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Tommy E. Murphy

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IGIER – Università Bocconi, Via Guglielmo Röntgen 1, 20136 Milano –Italy
http://www.igier.unibocconi.it

Persistence of Malthus or Persistence in Malthus?
Mortality, Income, and Marriage in the French Fertility Decline of the Long Nineteenth Century[♣]

Tommy E. Murphy
tommy.murphy@unibocconi.it
IGIER and Centro Dondena
Università Bocconi

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Abstract
This paper brings the French case into the current debate on Malthusian dynamics in early modern times. In particular, it studies the long-term evolution of aggregate variables, showing that nineteenth century France was hardly a Malthusian world in a strict sense. Homeostasis was maintained throughout the century and there were signs of a strong positive check, but if there was some sort of preventive check, this was not 'written in stone'. The results of both cointegrated VAR and short-run analysis grant a reading where departure from the Malthusian world (if there ever was one) is due to a secular change in the relationship between income, marriages, and births. If this interpretation is correct, the fertility decline was instrumental in the sustained decline in mortalit1y during the century.

Keywords: economic history, demographic history (Europe pre-1913), France, demographic economics, fertility, cointegrated VAR, short-run analysis.

JEL classification: N33, J13.

[♣] This paper originated in my doctoral studies at the Department of Economics at Oxford University, while being a member of Nuffield College, and completed while a research fellow at IGIER and Centro Dondena, at Università Bocconi. It has benefited greatly from discussions with my supervisor at Oxford, Bob Allen, with my thesis examiners, Knick Harley and Cormac Ó Gráda, and with various colleagues at Nuffield College, specially Roman Studer. Also Paul Sharp provided very useful comments. An earlier version of this article was presented at the Graduate Workshop on Economic and Social History (Oxford University), and I want to thank their participants for helpful comments. The responsibility for all remaining errors and omissions is, of course, entirely mine.

INTRODUCTION

Empirical studies on Malthusian dynamics have recently become a hot subject in economics and economic history. Although featuring from time to time in academic journals at least since the early 1970s [Lee, 1973], in the last few years papers on the topic have simply mushroomed [e.g. Lee and Anderson, 2002; Kelly, 2005; Clark and Hamilton, 2006; Nicolini, 2007; Møller and Sharp, 2008; Crafts and Mills, 2009]. This is partly due to the availability of new data, particularly on real wages [Allen, 2001; Clark, 2005b] but mainly to the growing interest in the conceptual implications of Malthus' model for our understanding of modern growth. Arguably, the increasingly influential discussion on unified growth theory is responsible for this. From the seminal contribution of Galor and Weil [1999, 2000] this literature has placed the classical Malthusian trap as the source of pre-industrial stagnation and its demise as one of the key forces behind the rise of economic growth. Hence, understanding the dynamics of the stagnation period becomes particularly relevant to learn about the transition to growth and, though still inconclusive in many aspects, the numerous works on the 'Malthusian world' have increased our knowledge of pre-industrial demography.

Of the scholars that have taken on the challenge of assessing the Malthusian hypotheses empirically few have done so with a country other than England. Although somewhat understandable given data constraints, this comes as a serious shortcoming in the way research is evolving in the area as the results on a country that history proved to be very unusual cannot be put into perspective. This deficiency is particularly clear with regard to another country with a peculiar history. Early modern France satisfied – as other countries in the region – many of the assumptions behind the Malthusian model but it had a rather atypical demographic history, which when confronted with the predictions of unified growth theory is even more puzzling.[1] Yet, except for a few early exceptions [e.g. Wrigley, 1985; Weir, 1984] France has been largely absent from the Malthusian debate, especially the one that began to take shape in the last few years.

[1] Whenever the French case is mentioned within the unified growth literature, it is merely treated as an anomaly [e.g. Galor and Moav, 2002: 1136; Galor, 2005: 201], not as a case that deserves explanation.

This paper attempts to place the French case in the current discussion on Malthusian dynamics in pre-industrial Europe, which so far has been concentrated almost exclusively on England. It contributes to the discussion by assessing the hypothesis of whether the long-term evolution of demographic and economic variables between 1740 and the end of the nineteenth century in France can be interpreted within a Malthusian model or not. In terms of methodology, recognising the empirical challenges assessing Malthusian systems impose, I use here two complementary approaches. On the one hand, I study the long-term equilibrium relationships between fertility, mortality, marriages and income using a vector-autoregressive (VAR) specification. In this I follow the initiative of Eckstein *et al.* [1985] that suggested the use of VARs to acknowledge the multivariate nature of these kind of systems, and the recent work of Møller and Sharp [2008] that provides a full explicit Malthusian model with a straightforward empirical correlation which, among other properties, has testable predictions for situations where the system is *out of equilibrium*, of particular relevance for a transitional period as the one analysed here. On the other hand, to gain further insight into the dynamics of the transition, I enhance and extend the analysis carried out by Weir [1983, 1984] on short-term variations of demographic and income variables in three ways. Firstly, Weir had to rely upon grain prices as a proxy for the standard of living, but here I exploit recently estimated series of real wages [Allen, 2001] to calculate responses to a more appropriate measure of income and to assess the accuracy of his estimations. Secondly, Weir studied three distinct periods ending in 1870. Since the transition was still going on in the late nineteenth century, I extended the period of analysis to include a fourth phase to shed some light on the overall dynamics of the transition. Lastly, as done by Guinnane and Ogilvie [2008], I also pay a closer look at the regressions producing the short-run estimates and show that something can be learned from those cases where regressions do not really fit the data well.

Overall, results suggest that during the period under study in France the assumption of a Malthusian model is not sustained, though many of its components were indeed present: there was equilibrium between births and deaths, the positive check was strong, and marriages still explained a large part of births (especially in the earlier period). The nature of the preventive check seems nevertheless to be changing during the nineteenth century, and the period between the start of the French revolution and the end of the Napoleonic wars appears to be the main watershed. By the end of the nineteenth century, the preventive check seems to revert in direction and the positive check virtually disappears.

PERSISTENCE OF MALTHUS

Arguably, two factors have decisively contributed to the persistence of Malthus' model in the academic literature: its relative simplicity and the fact that in many contexts it seems to work fairly well. As economists now understand it, [2] Malthusian theory is based upon two basic ideas: checks to population growth and diminishing returns in agriculture, concepts that are connected in a general equilibrium framework where the stability of the system is maintained in the long term. This is done through two basic mechanisms, a preventive and a positive check. The former imply some implicit interaction between births and means of subsistence whereas the second is associated with the higher mortality faced by families when such a restraint is not put into practice. Those ideas are usually formalised as relationships between income or real wages (i.e. the means of subsistence) and vital rates (i.e. fertility and mortality):

$$\text{(1)} \qquad \text{births} = b\left(\text{real wage}\right)$$

$$\text{(2)} \qquad \text{deaths} = d\left(\text{real wage}\right)$$

When in the current literature either the preventive or the positive checks are mentioned, these are technically meant to mean $b' > 0$ and $d' < 0$.[3] The interpretation of the shape of the death curve is straightforward. For very low income (here I use the terms 'income' and 'real wages' interchangeably), survival is impossible and death rates are extremely high. As real wage increases, chances of

[2] There is, of course, some debate on what Malthus said and what he actually meant [see e.g. Hollander, 1997: 15-18]. I will concentrate here on the modern (economic) interpretation of his theory.

[3] A careful reading of Malthus suggests also an interpretation where $b' > 0$ indicates a preventive check, but $b' = 0$ is what defines a positive check, as the assumption of $d' < 0$ is plausibly assumed to be valid throughout, but the other terminology is the one that is commonly used. Some authors refer to these alternative assumptions on the behaviour of birth rates in response to income as low- ($b' > 0$) or high-pressure ($b' \to 0$) equilibriums [e.g. Weir, 1984].

survival also increase and population death rate falls. But this fall eventually reaches a biological limit, as after some threshold death rates must become irresponsive to further increases in wages. The actual position of this curve with respect to the origin is dependent on external shocks (weather, natural disasters, etc.) and the technology to produce food or cure diseases. Things like good or bad harvests, or earthquakes can generate short-term movements of the curve, and the constant accumulation of medical knowledge is expected to drive it secularly towards the origin.

Biological factors also play a big role in shaping the birth curves. For very low levels of income this can happen at two levels. One direct influence has to do with malnutrition, which is associated with increases in the age of first menstruation, reduction in the age of menopause, increases in the rate of spontaneous abortion, an-ovulation and amenorrhoea.[4] For higher levels of income, biology puts a ceiling to fertility since families cannot have an infinite number of children. The positive slope in the birth curve is nevertheless normally seen as a consequence of fertility being channelled through marriages [Lee, 1997: 1065]. Malthus pointed out that *"a foresight of the difficulties attending the rearing of the family acts as a preventive check* " [Malthus, 1985 (1830): 89] and one way to interpret this is by assuming couples will not regularly control fertility once they are married, but they could delay marriage until they have the means to support a family. Under this view, it is *not* wages that directly affect the number of births, but wages influencing the number of marriages and *these*, by altering the stock of couples at risk of reproducing, affect fertility. In this context, (1) above should really be interpreted as the interaction of these two functions:

(3) $$\text{marriages} = m(\text{real wage})$$

(1)' $$\text{births} = \tilde{b}(\text{stock of marriages})$$

In this case, we have that $m' > 0$ and $\tilde{b}' > 0$. The distinction between high or low pressure equilibrium (i.e. more or less 'prudent' in Malthus' terms) could then well be a cultural one, depending on how social norms affect the relationships

[4] An indirect influence is also present through lower reproduction chances due to increase in abstinence, decrease of libido, or decrease in coital frequency as a result of psychological stress or because one or both partners search for means to obtain food.

between income and marriages (e.g. the presence of extended families or not), and stock of marriages to births (e.g. customs in breast-feeding, abstinence in certain periods of the year, etc.). Under high-pressure equilibrium marriages are relatively insensitive to income, hence the shape of the curve is mainly dominated by biological factors and some customs of married couples. Under low pressure, on the other hand, social norms can lower the rate at which births increase with income, generating an equilibrium with lower fertility and mortality and higher wages. In any case, the relevant thing about the interpretation of this low pressure situation is that it does not imply that families decide their size in an active sense: couples have just as many children as they can, but they decide whether or not they have enough resources to get married and support them.

The interaction of demographic variables determines an equilibrium (natural) wage where there is no population growth. Diminishing returns to agriculture then play the central role in putting a limit on the ever-increasing population, as food would become eventually less abundant and war, famine and misery would condemn societies to a meagre subsistence standard of living. This is modelled in terms of labour market dynamics as a completely inelastic labour supply (proportional to population size) that meets a labour demand and that shows a negative slope due to the presence of diminishing returns. If a positive technological shock shifts the labour demand outwards inducing an increase in real wages, higher incomes stimulate an increase in population diminishing returns to labour in the agricultural sector will drive income back to the natural level. As this last example shows, this equilibrium model depends on the characteristics of the labour market and, more specifically, depends on such a market being dominated by a limiting factor (land if agriculture is the main sector) that induces diminishing returns to labour. Its ultimate prediction is that technical and social progress cannot improve the human lot as long as population behaviour remains what it is; that is, while the death and the birth curves do not change.

Was Malthus right?

Many researchers have been trying to assess the Malthusian hypotheses empirically and even more have decided to debate about it.[5] The framework, as simple as it looks, has some predictions that seem to coincide with the data. One example comes from the comparison of different regions of the world, where the dichotomy 'high wages'-'late marriage pattern' (in Western Europe), and 'low wages'–'early marriage pattern' (elsewhere) seems to correspond with low- and high-pressure equilibria [Hajnal, 1965: 102-104].[6] The apparent long-term stagnation of real wages in pre-industrial times is yet another example of that. For the major part of human history income per capita seems to have remained at subsistence levels, with improvements in the standards of living (if any) only marginal or temporary [e.g. Galor, 2005: 178-179, Clark, 2005a: 507], which is consistent with a situation where diminishing returns in agriculture impose a strong constraint and technological change is 'eaten-up' through increased population while accompanied by fluctuation in vital rates. Following this logic, the model also provides an insightful interpretation of the curious pattern followed by England that showed a continuous increase of fertility in the second part of the eighteenth century. There is evidence that at least from the seventeenth century England experienced a secular increase in wages [Allen, 2001] which could have well been the consequence of a period of systematic improvements in technology [see, e.g. Mokyr, 1999: 17-28]. If, as suggested by Malthus, Britain was a low-pressure region, this increase in income lead to a real wage above that of equilibrium, generating both an upward pressure on births and a downward pressure on death, both consistent with the data [Wrigley and Schofield, 1981: 531-535].

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[5] Lately, Greg Clark has become one of the most avid supporters of a Malthusian interpretation of history. See his latest book [Clark, 2007] and the debate in the *European Review of Economic History* [Grantham, 2008; McCloskey, 2008; Persson, 2008; Voth, 2008; and Clark, 2008].

[6] Recently De Moor and van Zanden [2005] have suggested that in reality this preventive check through marriage appeared only in the fifteenth century as a consequence of Catholic ideology emphasising the role of individual choice in marriage and women having more bargaining power following the Black Death. Voigtlander and Voth [2009] have recently proposed an alternative reading.

Malthusian logic seems to do a good job describing 'big picture' dynamics but, at the moment of being formally tested, it has become elusive. To be sure, this does not come from an intrinsic complexity of the model, but from the empirical problems caused by the circularity of the implicit argument and consequent endogeneity of all variables involved. The literature has dealt with this in at least two ways. One has been to look into the short-run fluctuations in demographic and economic variables. By concentrating only on the reaction of series to short-run changes, this approach avoids identification problems by treating shocks (particularly to real wages, normally proxied using grain prices) as more or less exogenous. This was first explored by Lee [1981] using Wrigley and Schofield's [1981] data, but quickly replicated for other areas of the world.[7] Although estimation of effects varies from country to country, in general all these studies are very supportive of the Malthusian interpretation. Fertility is shown to have a negative reaction to shocks in prices whereas mortality has the opposite effect, hence providing strong evidence in favour of the preventive and positive checks.

The other way in which the literature has approached Malthusian dynamics has been by studying long-term relationships between variables using time-series econometric techniques, which in general have high data requirements and hence have limited the case studies basically only to England.[8] This approach goes back at least to the first studies of Lee in the early 1970s [e.g. Lee, 1973] but only in the last few years, stimulated by the unified growth debate and the advancements in econometric techniques that proved to be suitable for this kind of study, has the discussion intensified. Two general lines of research can be more or less identified. On the one hand, most works start from a structural model that defines the dynamics of the system and, though not identical, alternative formulations share most characteristics [Bailey and Chambers, 1993: 347-348]. They do tend to differ substantially, however, in the quantitative tools used to assess

[7] Galloway [1988] provides perhaps the most comprehensive study for pre-industrial Europe, and Weir [1984] remains as the main reference regarding France. Galloway [1986] also carried out this exercise for the smaller area of Rouen in France.

[8] The extensive work on parish registries done by Wrigley and Schofield [1981] has provided scholars with a substantial amount of information that helped to boost research on England, while recent estimates of real wages by Allen [2001] and Clark [2005b] have facilitated the assessment of Malthusian hypotheses. Sweden is one of the few other countries for which considerable datasets are available and have stimulated empirical studies in this direction [e.g. Eckstein *et al.*,1985; Hagnell, 1991].

those structural models in order to solve the endogeneity problem.[9] Some works, on the other hand, avoid imposing a particular structural model and apply instead a vector auto-regressive (VAR) approach to exploit regularities in the data to study the responses to shocks when all variables are endogenous.[10] Only recently a couple of (still unpublished) studies somewhat try to bridge these two approaches.[11] On one side or another, evidence is disappointingly ambiguous. Some strongly support the presence of Malthusian dynamics in the English case [e.g. Lee, 1981; Bailey and Chambers, 1993; Møller and Sharp, 2008], whereas some cast serious doubts [e.g. Lee and Anderson, 2002; Nicolini, 2007; Crafts and Mills, 2009]. Despite the inconclusiveness of the overall literature these studies have contributed to further our knowledge of demographic and economic dynamics in pre-industrial England and their extension to other parts of Europe appears to be a necessary exercise to put the results in perspective.

PERSISTENCE IN MALTHUS?

There are few examples of papers explicitly discussing the Malthusian hypotheses in France [e.g. Weir, 1983, 1984; or Wrigley, 1985] and, to my knowledge, none using the recently developed VAR techniques or any other multivariate approach. Since the growing debate on pre-industrial population dynamics has recently directed its attention towards this kind of analyses it makes sense to do the effort to place the discussion of the French case in those terms. Of all the multivariate formulations I pointed out in the previous section, at least a couple

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[9] Lee and Loschky [1987], for example, use three-stage least-squares to study the oscillations implied in the Malthusian model, while Stavins [1988] and Tsoulouhas [1992] choose two-stage least-squares to estimate an enhanced version of Lee [1973]. Bailey and Chambers [1993], recognising potential problems of non-stationarity in some of the series, opted for an error-correction specification for cointegrating relationships. More recently, Lee and Anderson [2002] used Kalman filtering to generate a state-space representation of the dynamic system. All these studies correspond to the English case.

[10] The seminal paper by Eckstein *et al.* [1985] on pre-industrial Sweden was followed by another on the same country by Hagnell [1991] and, lately, by Nicolini [2007] using English data.

[11] Crafts and Mills [2009] estimate both a structural model [Lee and Anderson, 2002] and a VAR specification [Nicolini, 2007] using new estimates of real wages by Clark [2005b]. On a completely different strategy, Møller and Sharp [2008] propose a structural model and relate the statistical implications of that model to a VAR specification that they subsequently test empirically.

of reasons made me follow the approach developed by Møller and Sharp [2008]. Different from many, it openly incorporates the role of marriages, absent in many other formulations and potentially very important to understand the intrinsic logic of the model (and eventual departures from it). Also, it discusses explicitly the statistical properties of the series implied by the theoretical model, a thing only done marginally in the literature [Møller, 2008], but of crucial importance for drawing conclusions from the empirical study. This is of particular relevance for the case analysed here, as the persistent decline could well indicate a departure from an equilibrium that only a well-specified model can properly recognise. In what follows, then, unless otherwise stated, I draw heavily on their work to assess the Malthusian hypotheses in France.

An empirical Malthusian model

In its most basic form, empirical formulation of the Malthusian model is done in terms of a linear system in the crude birth rate, crude death rate and the logarithm of real wages $(b_t, d_t, \ln w_t)$. Depending on the interest of the researcher, this vector is sometimes expanded to include as well some measure of marriages and the total population (N_t):

(4) $$b_t = \zeta \frac{M_t^f}{N_t}$$

(5) $$d_t = \eta^d - \phi^d \ln w_t + \varepsilon_t^d$$

(6) $$m_t = \eta^m + \phi^m \ln w_t + \varepsilon_t^m$$

As usual, ε_t^i indicates the error term at time *t* of variable *i*, and all the other are parameters. This specification is in essence no different from most other formulations [Bailey and Chambers, 1993: 346-348], but it makes explicit not only that the link between real wages and births is done *through* marriages, but also that the real determinant of the birth rate is the *stock* of fertile marriages (M_t^f).[12]

[12] This ignores the contribution of illegitimate births but, to be sure, their contribution to total births was only marginal, in the order of 2%-5% for the late eighteenth, early nineteenth century [Blayo, 1975: 68], perhaps somewhat higher only towards the end of the nineteenth century [INSEE, 1961: 32, 36].

This is somewhat of a departure from much of the literature that does not specify the way in which marriage affects the model or simply ignores the channelling through marriages [e.g. Lee and Anderson, 2002; Nicolini, 2007].[13] In practice, data on the stock of fertile marriages – if available – is relatively unreliable, so one counts only with marriages as a proportion of the population.[14] Møller and Sharp [2008: section 2.2] suggest that if some lags are included, (4) could be approximated with:

$$b_t \simeq \eta^b + \sum_{i=1}^{s}\left(\rho_i^{bb} b_{t-i} - \rho_i^{bd} d_{t-i} + \rho_i^{bm} m_{t-i}\right) + \varepsilon_t^b \tag{7}$$

Here, the coefficient ρ_i^{jk} indicates the size of the impact the *i*th lag of variable *k* has on variable *j* (in this case, births). The other part of the system is formed by the equations that determine the structure of labour demand and population dynamics:

$$\ln w_t = \eta^w - \lambda^w \ln N_t + \ln A_t \tag{8}$$

$$\ln A_t = c_A + \ln A_{t-1} + \varepsilon_t^A \tag{9}$$

$$N_t = N_{t-1} + b_{t-1} - d_{t-1} \tag{10}$$

Here A_t stands for factors that affect the labour demand and could probably be best interpreted as the level of technology. Equation (9) then determines the way in which that technology evolves through time (in this case, following a random

[13] Bailey and Chambers recognise that it is the stock of marriage that determine births and not the flow when modelling the system they test, but given the lack of information on stocks they stick to a model where fertility is not affected by marriages [Bailey and Chambers, 1993: 346].

[14] David Weir has estimated the stock of married fertile women for France every five years between 1740 and 1900 [Weir, 1994] and with information on annual marriages and some assumption on the death rate of married women, some reasonable proxy could be generated by interpolation. Nevertheless, such interpolated series would be too ‘dirty’ to perform the kind of time series study I am pursuing here.

walk with a trend),[15] whereas the last equation simply establishes the demographic equilibrium (where, as usual, it is assumed that there is no migration).

One of the key contributions of Møller and Sharp [2008] was to show that the VAR version of that system implies a very specific dynamic structure. In particular, they showed that a stable Malthusian model with those characteristics *requires* a *cointegrating* equilibrium between the stock variables (N_t, A_t), but a *stationary* equilibrium in terms of the rates $(b_t, d_t, m_t, \ln w_t)$. This is an interesting result that has not been explicitly discussed in the literature.[16] Intuitively, it also makes sense. If the birth and death curves are deterministic in the sense that are a consequence of relatively external factors (such as the knowledge of medicine or cultural norms) and establish a stable subsistence level real wage, fluctuations of these variables through time must be mean-reverting. This does not need to be the case with population and technology. If there are reasons to believe technological change is progressive [see e.g. Mokyr, 1999], this will probably induce movements in the demand for labour that will translate in a similar non-stationary evolution in the size of population.

This is easy to illustrate with the simple graph in Figure 1.[17] For the sake of clarity, I included the corresponding scheme in the labour market in the lower panel. As I pointed out above, the birth and death curves are largely determined by factors exogenous to the system and in equilibrium (E_R, where B = D = V and there is no population growth) they determine a long run subsistence real wage (W). It is possible for shocks to affect this equilibrium (i.e. some $\varepsilon_t^i \neq 0$ for vari-

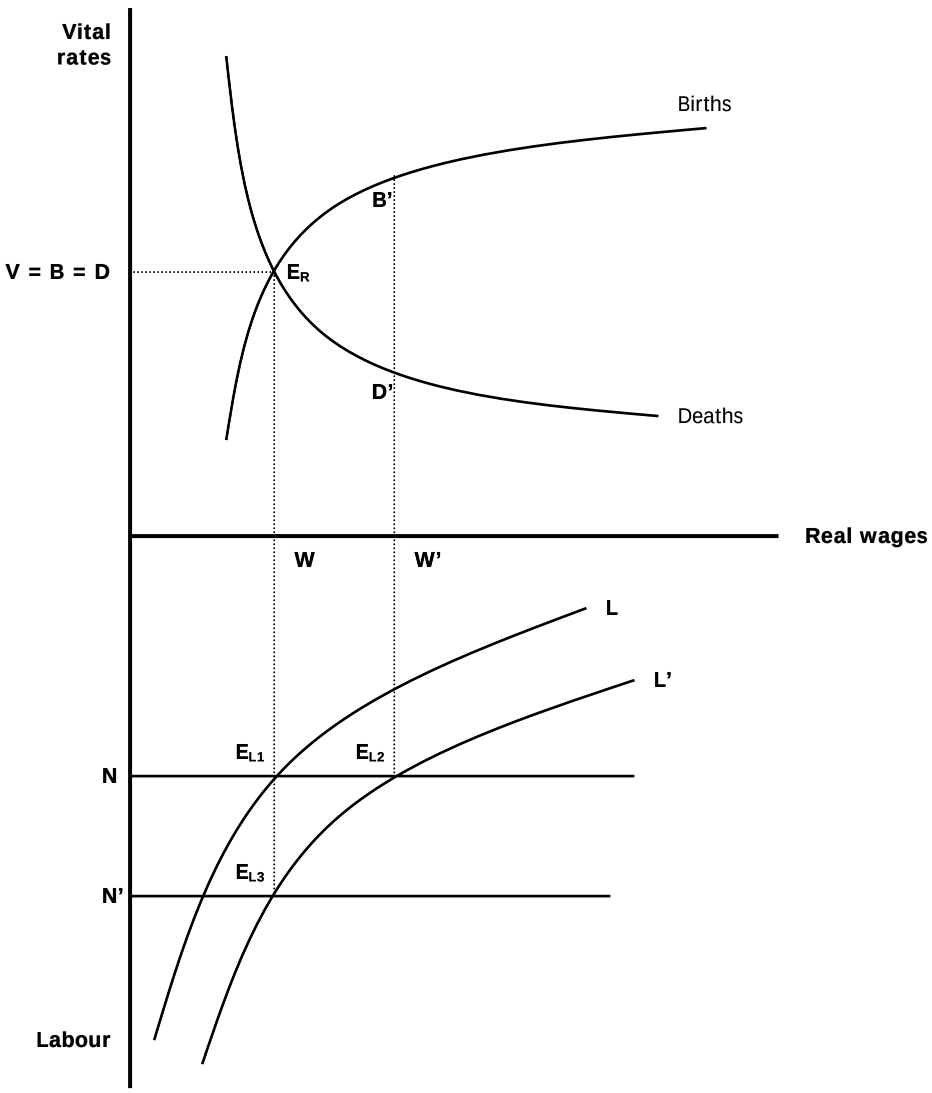
[15] Although Møller and Sharp decide to model the evolution of technology as a simple random walk [Møller and Sharp, 2008: 3], and their empirical results indeed seem to support that choice, they do discuss potential generalisations such as including a trend (i.e. a constant) or idiosyncratic technological shifts in the form of impulse dummies [Møller and Sharp, 2008: 23]. In the context of this work I opted instead for the random walk with a trend, which modifies only marginally the structure of the system they use (so my results remain comparable), but has a more intuitive appeal. As the results show later, this choice is supported by the French data.

[16] For example, Nicolini concludes that the series for England are stationary [Nicolini, 2007: 105] but he does not interpret this as an indication in favour of the Malthusian interpretation, which is what this formulation of the problem implies.

[17] This figure, or a variation of it, has been widely used in the recent literature to describe Malthusian dynamics [e.g. Lee, 1973: 593; Weir, 1984: 29; Clark and Hamilton, 2006: 2] and it goes at least back to Sauvy [1969: 23].

able *i* at time *t*), but – because it is exogenously determined - the system will tend to converge back to it through fluctuations in the rate of population growth. That is, unless something alters the intrinsic logic of those curves (like a breakthrough in medicine or a shift in social norms), in the long term all the series $\left(b_t, d_t, m_t, \ln w_t\right)$ in this system can be expected to be stationary.

Figure 1. Dynamics in a simple Malthusian system



The dynamics corresponding to the labour market are somewhat different. If a positive shock to technology drives out the labour demand, the presence of an inelastic supply will translate all the impact on wages and move the equilibrium to E_{L2}. At the level of W', birth are relatively high with respect to deaths (B' >

D'), so population will increase. It is clear that this population growth will translate into a shift (out) of the labour supply, driving wages down back towards W and leading the system back to the *same* equilibrium in rates (E_R), consistent with stationarity vital rates and real wages. These same dynamics, however, will lead to a *different* equilibrium in terms of population and technology, as the level of these variables in the initial state (E_{L1}) would be different of that in the final state (E_{L3}). Still, they are likely to move somewhat together. It is in this sense that the system (N_t, A_t) could be non-stationary, yet expected (perhaps) to be cointegrated. The dynamics of the system could then be studied by looking at the interaction of (N_t, A_t) or that of $(b_t, d_t, m_t, \ln w_t)$. Due to limitations in measuring A_t (i.e. the degree of technology) in any sensible way, one can then turn to study the properties the generalised empirical version of the system in rates, which is simply a reformulation of the system determined by (5) to (10):

(11) $$b_t \simeq \eta^b + \rho_0^{bm} m_t + \sum_{i=1}^{s} \left(\rho_i^{bb} b_{t-i} - \rho_i^{bd} d_{t-i} + \rho_i^{bm} m_{t-i} \right) + \varepsilon_t^b$$

(12) $$d_t \simeq \eta^d + \theta_0^d \ln w_t + \sum_{i=1}^{s} \left(\rho_i^{dd} d_{t-i} + \theta_i^d \ln w_{t-i} \right) + \varepsilon_t^d$$

(13) $$m_t \simeq \eta^m + \theta_0^m \ln w_t + \sum_{i=1}^{s} \left(\rho_i^{mm} m_{t-i} + \theta_i^m \ln w_{t-i} \right) + \varepsilon_t^m$$

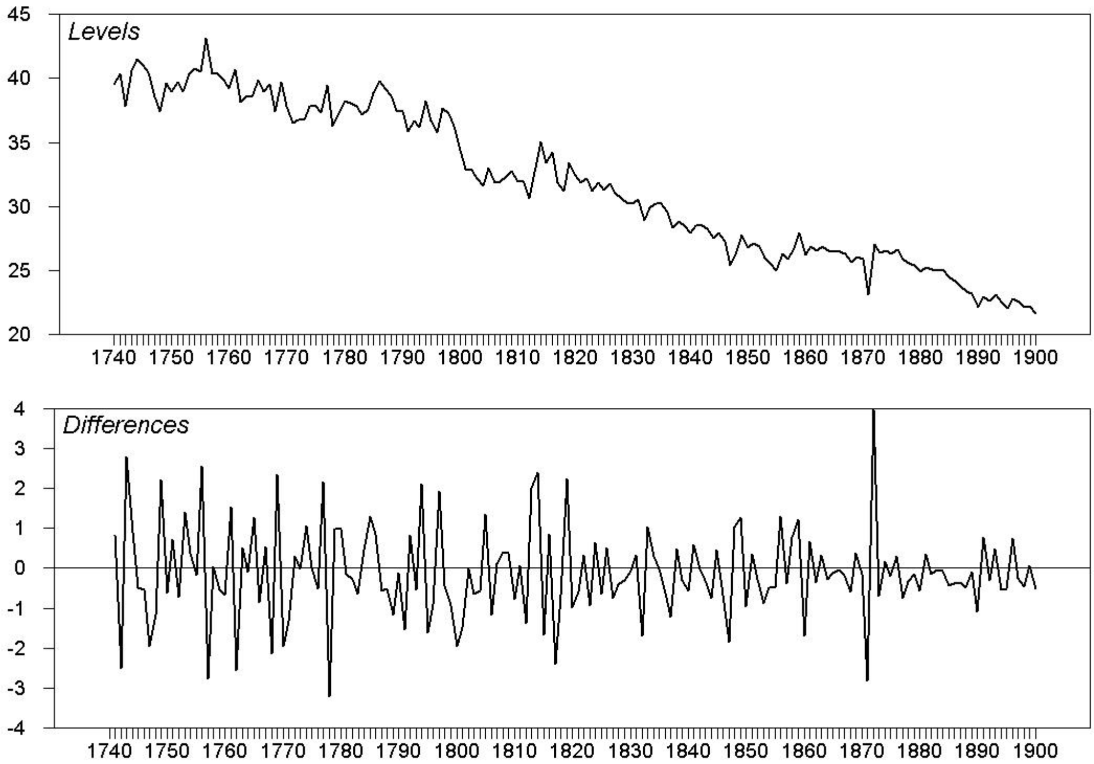
(14) $$\ln w_t = \ln w_{t-1} + c_A - \lambda^w \left(b_{t-1} - d_{t-1} \right) + \varepsilon_t^A$$

By studying the properties of the system defined by (11) to (14) in terms of the vector $(\Delta b_t, \Delta d_t, \Delta m_t, \Delta \ln w_t)$ – that is, in its error-correction form – we can assess if stationarity is fulfilled as expected or not and, if it is not, ask ourselves what determines the departure from it.

Evolution of series and specification of the model

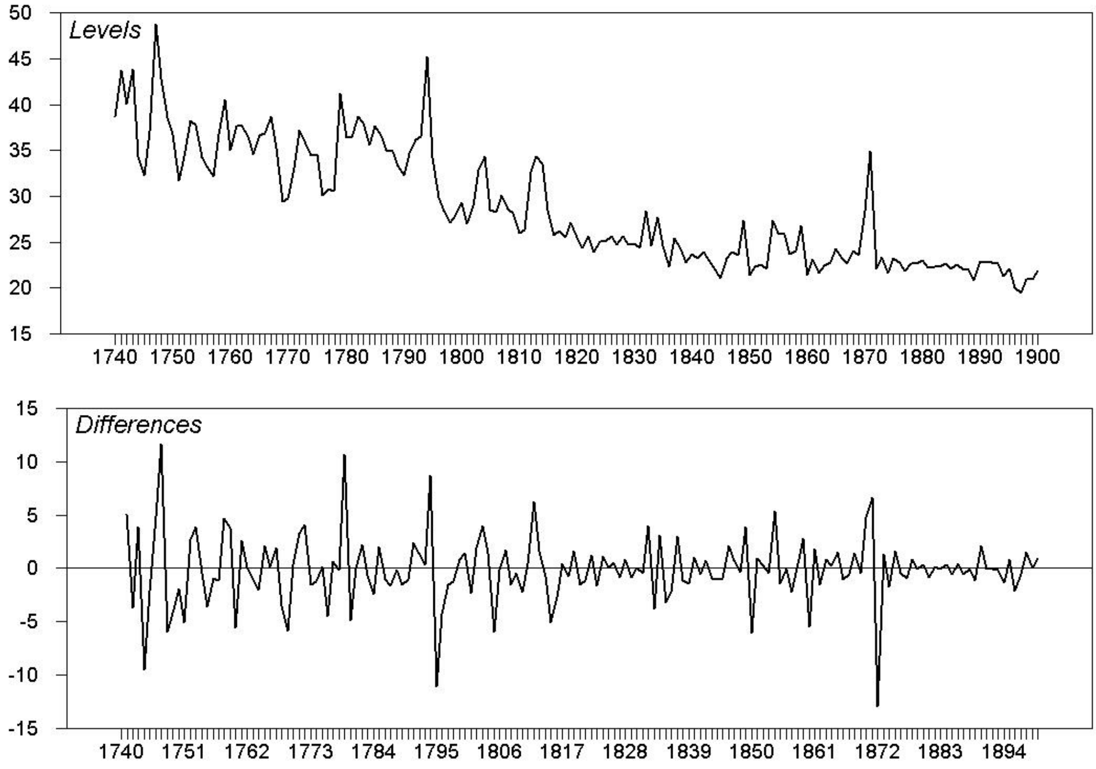
To evaluate the model described in the previous section we need to study the time series properties of the relevant series. Figures 2 to 5 show the evolution of crude birth, death and marriage rates, the natural logarithm of real wages and

Figure 2. **Fertility (crude birth rate, levels and first differences) in France, 1740-1900**



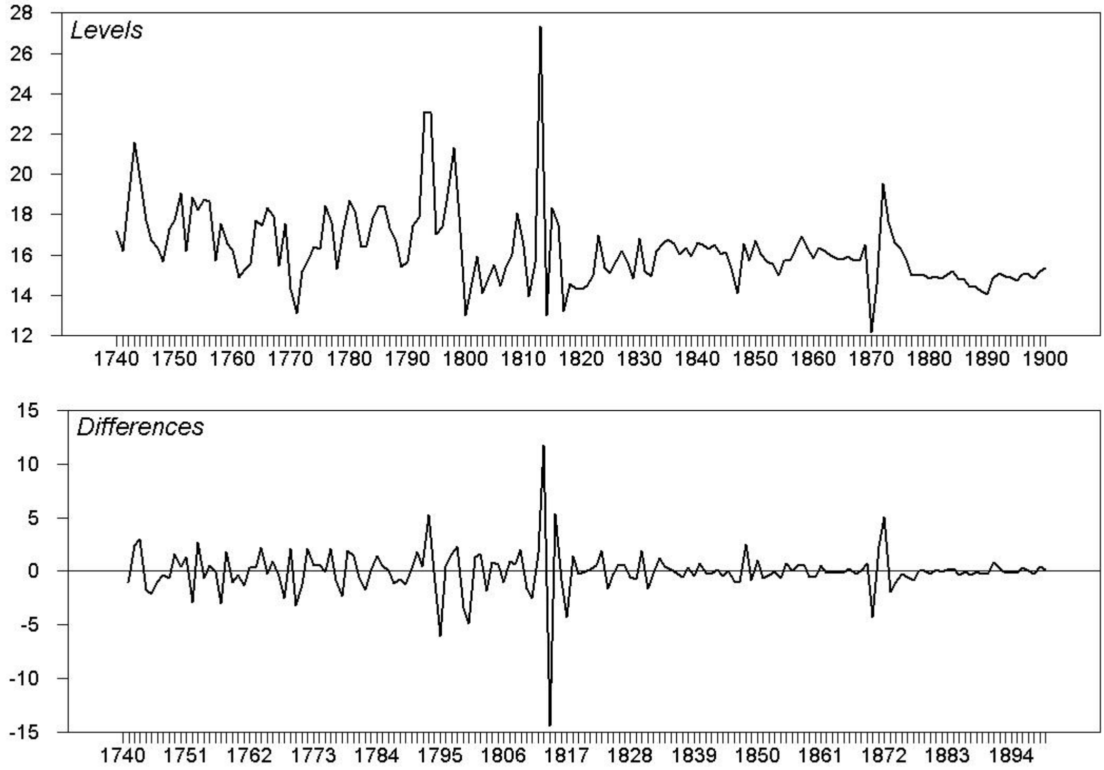
Sources: INED [1977] and Chesnais [1992]. See Appendix I for details.

Figure 3. **Mortality (crude death rate, levels and first differences) in France, 1740-1900**



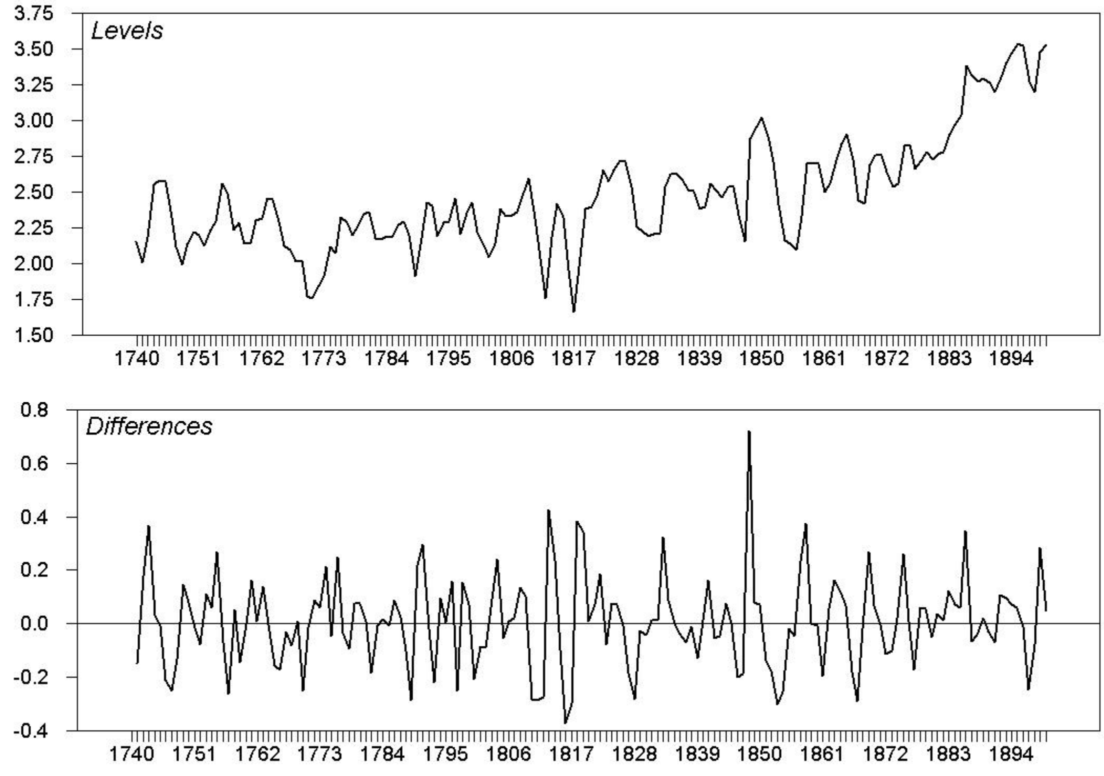
Sources: INED [1977] and Chesnais [1992]. See Appendix I for details.

Figure 4. **Nuptiality (crude marriage rate, levels and first differences) in France, 1740-1900**



Sources: INED [1977] and Mitchell [1998]. See Appendix I for details.

Figure 5. **Real wages (levels and first differences) in France, 1740-1900**



Sources: Allen [2001] and Labrousse *et al.* [1970]. See Appendix I for details.

their first differences.[18] In broad terms, visual inspection suggests a degree of nonstationarity in some series (most notably birth rates), but not so clearly in others, such as marriage rates. In any case, first differences look stationary for all four series.[19] Volatility appears to change somewhat after the Napoleonic era, especially in the marriage rate, and perhaps in the death rate as well, so there could be some issues of heteroscedasticity. Births and deaths both show a long-term decline that is accompanied by a secular increase in real wages, which is consistent with the basic prediction of homeostasis suggested in (14) by which we would expect that births and deaths remain in equilibrium, and a secular decline in their values could be associated with an increase in real wages.

Many well-known historical events appear to have had an impact in some of the variables. The French Revolution seems to have temporarily depressed birth and increased mortality, while having a positive effect on marriages, and the Franco-Prussian War had a considerable impact on all series. The marriage series has some rather substantial outliers, in particular during the years between the Revolution and of the fall of Napoleon. Revolutionary legislation facilitated marriage in several ways, hence induced a positive shock in the early 1790s.[20] The introduction of conscription by the Jourdan-Delbrel law in 1798 exempted married men, which perhaps explains the upsurge in the marriage rate that year and the unusual rate of 1813 when, after the Russian campaign, conscription was exceptionally extended (i.e. including more than one annual class).

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[18] See Appendix I for details on data sources.

[19] Later, I will show formally that the series are indeed non-stationary, but to maintain consistency with Møller and Sharp [2008] I will do so by testing stationarity as a system property within the VAR formulation [Juselius, 2006] instead of the univariate approach followed by others [e.g. Bailey and Chambers, 1993; Nicolini, 2007]. Nevertheless, if assessed with the standard Dickey-Fuller or Phillips-Perron tests, stationarity is rejected in the case of births (DF = -0.937, PP = -0.319) and real wages (DF = -2.159, PP = -1.909), not rejected for marriages (DF = -8.529, PP = -8.747), and rejected at the 1% level, but not at the 10% level for the deaths (DF = -3.308, PP = -2.697). Non-stationarity of first differences is strongly rejected for all series.

[20] This was done in several ways. Revolutionary laws lowered the age before which parental consent was needed, authorised divorce and, by making it a civil contract independent of the Church, it avoided the prohibitions of marriage in certain periods such as Advent and Lent [Bergeron, 1981: 110].

Empirical analysis of these series can be done using a VAR specification such as that defined by the system (11) to (14) that, without loss of generalisation, could be written in the error correction form [Hendry, 1995: 330-331; Hendry and Juselius, 2001: 90; Juselius, 2006: 61]:

$$\Delta X_t = \Pi_0 + \Pi X_{t-1} + \sum_{j=1}^{k} \Gamma_j \Delta X_{t-j} + \varepsilon_t \qquad (15)$$

As usual, X_t is the vector of variables, Π the long-term matrix, Γ_j the short-run response matrix for lag *j*, ε_t is the vector of normally distributed errors, and Π_0 the vector of constants. In this particular case we have that $X_t = [b_t, d_t, m_t, \ln w_t]$, and the empirical assessment begins with the estimation of this unrestricted model.[21] In general, it has been suggested that a model including only two lags should be sufficient to describe most dynamic structures [Juselius, 2006: 72], so I begin estimating such a model, and then apply different tests of lag length determination. Both the Schwartz and the Hannan-Quinn criterion were minimised when using two lags, hence favouring such structure, but the lag reduction test rejected all reductions to one or two lags, suggesting three instead.[22] Since using three lags also makes sense in other respects when dealing with a model that tries to explain fertility behaviour,[23] I chose $k = 3$. Running the baseline model with three lags produced a residual structure that violated some of the basic assumptions of the VAR model to produce meaningful estimation. Although autocorrelation of errors did not seem to be a major problem, joint nor-

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[21] Most relevant empirical results are mentioned in the text, but a detailed account of statistical output omitted here appears in Appendix II.

[22] For VAR(1), VAR(2) and VAR(3) the Schwartz criterion was -0.076, -0.494 and -0.199, whereas the H-Q criterion gave -0.319, -0.931, and -0.830, in both cases minimised for VAR(2). Reduction from five lags to one or two was strongly rejected, but reduction from five or four to three, was not (*p*-values 0.3879 and 0.3221).

[23] As pointed out in the literature [e.g. Yule, 1906: 125-126; Lee and Anderson, 2002: 207] birth dynamics imply some particular sort of delay. On the one hand, there is a nine-month lag between conception and actual birth. On the other, as there is a period of sterility after delivery that varies with biological factors (as nutrition) and cultural ones (as breastfeeding practices) spikes of fertility substantially lowers the population at risk of having children the following year, inducing cycles of more than two years.

mality was clearly rejected.[24] This latter is usually a serious problem, as lack of normality – especially if due to skewness – could make results meaningless [Hendry and Juselius, 2001: 83].[25] Looking into the residuals of individual series it becomes apparent that this non-normality is due to the presence of some of the outliers I pointed out when discussing the evolution of the series, most notably during the period between the Revolution and the fall of Napoleon, and the Franco-Prussian War. One way to control for these is to use a series of dummies and, despite the inherent instability of the period, only eight of them allowed me to achieve reasonable results.[26] The inclusion of dummies then generates a model that passes most normality tests with only minor impact on autocorrelation of errors.[27] In other respects the model also seem to be well specified as the tests of exclusion, stationarity and weak exogeneity are passed (see Table A5 in Appendix II).

The next step is to determine the cointegration rank. Only a correct choice of cointegration rank will allow making meaningful inference [Hendry and Juselius, 2001: 101], and a reduced rank would imply that the system in rates is non-stationary, hence *rejecting a crucial assumption* of the Malthusian model. Figuring out the right rank is nevertheless a difficult task and one normally has to rely on several tools [Hendry and Juselius, 2001: 106]. As shown extensively in Appendix II, all tests to determine cointegration rank indicate the presence of a

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[24] The LM tests did not suggest problems of autocorrelation of any order. The overall test of Normality gave $\chi^2(8) = 117.49$ (p-value = 0.04) and individually all but the births series strongly rejected the hypothesis of normality.

[25] There were also some issues regarding residual heteroscedasticity. The multivariate LM tests for no conditional heteroscedasticity were rejected for every order. Also, as expected after seeing the graphs of first differences, in individual series the most serious problem was with the marriage rate (p-value = 0.01), whereas the death rate only showed moderate, non-significant ARCH effects (p-value = 0.10). Nevertheless, it is agreed in the literature that statistical inference is moderately robust to residual heteroscedasticity [Hendry and Juselius, 2001: 83; Rahbek *et al.*, 2002].

[26] See a detailed description of how these dummies were constructed in Appendix II.

[27] Now the LM test shows there is some autocorrelation of first order left: $\chi^2(16) = 33.21$ (p-value = 0.01), but overall Normality improves: $\chi^2(8) = 14.32$ (p-value = 0.07). All individual series now pass the Normality tests, without showing any substantial skewness or kurtosis, and overall ARCH effects are reduced. Also, lag length determination test with this specification provides further support on the choice of VAR(3), as now the H-Q criterion also points in that direction.

unit root.[28] This suggests that the system has a reduced rank, implying that the data cannot really support the assumption of a Malthusian equilibrium. Under the assumption of *rank = 3* the dynamic structure predicts persistence in the rates model, [29] and this is simply not consistent with the model described by the system (11) to (14).

In other words, the analysis carried out so far suggests that within the framework specified by the model, the data does not support the hypothesis of stationarity in rates. We can, nevertheless, study the potential sources of this persistence to learn some things about the dynamic structure of the system, especially if there are cointegration relationships hidden in Π.

Sources of persistence in a Malthusian model

The presence of reduced rank implies that (15) could be rewritten as:

(16) $$\Delta X_t = \alpha\gamma' + \alpha\beta' X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + +\Theta D_t + \varepsilon_t$$

By looking at the characteristics of α and, specially, β we can learn about the dynamics of the system:

(17) $$\alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{21} & \beta_{31} & \beta_{41} & \gamma_1 \\ \beta_{12} & \beta_{22} & \beta_{32} & \beta_{42} & \gamma_2 \\ \beta_{13} & \beta_{23} & \beta_{33} & \beta_{43} & \gamma_3 \end{pmatrix} \begin{pmatrix} b_{t-1} \\ d_{t-1} \\ m_{t-1} \\ \ln w_{t-1} \\ 1 \end{pmatrix}$$

One of the interesting features of this formalisation of the Malthusian model is that it also suggests *ways* in which deviations from the theoretical equilibrium

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[28] The roots of the companion matrix, the trace statistic, the evolution of the recursive trace statistic, the graph of the cointegration relationships and the significance of the coefficients of the α matrix all point towards a rank of at least two, and most likely three. See Appendix II.

[29] In the context of time series analysis, series are said to be persistent if they are found to have a unit root [Møller, 2008: 5].

could be assessed and tested. In particular, Møller and Sharp [2008] showed that persistence can have, within this model, two alternative sources:

(18) $$\lambda^{w} = 0$$

(18) $$\left(\frac{\theta^{m}}{1-\rho^{mm}}\right)\left(\frac{\rho_0^{bm}+\rho^{bm}}{1-\rho^{bb}-\rho^{bd}}\right)=\left(\frac{\theta_0^{d}+\theta^{d}}{1-\rho^{dd}}\right) \qquad \text{with } \theta^{g}=\sum_{i=1}^{s}\theta_i^{g},\ \rho^{gh}=\sum_{i=1}^{s}\rho_i^{gh}$$

The first of these simply implies that real wages are not responsive to population increases; that is, the labour demand is flat. To test this hypothesis, we should simultaneously impose weak exogeneity of real wages (that is, the row α_{4i} should be zeros) and the presence of the standard (positive and preventive) checks [Møller and Sharp, 2008: 17]:

(20) $$\begin{pmatrix}\beta\\ \gamma\end{pmatrix}^{'}\begin{pmatrix}X_{t-1}\\ 1\end{pmatrix}=\begin{pmatrix}0 & \beta_{21} & 0 & \beta_{41} & \gamma_1\\ 0 & 0 & \beta_{32} & \beta_{42} & \gamma_2\\ \beta_{13} & 0 & \beta_{33} & 0 & \gamma_3\end{pmatrix}\begin{pmatrix}b_{t-1}\\ d_{t-1}\\ m_{t-1}\\ \ln w_{t-1}\\ 1\end{pmatrix}$$

In the equation above, the first cointegration relationship defines the positive check, and the other two the preventive check (first impact of real wages on marriages, and then of marriages over births). This restriction was convincingly rejected by the data (p-value was 0.001), hence suggesting the first potential source of persistence (i.e. $\lambda^{w}=0$) is not really valid. This particular result is probably not very surprising, as in a context like eighteenth and nineteenth century France, a relatively populated rural country with land as a limiting factor,[30] it is hard to imagine not finding evidence of diminishing returns (that would imply instead $\lambda^{w}>0$).

[30] By 1850 three-quarters of the country was still rural and even on the eve of the First World War the main source of income was the agricultural sector [O'Brien, 1996]. All through the nineteenth century agriculture dominated the economy, so the population feedback on wages probably played a major role.

The other source of persistence, that suggested by (19), looks complex but has a rather intuitive meaning. The terms on the left-hand-side measure somehow the impact of wages on births, via marriages. Similarly, the term on the right-hand-side measures the overall impact of wages on deaths. In terms of Figure 1 what that relationship suggests is that both the birth and the death curves have the same slope, so they are either parallel or the two curves overlap. For this we do not need to impose weak exogeneity, but one of the 'checks' should be replaced by the homeostasis assumption (i.e. that in equilibrium birth rate should equilibrate with death rates). The estimates of α and β given these restrictions are the following (t-values appear in parenthesis below the coefficients):

$$\alpha\begin{pmatrix}\beta\\ \gamma\end{pmatrix}'\begin{pmatrix}X_{t-1}\\ 1\end{pmatrix} = \begin{pmatrix} \underset{(-6.174)}{-0.15} & \underset{(-2.270)}{-0.03} & \underset{(-2.333)}{-0.01} \\ \underset{(4.728)}{0.39} & \underset{(1.695)}{0.06} & \underset{(0.758)}{0.01} \\ \underset{(-5.941)}{-0.22} & \underset{(-0.534)}{-0.01} & \underset{(5.624)}{0.05} \\ \underset{(-3.144)}{-0.02} & \underset{(-4.076)}{-0.01} & \underset{(1.867)}{0.00} \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 & 0 & \underset{(-6.363)}{-2.35} \\ 0 & 1 & 0 & \underset{(6.767)}{25.7} & \underset{(-10.06)}{-92.7} \\ 1 & 0 & \underset{(-9.263)}{-9.65} & 0 & \underset{(7.499)}{129.5} \end{pmatrix} \begin{pmatrix} b_{t-1} \\ d_{t-1} \\ m_{t-1} \\ \ln w_{t-1} \\ 1 \end{pmatrix}$$

These restrictions are now accepted with a p-value of 0.39. All coefficients in β are significant, and those in α have the expected sign (except that corresponding to death rates in the second relationship, which is positive instead of negative, but is insignificant). The estimates imply the following long-term relationships: [31]

(21) $b_t - d_t = 2.36$

(22) $d_t = 92.7 - 25.7 \ln w_t$

(23) $b_t = -129.5 + 9.6 m_t$

The first gives some support to the homeostasis hypothesis suggested by Malthusian theory under a relatively constant rate of growth of population. Figure 6,

[31] For someone not familiar with the VAR literature, notation can be confusing in this respect. In the way the □ matrix is expressed, each line corresponds to a vector that multiplied by the vector of variables will determine a linear combination that is stationary. Hence, each relationship can be 'solved' in terms of a particular variable, making the coefficients of all other variables switch signs.

which plots the difference between French birth and death rates throughout the period, seems to corroborate that this was the case.[32]

Figure 6. Difference between crude birth and death rates, 1740-1900

Sources: INED [1977] and Chesnais [1992]. See Appendix I for details.

The second relationship, on the other hand, indicates that the positive check was still large and significant during this period, and the third that marriage indeed had an impact on births.[33] Recursive estimations suggested the positive check was more or less stable throughout, but a comparison of forward and backward estimation for the coefficient of marriage on births indicates that results are highly dependent on the sample we take. Figure 7, for example, illustrates how that coefficient would have not been very different if we took a sample from the early period, but it would have been considerably smaller (and very close to zero) if starting from the later period.

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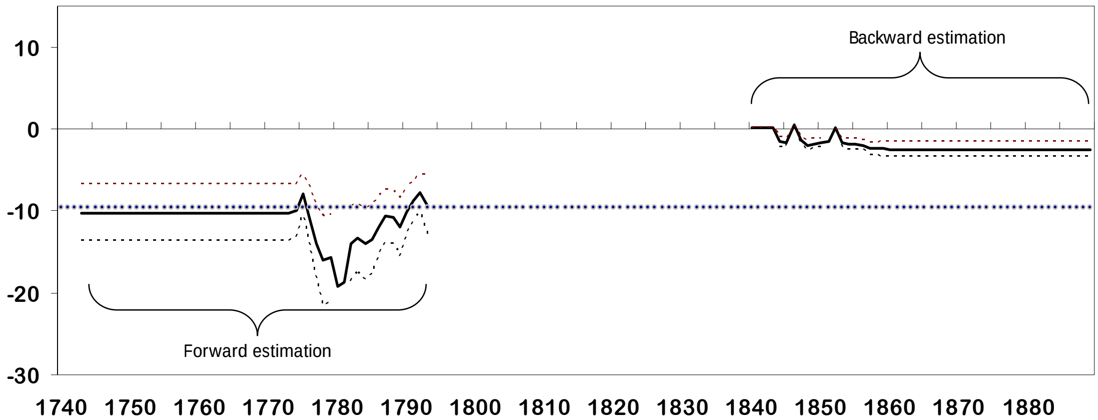
[32] This is not necessarily the case in other regions through the transition. In the case of England, for example, the process was clearly led by a simultaneous increase in fertility and decrease in mortality, leading to a non-stationary series.

[33] Compared to the results of Møller and Sharp the coefficients are large, but perhaps that is due to the fact that they included some dummies in the cointegrating space, hence taking some of the variation out of the slopes, or simply that French data for this period include (too) many shocks.

Figure 7. Forward and backward recursive estimation of the impact of marriages on births, 1740-1900



Notes: The forward estimation takes as the initial sample the period 1743-1773 and adds an observation per year (here depicted until 1793). Similarly, the backward estimation takes as reference the period 1860-1890 and estimates adding back a year (here until 1840). The dotted grey line indicates the value estimated for the full sample (-9.6).

The analysis in this section suggests that, during the period studied, France does not really fit into the Malthusian model in a strict sense. In fact, it actually suggests a specific way in which it does not fit. The system not being stationary implies that there was not an equilibrium such as the one described by the model. Some other aspects of the empirical analysis nevertheless support some parts of the Malthusian model. Homeostasis is indeed supported and the positive check is there as well, but some changes take place within the functioning of the preventive check. Given that we can identify only three cointegration relationships we cannot really say much about what happens with the link of wages to marriages, but in the following section I discuss some results on short-run variation that partly address that issue.

INTERPRETING THE RESULTS

Lack of suitable data to extend my analysis back into the past limits somehow the conclusions we can make about the presence or absence of a pre-industrial Malthusian equilibrium. We can say that from the mid-eighteenth to the late nineteenth century some components of that model were present, but also that the persistence in the series strongly suggests that a stable equilibrium was hardly maintained. One plausible reading of this, even in concordance with some of the literature on the English case reluctant to accept at face value the Malthusian hypothesis [e.g. Nicolini, 2007], is that though present in the past Malthusian dynamics were fading away. If this is the case, it might be interesting to ask ourselves how this happened.

Going back to the result of persistence in the dynamic model described in the previous section, does it make sense to think that the death and the birth curves have the *same* slope (which, given that the positive check is identified, would imply a negative slope on the birth curve)? Although they disagree on the specific values, most of the studies on short-run dynamics indeed suggest that preventive and positive checks were present in pre-industrial France [Weir, 1984; Galloway, 1988]. How can we rationalise the fact that in terms of the model the data suggest the persistence coming from that source? One way is to think that throughout the period studied there was a secular change in the characteristics of the birth curve (i.e. changes in the link between income and marriages, and changes in the link between marriages and births) that drove it *down*, making the equilibrium rate go down (and the real wage go up) as well, *along* the death curve.[34] Intertemporally, a data generating process that produces such a series might not be easily distinguishable from one that is based upon a negative relationship between income and births. In the rest of this section I will try to show that this is indeed a plausible reading of the results and might be able to explain the particular kind of persistence found in the data.

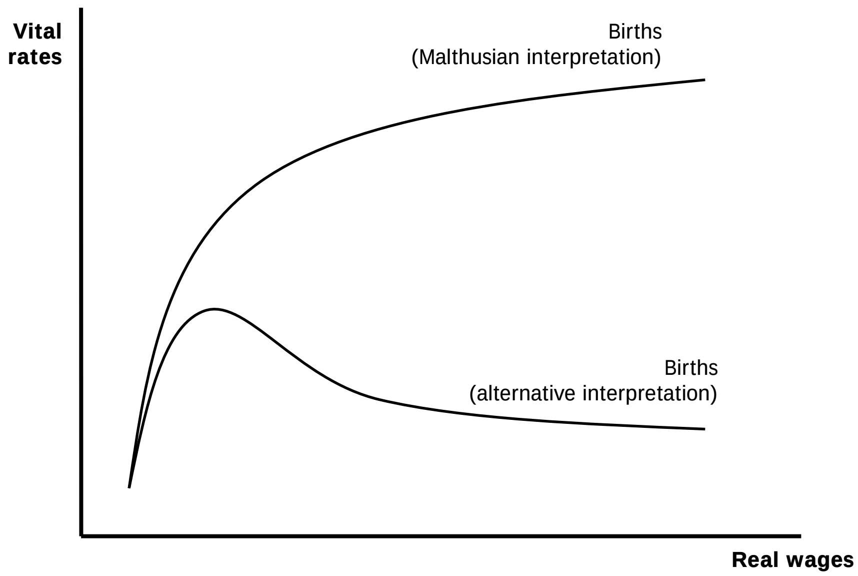
What makes a society not Malthusian?

There are several arguments that can explain why Malthusian dynamics do not seem to work in modern societies, but the one that is perhaps more obvious has to do precisely with the link between income and fertility. A modern reader will certainly find uncomfortable the idea that wages have a positive influence on births and that couples are passive players in this dynamic [Becker, 1991]. One is now prone to believe that families decide their size actively and that that decision involves a relationship much more complicated than that provided by simple

[34] In terms of Figure 1 this would translate into a *systematic* down-shift of the birth curve that, by the relative stability of the death curve, would induce a decline in the vital rates (i.e. fertility and mortality) and a secular increase in wages.

Malthusian logic. The obvious place to start, then, is to ask why people have children. Malthus' answer was in the line of 'because they cannot help it'. A modern economist, on the other hand, would answer more in the line of 'well, that depends'. And that depends on income as well as on many other things [e.g. Schultz, 1997]. The main arguments suggest that, once the minimum biological requirements are met, increasing wages will induce people to have more children just as Malthus suggested. Ignorance and incapability to control sexual urges, children being considered as a source of labour or social security for the parents, or a Darwinist need to maximise the representation in the next generations are among the reasons for that. This same literature indicates that, beyond a certain threshold, further increases in income might induce families to have fewer children [e.g. Becker *et al.*, 1990; Kremer, 1993]. Wealthier families not only have better access to education (hence having more information on family planning) and capital markets (hence not needing informal methods to obtain income or social security), but also higher wages that imply a higher opportunity cost of raising a child (hence, inducing fewer offspring). Also, within Darwinist logic, a less restrictive understanding of 'representation' can allow for a trade-off between having more children of 'low quality' (underfed, undereducated, etc.) or fewer children of 'higher quality'. What happens with further increases in income is even more speculative. Children might become once more a normal good (since the increase in quantity does not lower the probabilities of producing better educated individuals and the cost of raising a child becomes marginally small), but it seems fair to assume that beyond a certain point fertility becomes nonresponsive to income and the curve 'flattens out'. These points have been made in the literature and Kremer [1993: 693-695] gathered them to suggest an alternative shape of the relationship between income and fertility, as depicted in Figure 8.

<u>Figure 8</u>. Transition away from Malthusian checks in modern societies



Then, if a Malthusian logic dominated pre- and early-modern birth dynamics, and current family decisions are better described with this alternative logic, an explanation of the transition *must account for the movement from one to the other*. Many historical accounts, relying upon scattered information and anecdotal evidence, suggest that France indeed experienced a movement from a passive attitude towards family planning (in the sense that is endogenous to the decision on the age of marriage) to an active one [e.g. Flandrin, 1979: 174-242]. Of course, it is very difficult to perceive this change in attitude quantitatively, but with the analysis below I show that actual data is consistent with this interpretation. If one believes this transition took place without any substantial change in the other assumptions of the Malthusian model, the movement must have been *along* the death curve.

Short-run analysis and responsiveness to income

The movement from a classical Malthusian equilibrium to one where actual parental choice dominates suggests a series of hypotheses regarding the short-term effects of income on births and marriages. Under a preventive check equilibrium one expects a significant and positive response of marriage and a not necessarily significant but positive (if any) response of fertility (because it is supposed to operate through marriage, which implies some lag), and a negative response of deaths. Under the alternative equilibrium many things could happen, but one can expect a higher responsiveness (positive or negative) of fertility, and a lower or non-responsiveness of marriages. There are several ways to test these hypotheses, but probably the most standard in this Malthusian framework is short-run analysis.[35] This approach involves studying short-run fluctuations of detrended variables to infer the slopes of the Malthusian relationships. Its main appeal relies upon the possibility of treating wages as exogenous (because the long-term effects have been removed) and, hence, eliminating the identification problem.[36] In his study of French population dynamics, Weir [1984] used short-run variation to estimate the effect of price shocks on current and future birth, marriage and death rates. He began by constructing autoregressive distributed lag models of those variables and grain prices. To generate the effect of a price shock he simulates the impact of a 1% increase in prices in a single year through the estimated system of coefficients.[37] The numbers obtained are the cumulative elasticities and represent the area between the path followed by the variable after the shock and the trend it would have had if that shock never happened, so they measure to some degree the responsiveness of the variable to the price shock.

I think Weir's approach, with some amendments, can be useful in evaluating some of the hypotheses posed before, so I decided to extend some of his results. Firstly, Weir (as well as most researchers performing historical short-run analysis) relied upon grain prices as a proxy for the standard of living. Short-run analysis depends on the variation of series and since Weir's interest was focused on a historic period where the available data on real wages lacked any substan-

[35] Short-run analysis has become quite a popular technique to evaluate historical population dynamics using time series data [Lee, 1997; 1079-1086] and has been used, for example, by Lee [1981: 356-401] to study the case of England, by Weir [1984] for France, Guinnane and Ogilvie [2008] for Germany, and by Galloway [1988] for various countries in Europe.

[36] The typical transformation of the series to remove the longer-term variation is accomplished by dividing each term in the series by a 11-year moving average centred in that term. See the discussion in Lee [1981: 357] and in Galloway [1988: 283].

[37] That is, regressing the detrended birth, marriage and death rates against their own lagged values and lagged values of detrended wheat prices. For the birth and marriage equations he also included lagged values of death rates, something that is quite standard in the literature, and that it is used to net out the effect of death rates.

tial variation, he did not have many alternatives but to rely on commodity prices. Recent estimations of real wages indices [Allen, 2001] allowed me to calculate the responses to a more appropriate measure of income and to assess the accuracy of his estimations. Secondly, Weir elaborated on three distinct periods ending in 1870. For reasons that will become apparent later, the transition was still going on in the late nineteenth century, so I extended the period of analysis to include a fourth phase that could shed some light on the overall dynamics of the transition.[38] Lastly, as done recently by Guinnane and Ogilvie [2008], I also take a closer look to the regressions used to estimate these short-run parameters as, in some cases, the lack of overall significance of those regressions could indeed indicate that the relevant coefficient is not different from zero.

The short-run analysis I perform simply replicates Weir's [1984] that, in turn, closely follows Lee [1981: 356-401]. The regressions used to construct the simulations are those in Tables 2 and 3. Later, I will draw my attention to these, but for the moment my results – together with those of Weir – are reported in Table 1. To make assessment and comparison easier, I reported the inverse of Weir's results (that is, the effect of a *decrease* instead of an *increase* in prices), which is comparable with that of an increase in wages.

Again, the coefficients that appear in the table are the result of simulating the impact of a 1% increase in prices in a single year and represent the area between the path followed by the variable after the shock and the trend it would have had if that shock had never happened. They measure, then, the responsiveness of the variable to the income shock. Looking at the first two periods, one thing that comes out of the comparison is that my estimates for the response of fertility in the early periods are relatively similar to those of Weir. I obtain only slightly higher values, but the story told is basically the same and suggests that the influence of income on fertility was positive (hence, showing evidence of a preventive shock) but decreasing through time. Regarding nuptiality, my estimates for the pre-transitional period are substantially higher implying that a doubling in

[38] Weir is clear on the fact that he chooses those three periods for purely historical reasons [Weir, 1984: 36] but, besides commenting on the implied short-run responses, he never indicates whether that division actually makes sense in statistical terms. In order to address this issue, along with the discussion on the econometric output, I provide at the end of Appendix II diverse Chow-tests for the relevant variables.

wages would create an expansion of marriages over the ensuing five years equal to 89% of the number of marriages in an average year, instead of the 61% obtained by Weir. This reinforces the preventive check argument.

Table 1. Impact of income shocks on current and future vital rates in France, 1747-1906

Elapsed years	Decrease in prices (Weir)			Increase in real wages			
	1747-1789	1790-1829	1830-1865	1747-1789	1790-1829	1830-1865	1866-1906
			Fertility (crude birth rate)				
0	0.085*	0.047*	0.021**	0.178	0.035	0.026*	-0.006**
1	0.172*	0.116*	0.122**	0.141	0.181	0.170*	-0.052**
2	0.094*	0.060*	0.031**	0.166	0.087	0.127*	-0.118**
3	0.185*	0.082*	0.054**	0.208	0.091	0.014*	-0.156**
4	0.172*	0.077*	0.053**	0.217	0.082	0.024*	-0.139**
			Nuptiality (marriage rate)				
0	0.485*	0.476	0.126**	0.752*	0.429	0.162*	0.012**
1	0.471*	0.059	0.079**	0.404*	-0.187	0.141*	-0.187**
2	0.378*	0.435	0.074**	0.780*	0.727	0.083*	-0.194**
3	0.532*	0.411	0.098**	0.918*	0.382	0.105*	-0.118**
4	0.608*	0.306	0.092**	0.890*	0.173	0.105*	-0.121**

Sources: Effects after the decrease in prices are the negative of those reported by Weir [1984; 38-40]. The effects of an increase in wages are from my calculations, based on the techniques suggested in that same paper, and the sources are described in Appendix I. Stars indicate whether the F-test for the regression used to construct the estimate is significant to the 1% (**) or 5% (*). Underlining indicates whether in those same regressions the coefficients corresponding to real wage and their lags were jointly significant to the 5%. See Tables 3 and 4 for details.

The two last periods show some interesting results. After 1830 the preventive check affecting fertility is only slightly positive, and the sensitivity of marriage to income decreases considerably, in line with the results I obtained from the cointegration analysis. Interestingly enough, in the last interval the influence of income on fertility becomes negative. All these results support the hypotheses I suggested before and that has been already pointed out sometimes in the literature [e.g. Wrigley, 1985]. Fertility becomes increasingly sensitive to short-run changes in wages, but its response turns from being slightly positive to being strongly negative. And nuptiality, which in the early stages played a crucial role, loses relevance, eventually becoming affected in a different way (which, in this particular case could be influenced by the strong fall in marriage that occurs during the Franco-Prussian War). The transition changed the situation in the sense that couples could get married and not have a family immediately. This could have generated a period of relative independence between marriage and wealth.

All these estimations rely upon a number of autoregressive distributed lag regressions that I have not described yet. I will show that those regressions have in general a bad fit and that some of the values in Table 1 should be treated with a lot of caution. The reason why I still report them is that they remain comparable to Weir's results (which, by the way, do not rely upon substantially better regressions) and are still illustrative of what the data (weakly) suggest. But, interestingly enough, these relatively unsuccessful estimations tell a story along the lines of what I have claimed earlier about the characteristics of the transition, in a similar way as Guinnane and Ogilvie [2008] have done recently for Germany. I will begin by looking at the fertility equations.

Table 2. Modelling fertility (short-run analysis) in France, sample 1748-1906

	Coefficient (t-value)			
	1748-1789	**1790-1829**	**1830-1865**	**1866-1906**
	Crude birth rate			
Constant	-0.0007 (-0.18)	0.0002 (0.05)	0.0010 (0.27)	-0.0011 (-0.54)
crude birth rate $_{t-1}$	0.1456 (0.74)	0.2776 (1.47)	-0.0407 (-0.21)	-0.0540 (-0.29)
crude birth rate $_{t-2}$	0.0894 (0.47)	0.1056 (0.54)	-0.1405 (-0.77)	-0.2311 (-1.22)
adjusted death rate	-0.0009 (-0.01)	0.1023 (1.60)	-0.0501 (-0.75)	-0.2534 (-8.68)
adjusted death rate $_{t-1}$	-0.0571 (-0.96)	-0.0133 (-0.20)	-0.0497 (-0.71)	0.0341 (0.66)
adjusted death rate $_{t-2}$	0.0678 (1.14)	0.0674 (1.07)	0.0202 (0.31)	-0.0491 (-0.94)
adjusted death rate $_{t-3}$	0.0004 (0.01)	0.0601 (1.08)	-0.0081 (-0.12)	-0.0169 (-0.65)
real wage (unskilled)	0.1784 (2.07)	0.0353 (0.56)	0.0261 (0.46)	-0.0057 (-0.13)
real wage (unskilled) $_{t-1}$	-0.0638 (-0.62)	0.1356 (1.68)	0.1445 (1.94)	-0.0469 (-1.01)
real wage (unskilled) $_{t-2}$	0.0145 (0.14)	-0.1377 (-1.58)	-0.0327 (-0.40)	-0.0696 (-1.52)
real wage (unskilled) $_{t-3}$	0.0427 (0.52)	0.0147 (0.21)	-0.0953 (-1.35)	-0.0525 (-1.15)
sigma	0.025	0.026	0.021	0.013
R^2	0.286	0.388	0.477	0.788
F (10,T) =	1.24 [0.306]	1.84 [0.097]	2.28 [0.046]*	11.1 [0.000]**
F –birth- (2,T) =	0.47 [0.631]	1.40 [0.263]	0.32 [0.728]	0.86 [0.433]
F –death- (4,T) =	0.72 [0.586]	1.96 [0.127]	0.35 [0.845]	23.6 [0.000]**
F –real wage- (4,T) =	1.24 [0.315]	2.19 [0.095]	2.95 [0.040]*	3.81 [0.013]*
DW	2.01	2.12	2.00	2.14
	Marital fertility (Ig)			
Constant	-0.0014 (-0.38)	0.0013 (0.33)	0.0009 (0.27)	-0.0016 (-0.83)
marital fertility (Ig) $_{t-1}$	0.1637 (0.84)	0.1182 (0.61)	-0.0599 (-0.31)	-0.1214 (-0.66)
marital fertility (Ig) $_{t-2}$	0.1143 (0.61)	-0.0813 (-0.41)	-0.1831 (-0.99)	-0.2691 (-1.45)
adjusted death rate	0.0080 (0.12)	0.0511 (0.84)	-0.0522 (-0.80)	-0.2371 (-8.54)
adjusted death rate $_{t-1}$	-0.0723 (-1.25)	-0.0256 (-0.40)	-0.0564 (-0.83)	0.0275 (0.56)
adjusted death rate $_{t-2}$	0.0681 (1.14)	0.0679 (1.13)	0.0182 (0.29)	-0.0477 (-0.97)
adjusted death rate $_{t-3}$	-0.0018 (-0.03)	0.0578 (1.09)	-0.0026 (-0.04)	-0.0144 (-0.56)
real wage (unskilled)	0.1925 (2.27)	0.0496 (0.82)	0.0230 (0.41)	0.0068 (0.16)
real wage (unskilled) $_{t-1}$	-0.0541 (-0.52)	0.1228 (1.59)	0.1413 (1.96)	-0.0477 (-1.06)
real wage (unskilled) $_{t-2}$	0.0102 (0.10)	-0.1088 (-1.29)	-0.0274 (-0.35)	-0.0761 (-1.71)
real wage (unskilled) $_{t-3}$	0.0086 (0.11)	0.0304 (0.45)	-0.0840 (-1.23)	-0.0465 (-1.04)
sigma	0.025	0.025	0.020	0.012
R^2	0.334	0.335	0.472	0.786
F (10,T) =	1.56 [0.167]	1.46 [0.204]	2.24 [0.050]*	11.0 [0.000]**
F –marital fer.- (2,T) =	0.65 [0.528]	0.27 [0.764]	0.54 [0.588]	1.34 [0.276]
F –death- (4,T) =	0.82 [0.521]	1.42 [0.254]	0.42 [0.792]	22.4 [0.000]**
F –real wage- (4,T) =	1.59 [0.201]	2.11 [0.105]	2.87 [0.044]*	4.17 [0.008]**
DW	2.01	2.10	1.99	2.17

Sources: See Appendix I for sources. Variables are percentage deviations from 11-year moving averages. * denotes the 0.05 significance level and ** the 0.01 significant level.

In Table 2 I report the regressions for crude birth rate and for marital fertility (I_g), corresponding to the four periods, that I used to construct the elasticities above. Estimations for both variables are similar and can be interpreted as a robustness check of my results. Significant coefficients are not different from one specification to another, so evaluating either of them will provide basically the same outcome. As I mentioned before, the fit of these regressions is disappointing. In the first period, only changes in current wage seem to explain changes in fertility (and their influence is positive, arguing in favour of the preventive check), but the joint impact including the effect of lagged variables is not really significant.

For the second and third periods results are even weaker: only the first lag of wages is somewhat significant (at the 15% or 10% level, depending on the variable), but the relationship is still positive. In the fourth period, however, this relationship reverts and becomes more significant. Coefficients are not really different from zero under the standard significance levels, but if one considers as relevant values at 15% level, it is interesting to note that the second lag of the wages turn to have negative coefficients as well as the (less significant) first and third lags.

The results from this table, though weak in the strict econometric sense, suggest some transition towards a different relationship between income and fertility, especially in the last period. This can be complemented with the results on nuptiality in Table 3. In this case, the results are very suggestive. The first period regression is actually reasonably good and shows a strong relationship between income and marriage rate. This evidence points toward a classic Malthusian period of preventive checks acting through nuptiality and could explain the small reaction of fertility: all the action is happening in marriages. What the successive periods show is simply a fading of this relationship which, again, is supportive of my story. The negative elasticities reported in Table 1 for the last period are sustained only by the second lag of the real wage, which is only marginally significant.

Table 3. Modelling nuptiality (short-run analysis) in France, sample 1748-1906

	Coefficient (t-value)			
	1748-1789	**1790-1829**	**1830-1865**	**1866-1906**
Constant	0.0015 (0.15)	0.0026 (0.11)	-0.0022 (-0.42)	0.0021 (0.33)
crude marriage rate $_{t-1}$	0.4684 (2.56)	-0.2229 (-1.30)	-0.1294 (-0.67)	-0.0261 (-0.12)
crude marriage rate $_{t-2}$	-0.2459 (-1.43)	-0.3128 (-1.79)	-0.0495 (-0.28)	0.1494 (0.69)
adjusted death rate	0.1565 (0.89)	0.3475 (1.11)	-0.0516 (-0.53)	-0.3909 (-3.64)
adjusted death rate $_{t-1}$	0.1342 (0.91)	0.6425 (1.80)	0.0822 (0.81)	0.3842 (3.16)
adjusted death rate $_{t-2}$	-0.1373 (-0.90)	-0.2463 (-0.73)	0.1586 (1.63)	0.2545 (1.85)
adjusted death rate $_{t-3}$	0.1968 (1.28)	0.3794 (1.30)	0.1628 (1.66)	0.0164 (0.15)
real wage (unskilled)	0.7517 (3.34)	0.4286 (1.24)	0.1620 (1.82)	0.0119 (0.09)
real wage (unskilled) $_{t-1}$	-0.7000 (-2.48)	-0.5201 (-1.15)	0.0003 (0.00)	-0.1987 (-1.39)
real wage (unskilled) $_{t-2}$	0.7244 (2.64)	0.9109 (1.96)	-0.0527 (-0.42)	-0.0143 (-0.09)
real wage (unskilled) $_{t-3}$	-0.1244 (-0.55)	-0.3337 (-0.91)	0.0130 (0.13)	0.1060 (0.80)
sigma	0.066	0.143	0.031	0.040
R^2	0.468	0.311	0.484	0.678
F (10,T) =	2.73 [0.016]*	1.31 [0.271]	2.35 [0.041]*	6.31 [0.000]**
F –marriage- (2,T) =	3.53 [0.042]*	2.13 [0.137]	0.25 [0.780]	0.24 [0.788]
F –death- (4,T) =	0.69 [0.604]	2.29 [0.084]	1.57 [0.214]	8.97 [0.000]**
F –real wage- (4,T) =	4.63 [0.005]**	1.25 [0.311]	1.57 [0.214]	1.11 [0.372]
DW	2.14	2.10	1.80	2.06

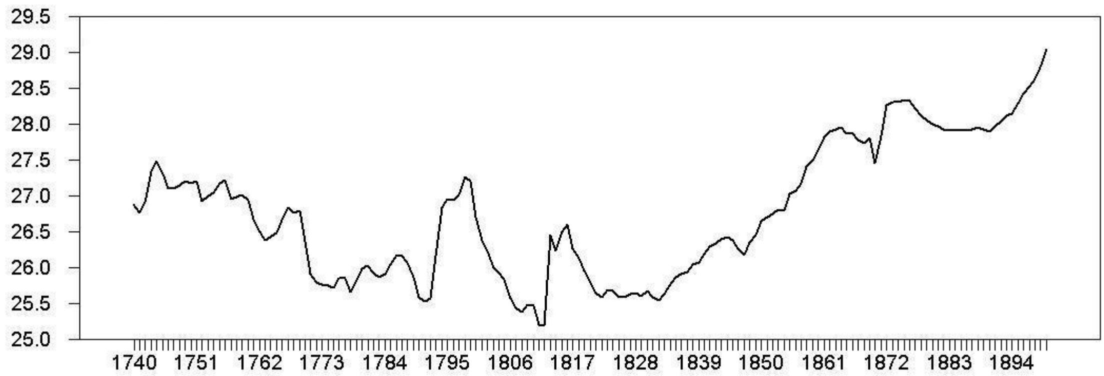
Sources: See Appendix I for sources. Variables are percentage deviations from 11-year moving averages. * denotes the 0.05 significance level and ** the 0.01 significant level.

This result, combined with that of the increasing relevance of fertility, is consistent with the evolution of the stock of marriage that I have roughly estimated on a yearly basis using Weir‘s quinquennial numbers [Weir, 1994: 328] and that I plot in Figure 9. Up to the early nineteenth century the evolution of the stock of marriages follows more or less well the evolution of fertility. This is consistent with several authors' results that age of marriage seems to be a good predictor of fertility during the *ancien régime* and somewhat supportive of a Malthusian interpretation of that period.[39] Following the Empire, nevertheless, this relationship is broken and fertility does become independent from marriages.

Whether this change in relationship is read as a special kind of Malthusianism, as Wrigley [1985] does, or a flat rejection of a 'Malthusian transition', as Weir [1984] partly does, is perhaps only a question of semantics. The analysis in this paper suggests that several components of the Malthusian model are valid throughout the period of the transition, but the interrelationship of income, marriages and fertility was undoubtedly, and fundamentally, disrupted somewhere in the early nineteenth century.

[39] Most notably, Henry and Houdaille's studies of the INED sample [Henry, 1972, 1978; Henry and Houdaille, 1973; Houdaille, 1976] and Weir [1995].

Figure 9. Proportion total female population that is married and of fertile age in France, 1740-1900



Sources: My calculations, based on Weir [1994: 328], INED [1977] and Mitchell [1998]. To estimate yearly fluctuations I considered the number of new marriages and assumed that within each five-year interval a constant proportion of women were leaving the fertile age group (20-49).

LEAVING THE MALTHUSIAN WORLD

To my understanding this is the first attempt to bring the French case into the current Malthusian debate. In some way, this paper brings support to Weir's conclusion that the transition was not a Malthusian response to population pressure [Weir, 1983: Chapter VII; 1984: 44], though it also reveals a plausible Malthusian reading. The implicit message behind Malthus' contribution is that people do not change. Whatever technological or environmental shock they have, they always go back to where they came from. If the new literature in unified growth is right, it was the technological advances associated with the Industrial Revolution that tipped the balance to make the trade-off between quantity and quality of children meaningful and changed the incentives of potential parents when deciding their fertility level. The events described here are talking perhaps of an alternative way of breaking Malthus' iron law. In France all the components of a standard Malthusian dynamic appeared to be present, and there were no substantial changes in the level of technology as across the channel, but something induced some movement of the initial equilibrium. Perhaps future research could further elaborate this hypothesis and study alternative ways of modelling some sort of inter-temporal dynamics for the birth curve.

On the one hand, we need to understand better pre-transitional dynamics, and this calls for more research aiming at extending back the period of analysis. Although data before 1740 is rather scarce, some information on baptism and other sacraments are available as early as 1670 [e.g. Rebaudo, 1979]. A more systematic study of these sources could provide the needed information to provide insight into the problem.[40] On the other hand, having elaborated some ideas on the peculiarities of France, some effort could be devoted to adapt the framework proposed by Møller and Sharp [2008] to them. From the analysis in this paper we can put forward the hypothesis that France engaged in a dynamic quite different to that of England, where the birth curve begins to move towards a modern shape quite early, with no apparent immediate economic motivation behind, decreasing the demographic pressure on income and probably making possible a softer transition to modernity. One way to approach this problem would be by modelling explicitly the apparent shift in the birth curve and assessing whether the data is indeed consistent with this alternative way of interpreting the history of France, though other modelling strategies could be explored.

[40] Lachiver has already done some work on estimating vital rates from this data [Lachiver, 1991].

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APPENDIX I : DATA SOURCES

For the period 1740-1839 crude birth, death and marriage rates were obtained from INED [1977: 332-333]. After 1840, birth and death rates are from Chesnais [1992: 518-541, 555-578] and marriage rates from Mitchell [1998: 93-119]. The figures for marital fertility correspond to the Coale index I_g [Coale and Watkins, 1986: 153-162], estimated for the whole period with yearly frequency by Weir [1994: 330-331]. For estimations of the short-run parameters, as suggested and also done by Weir [1984: 37], I employed a death rate that adjusts mortality for the structural impact of birth rate variations through the infant mortality rate. The formula I used for this transformation was the following:

$$\text{adjusted CDR}_t = \text{CDR}_t - \text{IMR}_t \cdot \left[s \cdot \text{CBR}_t + (1-s) \cdot \text{CBR}_{t-1} \right]$$

Where CDR is the crude death rate, IMR the infant mortality rate (that I obtained from INED [1977: 332-333] for 1740-1839, and from Chesnais [1992: 580-597] for 1840-1911), and CBR the crude birth rate. The coefficient *s* is a separation factor for the proportion of all infant deaths that occur in the same calendar year as the infant's birth. Following Weir [1984: 37], I assumed that for this sample that value is 0.74.

Nominal wages were obtained from Allen [2001]. As expected, the revolutionary period imposed some problems in terms of access to data, and that probably requires some clarification here. The series of nominal wages were comprehensive for skilled labour (craftsman), but for unskilled labour (building labourer) there was a gap in the period 1787-1804, so I completed it assuming the same rate of change that for skilled labour. The deficiency of data is more relevant for prices. Allen's consumer price index has a gap between 1787 and 1839, so I used instead only the price of wheat (which, in any case, still dominates the CPI when available). Wheat prices in francs per hectolitre were obtained from Labrousse *et al.* [1970]. In the data there is a gap in the "Terror" years.[41] Since Paris prices during those years, even if available, probably do not provide any relevant informa-

[41] Weir uses the same data, but I couldn't find in his thesis [Weir, 1983] or in his paper [Weir, 1984] how he dealt with the problem of this gap.

tion, I used the rates of changes of prices in the prices of wheat in Strasbourg [Hanauer, 1878]. For the actual regressions I constructed an index numbers (with 1890-99 = 100) for the daily income of a building labourer deflated by a price index.

APPENDIX II: EXTENDED STATISTICAL OUTPUT

Cointegrated VAR analysis

As mentioned in the paper, the output of the vector auto-regressive (VAR) analysis is not necessarily illuminating, so I have refrained from incorporating it in the core of the text, but here I provide a step-by-step account of all the graphs and statistics that are relevant for a complete study of such a model [see Juselius, 2006: 423-424].

The unrestricted VAR

In this study the main vector is defined by just four variables:

$$X_t = \left[\text{births}_t, \text{deaths}_t, \text{marriages}_t, \ln\left(\text{wages}\right)_t\right] = \left[b_t, d_t, m_t, \ln w_t\right]$$

VAR formulation allows a series of modelling alternatives that can incorporate the presence of different types of deterministic components, such as constants, trends and dummies. For modelling this demographic setting, the relevant parametric specification required restricted constant and empirical evaluation rejected the relevance of trends. The basic unrestricted VAR takes then the classic form:

$$\Delta X_t = \Pi_0 + \Pi X_{t-1} + \sum_{j=1}^{p} \Gamma_j \Delta X_{t-j} + \varepsilon_t$$

Where $\Pi_0 = \alpha\beta_0^{'}$ is the vector of constants (potentially part of the long-run relationships), $\Pi = \alpha\beta^{'}$ the long-run matrix, Γ_j the short-run matrices, and ε_t the error term.

<u>Table A1</u>. Lag determination tests for the unrestricted VAR model, effective sample 1745-1890

Model Summary

Model	k	Regr.	Log-Likelihood	Schwatrz Criterion	Hannan-Quinn Criterion	LM(1)	LM(k)
VAR(5)	5	21	160.894	0.663	-0.356	0.493	0.499
VAR(4)	4	17	153.089	0.224	-0.601	0.530	0.379
VAR(3)	3	13	144.074	-0.199	-0.830	0.411	0.233
VAR(2)	2	9	125.776	**-0.494**	**-0.931**	0.030	0.504
VAR(1)	1	5	55.387	-0.076	-0.319	0.000	0.000

Lag Reduction test

VAR(4)	<<	VAR(5)	:	ChiSqr(16)	=	15.609	[0.4805]
VAR(3)	<<	VAR(5)	:	ChiSqr(32)	=	**33.640**	**[0.3879]**
VAR(3)	<<	VAR(4)	:	ChiSqr(16)	=	**18.031**	**[0.3221]**
VAR(2)	<<	VAR(5)	:	ChiSqr(48)	=	70.235	[0.0199]
VAR(2)	<<	VAR(4)	:	ChiSqr(32)	=	54.625	[0.0076]
VAR(2)	<<	VAR(3)	:	ChiSqr(16)	=	36.594	[0.0024]
VAR(1)	<<	VAR(5)	:	ChiSqr(64)	=	211.013	[0.0000]
VAR(1)	<<	VAR(4)	:	ChiSqr(48)	=	195.404	[0.0000]
VAR(1)	<<	VAR(3)	:	ChiSqr(32)	=	177.373	[0.0000]
VAR(1)	<<	VAR(2)	:	ChiSqr(16)	=	140.779	[0.0000]

<u>Notes</u>: LM(k) indicates the LM-test for autocorrelation of order k. For the lag-reduction test, *p*-values are in brackets.

The first step is to determine the lag length and the test in Table A1 provide the relevant information to make that assessment. Both the Schwarz and Hannan-Quinn criterion suggest we should use a VAR(2) specification, but the lag-reduction test suggest instead a VAR(3). As I explain in chapter III, this latter result is further supported by theoretical considerations so I keep that second specification. As having three lags appear to be enough to describe the dynamic system, the basic model becomes:

$$\Delta X_t = \Pi_0 + \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \varepsilon_t$$

Estimating this basic unrestricted model gives the following results:

$$\left(\hat{\Pi},\hat{\Pi}_0\right)\begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} = \begin{pmatrix} \underset{(-5.688)}{-0.16} & \underset{(4.985)}{0.13} & \underset{(1.272)}{0.08} & \underset{(-2.499)}{-0.78} & \underset{(1.527)}{1.86} \\ \underset{(5.581)}{0.57} & \underset{(-5.509)}{-0.51} & \underset{(-1.286)}{-0.28} & \underset{(1.044)}{1.20} & \underset{(-0.436)}{-1.95} \\ \underset{(-2.368)}{-0.15} & \underset{(3.898)}{0.22} & \underset{(-5.536)}{-0.74} & \underset{(-0.692)}{-0.48} & \underset{(4.253)}{11.6} \\ \underset{(-2.575)}{-0.01} & \underset{(1.035)}{0.01} & \underset{(-1.639)}{-0.02} & \underset{(-4.394)}{-0.26} & \underset{(5.336)}{1.22} \end{pmatrix} \begin{pmatrix} b_{t-1} \\ d_{t-1} \\ m_{t-1} \\ \ln w_{t-1} \\ 1 \end{pmatrix}$$

$$\hat{\Gamma}_1 \Delta X_{t-1} = \begin{pmatrix} \underset{(-7.149)}{-0.61} & \underset{(-4.412)}{-0.11} & \underset{(3.557)}{0.18} & \underset{(5.624)}{2.31} \\ \underset{(-1.777)}{-0.56} & \underset{(0.519)}{0.05} & \underset{(1.701)}{0.31} & \underset{(-0.115)}{-0.17} \\ \underset{(0.187)}{0.04} & \underset{(-1.297)}{-0.07} & \underset{(-0.145)}{-0.02} & \underset{(-0.606)}{-0.56} \\ \underset{(-0.961)}{-0.02} & \underset{(-0.023)}{-0.00} & \underset{(1.548)}{0.01} & \underset{(4.865)}{0.37} \end{pmatrix} \begin{pmatrix} \Delta b_{t-1} \\ \Delta d_{t-1} \\ \Delta m_{t-1} \\ \Delta \ln w_{t-1} \end{pmatrix}$$

$$\hat{\Gamma}_2 \Delta X_{t-2} = \begin{pmatrix} \underset{(-1.198)}{-0.09} & \underset{(-1.571)}{-0.04} & \underset{(0.737)}{0.03} & \underset{(0.818)}{0.40} \\ \underset{(-0.003)}{-0.00} & \underset{(0.145)}{0.01} & \underset{(1.164)}{0.17} & \underset{(-2.812)}{-5.08} \\ \underset{(1.202)}{0.21} & \underset{(-0.995)}{-0.05} & \underset{(-1.358)}{-0.12} & \underset{(0.706)}{0.78} \\ \underset{(1.652)}{0.02} & \underset{(0.959)}{0.00} & \underset{(2.207)}{0.02} & \underset{(-1.623)}{-0.15} \end{pmatrix} \begin{pmatrix} \Delta b_{t-2} \\ \Delta d_{t-2} \\ \Delta m_{t-2} \\ \Delta \ln w_{t-2} \end{pmatrix}$$

$$\hat{\Omega} = \begin{pmatrix} 1.00 & & & \\ -0.08 & 1.00 & & \\ 0.48 & 0.07 & 1.00 & \\ 0.17 & -0.02 & 0.25 & 1.00 \end{pmatrix}, \quad \hat{\sigma}_{\varepsilon} = \begin{pmatrix} 0.7372 \\ 2.7000 \\ 1.6453 \\ 0.1382 \end{pmatrix}$$

However, when we look into the behaviour of residuals, described by statistics in Table A2 and Figure A1, we notice some problems.

Table A2. **Residual analysis of the unrestricted VAR model, effective sample 1745-1890**

Multivariate tests

Test for autocorrelation - Ljung-Box(37): ChiSqr(544) = 601.5234 [0.0440]

Test for Normality: ChiSqr(8) = 117.4878 [0.0000]

LM-tests:

	Test for Autocorrelation				Test for ARCH			
LM(1):	ChiSqr(16)	=	11.7195	[0.7631]	ChiSqr(100) =		249.2318	[0.0000]
LM(2):	ChiSqr(16)	=	15.0988	[0.5174]	ChiSqr(200) =		428.7744	[0.0000]
LM(3):	ChiSqr(16)	=	17.1872	[0.3736]	ChiSqr(300) =		525.5057	[0.0000]
LM(4):	ChiSqr(16)	=	14.1035	[0.5910]	ChiSqr(400) =		648.1098	[0.0000]

Univariate tests

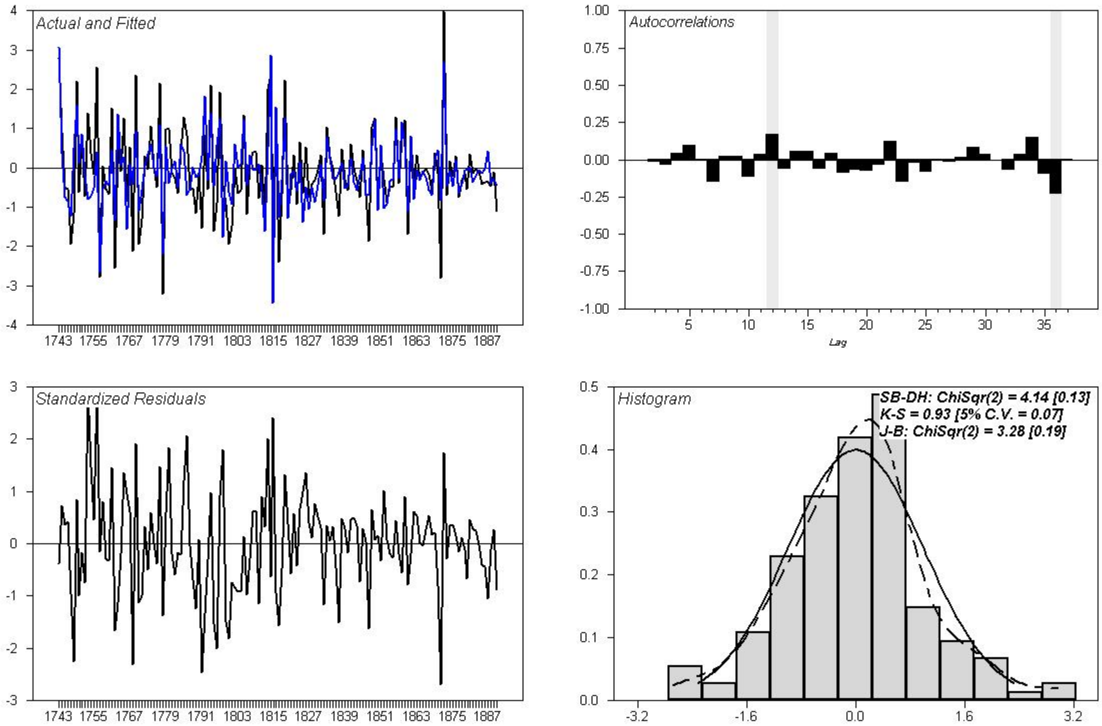
	ARCH (3)		Normality test		Skewness	Kurtosis
Δb_t	4.0429	[0.2569]	4.1445	[0.1259]	0.1053	3.6340
Δd_t	6.3598	[0.0954]	31.9478	[0.0000]	1.3753	7.2852
Δm_t	11.8609	[0.0079]	65.8215	[0.0000]	1.6436	13.491
$\Delta \ln w_t$	0.8351	[0.8411]	11.3871	[0.0034]	0.5926	4.4916

Notes: See Juselius [2006] for details on how tests are constructed. LM(k) indicates the LM-test for autocorrelation of order k. For all tests, *p*-values are in brackets.

As can be seen, there are no sign of autocorrelation problems, but normality is strongly rejected, as well as residual heteroscedasticity. Univariate tests and graphic analysis indicate that this problem is probably due to the presence of some outliers, so the next step is to account for them using a series of dummy variables.

Figure A1. **Series residuals for the unrestricted VAR model, effective sample 1745-1890**

Δb_t



Δd_t

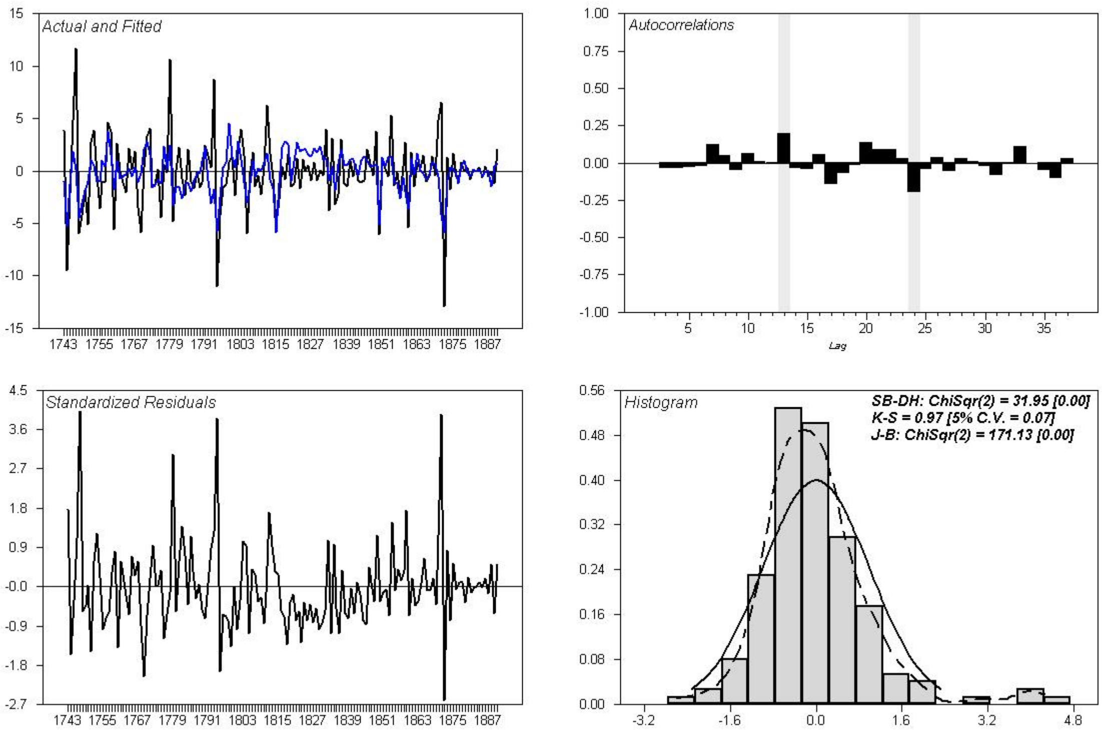
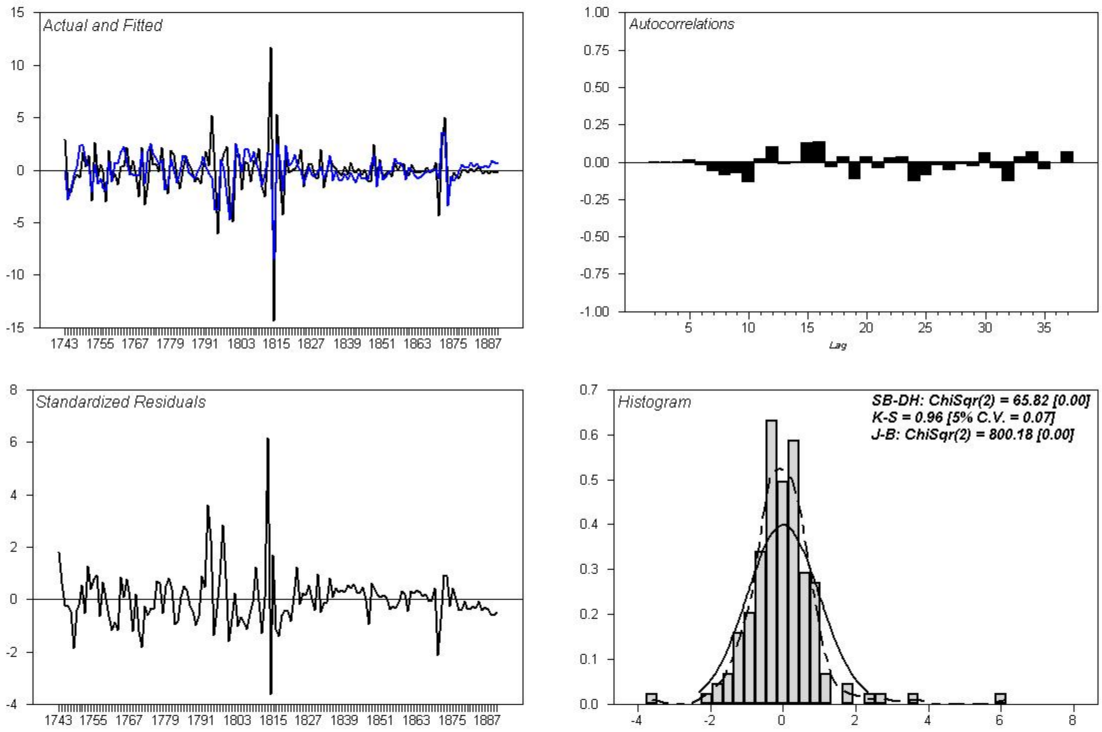
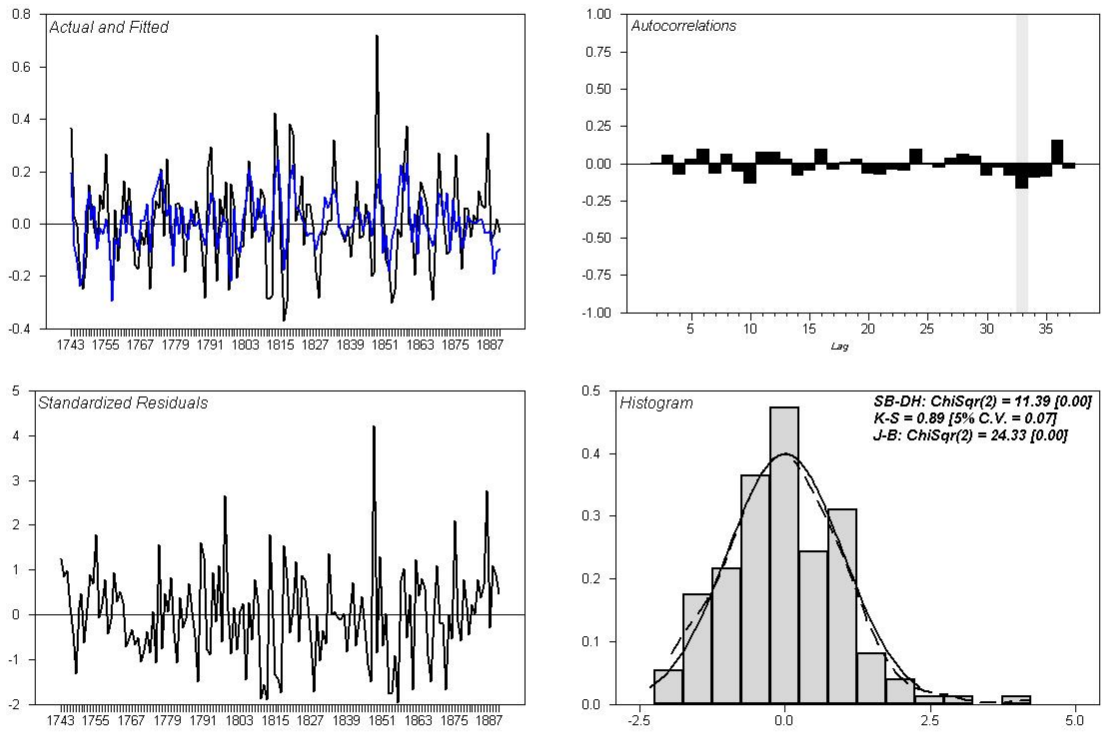


Figure A1. (cont.) Series residuals for the unrestricted VAR model, effective sample 1745-1890

Δm_t



$\Delta \ln w_t$



Model with dummies

As the residual analysis motivated the use of some dummies to control for the irregularities in the series, I extended the model above to account for the unusual events that I describe in the text as follows:

$$\Delta X_t = \Pi_0 + \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \Theta D_t + \varepsilon_t$$

The events were modelled as transitory ($D_t^{T:year}$) or permanent ($D_t^{P:year}$) shocks [Juselius, 2006: 104-109]. A series of outliers in the earlier periods in both death and birth rates that could be well related to climatic fluctuations where accounted for with transitory dummies:

$$D_t^{T:1747} = 1_{\{t=1747\}} - 1_{\{t=1748\}}$$
$$D_t^{T:1753} = 1_{\{t=1753\}} - 1_{\{t=1754\}}$$
$$D_t^{T:1779} = 1_{\{t=1779\}} - 1_{\{t=1781\}}$$

Perhaps unsurprisingly, from the outbreak of the French Revolution to the end of the Napoleonic era fluctuation in many of the variables was substantial, but only three dummies were able to reduce the distortion:

$$D_t^{T:1793} = 1_{\{t=1793\}} + 1_{\{t=1794\}} - 1_{\{t=1795\}} - 1_{\{t=1796\}}$$
$$D_t^{P:1798} = 1_{\{t=1798\}}$$
$$D_t^{T:1813} = 1_{\{t=1813\}} - 1_{\{t=1814\}}$$

Apparently, the period of the National Convention had a strong positive shock in some of the demographic variables, especially on mortality and marriage rates. As I pointed out in the text, the introduction in 1798 of a law regularising conscription but *exempting* married men probably had a long term effect on marriages, and I model that here with a permanent dummy. Following the Russian campaign there was an unusually large draft that, combined with the conscription law, generated a blip in marriage rates, and I controlled that with a dummy in the period 1813-1814. The Franco-Prussian war had also some considerable impact in all variables, and was modelled as a transitory dummy:

$$D_t^{T:1870} = 1_{\{t=1870\}} + 1_{\{t=1871\}} - 1_{\{t=1872\}} - 1_{\{t=1873\}}$$

After a series of bad harvest, the price of grain had gone up in the second part of the 1840s, and sharp fall in the price of wheat in 1848 seems to have generated a sudden upsurge in the real wage, that is controlled for with a transitory shock dummy:

$$D_t^{P:1848} = 1_{\{t=1848\}} - 1_{\{t=1849\}}$$

As Table A3 illustrates, results are substantially improved when introducing these dummies:

Table A3. Residual analysis of the unrestricted VAR model with dummies, effective sample 1745-1890

Multivariate tests

Test for autocorrelation - Ljung-Box(37): ChiSqr(544) = 561.9810 [0.2879]

Test for Normality: ChiSqr(8) = 14.3165 [0.0739]

LM-tests:

	Test for Autocorrelation	Test for ARCH
LM(1):	ChiSqr(16) = 33.2121 [0.0069]	ChiSqr(100) = 147.5378 [0.0014]
LM(2):	ChiSqr(16) = 12.6527 [0.6980]	ChiSqr(200) = 235.9933 [0.0414]
LM(3):	ChiSqr(16) = 18.7099 [0.2840]	ChiSqr(300) = 353.4309 [0.0183]
LM(4):	ChiSqr(16) = 16.4352 [0.4230]	ChiSqr(400) = 448.1179 [0.0484]

Univariate tests

	ARCH (3)	Normality test	Skewness	Kurtosis
Δb_t	3.5649 [0.3124]	2.7613 [0.2514]	-0.0218	3.4741
Δd_t	5.9239 [0.1154]	3.7564 [0.1529]	0.3779	3.0770
Δm_t	4.4253 [0.2190]	2.8276 [0.2432]	-0.1130	3.4741
$\Delta \ln w_t$	1.1068 [0.7754]	1.0770 [0.5836]	0.1816	2.7704

Notes: See Juselius [2006] for details on how tests are constructed. LM(k) indicates the LM-test for autocorrelation of order k. For all tests, *p*-values are in brackets.

Overall normality is (marginally) obtained and non-normality is rejected for all individual series, without any serious consequence on autocorrelation. Estimation of the lag length with this specification provides further support for the choice of 3 lags, as suggested by Table A4.

Table A4. Lag determination tests for the unrestricted VAR model with dummies, effective sample 1745-1890

Model Summary

Model	k	Regr.	Log-Likelihood	Schwartz Criterion	Hannan-Quinn Criterion	LM(1)	LM(k)
VAR(5)	5	29	288.4563	0.0081	-1.3992	0.0207	0.2133
VAR(4)	4	25	280.9174	-0.4348	-1.6480	0.0039	0.2340
VAR(3)	3	21	272.9414	-0.8716	**-1.8907**	0.0310	0.0908
VAR(2)	2	17	245.6600	**-1.0441**	-1.8691	0.0000	0.6315
VAR(1)	1	13	166.8589	-0.5108	-1.1416	0.0000	0.0000

Lag Reduction test

VAR(4)	<<	VAR(5)	:	ChiSqr(16)	=	15.078	[0.5189]
VAR(3)	<<	VAR(5)	:	ChiSqr(32)	=	**31.030**	**[0.5155]**
VAR(3)	<<	VAR(4)	:	ChiSqr(16)	=	**15.952**	**[0.4563]**
VAR(2)	<<	VAR(5)	:	ChiSqr(48)	=	85.593	[0.0007]
VAR(2)	<<	VAR(4)	:	ChiSqr(32)	=	70.515	[0.0001]
VAR(2)	<<	VAR(3)	:	ChiSqr(16)	=	54.563	[0.0000]
VAR(1)	<<	VAR(5)	:	ChiSqr(64)	=	243.195	[0.0000]
VAR(1)	<<	VAR(4)	:	ChiSqr(48)	=	228.117	[0.0000]
VAR(1)	<<	VAR(3)	:	ChiSqr(32)	=	212.165	[0.0000]
VAR(1)	<<	VAR(2)	:	ChiSqr(16)	=	157.602	[0.0000]

Notes: LM(k) indicates the LM-test for autocorrelation of order k. For the lag-reduction test, *p*-values are in brackets.

With a well-specified model, I tested different hypotheses in Table A5, without any negative result.

Table A5. **Tests of exclusion, stationarity and weak exogeneity for the unrestricted VAR model with dummies, effective sample 1745-1890**

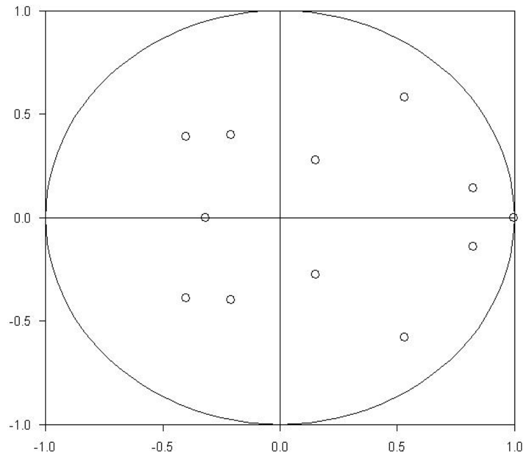
	Exclusion			Stationarity			Weak exogeneity		
r	1	2	3	1	2	3	1	2	3
d.f.	1	2	3	3	2	1	1	2	3
5% CV	3.84	5.99	7.81	7.81	5.99	3.84	3.84	5.99	7.81
Δb_t	11.86 [0.001]	38.63 [0.000]	43.26 [0.000]	50.97 [0.000]	33.58 [0.000]	7.21 [0.007]	6.35 [0.012]	34.66 [0.000]	34.66 [0.000]
Δd_t	15.56 [0.000]	39.23 [0.000]	44.95 [0.000]	47.33 [0.000]	31.59 [0.000]	7.51 [0.006]	7.92 [0.005]	19.46 [0.000]	19.48 [0.000]
Δm_t	3.76 [0.053]	31.05 [0.000]	37.67 [0.000]	24.44 [0.000]	8.70 [0.013]	7.05 [0.008]	15.44 [0.000]	40.51 [0.000]	45.94 [0.000]
$\Delta \ln w_t$	4.08 [0.044]	5.76 [0.056]	13.10 [0.004]	50.68 [0.000]	33.83 [0.000]	7.22 [0.007]	8.01 [0.005]	10.23 [0.006]	17.54 [0.004]
Constant	10.82 [0.001]	21.62 [0.000]	28.77 [0.000]						

Notes: See Juselius [2006] for details on how tests are constructed. For all tests, *p*-values are in brackets.

Rank determination

A first approximation to determine the rank is to look at the roots of the companion matrix. If a process is stationary, the eigenvalues of the companion matrix should be inside of the unit circle, whereas the presence of non-stationarity would lead some of those values close to (but inside) the unit circle. As Figure A2 shows, the unrestricted model has all the roots inside the unit circle, hence suggesting the process is not explosive, but at least one very close to the border (0.9989), which indicates the presence of a unit root. Setting the rank to three (i.e. imposing one unit root) leaves all other roots reasonably inside the circle.

Figure A2. **Eigenvalues of the companion matrix for the unrestricted VAR model with dummies, effective sample 1745-1890**



Notes: These are the roots of the characteristic polynomial for the VAR(3) model with dummies.

Table A6 shows the roots of the companion matrix with no restrictions in the rank, and then when we impose rank = 3, hence having a unit root.

Table A6. Roots of the companion matrix assuming full or restricted rank

	Full rank				Imposing restricted rank (r= 3)			
	Real	Imaginary	Modulus	Argument	Real	Imaginary	Modulus	Argument
Root1	0.9989	0.0000	0.9989	0.0000	1.0000	-0.0000	1.0000	-0.0000
Root2	0.8245	0.1404	0.8364	0.1687	0.8471	-0.1275	0.8567	-0.1493
Root3	0.8245	-0.1404	0.8364	-0.1687	0.8471	0.1275	0.8567	0.1493
Root4	0.5300	-0.5799	0.7856	-0.8303	0.5288	0.5775	0.7830	0.8294
Root5	0.5300	0.5799	0.7856	0.8303	0.5288	-0.5775	0.7830	-0.8294
Root6	-0.3990	-0.3898	0.5578	-2.3679	-0.4024	-0.3881	0.5591	-2.3742
Root7	-0.3990	0.3898	0.5578	2.3679	-0.4024	0.3881	0.5591	2.3742
Root8	-0.2090	0.3976	0.4492	2.0547	-0.2058	0.4070	0.4561	2.0390
Root9	-0.2090	-0.3976	0.4492	-2.0547	-0.2058	-0.4070	0.4561	-2.0390
Root10	-0.3169	0.0000	0.3169	3.1416	0.1563	-0.2913	0.3306	-1.0784
Root11	0.1531	0.2774	0.3168	1.0665	0.1563	0.2913	0.3306	1.0784
Root12	0.1531	-0.2774	0.3168	-1.0665	-0.3164	-0.0000	0.3164	-3.1416

The trace test depicted in Table A7 tells more or less the same story: it rejects the reduction of rank to two or below, but accepts the one to rank = *3*.

Table A7. Trace test for the unrestricted VAR model with dummies, effective sample 1745-1890

p-r	r	Trace	P-Value	Trace (S)	P-Value (S)
4	0	131.6420	0.0000	126.0287	0.0000
3	1	68.7206	0.0000	66.5170	0.0000
2	2	23.5400	0.0115	21.8653	0.0219
1	3	8.0760	0.0845	7.7857	0.0958

Notes: 'Trace' indicates trace test statistic and corresponding *p*-values. Values indicated with '(S)' correspond to the small sample corrected statistic.

The significance of the coefficients in the α matrix of the unrestricted model also provides clues on rank determination [Hendry and Juselius, 2001]:

$$\alpha = \begin{pmatrix} \underset{(-3.613)}{-0.04} & \underset{(-1.423)}{-0.02} & \underset{(3.581)}{0.04} & \underset{(0.268)}{0.00} \\ \underset{(-4.686)}{-0.26} & \underset{(5.064)}{0.28} & \underset{(-0.00)}{-0.00} & \underset{(1.396)}{0.08} \\ \underset{(4.063)}{0.72} & \underset{(-2.925)}{-0.52} & \underset{(0.040)}{0.01} & \underset{(2.282)}{0.40} \\ \underset{(-7.584)}{-0.61} & \underset{(-2.376)}{-0.19} & \underset{(-1.197)}{-0.10} & \underset{(0.772)}{0.06} \end{pmatrix}$$

The marginal significance of the fourth vector suggests a rank of three. This is further indicated by the evolution of cointegration relationships and recursive trace tests. In Figure A3 we can see how perhaps the last of the relationships shows a clear non-stationary trend, whereas the two clearly increasing trends, and potentially a third one in Figure A4 indicate as well a rank of 3.

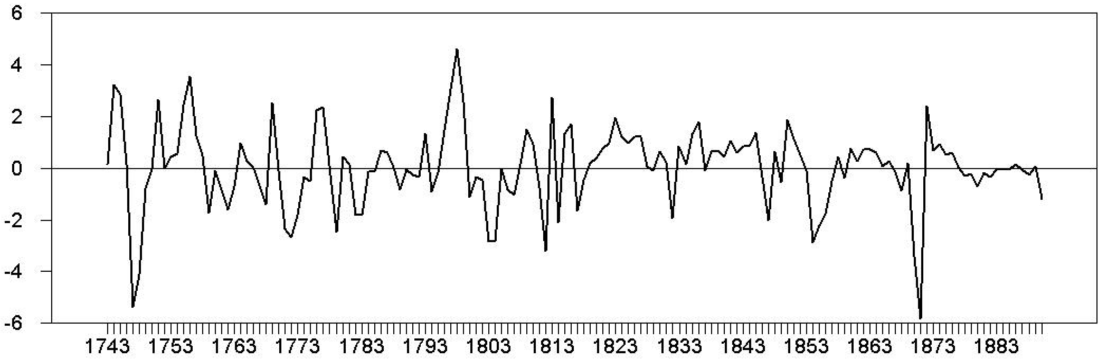
The restricted VAR

The full estimation of the model when we impose the restrictions is the following:

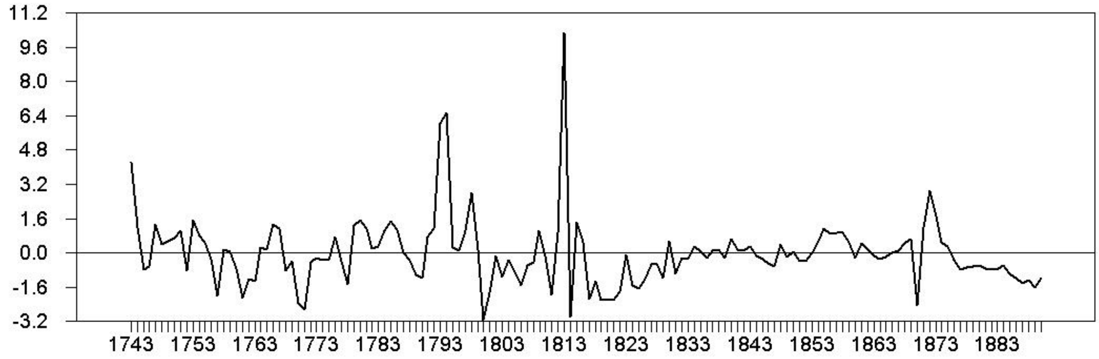
$$\left(\hat{\Pi}, \hat{\Pi}_0\right)\begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} = \begin{pmatrix} \underset{(-6.603)}{-0.17} & \underset{(5.689)}{0.13} & \underset{(2.333)}{0.13} & \underset{(-2.270)}{-0.66} & \underset{(0.903)}{1.02} \\ \underset{(4.826)}{0.41} & \underset{(-4.377)}{-0.33} & \underset{(-0.758)}{-0.14} & \underset{(1.695)}{1.60} & \underset{(-1.332)}{-4.87} \\ \underset{(-4.598)}{-0.17} & \underset{(6.285)}{0.21} & \underset{(-5.624)}{-0.45} & \underset{(-0.534)}{-0.22} & \underset{(4.541)}{7.37} \\ \underset{(-2.681)}{-0.01} & \underset{(1.464)}{0.01} & \underset{(-1.867)}{-0.02} & \underset{(-4.076)}{-0.22} & \underset{(5.210)}{1.08} \end{pmatrix} \begin{pmatrix} b_{t-1} \\ d_{t-1} \\ m_{t-1} \\ \ln w_{t-1} \\ 1 \end{pmatrix}$$

<u>Figure A3.</u> Evolution of the cointegration relationships for the unrestricted VAR model with dummies, effective sample 1745-1890

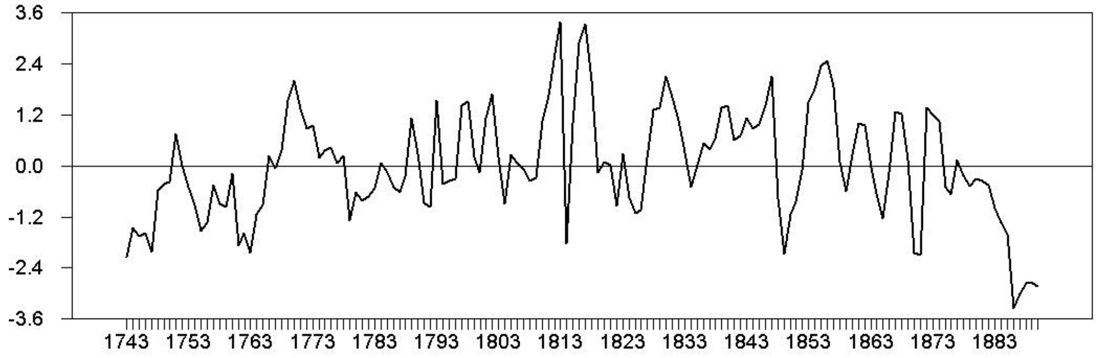
$\beta_1' X_t$



$\beta_2' X_t$

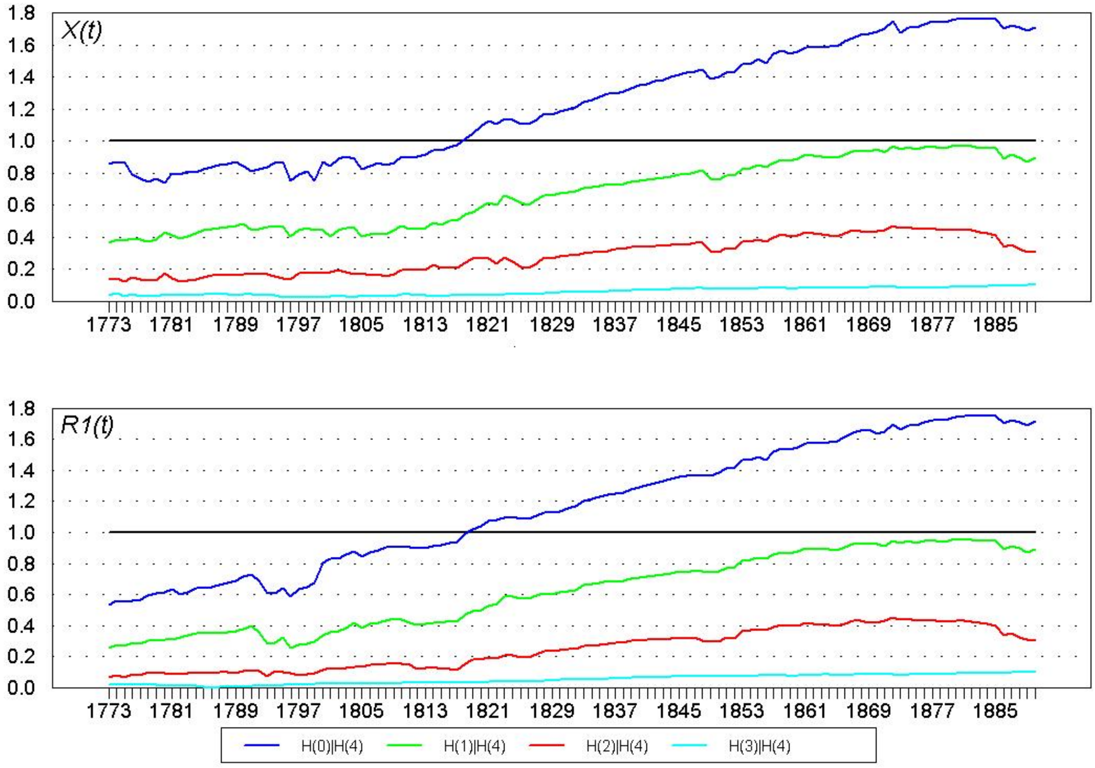


$\beta_3' X_t$



$\beta_4' X_t$

Figure A4. Recursive trace test statistics for the unrestricted VAR model with dummies, 1773-1890



Notes: The test statistics are scaled by the 5% critical values of the basic model.

$$\hat{\Gamma}_1 \Delta X_{t-1} = \begin{pmatrix} \underset{(-6.729)}{-0.55} & \underset{(-4.215)}{-0.10} & \underset{(2.249)}{0.11} & \underset{(7.235)}{2.99} \\ \underset{(-0.658)}{-0.17} & \underset{(-0.680)}{-0.05} & \underset{(1.232)}{0.19} & \underset{(-1.182)}{-1.58} \\ \underset{(0.498)}{0.06} & \underset{(-1.517)}{-0.05} & \underset{(-2.878)}{-0.19} & \underset{(4.042)}{2.40} \\ \underset{(-0.899)}{-0.01} & \underset{(-0.127)}{-0.00} & \underset{(1.993)}{0.02} & \underset{(6.555)}{0.50} \end{pmatrix} \begin{pmatrix} \Delta b_{t-1} \\ \Delta d_{t-1} \\ \Delta m_{t-1} \\ \Delta \ln w_{t-1} \end{pmatrix}$$

$$\hat{\Gamma}_2 \Delta X_{t-2} = \begin{pmatrix} \underset{(-0.388)}{-0.03} & \underset{(-0.903)}{-0.02} & \underset{(-0.019)}{-0.00} & \underset{(-0.428)}{-0.20} \\ \underset{(0.580)}{0.14} & \underset{(1.163)}{0.08} & \underset{(0.678)}{0.08} & \underset{(-4.554)}{-6.91} \\ \underset{(2.622)}{0.27} & \underset{(0.410)}{0.01} & \underset{(-2.525)}{-0.14} & \underset{(-1.219)}{-0.82} \\ \underset{(2.136)}{0.03} & \underset{(1.384)}{0.01} & \underset{(2.655)}{0.02} & \underset{(-2.648)}{-0.22} \end{pmatrix} \begin{pmatrix} \Delta b_{t-2} \\ \Delta d_{t-2} \\ \Delta m_{t-2} \\ \Delta \ln w_{t-2} \end{pmatrix}$$

$$\hat{\Theta}D_t = \begin{pmatrix} \underset{(1.138)}{0.56} & \underset{(1.362)}{0.68} & \underset{(0.794)}{0.40} & \underset{(3.019)}{1.12} & \underset{(2.280)}{1.63} & \underset{(2.269)}{1.22} & \underset{(1.047)}{0.54} & \underset{(-2.911)}{-1.06} \\ \underset{(4.613)}{7.41} & \underset{(1.317)}{2.14} & \underset{(2.861)}{4.69} & \underset{(5.040)}{6.04} & \underset{(-0.929)}{-2.15} & \underset{(0.798)}{1.39} & \underset{(-1.320)}{-2.20} & \underset{(5.201)}{6.14} \\ \underset{(1.918)}{1.37} & \underset{(1.651)}{1.19} & \underset{(0.867)}{0.63} & \underset{(7.654)}{4.07} & \underset{(5.560)}{5.71} & \underset{(12.87)}{9.97} & \underset{(2.125)}{1.57} & \underset{(-4.254)}{-2.23} \\ \underset{(0.831)}{0.08} & \underset{(0.458)}{0.04} & \underset{(0.664)}{0.06} & \underset{(-0.183)}{-0.01} & \underset{(3.210)}{0.42} & \underset{(2.027)}{0.20} & \underset{(4.491)}{0.42} & \underset{(0.648)}{0.04} \end{pmatrix} \begin{pmatrix} D_t^{T:1747} \\ D_t^{T:1753} \\ D_t^{T:1779} \\ D_t^{T:1793} \\ D_t^{P:1798} \\ D_t^{T:1813} \\ D_t^{P:1848} \\ D_t^{T:1870} \end{pmatrix}$$

$$\hat{\Omega} = \begin{pmatrix} 1.00 & & & \\ -0.14 & 1.00 & & \\ 0.36 & -0.05 & 1.00 & \\ 0.13 & -0.03 & 0.18 & 1.00 \end{pmatrix}, \quad \hat{\sigma}_\varepsilon = \begin{pmatrix} 0.6727 \\ 2.1551 \\ 0.9729 \\ 0.1245 \end{pmatrix}$$

Chow-Tests

Although it is clear that these periods make historic sense [Weir, 1984: 36], it is worth asking whether they also make statistical sense. A way to address this issue is to use a Chow test, which is simply a Wald-test of whether the coefficients estimated over one group of data are equal to those estimated over other group of data. When testing whether the four groups were the same (in a Wald-test sense) when running the regressions on crude birth rates, the overall statistic suggest they were (F(33, 115) = 1.63 [0.032]). Further analysis, however, suggests this result is mainly driven by the inclusion of the last period, as can be seen in Table A8.

Table A8. Chow-tests for the crude birth rate model in the short-run analysis

	1748-1789	1790-1829	1830-1865	1866-1906
Respect to whole period				
all coefficients	0.94 [0.505]	1.50 [0.138]	0.62 [0.813]	3.34 [0.000]***
crude birth rate	0.26 [0.769]	1.31 [0.273]	0.53 [0.591]	0.55 [0.577]
adjusted death rate	0.76 [0.553]	2.97 [0.022]**	0.33 [0.861]	6.56 [0.000]***
real wage (unskilled)	0.95 [0.435]	0.53 [0.717]	1.10 [0.358]	2.02 [0.095]*
Respect to previous period				
all coefficients		0.78 [0.662]	0.44 [0.933]	3.01 [0.003]***
crude birth rate		0.14 [0.870]	1.03 [0.362]	0.02 [0.977]
adjusted death rate		0.47 [0.757]	0.62 [0.650]	2.40 [0.061]*
real wage (unskilled)		0.94 [0.447]	0.44 [0.781]	4.16 [0.005]***

Sources: See Appendix I for sources. Chow-test statistics corresponding to the regressions used to explain crude birth rate. *, **, and *** denotes the 0.10, 0.05, and 0.01 significant level, respectively.

The set of numbers on top assess whether the coefficients (either all of them or those for each relevant variable) of a regression on the whole period 1745-1906 are statistically different from those of each sub-period. Those on bottom, on the other hand, compare one period with the previous one (again, for each group of variables). The general picture one gets from looking into these figures is that indeed coefficients are clearly different only in the last period. This result does not change if we look instead to the index of marital fertility, as in Table A9. For that the overall test also suggests the coefficients were not the same across the periods (F(33, 115) = 1.55 [0.047]), but a closer look indicate that this result is driven by the last period.

Table A9. Chow-tests for the marital fertility model in the short-run analysis

	1748-1789	1790-1829	1830-1865	1866-1906
Respect to whole period				
all coefficients	1.19 [0.297]	1.17 [0.313]	0.58 [0.845]	3.27 [0.001]***
marital fertility (Ig)	1.00 [0.369]	0.53 [0.591]	0.52 [0.598]	0.54 [0.587]
adjusted death rate	0.98 [0.419]	1.79 [0.135]	0.31 [0.874]	5.79 [0.000]***
real wage (unskilled)	0.96 [0.434]	0.48 [0.754]	0.95 [0.435]	2.36 [0.057]*
Respect to previous period				
all coefficients		0.74 [0.696]	0.26 [0.991]	3.04 [0.003]***
marital fertility (Ig)		0.24 [0.786]	0.22 [0.802]	0.05 [0.954]
adjusted death rate		0.25 [0.912]	0.35 [0.840]	2.19 [0.083]*
real wage (unskilled)		0.72 [0.579]	0.37 [0.830]	4.21 [0.005]***

Sources: See Appendix I for sources. Chow-test statistics corresponding to the regressions used to explain crude birth rate. *, **, and *** denotes the 0.10, 0.05, and 0.01 significant level, respectively.

Although further weakening the relevance of these regressions to construct the responses, the results of these tests support the idea I develop in the text that the substantial shift in the relationship takes place relatively late in the period.

For the case of marriage rate, described in Table A10, we have that the constancy of parameters is only slightly rejected under the standard values (F-test = 1.39 [0.104]), but in-sample some breaks are clear, especially during the revolutionary period.

<u>Table A10</u>. Chow-tests for the crude marriage rate model in the short-run analysis

	1748-1789	1790-1829	1830-1865	1866-1906
Respect to whole period				
all coefficients	1.35 [0.202]	3.00 [0.001]***	0.41 [0.952]	1.31 [0.227]
crude marriage rate	4.09 [0.019]**	7.65 [0.001]***	0.00 [0.996]	0.31 [0.737]
adjusted death rate	0.63 [0.640]	2.46 [0.048]**	0.19 [0.946]	1.85 [0.123]
real wage (unskilled)	1.04 [0.389]	1.52 [0.199]	0.65 [0.627]	1.14 [0.340]
Respect to previous period				
all coefficients		1.14 [0.349]	0.60 [0.823]	1.70 [0.099]*
crude marriage rate		3.43 [0.039]**	0.05 [0.950]	0.42 [0.662]
adjusted death rate		0.73 [0.575]	0.93 [0.451]	2.80 [0.035]**
real wage (unskilled)		0.18 [0.949]	0.74 [0.567]	1.76 [0.151]

<u>Sources</u>: See Appendix I for sources. Chow-test statistics corresponding to the regressions used to explain crude birth rate. *, **, and *** denotes the 0.10, 0.05, and 0.01 significant level, respectively.