. Substituting $T_A\prime$ in eq. 7.9 yields the following inequality

$$\phi R \sqrt{1 - \frac{4\,(1+n)}{\phi S}} < \phi R - 2.$$

Clearly, for $\phi < 2/R$, the above inequality is not satisfied, and thus $T_A\prime$ is not part of an interior equilibrium solution. For $\phi > 2/R$, we can elaborate on the above expression to obtain the following inequality: $\phi > \frac{S/R}{S-R(1+n)}$. Simple algebra shows that for $\sigma^2/R^2 > 1$ (see assumption 1), this inequality cannot hold for $\phi > 2/R$. Hence, candidate solution $T\prime = T_A\prime$ cannot be part of an interior equilibrium solution.

Let's now turn to the candidate solution $T\prime = T_B\prime$. Substituting $T_B\prime$ in eq. 7.9 yields the following inequality

$$\phi R \sqrt{1 - \frac{4\,(1+n)}{\phi S}} > 2 - \phi R.$$

Clearly, for $\phi > 2/R$, the above inequality is always satisfied, and thus $T_B\prime$ can be part of an interior equilibrium solution. For $\phi < 2/R$, the above inequality can be rewritten as $\phi > \frac{S/R}{S-R(1+n)}$. Notice that for $\sigma^2/R^2 > 1$ (see assumption 1) $\frac{S/R}{S-R(1+n)} < 2/R$. Hence, candidate solution $T\prime = T_B\prime$ is part of an interior equilibrium solution if $\phi > \frac{S/R}{S-R(1+n)}$.

With $T' = T_B'$, we can now solve the expression at eq7.6 for θ. Simple algebra shows that $\theta = \frac{2(\gamma-(1+n)) - \phi(\gamma R - S)\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}{\phi[S-R(1+n)]\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}$.

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, ϖ. That is, we require $T^P(\varpi) < 1 \; \forall \varpi$. To guarantee that this condition holds, we need to impose $T^P(\varpi) < 1$ for $\varpi = 0$, i.e., $\theta < 1$. Simple algebra yields $\phi > \frac{S/R}{S-R(1+n)}$. Additionally, we require risk sharing to take place, and thus a transfer from the young to the old to occur at least in some state. The more favorable case for this transfer to occur is for $\varpi = 0$. We hence need to have $\theta > 0$. Simple algebra shows that the denominator is always positive, while the numerator is positive for $\phi < \frac{S(\gamma-(1+n))^2}{(S-R(1+n))\gamma(\gamma R-S)}$. Q.E.D.

Proof of Proposition 4.3

For $\phi \in \Phi$, recall that the time-consistent linear policy function is $T^P(\varpi) = \theta + T'\varpi$ with θ and T' defined in proposition 4.2. Consider a change in the average rate of return, R. It is easy to see that $\frac{\partial T'}{\partial R} = 0$ and $\frac{\partial \theta}{\partial R} < 0$. Hence $\frac{\partial T^P(\varpi)}{\partial R} < 0$.

Notice that θ can be written as

$$\theta = \frac{\phi S Q^2 + \gamma (1 - \phi Q R)}{\phi S Q^2 + (1+n)(1 - \phi Q R)} \tag{7.10}$$

with $Q = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)$ and $\phi Q R > 1$. It is easy to see that $\frac{\partial T'}{\partial \phi} = \frac{1}{\phi^2 S \sqrt{1 - \frac{4(1+n)}{\phi S}}} > 0$. Using the above expression, we have

$$\frac{\partial \theta}{\partial \phi} = -2 \frac{(\gamma - (1+n))\left[2(1+n) + S(1+\sqrt{\Delta})\sqrt{\Delta}\phi\right]}{S(S-(1+n)R)(1+\sqrt{\Delta})^2\sqrt{\Delta}\phi^3} < 0$$

with $\sqrt{\Delta} = \sqrt{1 - \frac{4(1+n)}{\phi S}}$. Q.E.D.

Proof of Proposition 5.1

Comparing the first order condition respectively for the social planner (eq. 3.3) and for the politicians (eq. 4.5), we have that, in an interior equilibrium, the social planner and the politicians will adopt the same policy if $\delta = \phi(1 + (1+n)T')$. Recall that interior equilibrium policies, involving risk sharing at least when $\varpi = 0$, require respectively, $\delta \in \Lambda$ and $\phi \in \Phi$. Using the expression for $T\prime$ at proposition 4.2, the above expression can be rewritten as $\phi = f(\delta) = \delta\left[1 - \frac{1+n}{\delta S}\right]^{-1}$. Furthermore, it is trivial to see that for δ and $T^S(\varpi)$ that solve the social planner problem (for an interior equilibrium), if $\phi > f(\delta)$, at $T^P(\varpi) = T^S(\varpi)$ the first order condition of the politicians is negative, so that $T^P(\varpi) < T^S(\varpi)$. And viceversa for $\phi < f(\delta)$.

Proof of Proposition 6.1

Recall that for $\delta \in \Lambda$, the time-consistent linear policy function chosen by the social planner is $T^S(\varpi_t) = A + B\varpi_t$ with A and B defined in proposition 3.1. To see the effect of population aging, i.e., a reduction in n, notice that $\frac{\partial B}{\partial n} = 0$ and that $\frac{\partial A}{\partial n} = -\frac{(\delta R-1)(R\gamma - R)}{\delta[S-R(1+n)]^2} < 0$. Hence, $\frac{\partial T^S(\varpi_t)}{\partial n} < 0$.

Recall that for $\phi \in \Phi$, the time-consistent linear policy function chosen by the politicians is $T^P(\varpi) = \theta + T'\varpi$ with θ and T' defined in proposition 4.2. To assess the effect of population aging, we need to determine $\frac{\partial T'}{\partial n}$ and $\frac{\partial \theta}{\partial n}$, and hence $\frac{\partial T^P(\varpi)}{\partial n}$. We have

$$\frac{\partial T'}{\partial n} = \frac{2(1+n) - \phi S\left(1 - \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)}{2(1+n)^2 \phi S \sqrt{1 - \frac{4(1+n)}{\phi S}}} < 0$$

since it is easy to show that the numerator is negative. To calculate the change in θ, it is convenient to use the expression at eq. 7.10 and to recall that $Q = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)$ and $\phi QR > 1$. We have

$$\frac{\partial \theta}{\partial n} = \frac{(\phi QR - 1)(\phi SQ^2 + \gamma(1 - \phi QR))}{[\phi SQ^2 + (1+n)(1 - \phi QR)]^2} + \frac{\partial \theta}{\partial Q}\frac{\partial Q}{\partial n}$$

where $\frac{\partial Q}{\partial n} = -\frac{1}{\phi S\sqrt{1 - \frac{4(1+n)}{\phi S}}} < 0$. As in the proof of proposition 4.3, we have that $\frac{\partial \theta}{\partial Q} = \frac{\phi SQ(\gamma - (1+n))(\phi QR - 2)}{[\phi SQ^2 + (1+n)(1-\phi QR)]^2}$. Thus $\frac{\partial \theta}{\partial Q} < 0$ for $\phi QR < 2$, that is, for $\phi < \frac{4S/R}{2S - R(1+n)}$. It follows that $\frac{\partial \theta}{\partial n} > 0$ for $\phi < \frac{4S/R}{2S-R(1+n)}$.

Finally, population aging affects the function $f(\delta) = \delta\left[1 - \frac{1+n}{\delta R}\right]^{-1}$. In particular, $\frac{\partial f(\delta)}{\partial n} = \left(\frac{\delta}{\delta - \frac{1+n}{S}}\right)^2 \frac{1}{S} > 0$. Thus, for a given value of δ, aging reduces the value of ϕ such that the transfer policy chosen by the politicians is socially optimal. Since by proposition 5.1 we have that $T^P(\varpi) > T^S(\varpi)$ for $\phi < f(\delta)$ aging reduces the set of parameters (ϕ, δ) such that politicians "overspend", that is, such that $T^P(\varpi) > T^S(\varpi)$.

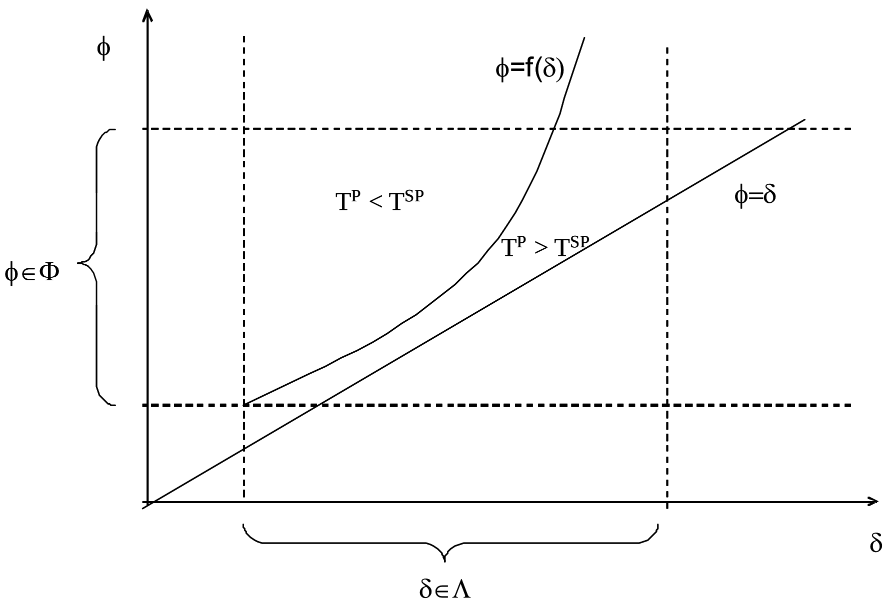

Figure 7.1: Comparing Intergenerational Risk Sharing by Social Planner (SP) and by Politicians (P)

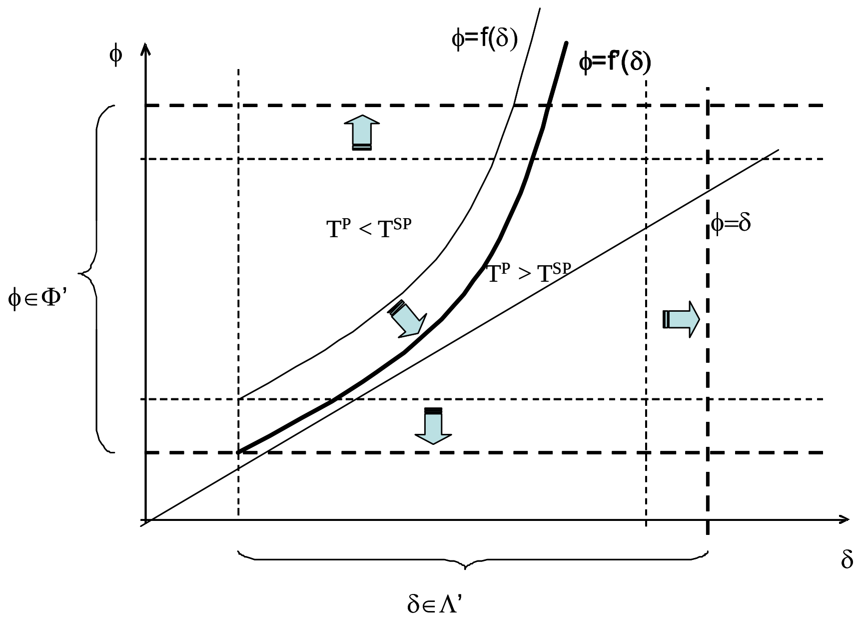

Figure 7.2: Comparing Intergenerational Risk Sharing by Social Planner (SP) and by Politicians (P): the Effect of Aging