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Theory and Experiment[*]

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Abstract

We explore theoretically and experimentally the general equilibrium price and allocation implications of delegated portfolio management when the investor-manager relationship is non-exclusive. Investors transfer their securities allocations to managers, managers trade in a competitive marketplace to achieve new allocations, and payoffs are distributed back to investors after subtraction of a portfolio management fee. Our theory predicts that competition forces managers to promise portfolios that mimic Arrow-Debreu (AD) securities, which investors then combine to fit their preferences. As such, individual manager portfolios do not match investor preferences, but certain combinations will, and consequently, prices in the inter-manager market still reflect investor preferences. In particular, state prices (relative to state probabilities) implicit in prices of traded securities will be inversely ranked to aggregate wealth across states, which means that a weak version of the Capital Asset Pricing Model (CAPM) obtains, as in a world without delegated portfolio management. Our experiment broadly corroborates the price and choice predictions of the theory. However, pricing quality deteriorates when only a few managers end up attracting most of the available wealth. Fund concentration increases because funds flow towards managers who offer portfolios closer to replicating AD securities (as in the theory), but also because funds flow to managers who had better performance in the immediate past (an observation unrelated to the theory).

1 Introduction

Participation in investment markets has steadily shifted away from individuals toward institutions. With 93% of US equity directly held by individuals in 1950, this figure was estimated to be as low as 25% by 2009.[1] At the same time, households' holdings of funds has risen from a mere 3% in 2000 to 23% of their total financial assets in 2010, with mutual funds consequently owning 27% of all US equity, 45% of all commercial paper, and 11% of US treasury securities.[2] While there is no doubt that the participation of delegated portfolio managers who invest for others is one of the key characteristics of both modern equity and debt markets, it remains an open question, and a subject of a growing literature, how delegation impacts asset prices and market efficiency.

In this paper, we explore the consequences for asset prices and allocations of competition between portfolio managers within a stylized general equilibrium framework, and we offer experimental evidence on the key principles underlying our theory. Absent delegation, the setup is one that had been tested extensively in the laboratory and the experiments had provided robust support for the theory [6, 7, 3]. As such, there exists a solid benchmark against which to calibrate an experiment with delegation.

Investors in the real world have choices of both direct and indirect investment. To cleanly isolate the effects of delegation and competition, we shut down the direct investment channel: our investors hold securities but they cannot directly trade. Instead, they need to engage managers to trade on their behalf. Managers each offer a portfolio contract and stand ready to deliver any number of units of it in exchange for a fixed fee per unit. Importantly, managers compete for investors and investors can hold contracts from multiple managers. We are interested in what portfolios managers offer, what portfolios investors choose, and what asset prices emerge in the inter-manager market.

To further simplify our theory and experiment, we consider an environment where both skill and information are homogeneous across managers. We also assume a basic fee structure, stripping out complications, even at the cost of reducing realism. We do not doubt the importance of skill and informational advantage that some managers may possess. Nor do we under-estimate the impact of more complex fee structures. Still, even with added realism, the research question would continue to be about the effects of competition for fund flows, and the layers of complexity one would have to add to accommodate skill or information advantages or to make fee structure more realistic would only make it more difficult to isolate the specific impact of manager competition on prices and allocations.

Much of the existing literature has focused on contract design in the resolution of the conflicts of interest between investors and (heterogeneously skilled or informed) portfolio managers [5, 19, 14,

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[1]See [11]

[2]Except when specified, all data reported in this introduction are taken from the 2011 Investment Company Fact Book, [15].

16, 9, 13]. Another strand takes as given the conflicts of interest and their imperfect resolution (e.g., through benchmarking), and studies the impact on market behavior [8, 10, 12]. Our approach differs from both in that we do not consider the relationship between manager and investor as one-to-one (between a principal-investor and agent-manager). Instead, in our approach, the relationship may be many-to-many: investors may engage several managers, and managers may offer services to multiple investors.

As a result, our theory is essentially an extension to delegated portfolio management of the analysis of competition in contracts pioneered by Rothschild & Stiglitz [18]. Our application goes one step further, though: it not only analyzes what contracts will emerge in equilibrium, but also whether pricing in the securities markets is affected because portfolio management is delegated. Our equilibrium notion is thus closely related to that of Zame [20], who investigates the interaction between, on the one hand, prices in markets for goods, and on the other hand, competition between contracts that arrange the production of the goods to be supplied to these markets. There are a number of reasons why we follow the tradition of competitive analysis of contracts. First, it provides a manageable notion of competitive equilibrium. Second, in the field, there is plenty of casual support for its main predictions.[3] Third, these and other predictions have recently been confirmed in experimental testing [2].

Berk & Green [4] also consider competition between fund managers, but their model focuses on fee competition in the face of learnable talent differences. Closer to our analysis, Agranov, Bisin & Schotter [1] investigate strategic portfolio choices when two managers Bertrand-compete for a single investor's money. Our setting is competitive: managers may enter freely, and may attract multiple investors. Moreover, [1] does not study the consequences for pricing in the inter-manager market.

Our theory adds delegation to the standard asset pricing model in finance. In the standard model, investors engage directly with the market. Investors are risk averse and hence, equilibrium prices and allocations reflect investors' desire to smooth wealth across states. In equilibrium, investors hold well-diversified portfolios, and, provided markets are complete, the price of insurance against poor states (relative to the corresponding state probability) is larger than that in rich states. In other words, *state-price probability ratios are inversely related to aggregate wealth across states.*[4] This standard equilibrium pricing result will be referred to as *weak CAPM* (Capital Asset Pricing Model).

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[3]In the context of lending under adverse selection, for instance, this theory anticipated the inevitability of zero-deductible insurance contracts or zero-downpayment "subprime" loan contracts, and it explained why banks could at times be casual about creditworthiness checks (because risks could easily be inferred from contract choice).

[4]Uncertainty in an economy is represented by *states* of the world. States differ in that they entail varying aggregate or private wealth. The price of a state is the value of claims to state-contingent payoffs referred to as Arrow-Debreu (AD) securities. An AD security for state s pays one unit of cash in state s and 0 in all other states.

We first introduce delegation as follows. Fund managers compete for investors who hand over (part) of their wealth, and obtain shares proportional to the value of the assets handed over. Investors compensate managers with a nominal fee also proportional to the value of the contributed assets. This base setup, however, leads to unsatisfactory predictions because a wide variety of choice patterns (investor choice of managers and manager choice of portfolios) is consistent with equilibrium. To sharpen predictions, we add a limited liability provision: investors are liable for the fee only to the extent that the manager's final portfolio value covers it. The result is that managers now compete to reduce the *effective* fee paid by investors. Sharp equilibrium predictions ensue. Managers will pattern their portfolios to mimic AD securities and investors obtain diversification by investing in many managers simultaneously. This "home made" diversification eventually is reflected in pricing: prices in the inter-manager asset market are identical to those in the standard model (weak CAPM obtains), except for a lower risk premium.

There is a problem with implementation of the above scenario, however. Both fees and shares in a manager's fund are determined by the *value* of the assets an investor contributes. Computation of value requires asset prices, which do not become available until the inter-manager market opens. To avoid having to guess prices, and inspired by practice in the field, we study a variation of our theory where the share in a fund (and resulting fees) is determined by *expected final payoffs*. In this case weak CAPM still obtains under mild additional assumptions on investor initial allocations.

We report the results of a large-scale experiment designed to test the predictions of our theory. The design closely followed that of earlier experiments that had reliably generated weak CAPM pricing in the absence of portfolio delegation [6, 7, 3]. Thirty two subjects participated as managers during all replications. On average seventy subjects participated as investors (not all investors were the same across replications). At the start of each replication, investor-subjects (investors) allocated their initial endowment to manager-subjects (managers). Managers could then trade the assets, during a certain time, through anonymous, online continuous open-book markets. When trading concluded, investors were paid their share of the liquidating fund payoff, net of fees. Managers received a payment proportional to the expected payoff of initial contributions, while investors' obligation to pay the fee was limited by the final value of a manager's portfolio. Basic performance indices were reported, and investors were given new endowments of securities and cash, and a new replication started. This process was replicated six times (over six "periods").

Consistent with the theory, we found strong evidence that investors preferred managers whose portfolios mimicked more closely AD securities. Still, investor choices were also partly determined by performance in the immediate past. Over time, these two elements contributed to increased concentration of wealth across managers. Price dynamics appeared to be driven by weak CAPM, but

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pricing quality was negatively affected as fund concentration increased. In contrast to manager portfolios, which became more "polarized" over time (better mimicked AD securities), investor portfolios (of manager funds) were well diversified.

In sum, our results are suggestive that competition between fund managers along with the ability of investors to distribute wealth across multiple funds preserve the main predictions of asset pricing theory because investors can engage in home-made diversification, and hence prices continue to reflect the demand for diversification on which (weak) CAPM is based. Managers do not offer portfolios that are optimal on their own; managers instead specialize in providing components of optimal portfolios if this is advantageous to investors. Our experiment corroborated the theoretical developments. From a policy point of view, our theory advises that competition be encouraged in the mutual fund industry, by allowing investors access to multiple fund managers, and ensuring free entry. The current practice of many employers, to contract retirement savings of their employees with only one manager (TIAA-CREF, Fidelity, Vanguard, etc.), is to be shunned. At the market level, this not only improves competition, but it also enhances price quality, as is clear from our experimental results. If concerned with fostering investors' effective benefit from fund markets, use of fund performance measures that assume that the investor can only acquire shares in a single fund (e.g., Sharpe ratios) should be discouraged. Our theory (and experiment) illustrates this in an extreme fashion: it is optimal for managers (and their customers agree) to take huge risks, namely, to promise payouts in only one state, as if betting that only one state will occur. Finally, in a competitive fund industry, management compensation schemes may distort incentives less than previously thought because managers compete for patronage in another dimension, namely, composition of the portfolios they offer.

The remainder of the paper is organized as follows. The next section provides further motivation for the structure behind our theory and our experiment. The theory is developed in Section 3. Section 4 presents details of the experimental design. Section 5 discusses the results, and Section 6 concludes.

2   Motivating the Theory

Before presenting our theory, we elaborate on some of the issues raised in the Introduction concerning its approach and its scope.

We focus on aspects of delegated portfolio management that have not been investigated much, yet may be important drivers of outcomes as we see them in the real world. Specifically, we study here the consequences of the ability of investors to allocate wealth across several managers, and of

the need for managers to compete with others for the patronage of investors. We deliberately limit the scope, isolating the forces we choose to focus on, in order to understand, and experimentally corroborate, the impact of these forces on outcomes.

Furthermore, we insist on studying not only pricing, but perhaps more importantly, choices (of investors and managers). While much empirical work analyzes only prices because choice data are often absent in historical data, our experimental approach affords equal access to prices and choices. To make the exercise meaningful, we introduce forces so that choice predictions become sharp.

With these considerations in mind, we first discuss the general environment. Subsequently, we introduce the necessary refinements that generate sharper choice predictions. Finally, we present modifications that facilitate laboratory testing of the theory.

The Environment

We envisage a situation where investors face both idiosyncratic and aggregate risk in a one-period investment setting. Investors are not allowed to manage their own wealth; they are required to delegate its management to one or several funds, for a fee. Specifically, today ($t = 0$), investors can acquire contracts offered by investment managers so that their payoff tomorrow (at $t = 1$) has a desirable distribution of risk. Managers supply the contracts demanded by investors through trading in a complete set of financial markets. They use the resources entrusted to them by the investors to pay for the securities that could provide the promised payoff distribution. Resources consist of a portfolio of state contingent "Arrow-Debreu" (AD) claims. We assume that the investors are risk averse and have a concave utility over wealth tomorrow and maximize the expected value of that utility today. Managers are assumed to be risk neutral and only care about the payment they receive from investors in the form of a management fee. The fee owed to a manager and shares held in a fund are proportional to the value of assets entrusted with that manager.

This baseline setting allows us to readily make sharp predictions about asset prices, but choices (of investors and managers) are far less constrained, certainly in a setting where one does not control or even observe risk aversion.[5] Moreover, in this baseline scenario, fees and shares depend on an endogenous variable, namely, the value of assets entrusted by the investor. We next motivate refinements on the baseline that generate sharper choice predictions and facilitate experimental implementation.

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[5]In experiments risk aversion is only observed indirectly, through choices. But inferring risk attitudes from choices requires one to know the right choice predictions.

Investor Preferences

It is tradition in finance to restrict preferences in order to generate asset pricing restrictions that lead to convenient statistical tests on historical data. The CAPM (Capital Asset Pricing Model), for instance, assumes quadratic utility, and can readily be tested by checking the significance of intercepts in least-squares projections of historical average excess returns of individual assets/portfolios onto those of a market benchmark. The spirit of the main parametric asset pricing models, however, carry over to more general models. All models somehow restrict returns, and hence prices, as a function of the covariance (across states) between asset payoffs and aggregate wealth. We shall refer to the general restrictions as the "weak CAPM." The general restrictions are hard to test on historical data, because relevant states have to be specified unambiguously, their probabilities have to be assessed, and aggregate wealth in each of them has to be identified. In an experimental setting, in contrast, states, their probabilities, and aggregate wealth are set by the experimenter, and hence, form part of experimental control. Therefore, we can aim for general asset pricing restrictions, obviating the need to make (potentially unrealistic) assumptions about preferences. Nevertheless, in the Appendix, we briefly discuss how and when more popular parametric asset pricing equations, in particular (strict) CAPM, obtain.

Manager Competition

In our baseline environment, a multitude of choice scenarios support the pricing predictions. Investor choices could be exactly pinned down if we had information on investor preferences (risk aversion). However, even in such an idealized scenario, we would still have multiple manager choices consistent with equilibrium pricing predictions.

We thus add restrictions that induce a form of competition with strong implications for equilibrium choice of managerial contracts, while inducing transparent incentives for investors in their choices of managers regardless of how risk averse they are, and while keeping intact the pricing implications. Specifically, we provide investors with *limited liability* for fees: they are liable for fees to a manager only to the extent that the final value of this manager's contract covers the fees. Limited liability renders manager payment more realistic than in the baseline because fees become correlated with manager performance across states of the world. Indeed, real-world fee schemes typically do change over time, if only because the value of assets under management fluctuates, and hence, manager compensation is state-dependent.

Limited liability introduces a specific form of competition among managers since they can offer contracts that allow investors to save on fees in some states of the world. It is an alternative to direct fee competition that is advantageous for experimental purposes. In our setting, if managers

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were allowed to choose fees, competition would force fees down to zero. As a consequence, subjects who play the role of manager would become dis-incentivized. Without proper incentives, we cannot expect manager-subjects to compete aggressively in a (for this study) more important dimension, namely, portfolio composition. In any event, direct fee competition would not solve the problem of multiplicity of the type of contracts offered by managers in equilibrium.

Unlike competition in fees, limited liability allows managers to earn positive profits in equilibrium, and hence, provides the right incentives to manager-subjects. A further experimental rationale for limited liability is that it prevents investors from going bankrupt. Bankruptcy has distorting effects on experimental studies since human subjects protection protocol prohibits experimenters from enforcing negative earnings, thus effectively truncating the distribution of earnings from below.

Manager-Subject Preferences

With limited liability, fee income becomes potentially state-dependent, and hence, we need to worry about risk attitudes of manager-subjects. Our theory assumes that managers are risk neutral. Yet, ample evidence exists that subjects are averse to risk in the laboratory (as mentioned before), and therefore choices of manager-subjects would be different from those under risk neutrality if fees are state-dependent. In the experiment, we addressed this issue by bailing out the managers in case their portfolio value did not cover fees. As a result, manager income became state-independent and hence risk-free (while investor income generally remained state-dependent), so manager risk aversion was of no consequence. Predicted outcomes would be the same as if fee income were state dependent and managers were risk neutral. Optimal choices of investor-subjects are unaltered.

Share And Fee Determination

In the baseline scenario, investors hand over to the chosen managers assets with a worth sufficient for managers to trade to the contractually promised portfolios. The worth is valued at inter-manager market prices. Unfortunately, these prices are unavailable at the moment the investors make their choices of managers.

The field suggests a practical way to solve this problem while at the same time illustrating that the solution is not perfect. Consider a new investor who contributes cash to an existing fund. Shares are traditionally assigned based on the value of all assets (both already in the fund and contributed) at the close of the market after contribution. Consider the fraction of the manager's portfolio that the contributed cash will buy if stock prices increase the subsequent day when the manager aims for a particular portfolio composition, say, an equally weighted index of two types of common shares. This fraction will be lower than the share allocated to the new investor, which was based on the lower

stock prices at the close the previous day.[6] Conversely, the cash contribution of the new investor will buy a higher fraction of the manager's portfolio if stock prices decline, compared to the share that was allotted. That is, the share in the eventual portfolio which the new investor's cash contribution generates generically does not correspond to the share he is being assigned.

Notice that, in the field, shares are effectively determined by the *expected payoff* of the asset contributed, namely, cash. We decided to apply the same rule. In our experiment, however, investor-subjects contribute assets not only in the form of cash, but also in the form of risky securities. Still, the rule can be applied: shares (and consequently, fees) can be determined based on the expected payoff of the contributed assets. Since the experimenter controls and announces the state probabilities, expected payoffs, and hence shares and fees, are readily computed and immediately verifiable.

In principle, investor-subjects could choose many combinations of asset holdings to hand over to managers. To simplify the theoretical analysis, we required investor-subjects to hand over fractions of their entire endowment, so that different managers received portfolios with the same composition. E.g., if a given manager received 7% of a certain investor's holdings of one security, she received 7% of this investor's holdings of all other securities as well. In the theory, we shall also assume that every investor starts with the same initial portfolio, so that expected payoffs and values are the same across investors. For reasons discussed in Section 4, we did not implement this assumption literally in the experiment; instead, we allocated securities in ways that ensured that expected payoffs and values were approximately equal, which was all we needed for the theory to apply.

3   The Model

We now develop the theory, starting from the baseline setting, and subsequently introducing amendments that generate sharper (choice) predictions and that simultaneously admits of better experimental control.

Baseline Scenario

The baseline is a variation of an Arrow-Debreu (AD) economy with I investors. States are indexed by s $(= 1, \ldots, S)$ and π_s denotes the chance that state s occurs. An AD security, or a state security, s, pays \$1 in state s and \$0 otherwise. Investor i $(i = 1, \ldots, I)$ has an initial endowment of state-

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[6]If he contributed \$5 in cash and the existing fund was worth \$10 at the close, he is assigned a share of 1/3. If prices subsequently increase 20%, the existing fund will be worth \$12, but the cash contribution still only allows the manager to buy \$5 in stock, and hence, the contributed assets buy only 5/17 ($<$ 1/3) of the shares of the fund as intended by the manager.

dependent wealth stream, expressed as a collection of state securities with $x_s^{i,0}$ units of security s. Let $x_s^0 = \sum_i x_s^{i,0}$ be the aggregate wealth in state s.

There are M, $M > S$, managers indexed by m $(= 1, \ldots, M)$, each of whom can offer investors units of a single contract denoted θ^m. This contract, gross of management fees, is either an AD security or a convex combination of AD securities. Thus, a manager m's contract is a vector $\theta^m = (\theta_1^m, \ldots, \theta_S^m)$, $(\theta_s^m \geq 0, \sum_s \theta_s^m = 1)$.[7] Managers can sell any non-negative number of units of their contract to different investors. To deliver the promised payouts, managers trade in a set of complete securities markets. Managers enter the markets endowed with assets given to them by investors. Markets are competitive and both borrowing and short selling are allowed. Prices of securities are denoted p_s^d $(s = 1, \ldots, S$; the superscript d indicates that these prices are from the inter-manager market).[8]

Managers are paid a fixed fee, f $(0 < f < 1)$, for each unit of contract they sell, charged as a back-end load fee. Let x_m^i denote the number of contracts investor i acquires from manager m $(x_m^i \geq 0)$. As a result, investor i owes a fee of $x_m^i f$ to manager m. To acquire x_m^i units of manager m's contract, investor i has to hand over the necessary holdings of state securities so that manager m can trade to positions that allow her to deliver the contractually-specified payoffs. This means that i must hand over to m state securities with a total value of $\sum_s p_s^d x_m^i \theta_s^m$.

Investors are assumed to have a standard concave utility over final wealth after fee payment. The wealth gross of fees of investor i in state s is equal to $x_s^i = \sum_m x_m^i (\theta_s^m - f)$. Investor i is assumed to be maximizing expected utility with respect to his contract holdings. Investor i's objective function is therefore

$$U_i(\{x_m^i\}) = \sum_{s=1}^S \pi_s u_i \left(\sum_m x_m^i (\theta_s^m - f) \right).$$

Investor i's budget constraint is derived as the sum of i's constraints with respect to each manager:

$$\sum_s p_s^d \sum_m x_m^i \theta_s^m = \sum_s p_s^d x_s^{i,0}.$$

The economy is thus comprised of the set of investors I, their (concave) utilities over final wealth, u_i, and their initial endowments, $x_s^{i,0}$, and the set of managers M and their fees f.

An <u>equilibrium</u> for this economy is given by:

(i) Managers' offered contracts, $\theta^m, m = 1, \ldots, M$, such that no contract outside the equilibrium set can make at least one investor better off;

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[7]If the manager chooses to offer an AD security, $\theta_s^m = 1$ in some state s and $\theta_{s'}^m = 0$ in all other states, $s' \neq s$.

[8]In the experiment, investors are endowed with, and managers trade in, assets that are not state securities. Because our markets are complete, however, all trades and prices can always be re-expressed in terms of state securities.

(ii) Investor holdings, $x^i, i = 1, \ldots, I$ ($x^i = (x_1^i, \ldots, x_M^i)$), such that expected utility, U^i, is maximized subject to the budget constraint;

(iii) Inter-manager market prices, $p_s^d, s = 1, \ldots, S$, such that all markets clear.

This equilibrium notion follows the tradition established in [18]. Conditions (ii) and (iii) above are standard. Condition (i) states that in equilibrium *no investor can benefit from the addition* of a contract to the equilibrium set. This requirement relates to subgame perfection. It pre-supposes that making some investor better off is profitable for an entering manager. This can only be the case if investors choose optimally along each subgame, immediately reacting to the entry of a beneficial contract. Even with subgame perfection, a *deviation* that improves the situation of some investor may not be beneficial for a manager, since she may lose other "clients" (investors). Hence our requirement that no *additional* contract be beneficial. With sufficiently many managers, there will almost certainly be redundant contracts, in which case a deviation by an existing manager can be interpreted as an entry.[9]

If $M > I$, an equilibrium where managers coordinate and offer investors' optimal portfolios always exists. Non-trivial equilibria where managers offer portfolios different from each investor's optimum exist if managers offer a complete set of securities such that all investors can achieve their optimum by acquiring a conical combination of those securities. One such equilibrium, requiring that $M > S$, is where each manager offers an AD security.

Notice that any equilibrium with a complete set of contracts is equivalent in state prices and investor allocations to an equilibrium where only the S AD securities are offered. If managers only offer AD securities, we can assume that there are S representative managers indexed by s, the AD security they offer. The investor problem becomes one where he directly chooses his state s' wealth (gross of fees) x_s^i. As a result, the program for investor i in this setting is:

$$\max_{x^i} \sum_{s=1}^S \pi_s u_i \left(x_s^i - f \sum_{s'=1}^S x_{s'}^i \right)$$

$$\text{s.t. } \sum_{s=1}^S x_s^i p_s^d = \sum_{s=1}^S x_s^{i,0} p_s^d.$$

Investors pay a fee, f, on the total of contracts they acquire, $\sum_s x_s^i$. Hence, their wealth net of fees is $z_s^i = x_s^i - f \sum_{s'} x_{s'}^i$. Normalizing market prices so that they add up to one, the first-order

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[9]Consider a setting, as in our experiment, where there are 3 states of the world and 32 managers. Three linearly independent contracts suffice for investors to achieve their optimal portfolios. Suppose only two linearly independent contracts are offered. It is very likely that some manager can make a profit by offering a third linearly independent contract instead of the redundant contract she is currently offering.

condition (FOC) of the investor program generates the following condition for relative prices:

$$\frac{u_i'\left(z_s^i\right)}{u_i'\left(z_t^i\right)} = \frac{\left[(1-Sf)\,p_s^d + f\right]/\pi_s}{\left[(1-Sf)\,p_t^d + f\right]/\pi_t}.$$

With complete markets, the right hand side of the above expression is equal for all i, which implies the existence of a representative consumer with utility u, such that

$$\frac{u'\left(z_s^0\right)}{u'\left(z_t^0\right)} = \frac{\left[(1-Sf)\,p_s^d + f\right]/\pi_s}{\left[(1-Sf)\,p_t^d + f\right]/\pi_t}. \tag{1}$$

Clearly market prices are different than when portfolio delegation is absent.[10] However, the powerful relation that the state price-probability ratios are inversely ranked to the wealth in the states continues to hold. In line with previous experimental work, we call this relation "weak CAPM." State-price probability ratios are the ratios of state prices to the probability of occurrence of the state. For general state probabilities, the relation (1) implies that weak CAPM holds for fee-adjusted prices, $\tilde{p}_s^d = (1-Sf)\,p_s^d + f$, but not necessarily for the inter-manager trading prices, p_s^d. When states are equally likely, weak CAPM also holds for inter-manager trading prices, since

$$\frac{\tilde{p}_s^d/\pi_s}{\tilde{p}_t^d/\pi_t} = \frac{\tilde{p}_s^d}{\tilde{p}_t^d} = \frac{(1-Sf)\,p_s^d + f}{(1-Sf)\,p_t^d + f},$$

and clearly, $\tilde{p}_s^d > \tilde{p}_t^d$ if and only if $p_s^d > p_t^d$.

Introducing Limited Liability

Adding limited liability for fee payments, the investor program with AD contracts becomes as follows.

$$\max_{x^i} \sum_{s=1}^{S} \pi_s u_i\left(x_s^i(1-f)\right)$$

$$\text{s.t.} \ \sum_{s=1}^{S} x_s^i p_s^d = \sum_{s=1}^{S} x_s^{i,0} p_s^d.$$

Notice that investor i's fee-adjusted wealth in state s is now $z_s^i = x_s^i\,(1-f)$. This is because a manager offering state security s will only receive her fee payment in state s. Hence, investor wealth in any given state s is reduced by the fee payment made to state s managers only.

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[10]Without delegation and delegation fees, the condition analogous to condition (1) would be

$$\frac{u'\left(x_s^0\right)}{u'\left(x_t^0\right)} = \frac{p_s/\pi_s}{p_t/\pi_t}.$$

As before, market completeness and the first order conditions of investors imply that there exists an aggregate investor with utility u such that

$$\frac{u'(z_s^0)}{u'(z_t^0)} = \frac{p_s^d/\pi_s}{p_t^d/\pi_t}.$$

As before, state price-probability ratios are inversely ranked to the wealth across states. In other words, weak CAPM continues to hold.[11]

With limited liability, managers will want to offer AD securities. To see this, refer to Fig. 1, and imagine that an investor would like to acquire a portfolio which, including fees, brings him to position a post-fee. This can be accomplished by acquiring ξ units of the contract of a manager who offers the AD security 1 (which pays \$1 in state 1) and $1 - \xi$ units of 2. The position gross of fees is denoted a'. The total fee that our investor pays equals ξf in state 1, and $(1-\xi)f$ in state 2. The AD securities expire worthless in complementary states, and hence, the investor does not owe any fee for those states. Can a manager enter and make the investor better off when offering a contract which is a strict convex combination of AD securities, like E in Fig. 1? Our investor could now generate a' with γ units of AD contract 1 and $1 - \gamma$ units of contract E. (Investors can only acquire positive fractions of manager contracts, so our investor cannot obtain a' by combining E with AD security 2.) Necessarily, $\gamma < \xi$, but because of this, the fee cost to our investor is strictly higher in both states: it is $\gamma f + (1 - \gamma)f = f \ (> \xi f)$ in state 1 and $(1 - \gamma)f \ (> (1 - \xi)f)$ in state 2. So, *net of fees*, our investor receives strictly less in both states than required for his desired allocation a. The investor will not be enticed by the manager with contract E; he will stay with the manager with contract 2. The argument is not specific to two states.

Hence, when there are a sufficient number of managers, we obtain an equilibrium where a complete set of AD securities is offered to satisfy the demands of all investors. There can be other equilibria where a complete set of contracts different from AD securities is offered. Such equilibria are not intuitive, though. More importantly, not every complete set of securities survives the entry of even a single manager offering an AD security. One can push this and allow for simultaneous entry of multiple managers and refine our equilibrium notion to require that "no set of contracts outside those offered in equilibrium makes at least one investor better off." *With this strong form of equilibrium, only equilibria where managers offer a complete set of AD securities survive.* Other equilibria will be ruled out because a set of AD contracts would make investors better off.

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[11]See the Appendix for a numerical example where investors hold quadratic utility and parameters emulate those of our experimental design. The example is worked out both with and without limited liability.

Share And Fee Determination Based On Expected Payoffs

We now study the impact of assessing the management fees as a fixed percentage ϕ of the *expected payoff* rather than the value of the contributed assets. At the same time, shares in the fund will be determined by the expected payoff of contributed assets as well.

First, we address whether this changes the desire of investors to solely choose managers who intend to trade to AD securities, and hence, to whom they owe the management fee in only one state. The answer is no, and here is why. The investor allocates fractions of his initial endowment to each of the managers, so the fees he pays in each state will only be a *fraction of the fraction* of the expected value of his initial endowment he allocates to the managers who invest for the corresponding state. Imagine that an investor's initial endowment is expected to generate \$10 in payoffs. The investor allocates 1/3 of his wealth to managers who intend to buy AD securities for each of three possible states. If the fee rate is $\phi = 0.2$, then he owes managers \$0.67 (=10*0.33*0.2) in each state. If instead the investor decides to assign his entire wealth to a manager who intends to buy to a portfolio that pays in each state, he owes the manager \$2 (=10*0.2) in all states (provided the manager always generates enough payoff). The latter is obviously larger, so the investor should prefer to allocate his wealth over managers who commit to trading to AD securities.

As to the impact on pricing, we envisage that investor i contributes a share α_s^i of his entire portfolio to the manager offering AD security s. Let $P_i = \sum_s \pi_s x_s^{i,0}$ be the expected payoff of investor i's initial endowment, and $W_i = \sum_s p_s^d x_s^{i,0}$ be the market value of investor i's initial endowment. Since all securities are distributed equally to a given manager, it follows that manager s receives a fraction of investor i's expected payoff equal to $\alpha_s^i P_i$ and a fraction of the market value of investor i's endowment equal to $\alpha_s^i W_i$. The fee owed to manager s is $\alpha_s^i \phi P$. Investors face two constraints in their choice of manager contracts: they distribute all their resources, meaning that $\sum_s \alpha_s^i = 1$, and their desired amount, x_s^i, must equal their share in the fund for AD security s.

For the remainder we require that all investors have equal expected payoff ($P_i = P$) and equal market value of their initial endowment ($W_i = W$). We therefore assume that all investors have the same initial endowment of assets, $x_{i,0}$. This assumption implies both $P_i = P$ and $W_i = W$. Equal expected payoff implies that the share of investor i in fund s is $\tilde{\alpha}_s^i = \frac{\alpha_s^i}{\sum_{j=1}^I \alpha_s^j}$. Thus, $x_s^i = \tilde{\alpha}_s^i x_s^0$ (the latter employs the argument that the entire social supply of x_s^0 will be demanded and supplied in equilibrium). It then follows that the dollar fee per unit of security s for investor i is

$$f_s^i = \frac{\alpha_s^i \phi P}{x_s^i} = \frac{(\sum_{j=1}^I \alpha_s^j)\phi P}{x_s^0} = \frac{A_s \phi P}{x_s^0},$$

where $A_s = \sum_{j=1}^I \alpha_s^j$. Thus, the fee is now endogenous (state dependent), though it remains the

same for all investors.

Investor i's program in this setup is:

$$\max_{x^i} \sum_{s=1}^{S} \pi_s u_i \left(x_s^i (1 - f_s) \right)$$

$$\text{s.t.} \sum_{s=1}^{S} A_s \frac{x_s^i}{x_s^0} = 1,$$

where the latter constraint is derived from $\sum_{s=1}^{S} \alpha_s^i = 1$ and $x_s^i = \frac{\alpha_s^i x_s^0}{A_s}$. Like before, fee-adjusted wealth is simply $z_s^i = x_s (1 - f_s)$, with the difference that now the fee is state-dependent. Managers, on the other hand, face a budget constraint that ensures that they can acquire the promised amounts x_s^0 with the resources provided by investors:

$$p_s^d x_s^0 = \sum_{i=1}^{I} \alpha_s^i W_i = W A_s.$$

Investors' FOC imply the existence of a representative investor with utility u. Combining the representative investor's FOC and managers' budget constraints, we obtain the following result for relative prices:

$$\frac{u'(z_s^0)}{u'(z_t^0)} = \frac{\frac{p_s^d}{\pi_s(1 - f_s)}}{\frac{p_t^d}{\pi_t(1 - f_t)}}. \tag{2}$$

Equation (2) states that fee-adjusted prices, $\tilde{p}_s^d = \frac{p_s^d}{1 - f_s}$, satisfy weak CAPM. Since fees are state-dependent, it does not immediately follow that weak CAPM will also hold for trading prices, p_s^d. We show in the Appendix, however, that weak CAPM follows through for trading prices as well. That is, if state s has a larger fee-adjusted wealth (z_s^0) than state t (z_t^0), the price of state s (p_s^d) will be smaller than the price of state t (p_t^d).

4 Details of the Experimental Setup

The experiment consisted of a multi-period main session followed by a one-period end session. The purpose of the end session was to eliminate possible unraveling of manager incentives. The entire experiment including the end session lasted approximately two months. Each week, one period of the main session was conducted, for a total of six periods. The end session lasted one period (one week).[12] Subjects in the experiment were divided into *investors* and *managers*. During the entire

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[12]The original design aimed for eight periods in the main session followed by the end session. Here we report only six periods since an accounting error in the seventh and eighth periods caused inaccurate reporting in the information

experiment managers were the same 32 subjects. Investors needed not be the same individuals each period and their number changed slightly from period to period, but equalled 70 on average.

As in the theory, investors were endowed with initial holdings of securities which they were forced to distribute among managers. Investors could not buy, sell, or store assets directly, and we asked them to assign *all* of their initial resources to one or more of the managers, who would subsequently trade on their behalf and collect the liquidating dividends. In what follows, we outline the design of the main session of the experiment. First we describe the assets, their supply and dividends (the "economy") and next we describe the timeline and subjects' payoffs in each experimental period.

Assets and Dividends

Investors were endowed with units of two risky assets, called A and B, and some cash. In addition to trading assets A and B, managers could trade a risk-free security called "Bond," which was in zero net supply. Assets A and B were risky because their end-of-period liquidating dividends depended on the realization of a random *state of the world* that could take three values, X, Y, and Z. Security Bond paid the same dividend of $1 in every state of the world. Because of the presence of cash, Bond was a redundant security. However, managers were allowed to short sell Bond if they wished. Short sales of security Bond corresponded to borrowing. Table 2 summarizes the dividends of the three traded assets, expressed in cents (all accounting in the experiment was done in U.S. currency). The three payoff-relevant states, X, Y, and Z, were equally likely and this was known to both managers and investors.

Unlike in the theory, investors were not originally endowed with AD securities. This did not affect equilibrium predictions since the markets created in the experiment were complete (security payoffs across states were linearly independent) and managers were allowed to short-sell securities. By forcing managers to construct AD securities through short sales, we know that if their portfolios were AD securities, it was not by accident. Managers had to carefully construct their portfolios in order to emulate different AD securities. To emulate an AD security paying $1 in state X and 0 in the other states, they had to short-sell both risky securities. Similarly, to pay $1 in state Z and 0 in other states, managers had to short-sell security A. Finally, to emulate an AD security that paid $1 only in state Y, managers had to go long in both risky securities, while shorting Bond.[13]

In the tradition of CAPM experiments [6], we split investors in two groups and gave them different endowments. An investor of type A held 100 units of asset A and $6 of cash, while an investor of

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disclosure stage. As such, periods 7 and 8 effectively became periods in the end session, which absorbed possible end-of-game effects in manager behavior.

[13]The exact portfolios in order to attain a payoff of $1 in state s and zero in all other states, are, for $s = X$, -0.81 units of A, -1.3 units of B, and 1.04 bonds; for $s = Y$, 1.3 units of A, 0.081 units of B, and -0.065 bonds; and for $s = Z$, -0.49 units of A, 1.22 units of B, and 0.024 bonds.

type B held 70 units of asset B and \$9 of cash. Nobody started out with bonds.

In the variant of our model where fees and shares are determined using expected payoffs, as in the experiment, we assumed that all investors started with the same endowment of securities and cash. We did not fully implement this assumption because it is bad experimental design: absent strong indications of differences in preferences across subjects (though admitting that heterogeneity must somehow exist), heterogeneity in initial allocations provides more opportunities for exchange, and hence, facilitates price formation. In the theory, the assumption of equal initial allocations was only made to ensure that expected payoffs and market values of initial allocations were equal. By cleverly assigning initial allocations, we ensured that investors' expected payoffs (sum of total dividends per state weighted by state probabilities) were the same to within \$0.50 (\$34.3 and \$34.7 for type A and type B, respectively). It was more difficult to ensure that market values were equal because we did not have *ex ante* information about prices in the inter-manager market. However, by choosing allocations with almost equal expected payoffs for all investors, we ensured that market values would be similar if risk premia were not too large.[14]

Endowing investors with securities instead of cash, and making these endowments different across investor, creates idiosyncratic risk beyond mere differences in risk aversion. This is an important element of our model and – as mentioned above – provides more opportunities for exchange, both in the experiment, as well as in practice. In our setup, idiosyncratic risk provides strong incentives for investors to correctly construct their portfolio of funds so to move from a position where wealth is highly variable across states to a smoother, well-diversified position. Moreover, endowing investors with securities is the only way for pricing to be endogenous. If investors were endowed with cash instead, then the assets would have to be supplied exogenously, e.g., by a market maker. The incentives of such exogenous supplier of assets would be different from the incentives of other market participants, and the analysis would then no longer be one of general equilibrium but one of partial equilibrium.

The market portfolio is the aggregate endowment of assets A and B. In our setup, the total number of investors as well as the fraction of investors of each type varied slightly from one period to the next, so that the composition of the (per capita) market portfolio also varied. Table 3 provides period-by-period details on the distribution of investor types and the corresponding market portfolio composition as well as the resulting aggregate wealth (dividend payments) across states. As one can

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[14]Closer inspection of the pricing equations derived in the theory reveals that they rely on being able to write $\sum_{i=1}^{I} \alpha_s^i W_i = K \sum_{i=1}^{I} \alpha_s^I$ (where K is a constant). A sufficient condition is that $W_i = W$, which is what we assumed (values of initial endowments are the same across investors). An alternative approach that approximates the result as the number of investors increases is to assume that initial endowments and preferences are randomly and independently distributed across investors. This would imply that $E\left(\alpha_s^i W_i\right) = E\left(\alpha_s^i\right) E\left(W_i\right)$, and hence $\sum_{i=1}^{I} \alpha_s^i W_i \approx \overline{W} \sum_{i=1}^{I} \alpha_s^I$, where $\overline{W}$ is the average wealth across investors. In our experiments it is very plausible that preferences and initial endowments were independent, since the latter were randomly assigned upon subject sign-up.

discern, the aggregate wealth was always highest in state Y and lowest in state X.

Timeline and Payoff

One period of the main session spanned over one calendar week and was roughly divided up into three stages: the asset allocation stage, the trading stage, and the information disclosure stage. Table 1 gives a schematic overview of a single weekly period. Effective time commitment of subjects was variable, since investors could freely dedicate time to planning their allocation strategies, while managers could freely allocate time to planning their trading strategies. However, by agreeing to participate, managers committed to logging in to the trading session during 30 minutes once per week, and investors committed to submitting their choice of allocation to managers within a fixed time window of 11 hours. We begin by describing this asset allocation stage.

Asset Allocation Stage

Investors signed up weekly. Once signed up, investors were granted access to the allocation interface which opened at 7am on Tuesday and remained open until 6pm on the same day. Investors could log in to the interface at any moment, for any duration of time, and possibly repeatedly, during this window of time. During the asset allocation stage, an investor could choose the number of units of his risky asset (A or B, depending on the investor's type) to allocate to each manager. If a manager was allocated a fraction of an investor's risky asset, the same fraction of the investor's cash was also allocated to that manager (this is in accordance with the theoretical assumption that investors allocate fractions of their entire portfolios to each manager).

Investors' allocations to managers determined each manager's initial portfolio, each investor's share in each manager's fund, and each managers' fee. Managers were informed of their own initial portfolio but not that of other managers, at the beginning of the trading stage. An investor's share in the manager's fund was determined by the expected dividend of his contribution to the total expected dividend of the manager's initial portfolio. Management fees were determined as a fixed fraction (40%) of the expected dividend of the contributed assets. The fee was back-end loaded, which means that it came out of the liquidating dividend of the manager's final portfolio. If this dividend was insufficient, investors did not owe the fee (investors had limited liability, as assumed in the theory); instead, we, the experimenters, paid out the managers.

A 40% management fee on assets under management is not realistic, but it allowed us to compensate subjects appropriately for the time and effort invested in the experiment. A lower fee, especially a realistic 2%, would have meant that investors, who were not investing their own money and did not need to engage in trade would have earned disproportionately more than managers. This problem

would have been exacerbated in our experiment because the number of investors with respect to the number of managers was relatively small. Our choice to have a large set of managers (32) was based on evidence from previous asset pricing experiments on the relation between the number of market participants and convergence to competitive equilibrium (see, e.g., [6]). Put simply, one needs sufficiently many, relatively small, market participants, in order to maintain a competitive market.

We already discussed earlier why we decided to pay managers their full fee even if dividends were insufficient and investors' limited liability was binding. This bailout does not affect managers' incentives to obtain as many funds as possible from investors and, as such, to supply contracts preferred by these investors (AD securities). And it ensures that managers effectively face no risk, since they receive their full fee regardless of the realized state of the world. Thus, the bailout clause allows us to have a set of investor-subjects who are risk averse and simultaneously a set of manager-subjects who are risk neutral, since they effectively face no risk.

The average fee per manager per period equaled $30. The dispersion of fees among managers varied across periods, and was most extreme in period 5, when a single manager collected over $200 in fees. As we shall see later, this is because of changes in concentration of allocation of funds to managers in response to the type of portfolios they historically had invested in and their performance records in the prior round.

Trading Stage

Managers participated in a 30-minutes trading session once per week (Tuesday at 10pm). During the trading stage, managers could trade through a web-based, electronic, anonymous, continuous open-book limit-order system called *jMarkets*.[15] A snapshot of the trading screen is provided in the electronic companion to this article.

Managers' initial portfolio of risky assets and cash was given by investors' allocation during the asset allocation stage. All managers had zero initial holdings of security Bond (Bond was in zero net supply). Managers could buy and sell the two risky assets and Bond in exchange for cash. Short sales of all three tradable assets were allowed. To avoid bankruptcy (and in accordance with classical general equilibrium theory), our trading software constantly checked subjects' budget constraints. In particular, a bankruptcy rule was used to prevent managers from committing to trades (submitting limit orders) that would imply negative cash holdings at the end of the period.[16]

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[15]This open-source trading platform was developed at Caltech and is freely available under the GNU license. See http://jmarkets.ssel.caltech.edu/. The trading interface is simple and intuitive. It avoids jargon such as "book," "bid," "ask," etc. The entire trading process is point-and-click. That is, subjects do not enter numbers (quantities, prices); instead, they merely point and click to submit orders, to trade, or to cancel orders.

[16]Whenever a manager attempted to submit an order, her cash holdings after dividends were computed for all states, given current asset holdings and outstanding orders that were *likely* to trade (an order was considered likely to trade if

The final portfolio of a manager generated a liquidating dividend according to the random realization of a state variable that became known only after the conclusion of trading. The dividend, with the management fee subtracted from it, was distributed to the investors according to their shares in the fund. If this residual dividend was negative, the distribution to the investors was equal to $0, which means that investors had limited liability with respect to management fees. The average payoff to investors was $26 ($3.50 when the realization of the state variable was X, $42 when Y, and $23.50 when Z).

Information Disclosure Stage

Each week, on Monday morning, a series of performance indicators were published on the experimental webpage as well as the weekly university newspaper. To preserve the privacy of participants in the experiment, all managers were assigned pseudonyms used for all announcements.

We reported four performance indicators for all managers as well as for an index called the *Dow-tech* Index, composed of one unit of asset A, one unit of asset B, and $1 cash. The following performance indicators were used: *Portfolio Return, Market Share* (called "Volume" in the published reports), *Residual,* and *Risky Share.* Portfolio return is the (pre-fee) final value (based on liquidating dividends) of a manager's portfolio as a percentage of the market value of the initial portfolio. Market value was determined using average transaction prices in the period. Market Share is the ratio of the expected liquidating value of manager j's initial portfolio and the expected liquidating value of the portfolio comprised of all assets and cash available to all investors. Expected liquidating value is determined using the expected value of asset dividends. The Residual is the liquidating value of a manager's portfolio (based on paid dividends) minus the fees paid to the manager. The Risky Share provides an indication of the amount of risk the portfolio manager was taking. It is given by the market value of risky securities (A and B) as a fraction of the market value of the manager's entire portfolio.[17] The experimental webpage dedicated to weekly performance contained a succinct but complete explanation of the meaning of each of the indicators. This was always available for

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its price was within 20% of the last transaction price), including the order she was submitting. If these hypothetical cash holdings turned out to be negative for some state, the order was automatically canceled and the trader was informed that her order did not go through because of risk of bankruptcy. However, in trading sessions when prices changed a lot, it was possible that orders that originally passed the bankruptcy check (because they were not considered likely to trade) eventually were executed and led to negative value of final holdings. Thus, despite the best attempts to avoid bankruptcy, rates of return at times were below -100%.

[17] The electronic companion contains further details, including the list of pseudonyms, experimental instructions and computational details for the variables used in the reports, as well as in our Results section. The end session is also described in detail in that document. The end session, with changed incentives for managers, was included to prevent managers from disregarding performance in the final period and, in this way, unravel their incentives as well as investors' trust all the way back to the initial period. *Ex-post* the terminal period was of little importance since periods 7 and 8 were excluded from our data. Period 6 could thus safely be used with no concerns regarding possible last-period effects.

investors to consult.[18]

Notice that we did not have a stage where managers announced the portfolios they planned to supply to investors. Allowing for such announcements would have been problematic, since managers' ability to stick to their promises depended on trading prices, in turn determined by demand and supply forces. Hence, the possibility that all managers simultaneously offered feasible portfolios to investors constituted a complex coordination problem that very likely would have led managers to default on their promises in early periods of the experiment. Instead, we counted on managers building a reputation for offering certain types of contracts that investors could learn via the announced indicators for previous periods. We checked this reputation-building assumption ex-post with regressions of managers' final holdings of each security in period t on the final holdings in period $t-1$ (holdings are normalized using the value of the initial endowment, which could vary a lot from one period to the next). The effect of holdings in $t-1$ is significant for all three securities ($p < 0.001$). We did not announce exact portfolio composition for each manager in past periods, since this would have led subjects to learn the market portfolio in the experiment, up to the small changes in the market portfolio from one period to the other. Subjects' knowledge of the market portfolio would have weakened our results since such information can be used to compute theoretical equilibrium price rankings.

5 Results

The main implication of our theory when investors have limited liability is the coexistence of managers who offer extreme portfolios, emulating AD securities, with investors that ultimately hold well-diversified portfolios. We thus first present results regarding investor preferences for managers who offer portfolios that mimic AD securities. Next we show that investors hold well-diversified portfolios throughout the experiment while manager portfolios become less diversified (more AD-like) as the experiment progresses. Finally we turn to results regarding prices and the ranking of state price probability ratios (weak CAPM).

Before we turn to the relevant results, we present some descriptive statistics, namely, asset turnover and state realizations. Table 4 suggests that asset turnover was high: often more than 50% of the outstanding securities changed hands in the inter-manager market. Across the six periods, each state was realized at least once, with the worst state (state X, with the lowest aggregate wealth) being drawn in periods 3 and 6.

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[18]Anecdotal evidence suggests investor-subjects followed and used the weekly reports. The experimental email account received emails from investor-subjects asking when information would be disclosed, requesting the indicators in formats useful for data processing, or requesting more detailed indicators.

Investors' Choice of Managers

According to our theory, in equilibrium investors allocate funds across managers who offer portfolios that mimic AD securities. We shall determine to what extent this preference is revealed in investor choices by studying manager market shares and correlating those with a variable that measures how far manager portfolios were from AD securities.

First consider individual manager market shares, i.e., percentage of the expected liquidating value of all assets that is received by a given manager. In the first round, investors allocated their initial endowment to managers very much randomly, so that each manager received an approximately equal share (1/32, or about 3%). This reflects absence of knowledge of the investment plans of the managers. Only in period 2 could investors gauge the intentions of managers, by observing past performance, as reported on the experiment web site and in the university newspaper. Already in period 2, investors preferentially allocated their wealth to a few managers. This pattern continued over time, and led to relatively high concentration of investor wealth among a few portfolio managers. Indeed, Table 5 confirms that both, the market share of the largest manager in terms of value of assets under management, and the Gini index of concentration,[19] increased in period 2 and remained high (double-digit market share of the largest managers, and Gini Indices around 0.50) for the remainder of the experiment.

Of course, the main issue is whether these preference patterns reflect the theoretical prediction that investors prefer to go with managers who invest in portfolios with payoffs that mimic those of AD securities. That is, were managers who attracted a disproportionate share of the flow of funds also the ones that invested in portfolios with AD-like payoffs? To verify this, we ran a number of regressions with manager market share as dependent variable, and as explanatory variables: (i) a measure of distance of final manager portfolio holdings from an AD security in the prior period, (ii) prior-period residual payoff after managerial fees (the "Residual" indicator reported to subjects). The first explanatory variable, called LagDistanceAD, was measured as follows. Let ς^m denote the state where manager m's final portfolio paid the highest dividend, and let $\bar{\varsigma}^m$ denote the collection of remaining states. LagDistanceAD was computed as the ratio of the state-price-weighted average total dividend in all states in $\bar{\varsigma}^m$ over the total dividend times the state price in state ς^m. This measure is minimal (0) when the manager perfectly mimics the payoff on AD security ς^m. It increases as

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[19]The Gini index measures the difference between the area under the *cdf* of a uniform distribution of (manager) wealth and the actual, empirical, distribution. A large difference indicates high concentration. In our case it was computed as follows. First, record the expected value of a manager's initial endowment v^m. Next rank managers so that $v^1 \geq v^2 \geq \cdots \geq v^M$. Then, for each m, let $w_m = mv^m$ and compute the ratio $r = \sum_m w_m / \sum_m v^m$. Finally, transform r as follows to get the Gini index: $G = (I + 1 - 2*r)/I$. A list of market shares of individual managers across all periods is available on the experiment web site (see footnote 13 for link).

managers buy portfolios that pay more total dividend in states in $\bar{\varsigma}^m$.[20] We will refer to ς^m as manager m's *type*.

Many variations of this base regression were tried. In these variations we either slightly varied the definition of the distance of a manager's final portfolio payoff pattern from the payoff on an AD security or we added potentially confounding factors such as payoff variance of the manager's final portfolio in the previous period, expected payoff of the manager's final portfolio, and period dummies. While addition of confounding factors at times had an effect on the significance of the effects of the two main explanatory variables [(i) and (ii) above], the signs remained robust.

Table 6 displays the regression results. Consistent with the theory, the distance of a manager's portfolio from mimicking an AD security ("LagDistanceAD") has a highly significant ($p < 0.001$ univariate and $p = 0.003$ multivariate) negative effect on a manager's market share, suggesting that investors preferred to allocate their wealth across managers who offered portfolios that looked more like AD securities, and hence, exploited investors' limited liability for the manager fee. The regression results are based on 31 (number of managers minus 1) times 5 (number of periods minus 1), i.e., 155, observations.

Table 6 also shows that the realized return minus fee in the previous period ("LagResidual") has a significantly positive effect ($p < 0.001$) on market share. Lagged residual is an indicator that combines a manager's realized return in the previous period, given the state of the world, with the ability of this manager to charge relatively low fees by concentrating on one state's payoffs only. There are a number of interpretations of this result.

First, as a proxy for realized returns, lagged residual could be a way for investors to resolve indifference between two managers who offered portfolios with the same distance from an AD security. There is no *a priori* reason to believe in differential managerial skill in our experimental setting, but investors may have taken lagged residual as an indicator that there was, and may have allocated funds accordingly. This phenomenon would be consistent with the theory of (and evidence of real-world) fund flows in [4].

Second, keeping fixed the distance from AD securities, lagged residual together with information on the realized state was the best index that investors had about what state of the world a given manager was focusing on. Investors could subsequently react by allocating more funds to those managers focusing on the wealthier states so that they (investors) could successfully mimic the

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[20]With large trading volumes, the automated bankruptcy check used in the experiment did not always prevent managers from holding portfolios with negative payoffs in some states (see footnote 16). Whenever this happened, we reset the negative payoff to zero for the computation of the indicator LagDistanceAD and its un-lagged variant DistanceAD. This was done to prevent the indicator from being biased toward zero (small DistanceAD). For example, a manager that yields the largest payoff in state Z, closely followed by the payoff in state Y, should have a large DistanceAD. But if her payoff in state X is negative, the numerator in the formula of DistanceAD will be close to zero, leading to an artificially-low DistanceAD.

market portfolio.

Third, although less likely, absent transparent information about the precise composition of a manager's portfolio (we only reported the weight on risky securities), lagged residual could be another, albeit noisy, indicator of how far a manager's portfolio was from mimicking an AD security. Indeed, the correlation between our measure of distance ("LagDistanceAD") and the lagged residual, is a significant -0.23 ($p = 0.003$). In part this correlation arises because managers with a small distance from AD securities have low fees and, hence, higher residuals. However, in our experiment this correlation is also driven by the realization of the richest state (state Y) in three out of five lagged periods, leading to a strong positive correlation between being an AD security and obtaining a large return. As such, "LagResidual" provided information about "LagDistanceAD."

One may wonder whether lagged distance from AD securities affected market share only via its correlation with realized returns whenever state Y was realized: this state was so much wealthier than other states that even managers offering the market portfolio would appear to have a small distance from AD securities. With return-chasing investors, such managers would also receive a large flow of funds because of the large returns they generated when state Y was realized. We consider an important variant of our main regressions to address this concern.

We add indicators (dummy variables) for $\varsigma^m = X, Y, Z$, denoting the state where the final manager portfolio paid the highest dividend, to the above-considered regressions, and report this in table 7. LagDistanceAD has a significant negative effect on market share also after controlling for ς^m (this is so also when D_Y or D_Z – instead of D_X – are omitted). Thus, even after taking into account a manager's "type" (ς^m), lagged distance from AD securities has a negative effect on market share.

Notice that all managers were of some type, regardless of their DistanceAD indicator. A manager of type X held a portfolio that yielded the highest payoff in state X ($\varsigma^m = X$). If her payoff in other states was very low or zero, she additionally had a low DistanceAD indicator, while if her payoff in other states was substantial, she had a high DistanceAD. Since managers often received very balanced allocations, in 90% of the cases they started out the trading stage being of type Y, like the market portfolio. Becoming a type X manager required them to short sell asset B. To further become an AD security for state X, they also had to short sell asset A. Although being of a given type is not necessarily equal to holding a portfolio mimicking the AD security of the corresponding state, the fact that managers divided up into different type groups, as opposed to all keeping well-diversified portfolios, gives an indication of movement toward the focal equilibrium with AD securities.

To further clarify the above point, recall that in theory investors can achieve diversified portfolios even if all managers offer funds that are close to the market portfolio. However, in that case, they

pay higher fees than if managers split up in types corresponding to states of the world. Fee payments are minimal if managers become AD securities for each state of the world. In the equilibrium where managers offer AD securities, the market share of funds offering each AD security perfectly matches the market portfolio. That is, managers offering the state-X AD security have a market share equal to the ratio of state-X wealth over the sum of wealth in all states, and analogously for states Y and Z. The closer our data are to such an equilibrium, the closer the market share of each type should be to the wealth shares of the market portfolio. This is why we now ask whether managers split into types and, if yes, what the market share was for each type group.

The market portfolio changed very slightly from period to period, and on average had wealth split up in 3.06% for state X, 60.46% for state Y, and 36.48% for state Z. Table 8 shows the market shares of managers of each type in each period. The share of type Y managers decreases almost steadily, which is consistent with a situation where managers start out offering funds that are close to the market portfolio (the market portfolio is of type Y), but move to more polarized funds in later periods. In the final period market shares closely resemble the wealth shares of the market portfolio, with an imbalance giving too large a share to type X and too small shares to types Y and Z. Such an imbalance fits with the choices of managers within each type group, to which we turn next.

We ask whether within each type group investors chose those managers that most resembled AD securities. The answer given by the regression in table 7 was yes, but a closer look reveals that it is mainly driven by competition between managers of type Y. The correlation between market share and distance from AD securities is strongest for type Y managers and, in general, the distance from AD securities is larger for type X managers. The former is mainly explained because there are more type Y managers to start with, who persist in being of that type, thus accumulating a history that allows investors to choose those generating lower fees.[21] The latter phenomenon is driven by the difficulty of creating a state-X AD security, since it requires short sales of both risky assets. This in turn contributes to the observed fact that managers that are of type X in one period, are not of type X in every period, further leading to little selection by investors of AD-like managers. The disproportionate market share of type X in the last period is a direct consequence of the observation that managers of types Y and Z are more polarized than those of type X.[22] In fact, in periods 4

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[21]Managers 37, 16, and especially, 32, carry out the strongest competition. They are persistently of type Y and have very low DistanceAD (manager 32 always has either the lowest or the second lowest distance). Jointly they hold a market share of approx. 9% in period 2, which steadily increases to 36% in period 6. Manager 12 attains the highest market share of the experiment in period 5, but he is an outlier capturing investors' chasing of returns (possibly wrongly interpreted as quality): he changed types every period, by chance matching the realized state in the first four periods of the experiment. In fact, when he failed to match the realized state in period 5, his market share abruptly decreased in the following period.

[22]The median DistanceAD across managers and periods was 1.23 for type X, while it was 0.83 for type Y and 0.77 for type Z. The managers with the lowest DistanceAD across periods were of type Y, with a minimum median across periods (for subject 32) of 0.004.

and 5, when the share of X is lower, but still above the wealth share of state X, state prices are not correctly ranked. We return to this result in section *Pricing* below.

Investor and Manager Holdings

In the theory section we showed that investors in our model will hold equal ratios of marginal utilities across states (these ratios equal relative prices and prices are equal for all investors). This implies that all investors will hold similar distributions of wealth across states. Consequently, investor final holdings are predicted to be well-diversified – approximately proportional to the wealth distribution of the market portfolio.[23] One of the main points illustrated by our model is that because investors can hold diversified portfolios of contracts offered by managers, they can hold well-diversified portfolios, while managers do not (will not). With limited liability of investors, we actually expect managers to completely specialize and hold AD securities. Figure 2 illustrates the extent to which this important result is verified in the experiment. It displays the histogram of the ratio of wealth in state Y to the total wealth across all states for both investors and managers, given investors' shares in the managers' funds and given these funds' post-fee payoffs in every state. For both groups of subjects, the wealth in every state takes into account limited liability and truncates at zero in states where post-fee payoff would have been negative.

As is clear from Figure 2, not all investors hold the market portfolio, but the histogram of relative final wealth is unimodal, centered around the (post-fee) market portfolio. This replicates the findings in past asset pricing experiments where investors trade assets directly (e.g., [6, 7]). The coexistence of CAPM prices and a unimodal distribution of investor holdings centered at the market portfolio is theoretically founded by a variant of the CAPM that relaxes the assumption of quadratic utility, allowing for individual deviations that wash out in aggregate (CAPM+ε, [7]). Figure 2 also shows, importantly, that this unimodal distribution of investor final wealth is maintained to a large extent across all periods of the experiment. A significant deviation from a unimodal distribution centered at the market portfolio is observed only in period 5, coinciding with the observed violation of weak CAPM in price predictions (this is in agreement with the theory of CAPM+ε mentioned before).

On the other hand, the distribution of final relative post-fee wealth for managers changes over the course of the experiment. Already in the second period a mode at 1 emerges. This means some managers quickly start offering contracts that mimic state-Y AD securities. As periods pass, the histogram becomes more dispersed, closer to bimodal (second mode at 0). This is a very sharp illustration of the intuition lying at the base of our theory: investors achieve their portfolio goals by

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[23]With the additional assumption of quadratic utility over final wealth, the relation becomes exact. This is the CAPM property discussed in the Appendix.

mixing different managers, whom they choose based on their ability to handle costs (fees). In our setup, managers' correct reply to investors' objectives is to become polarized (mimic AD securities), which they do, while prices still satisfy weak CAPM.

At the same time as managers become more polarized, investors reduce the number of managers they include in their portfolio. The median investor holds 18.5 managers in the first period – basically, investors randomly allocate their assets, 8 in the third period, and 6 in the last period. Our theory indicates that three managers would suffice to obtain any desired portfolio. There are several reasons to not expect this number to arise in our experimental implementation, not least of all, that managers – unlike in the theory – do not announce the contracts they will offer but rather that investors deduce the contract a manager will offer from the manager's past performance.

Pricing

Figure 3 provides time series plots of transaction prices, per period. Prices of risky securities A and B are close to, and with few exceptions, below, expected payoffs (indicated with solid horizontal lines). The fact that prices are different from expectations suggests risk aversion. With risk aversion, weak CAPM predicts that when prices of traded securities are converted to prices of AD securities, their ranking (adjusted for state probabilities) should be inverse to aggregate wealth across states.

In our setting, state-price probability ratios should be highest in state X, where aggregate wealth is lowest (see Table 3), and lowest in state Y, where aggregate wealth is highest. Aggregate wealth is computed from the dividends on the outstanding assets (A, B, and Bond), added to the total cash outstanding. Figure 4 provides evidence. Plotted are the time series of state-prices for the three states, implied from trade prices (since all states are equally likely, it is unnecessary to adjust state prices for state probabilities). While the evidence looks mixed at first, late in periods 2, 3, and 6, and during the first half of period 5, state-price probability ratios are ranked exactly as predicted by weak CAPM: the price for state X is highest, and that for state Y is lowest.

While not immediate from Figure 4, there is a statistically significant tendency for state-price probabilities to revert back to the right ranking (according to weak CAPM) when the ranking is incorrect. This is determined as follows. Following [6], we construct a statistic, called τ, that measures the frequency with which state-price probability ratios move in the right direction to restore ranking according to weak CAPM when ranking is incorrect. Specifically, τ is the average across transactions of a variable that assigns a value of 1 to a change in state prices in the direction of weak CAPM, and 0 otherwise. For each possible ranking of the state-price probabilities after transaction t, Table 9 presents the required subsequent changes (at transaction $t+1$) for the indicator variable determining τ to take the value of 1. To avoid bias, we eliminate observations where prices do not

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change (these would otherwise automatically be assigned a value of 1).

To determine correct rejection levels for τ at the 5% and 1% significance levels in each trading period under the assumption that state-price probability ratios change randomly, we bootstrapped the original time series of transaction prices (period by period for assets A, B and bond) after subtracting price drifts (so price series become martingales), inverted the resulting bootstrapped prices for state-price probability ratios, and constructed 200 series of levels and subsequent changes of state-price probability ratios with the same length as the original (period) series. For each of these 200 series, we computed the τ statistics and then determined the critical level of τ so that 5% (1%) of the outcomes were above this level, thus obtaining cut-off levels at $p = 0.05$ (and $p = 0.01$ respectively) for each of the six trading periods.

Table 10 displays the results. Shown are τ statistics for all periods, and corresponding critical values at the 5% and 1% levels. The null of no drift is rejected at the 1% in all but one (5th) period, where τ does not even reach the 5% cut-off level. Overall, Table 10 provides evidence that is difficult to discern from Figure 4, namely, that prices tend to change in the direction of weak CAPM in the instances where they do not conform to this theory.

In periods 1, 4, and 5 we observe state prices that often deviate from the weak CAPM prediction. While in period 1 this can be attributed to subjects' learning of the experimental situation and market composition, this does not apply to periods 4 and 5 – especially after observing perfect state price rankings in periods 2 and 3. In periods 4 and 5, type X managers have a lower market share than in periods 3 and 6. The market share is larger than in period 2, but in period 2 all type managers were less polarized than in later periods (larger DistanceAD). With a lower demand for state X wealth (lower than in other periods, and lower than in equilibrium), the price of this state is too low, so much so, that it leads to an incorrect ranking of state prices.[24]

The lower market share of type X managers in periods 4 and 5 may be a simple mis-coordination among managers in the offering of state securities. However, it is surprising that with the disproportionate increase in the price of security B, no manager chose to convert to a type X manager via short-sales of this security. It is more likely that the market share of type X was affected by market concentration, which was indeed highest in these periods. The successful managers, who obtained most of the market share, were not committed to supply diversified portfolios, but to supply AD-like securities. Since investors allocate too many funds to these managers, the idiosyncrasies of these large funds have substantial effects on prices. Table 5 shows that the Gini index of concentration could be as high as 0.55, and the market share of the largest manager as high as 20%. Moreover,

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[24]Indirectly, with fewer (or poorer) managers who are committed to be of type X, the volume of (short) sales of security B is reduced, thus increasing its price. This can be seen in Figure 3. Such an over-pricing of security B in turn leads to an under-pricing of state X, while the other states need not be mis-ranked with respect to each other.

we know that the largest fund in periods 4 and 5 was of type Y and Z, respectively, not type X. For CAPM pricing to obtain robustly, it is known that individual idiosyncrasies need to average out, and this can only occur if there are a sufficient number of participants all of whom only hold a small fraction of wealth [7].

Consistent with the hypothesis that concentration has a detrimental effect on pricing, we discovered that the correlation between concentration measures and the τ statistic is negative, albeit only marginally significant. The correlation across periods with the Gini concentration index is -0.51 (standard error 0.30); that with the market share of the largest manager is -0.42 (standard error 0.40). When computing τ based on all observations, including those where prices do not change, these correlations become significant: -0.54 (standard error 0.26) and -0.58 (standard error 0.23), respectively. As such, some of the poor pricing should be attributed to market concentration, which itself is the consequence of investors' preferences for managers who (i) acquire undiversified, AD security-like portfolios, (ii) had high returns in the prior period.[25]

We focus on weak CAPM but one may be interested to know whether also strong CAPM obtains. CAPM requires the assumption that investors have quadratic utility (or that a quadratic function is a good approximation of the true utility, see [7]). In our setting where shares and fees are computed using expected payoffs, even with the assumption of quadratic utility, prices gross of fees satisfy CAPM only approximately. However, as shown in the Appendix, numerical analyses suggest that the difference between our equilibrium prices and CAPM prices is minor. We could therefore test one of the main predictions of the CAPM. Namely, that the market portfolio should be mean-variance optimal.

An effective way to gauge the distance of the market portfolio from optimality in mean-variance space is to compute the difference between the Sharpe ratio of the market portfolio and that of the maximal (mean-variance optimal) portfolio (see [6] for an early application of this metric in the context of price data from experimental markets). One can re-compute this Sharpe ratio difference after each transaction and take the average across all transactions within a period.

If we then compare the average Sharpe ratio difference with τ (which measures the extent to which prices move in the direction of weak CAPM), we observe a correlation of 0.34 (standard error 0.51). When computing τ based on all observations, including those where prices do not change, this correlation becomes significant: 0.55 (standard error 0.25). This illustrates that our conclusions about pricing in the delegated portfolio management experiment would have been qualitatively the

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[25]Some direct evidence that large managers influence prices in an adverse way is found by looking at trade in the last 10 minutes of periods 4 and 5. In period 4, the two largest managers, jointly holding approx. 35% market share, *bought* asset B (approx. 400 units), with only one large manager on the sell side (approx. 8% market share). In period 5 the largest manager, holding about 21% market share again bought asset B (450 units), with no large managers selling.

same if we had instead used the strong version of the CAPM (which requires us to assume quadratic utility).

6 Conclusion

In a break with tradition in the analysis of delegated portfolio management, we here proposed a theory where investors can choose to distribute wealth over many managers, and where managers compete in terms of composition of portfolios offered to investors. To narrow our predictions, we assumed that investors have limited liability for manager fees. Fees are charged on initial funds under management, but paid out of final portfolio value. We find that manager incentives are aligned with those of investors. In equilibrium, managers offer portfolios that mimic Arrow-Debreu (AD) securities, and investors combine holdings in multiple funds in order to generate "home-made" diversification. Prices in the inter-manager market continue to reflect demand for diversification, as if investors had been able to invest directly. Specifically, weak CAPM obtains, which predicts state-price probability ratios that are ranked inversely with aggregate wealth.

We designed and ran a large-scale experiment involving approximately one hundred subjects over multiple weeks and studied manager portfolio choices, investor fund allocation decisions, and pricing in the inter-manager market. The three main predictions of the theory were confirmed, namely: (i) investors preferred managers that offered portfolios that came closer to mimicking the payoff on AD securities; (ii) investors engaged in "home-made" diversification, (iii) prices tended to levels that were in accordance with weak CAPM (and in fact, even in accordance with the traditional, strong version of CAPM).

We did observe an effect of return realization in the immediate past on subsequent fund flows, as in historical data from the field [4]. The resulting concentration of funds led to a deterioration of price quality; all else equal, prices reflected weak CAPM to a lesser extent than when funds were distributed more evenly across managers.

Our experiment should be considered a proof of concept: it shows that it is possible to run meaningful controlled experiments on delegated portfolio management in the laboratory. Our experiment was limited in many respects. Foremost, the same thirty-two managers participated in all the investment rounds, and many investors participated in multiple rounds. As such, we effectively ended up observing only one history of prices and choices. Independent replication, with a different cohort of managers and investors, is needed. Second, part of our set-up was unrealistic, but this was done deliberately in order to isolate the effects of competition for fund flows when investors could allocate wealth across multiple managers. Further experiments should relax some of our assumptions. In

particular, we assumed a very simple management fee structure. One can imagine theory and tests about joint determination of fee and portfolio structure. We leave this and other extensions for future work.

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Tables and Figures

Table 1: **Timeline For One Week-Long Period**

Wed	**Fri**	**Sat**	**Mon**	**Tue**
Investors and managers informed of payoffs	Performance indices published on website	Sign-up announcement for investors *(Only First Week)* Sign up for managers	Performance indices published in *Tech* Investors receive access to allocation software	6pm Close of investor allocation stage 10pm Managers see allocations and trading starts 10:30pm Trading ends

Table 2: **State-Dependent Liquidating Dividends (in US Cents Per Unit).**

	State		
	X	Y	Z
Asset A	5	80	0
Asset B	0	30	80
Bond	100	100	100

Table 3: **Number of Participants by Type and Corresponding Per Capita Market Portfolio and Aggregate Wealth (Dividend) Per State (in dollars).**

Period	Number of Participants		Per Capita Market Portfolio			Aggregate Wealth Per State		
	Type A	Type B	A	B	Cash	X	Y	Z
1 (061017)	30	34	46.9	37.2	$7.59	150	3114	1904
2 (061024)	28	38	42.4	40.3	$7.73	140	3038	2128
3 (061030)	37	34	52.1	33.5	$7.44	185	3038	2128
4 (061107)	37	35	51.4	34	$7.46	185	3695	1960
5 (061114)	34	33	50.7	34.5	$7.48	170	3413	1848
6 (061121)	35	35	50	35	$7.50	175	3535	1960

Table 4: **Asset Turnover And State Realization (Per Period)**

	Period					
	1 (061017)	2 (061024)	3 (061030)	4 (061107)	5 (061114)	6 (061121)
Turnover:[a]						
Asset A	0.37	0.53	0.45	0.47	0.48	0.57
Asset B	0.40	0.35	0.53	0.56	0.69	0.78
State Outcome	Z	Y	X	Y	Y	X

[a]Asset turnover is calculated by dividing the trading volume by the total number of units outstanding.

Table 5: **Market Concentration (Gini Index) and Market Share of the Largest Manager**

Period	Gini Index	Largest Manager Market Share
1 (061017)	0.1334	0.0524
2 (061024)	0.5039	0.1236
3 (061030)	0.3434	0.1032
4 (061107)	0.4905	0.1995
5 (061114)	0.4978	0.2034
6 (061121)	0.5491	0.1342

Table 6: **Manager Market Share Regressed On Lagged Distance Of Offered Portfolio From AD Security (Top) And On This Lagged Distance Plus Lagged Residual Divided By 100 (Bottom).**

Explanatory Variable	Coefficient	Standard Error	t	p value
Intercept	0.048	0.006	7.55	< 0.001
LagDistanceAD	-0.020	0.004	-5.14	< 0.001
Intercept	0.023	0.003	7.05	< 0.001
LagDistanceAD	-0.009	0.003	-3.23	0.003
LagResidual	0.023	0.002	10.52	< 0.001

Table 7: **Manager Market Share Regressed On Lagged Distance Of Offered Portfolio From AD Security And Lagged Residual Divided By 100, Controlling For Manager's Type (State Where Her Final Portfolio Generated The Highest Dividend). The Dummy For Type X Is Ommitted.**

Explanatory Variable	Coefficient	Standard Error	t	p value
Intercept	0.025	0.004	6.15	< 0.001
LagDistanceAD	-0.009	0.003	-3.16	0.003
LagResidual	0.023	0.002	10.69	< 0.001
D_Y	-0.0025	0.003	-0.94	0.353
D_Z	-0.0003	0.006	-0.04	0.966

Table 8: **Aggregate Market Share Of Managers With Highest Dividend Paid In Different States Of The World.**

Period	Manager Type – ς^m		
	X	Y	Z
1	0	77.5	22.5
2	0	67.4	32.6
3	13.3	82.66	4.04
4	4.79	69.91	25.3
5	4.99	63.98	31.03
6	14.94	54.28	30.78

Table 9: **List Of Changes That Suggest State-Price Probability Rankings Drift Back To Satisfying Weak CAPM.**

Ranking at t	Correct Subsequent Change In State-Price Probability Ratios
$u_X(t) > u_Z(t) > u_Y(t)$	Any Change
$u_X(t) > u_Y(t) > u_Z(t)$	$u_Z(t+1) - u_Y(t+1) \geq u_Z(t) - u_Y(t)$
$u_Z(t) > u_X(t) > u_Y(t)$	$u_X(t+1) - u_Z(t+1) \geq u_X(t) - u_Z(t)$
$u_Z(t) > u_Y(t) > u_X(t)$	$u_X(t+1) - u_Y(t+1) \geq u_X(t) - u_Y(t)$
$u_Y(t) > u_X(t) > u_Z(t)$	$u_X(t+1) - u_Y(t+1) \geq u_X(t) - u_Y(t)$
$u_Y(t) > u_Z(t) > u_X(t)$	$u_X(t+1) - u_Z(t+1) \geq u_X(t) - u_Z(t)$
	or $u_Z(t+1) - u_Y(t+1) \geq u_Z(t) - u_Y(t)$
	or $u_X(t+1) - u_Y(t+1) \geq u_X(t) - u_Y(t)$

Table 10: **Tests Of Whether Transaction Prices Follow A Martingale (H_0) Against Drift Towards Satisfying Weak CAPM (H_1).**

Period	τ	Critical Values 5%	1%
1 (061017)	*0.73*	0.52	0.62
2 (061024)	*0.69*	0.38	0.49
3 (061030)	*0.83*	0.60	0.70
4 (061107)	*0.68*	0.52	0.66
5 (061114)	*0.72*	0.98	0.98
6 (061121)	*0.68*	0.29	0.30

Figure 1: The optimal allocation a can be obtained, gross of fees, through a convex linear combination a' of managers offering AD securities 1 and 2. Blue dashed lines indicate fees charged. The gross-of-fee allocation a' can also be obtained through a convex linear combinations of managers offering 1 and E. Post-fee (i.e., net of fees), however, the latter produces a strictly inferior allocation, because the fees required in each state (green dashed lines) are strictly larger. Investors will prefer combining offerings 1 and 2.
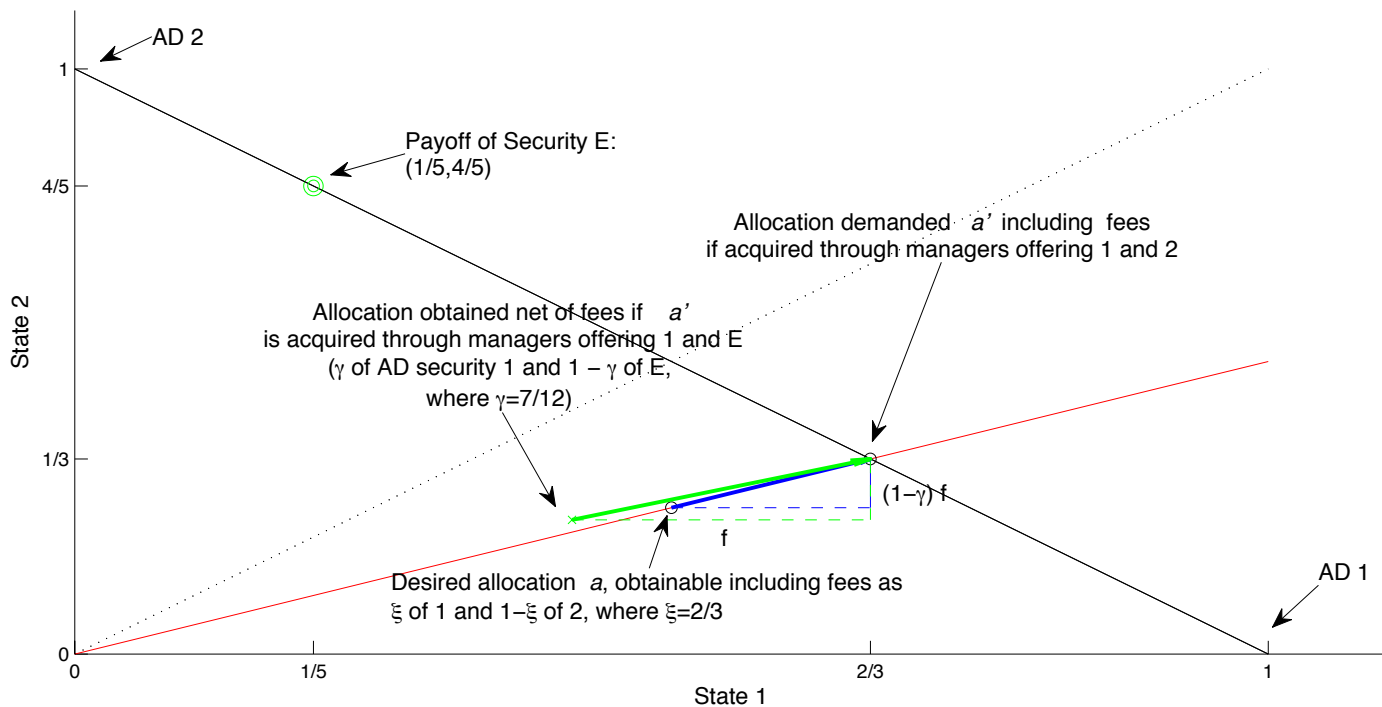
Figure 2: Histogram of final holdings of wealth in state Y as a proportion of the sum of wealth in all states for investors (after fee, left column) and for managers after fee, with a floor at zero (right column).
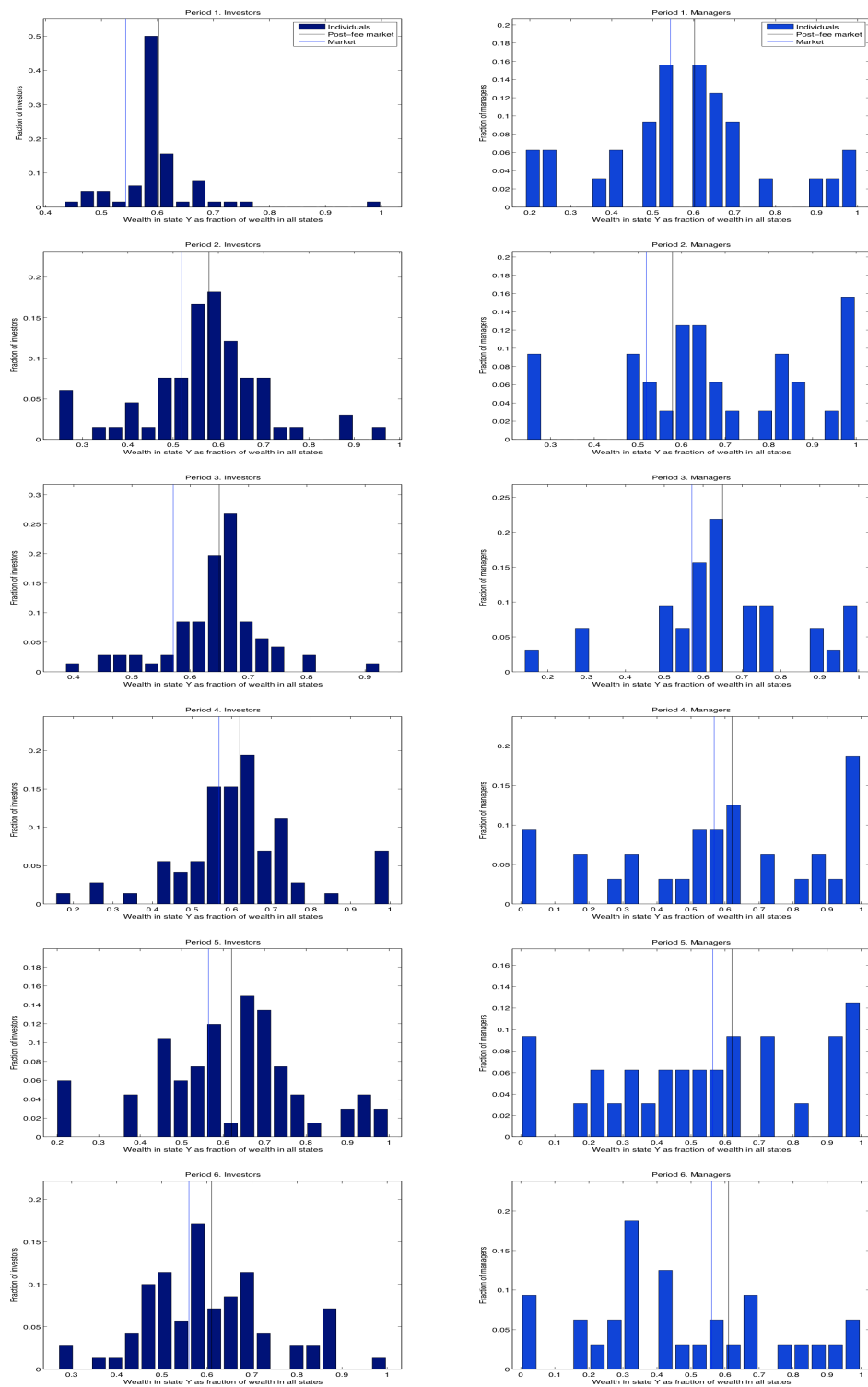
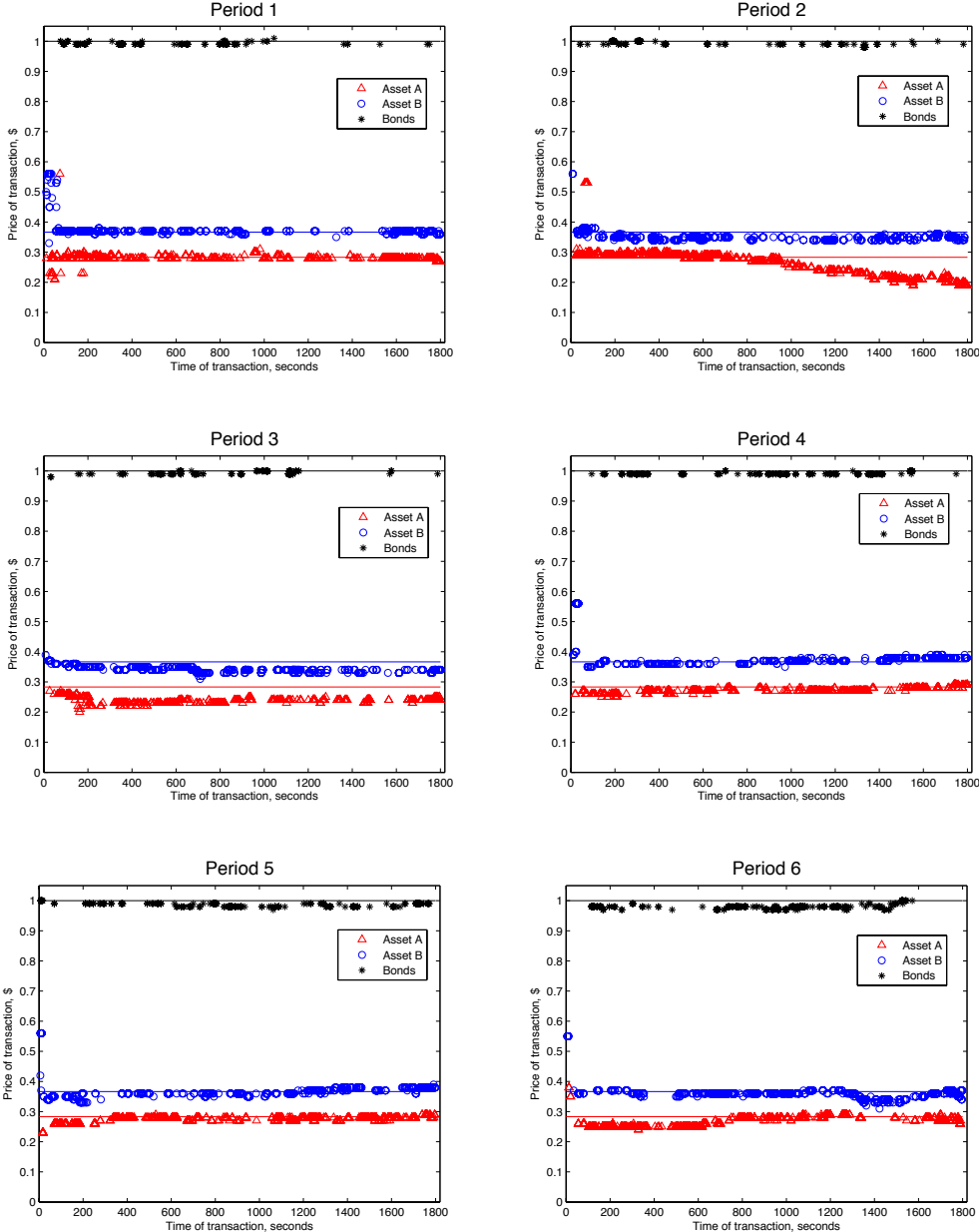Figure 3: Time series of transaction prices, per period. Horizontal lines indicate expected payoffs.

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Figure 4: Time series of state prices, per period. Weak CAPM predicts that the state-price probability ratios should be highest for state X (when aggregate wealth is lowest – see Table 3) and lowest for state Y (when aggregate wealth is highest). Since all states are equally likely, state prices preserve the ranking of state-price probability ratios; we show state prices because they are normalized to add up to 1. State-prices of X are indicated with red arrows pointing up; those for Y are displayed with blue arrows pointing down; and those for Z with black arrows pointing sideways. State prices are implied from transaction prices (see Figure 3) after each trade, based on the most recent transaction prices for each security, and on inversion using the payoff matrix (Table 2).

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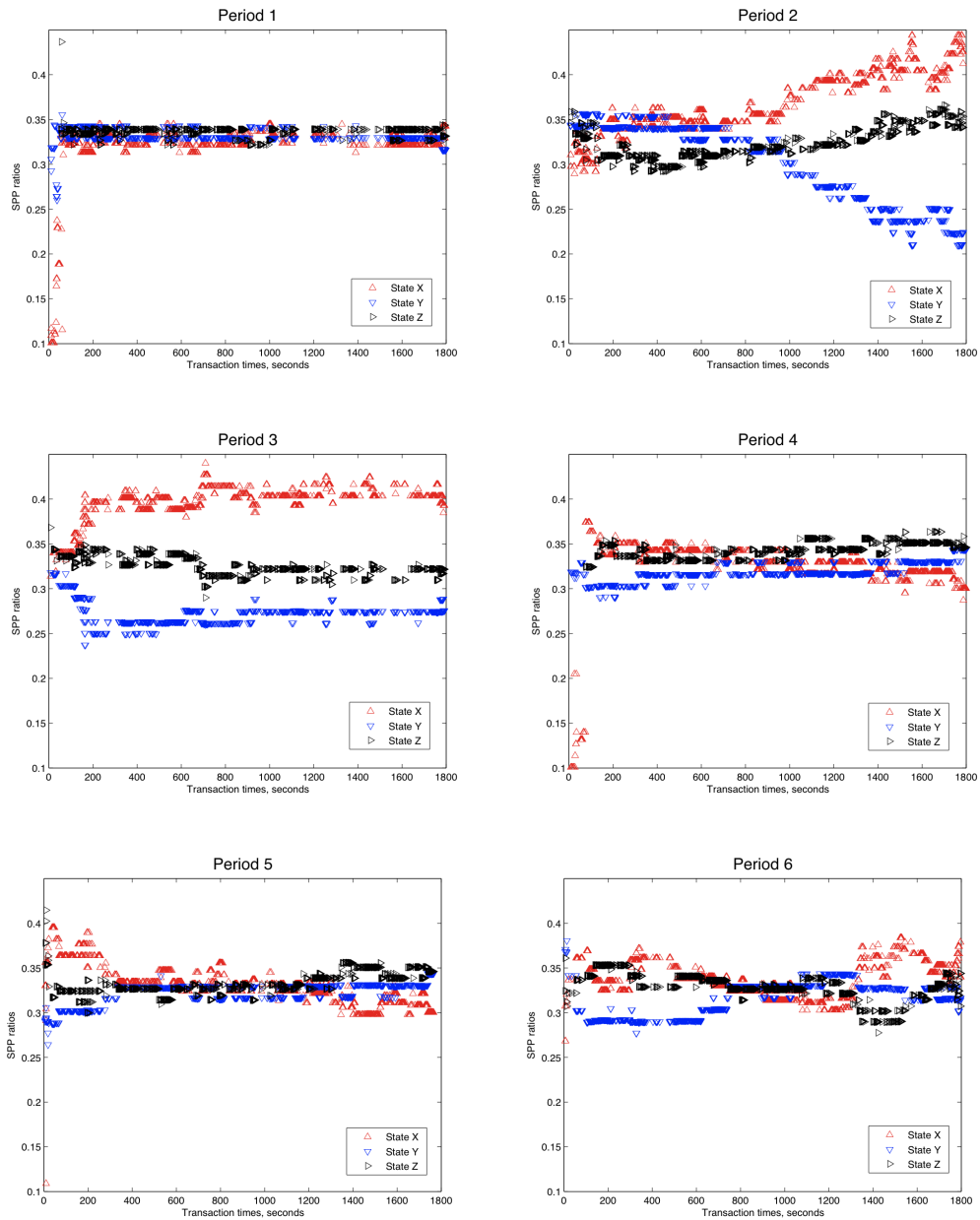
Appendices

A Quadratic Utility and the CAPM

In all variations of the theory we had first-order conditions for optimality of investors given by

$$u'_i\left(z^i_s\right) = \lambda \frac{\tilde{p}_s}{\pi_s},$$

where z^i_s is always the investor's final wealth after subtracting possible fee payments, and $\tilde{p}_s$ is the state price, possibly "fee-adjusted".

Assume, as in the CAPM, that all investors hold a quadratic utility over post-fee wealth, $u_i\left(z^i_s\right) = z^i_s - \frac{b_i}{2}\left(z^i_s\right)^2$. The above FOC becomes

$$1 - b_i z^i_s = \lambda_i \frac{\tilde{p}_s}{\pi_s}.$$

Solving first-order-conditions with the Lagrange multiplier λ_i, the following optimal demands obtain:

$$z^i_s = \frac{1}{b_i} - \frac{\lambda_i}{b_i}\frac{\tilde{p}_s}{\pi_s} = z^i_c + z^i_{e,s}.$$

Notice that demands can be decomposed in terms of a constant, z^i_c, and a term that depends on the states only through the state price probability ratio $\tilde{p}_s/\pi_s$, $z^i_{e,s}$. We call the part of the demand that is state dependent "risk-exposed" demand. This property is referred to as (two-fund) separation. The constant in the demands, z^i_c, can be accommodated by purchasing risk-free securities. Isolating the risk-exposed demand we get:

$$\frac{z^i_{e,s}}{z^i_{e,s'}} = \frac{\frac{p_s}{\pi_s}}{\frac{p_{s'}}{\pi_{s'}}} = \frac{z^j_{e,s}}{z^j_{e,s'}},$$

for any two states s and s' and any other agent j (with quadratic utility) in the economy. Thus, the vector of risk-exposed components, $\{z^i_{e,s}\}^S_{s=1}$, is the same for all investors, up to a constant of proportionality that depends on individual risk tolerance.

Two-fund separation implies that individual demands aggregate, and one can work with a representative agent with a quadratic utility with risk parameter $B = \frac{1}{\sum_i \frac{1}{b_i}}$. The aggregate demands $z_s = \sum_i z^i_s$ satisfy

$$z_s = \frac{1}{B} - \frac{\lambda}{B}\frac{\tilde{p}_s}{\pi_s} = z_c + z_{e,s}.$$

These aggregate demands are those of a representative agent with the same preferences and risk aversion coefficient B. That is aggregate demands are optimal as well. They satisfy the second important property of CAPM, which is that the market portfolio is mean-variance optimal. This means that the aggregate portfolio of all AD securities available in the marketplace is optimal for

some agent with quadratic utility. Hence, its Sharpe ratio (expected excess return divided by return standard deviation) is maximal, or equivalently, that the well-known "beta" pricing relationship holds (expected excess returns on all risky assets are strictly proportional to their "betas;" see [17]).

Imposing $\sum_s \tilde{p}_s = 1$, we can solve for the Lagrange multiplier (of the representative agent) after multiplying the optimal (aggregate) state-security demands by state probabilities (π_s) and summing:

$$\lambda = 1 - B \sum_s \pi_s z_s.$$

It is easy to show that $\lambda > 0$ as long as the representative agent is not satiated at optimal demands (i.e., marginal utility is still positive).

In equilibrium, demands should equal supplies ($z_s = z_s^0 = z_s^0 = \sum_i z_s^{0,i}$), and hence, the state price probability ratios should satisfy the following restriction:

$$\frac{\tilde{p}_s^*}{\pi_s} = \frac{1}{\lambda^*} - \frac{B}{\lambda^*} z_s^0, \tag{3}$$

where $\lambda^* = 1 - B \sum_s \pi_s z_s^0$. While not immediately transparent, this is the CAPM pricing equation. It implies that the market portfolio is mean-variance optimal, and hence, that its Sharpe ratio is maximal, and that expected excess returns on individual securities are strictly proportional to their "betas."

Hence, with the added assumption of quadratic utility, in all variants of our theory, the fee-adjusted wealth and fee-adjusted prices, jointly satisfy CAPM properties. In the baseline with limited liability, also the trading prices, p_s^d, satisfy CAPM, while this is not the case when expected payoffs are used to compute fees and shares. However, it will still be approximately true, as illustrated in the examples that follow.

B   Numerical Examples

For the numerical examples we use the pricing equation for a representative investor with quadratic utility over wealth, which we derived in the previous section.

Aggregate gross state wealth is assumed equal to the parameters used (on average) in our experiment. There are three states of the world, $s = 1, 2, 3$, corresponding to X, Y, and Z in the experiment. Per capita aggregate wealth is thus

$$\frac{1}{I} \left(x_1^0, x_2^0, x_3^0 \right) = (2000, 11600, 7100).$$

All states are equally likely, implying $\pi_s = 1/3$, and $1/\pi_s = 3$. We assume a *reasonable* value of aggregate risk aversion, $B = \frac{2 \times 10^{-5}}{I}$. We now consider each one of the variants of our theory, appropriately replacing values of z_s^0 and $\tilde{p}_s^d$ for each case.

No Delegation

No fees are paid and, therefore, $z_s^0 = x_s^0$ and $p_s = \tilde{p}_s$ for all s. Therefore,

$$\lambda = 1 - 2 \times 10^{-5} \frac{1}{3} \sum_s \frac{x_s^0}{I} = 0.8620,$$

and equilibrium prices are

$$(p_1, p_2, p_3) = (0.3712, 0.2970, 0.3318).$$

Delegation with Standard Liability

Consider a fee per unit of acquired contract of 8%, $f = 0.08$. Notice that this is not the same as the fee implemented in the experiment, which corresponds to ϕ in the theory. The total per capita fee payment is therefore $0.08 \times \sum_s \frac{x_s^0}{I} = 1656$, making the state wealth net of fees equal to

$$\frac{z^0}{I} = (344, 9944, 5444).$$

With these values, we obtain $\lambda = 0.8951$, and fee-adjusted prices are immediately computed to be

$$\tilde{p} = (0.3698, 0.2983, 0.3318).$$

Recall from section 3 that with standard liability, fee-adjusted inter-manager prices are $\tilde{p}_s^d = (1 - Sf)p_s^d + f$, and hence

$$p^d = \frac{\tilde{p}^d - f}{1 - Sf} = (0.3813, 0.2873, 0.3314).$$

Notice that in this case, $\sum_s \tilde{p}_s^d = 1 \Leftrightarrow \sum_s p_s^d = 1$, so we need not re-normalize gross prices.

As expected, since it holds generally when states are equally likely, state prices satisfy CAPM, which also implies weak CAPM. Indeed, prices are ranked inversely to wealth in each state (both before and after fee adjustment). In this example all three state prices move farther away from the state probability, but this is not a general result. In general, state prices of states with below-average wealth increase in the economy with delegation and standard liability (this is the case here), but nothing general can be said about states with above-average wealth.

Delegation with Limited Liability

Using the same 8% fee as before, we now have state wealth net of fees given by $z_s^0 = x_s^0 (1 - f)$, yielding

$$\frac{z^0}{I} = (1840, 10672, 6532).$$

Consequently, $\lambda = 0.8730$, and fee-adjusted prices are

$$\tilde{p}^d = (0.3678, 0.3003, 0.3319).$$

In this case, fee-adjusted prices are defined to be $\tilde{p}_s^d = \frac{p_s^d}{1-f}$. We solve for gross prices, and obtain $p^d = (0.3997, 0.3264, 0.3608)$. If we normalize these prices so that they add up to one, we get back $\tilde{p}^d$, hence we use $\tilde{p}^d$ for comparison to the case without delegation. Notice that all three prices get closer to state probabilities, implying that state price probability ratios are closer to 1 with delegation. This reflects higher risk tolerance under delegated portfolio management. The intuition behind this result is simple. Notice that with delegation we have

$$\frac{p_s^d}{\pi_s} = \frac{1}{\lambda} - \frac{B}{\lambda} z_s^0 = \frac{1}{\lambda} - \frac{B}{\lambda}(1 - f) x_s^0 = \frac{1}{\lambda} - \frac{\tilde{B}}{\lambda} x_s^0,$$

where $\tilde{B} = B(1 - f)$. Hence, the aggregate investor has a lower risk aversion with than without delegation (of course, also λ is lower, but not enough to counter the decrease in expressed risk aversion).

Shares And Fees Based On Expected Payoff

Take, as in the experiment, $\phi = 0.4$. We solve the following system obtained from the representative investor's optimization, budget constraints, and market clearing:

$$\lambda \frac{p_s^d}{1/3 (1 - f_s)} = 1 - B x_s^0 (1 - f_s), \quad s = 1, 2, 3,$$

$$f_s = \frac{p_s^d \phi P}{\sum_t p_t^d x_t^0}, \quad s = 1, 2, 3, \text{ and}$$

$$\lambda = 1 - \frac{IB}{3} \sum_t (1 - f_t) \frac{x_t^0}{I}.$$

We obtain $\lambda = 0.8804$. The state-specific per-contract fees are

$$f = (f_1, f_2, f_3) = (0.1502, 0.1273, 0.1384),$$

and gross prices, after re-normalizing so that $\sum_s p_s^d = 1$, are

$$p^d = (0.3611, 0.3062, 0.3327).$$

The above prices, while satisfying weak CAPM, do not satisfy strong CAPM. The fee-adjusted prices, on the other hand, do satisfy CAPM (for fee-adjusted state wealth). These prices are

$$\tilde{p}^d = (0.3657, 0.3020, 0.3323).$$

It is easily seen that the difference between the CAPM prices (fee-adjusted) and the equilibrium prices is very small, in fact, it equals

$$\tilde{p}^d - p^d = (0.0046, -0.0042, -0.0004).$$

This means that investors with quadratic utility, facing prices that closely resemble CAPM prices, will also hold holdings that are close to CAPM. Hence, mean-variance efficiency of the market portfolio and two-fund separation hold approximately.

C   Shares And Fees Based On Expected Payoff

We must show that when shares and fees are computed based on expected payoff, weak CAPM holds for **gross prices, p^d**, and not only for fee-adjusted prices, $\tilde{p}^d$.

Under the assumptions made for the feasible variation derivation (all investors have equal expected payoff and market value of their initial portfolio), we obtained that

$$\frac{u'\left(z_s^0\right)}{u'\left(z_t^0\right)} = \frac{\tilde{p}_s^d/\pi_s}{\tilde{p}_t^d/\pi_t},$$

where $\tilde{p}_s^d = \frac{p_s^d}{1-f_s}$, and therefore, $p_s^d = (1 - f_s)\tilde{p}_s^d$. For the representative investor, $\alpha_s = \frac{p_s^d x_s^0}{\sum_t p_t^d x_t^0}$, and the fee-adjusted wealth in state s is $z_s^0 = x_s^0 - \phi\alpha_s P$. Letting $W = \sum_t p_t^d x_t^0$, we obtain that

$$1 - f_s = 1 - \frac{p_s^d \phi P}{W}.$$

Therefore, gross state-price probability ratios are given by

$$\frac{p_s^d}{\pi_s} = \left[1 - \frac{p_s^d \phi P}{W}\right]\frac{\tilde{p}_s^d}{\pi_s}.$$

Solve the above for p_s^d/π_s, to obtain

$$\frac{p_s^d}{\pi_s} = \frac{\tilde{p}_s^d/\pi_s}{1 + \frac{\phi P}{W}\tilde{p}_s^d}.$$

Assume that all states are equally likely (as in our experimental setup), to obtain the following expression for the relative (gross) price of two given states, s and t:

$$\frac{p_s^d}{p_t^d} = \frac{\tilde{p}_s^d}{\tilde{p}_t^d}\left[\frac{1 + \frac{\phi P}{W}\tilde{p}_t^d}{1 + \frac{\phi P}{W}\tilde{p}_s^d}\right].$$

Assume that the fee-adjusted prices are such that $\tilde{p}_s^d < \tilde{p}_t^d$, which will be the case if the fee-adjusted wealth of state s is larger than that of state t. We want to show that in this case, also

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$p_s^d < p_t^d$. With the above expression, this will be the case if and only if

$$\frac{1 + \frac{\phi P}{W}\tilde{p}_t^d}{1 + \frac{\phi P}{W}\tilde{p}_s^d} < \frac{\tilde{p}_t^d}{\tilde{p}_s^d}.$$

Work with the above expression to see that it holds if and only if

$$1 + \frac{\phi P}{W}\tilde{p}_t^d < \frac{\tilde{p}_t^d}{\tilde{p}_s^d} + \frac{\phi P}{W}\tilde{p}_t^d \Leftrightarrow$$

$$1 < \frac{\tilde{p}_t^d}{\tilde{p}_s^d},$$

and the latter holds by the original assumption. Hence, we conclude that whenever $\tilde{p}_s^d < \tilde{p}_t^d$, it will also hold that $p_s^d < p_t^d$.