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Measuring the Impact of Longevity Risk on Pension Systems: The Case of Italy.

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May 2012

Abstract

This paper proposes to estimate the impact of longevity risk on pension systems by combining the prediction based on a Lee-Carter (1992) mortality model with the projected pension payments for different cohorts of retirees. An application to the Italian pension system is provided. Longevity risk for a pension system is represented by higher-than-expected longevity resulting into an unexpected increase in total old-age pension expense. We measure longevity risk by the difference between the upper bound of the total old-age pension expense and its mean estimate. This difference ranges between 0.06% in 2012 and 4.35% in 2050. Our estimate of total longevity risk over the period 2040-2050 is as high as 40 per cent of total GDP over the same period. The impact of longevity risk is sizeably reduced by the introduction of indexation of retirement age to expected life at retirement. Our results gives evidence in favour of a market for longevity risk and call for a closer scrutiny of the potential redistributive effects of heterogeneity in the impact of longevity risk on different groups of the retired and retiring population.

Keywords: stochastic mortality, longevity risk, social security reform

JEL Classification Numbers J11,J14

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1 Introduction

This paper estimates the impact of longevity risk on the Italian pension system by combining the predictions based on a Lee-Carter (1992) mortality model with the projected pension payments for different cohorts of retirees. The novelty in our approach is the use of the uncertainty generated by a mortality model to assess the impact of longevity risk on pension expedinture and the impact of social security reforms. The use of mortality models to assess social security policy has two main advantages. First, mortality models generate predictions for the evolution over time of population in each cohorts. This is the requirement needed to assess the impact of sequential social security reforms that, by usually not being retroactive, affect differently different cohorts of the population. Second, the parameters in a mortality model are very unlikely to be affected by the specific social security policy adopted by the government. Therefore, the econometric specification used for policy simulation analysis is robust to the Lucas'critique (1976). The framework proposed here is of general applicability. In our specific application we concentrate on Italy, as Italy represents one of the most interesting cases to ask the question to our interest. The Italian economy is characterized by one of the largest world public debt (both as a ratio of GDP and in an absolute terms), a traditionally very generous pension system and one the world's lowest fertility rate (Sartor, (1999)). Also, the Italian pension system has been subject to a number of reforms, and the most recent one, implemented initially in 2010 and completed in 2012, has introduced an automatic indexation of the retirement age to expected residual life at retirement.

There is by now a wide literature in economics on generational accounting, which measures directly the amount of net taxes that current and future generations are expected to pay under existing public policy. This literature examines the impact of the current deficit on the welfare and spending patterns of future generations (See Kotlikoff(1993), Kotlikoff and Burns (2004), Gomes et al. (2011), Auerbach et al. (1999)). Although the relevance of longevity risk is widely recognized, especially by supranational institutions (see Visco(2006),Visco(2009), IMF(2012)), the abundant literature in demography aimed at estimating statistical models of mortality rates to be able to project them with the associated uncertainty is not currently used to simulate the impact of longevity risk on pension systems. This literature is potentially important as pension providers face the risk that retirees might on average live longer than expected. Longevity risk can be decomposed in two underlying components: random variation risk and trend risk. Random variation risk is the risk that individual mortality rates differ from the outcome expected as a result of chance – some people will die before their life expectancy, some will die after. Trend risk is the risk that unanticipated changes in life-style behavior or medical advances significantly improve longevity. A state pension system naturally deals with random variation risk by pooling a large number of different individuals and relying on the law of large numbers to reduce its variability. Trend risk, similarly to any macroeconomic risk, is on the other hand an "aggregate risk" that cannot be diversified away by pooling and is therefore the relevant one for a state pension system. A first look at the data reveals immediately the possibility of sizeable longevity risk. Mortality rates, survival probabilities, frequencies of death and expected residual life have dramatically changed for individuals aged 65 and over in Italy[1]. Figure 1.1 illustrates the strong downward trend in the time series of mortality at age 65 over the period 1965-2008. Note, however, that the reduction in mortality is not uniform at all ages. Mortality improvements at old ages have been more drastic than the ones for individuals aged between 65 and 70. In 1980 an individual alive at age 65 had a probability of 90.5% of being alive at 69, a probability of 52.7% of being alive at 79, and a probability of 10.7% of being alive at 89; such probabilities have shifted respectively to 95.4%, 73.5% and 29% in 2008. Figure 1.2 illustrates this point by reporting survival probabilities for individuals aged 65 and over in 1980 and 2008. Note also that the survivor function tend to shift towards a rectangular shape over time as a consequence of the increasing concentration of death around the mode of the curve of deaths. Figure 1.3 illustrates how this "compression of morbidity" has made the profile of frequencies of death for ages above the mode closer and closer to a straight vertical line. Finally, Figure 1.4 shows that life expectancy at 65 has moved from about 13 years to nearly 20 years in 2010. Our strategy to assess the impact of longevity risk on the Italian pension system is based on three steps. First, we derive the numerosity of each cohort of retirees up to 2050 by using the Lee-Carter mortality model to project future mortality and by applying it to the current population pyramid. As future mortality rates are projected with uncertainty, a confidence

[1] $q(x,t)$ denotes the mortality for individuals of age x in year t, where mortality is the probability that a person aged x and alive at the beginning of the year dies within the end of the year. $s(x,t)$ is instead the survivor probability for individuals of age x in year t, which is the probability that an individual will be alive at age x given that he has survived up to age $x-1$. Survivor probabilities are derived recursively for individuals aged 65 and over: If $x = 65$ then $s(x,t) = 1 - q(x,t)$, if $x > 65$ then $s(x,t) = s(x-1,t)[1 - q(x,t)]$. Frequencies of death for individuals of age x at time t are determined as first differnces of survival probabilities: $fod(x,t) = s(x,t) - s(x+1,t)$. Finally, life expectancy at 65 is defined as follows $E_x = \sum_{t=1}^{\infty} s(x,t)$

interval is associated to future population at each age. Second, pension payments to each cohort in the future are projected using institutional information on the Italian pension system. Third, total old-age pension expenditure as a ratio of GDP is projected over the horizon 2012-2050 with its associated confidence intervals. The width of our confidence intervals reflects the impact of longevity risk. In fact, an analogue of the concept of Value at Risk in portfolio management can be applied to future pension expenditure by estimating the upper bound with a given probability (namely the upper limit of the 95 per cent confidence interval) of pension expenditure as a ratio of GDP in each year. In assessing the impact of longevity risk of the Italian Pension system we consider first a scenario that reflects all the current institutional details with the exception of the indexation of retirement age to expected residual life at retirement. We then devote a section to the simulation of the impact of such an indexation.

2 Methodology

Our proposed methodology is based on the computation of future pension payments based on the last observed population pyramid (2011, in our sample), projected mortality rates and projected future pension payments to each cohort. In practice, the following specification is adopted:

$$E\left(TP_{2011+j} \mid \Omega_{2011}\right) = \sum_{i=0}^{45} E\left(POP_{65+i,2011+j} \mid \Omega_{2011}\right) E\left(PD_{65+i,2011+j} \mid \Omega_{2011}\right)$$

$$E\left(POP_{65+i,2011+j} \mid \Omega_{2011}\right) = POP_{65+i-j,2011} \prod_{k=1}^{j} \left(1 - \widehat{q}_{65+i-j+k,2011+k-1}\right) \tag{1}$$

$$j = 1, ..., 39, \quad i = 0, ..., 45$$

where $E\left(TP_{2011+j} \mid \Omega_{2011}\right)$ is the expected total old age pension payments in year 2011+j, given the information set available in year 2011. Pension payments for each cohort from retirement age onward are computed by multiplying the projected number of individuals in each cohort $E\left(POP_{65+i,2011+j} \mid \Omega_{2011}\right)$ times the average projected pension payment to each individual in that cohort $E\left(PD_{65+i,2011+j} \mid \Omega_{2011}\right)$. The number of individuals in each cohort in the future is obtained by applying to the observed population pyramid in 2011 the mortality rates projected for each cohort over the period 2012- 2050. A demographic model is used to project future mortality rates, while simple time-series model for annual income growth and inflation, paired with a description of the Italian institutional system, deliver projected $E\left(PD_{65+i,2011+j} \mid \Omega_{2011}\right)$, i.e. the expected pension due to all cohorts in all future years. Using then the lower bound of the 95 per cent confidence interval on the projected mortality rates we compute the upper bound on future pension payments as follows:

$$VaR^{95}\left(TP_{2011+j} \mid \Omega_{2011}\right) = VaR^{95}\left[\sum_{i=0}^{45}\left(POP_{65+i,2011+j} \mid \Omega_{2011}\right) E\left(PD_{65+i,2011+j} \mid \Omega_{2011}\right)\right]$$

$$\left(POP_{65+i,2011+j} \mid \Omega_{2011}\right) = POP_{65+i-j,2011}\prod_{k=1}^{j}\left(1-\widehat{q}_{65+i-j+k,2011+k-1}\right) \qquad (2)$$

$$j = 1, ..., 39, \quad i = 0, ..., 45$$

$VaR^{95}\left(TP_{2011+j} \mid \Omega_{2011}\right)$,calculated by bootstrap and based on the Lee-Carter model, provides an estimate of the impact on pension payments of the realization of the lowest projected mortality rates from 2012 onwards. Note that, as in the computation the only source of uncertainty allowed for is that on demographic trends, the macroeconomic uncertainty affecting $E\left(PD_{65+i,2011+j} \mid \Omega_{2011}\right)$ is not considered in the computation of the upper bound for pension expenditure.

3 Projecting Retired Population over the period 2012-2050

To project retired population at each age over the period 2012-2050 we take the Italian population pyramid observed in 2011, available from the Italian National Agency for Statistics (ISTAT) website[2], and adopt a Lee-Carter mortality model to predict future mortality at all ages. Natality is irrelevant to our projections as individual born from 2012 onwards cannot retire before 2050. The adopted method also is based on the maintained hypothesis of future zero net immigration flows. In 2011 younger generations in the ages between 20 and 34 accounted for slightly more than 16% of the total population, while ages between 35-50 for more than 25%. Individuals above 65 years accounted for approximately 24% of the total. Data on Italian central mortality rates for the years 1872-2008 are available from the Berkeley Human Mortality Database website[3]. The mortality model is estimated on the sample annual data 1965-2008.We adopt the Lee-Carter (1992) model to project future mortality rates. The model consists of a system of equations for logarithms of mortality rates for age cohort x at time t, $\ln[m_{x,t}]$, and a

[2] http://dati.istat.it/
[3] www.mortality.org

time-series equation for an unobservable time-varying mortality index k_t:

$$\begin{aligned} \ln[m_{x,t}] &= a_x + b_x k_t + \epsilon_{x,t} \\ k_t &= c_0 + c_1 k_{t-1} + e_t \\ \epsilon_{x,t} &\sim NID\left(0, \sigma_\epsilon^2\right) \\ e_t &\sim NID\left(0, \sigma_e^2\right) \end{aligned} \quad (3)$$

where a_x and b_x are age-specific constants. The error term $\epsilon_{x,t}$ captures cross-sectional errors in the model based prediction for mortality of different cohorts, while the error term e_t captures random fluctuations in the time series of the common factor k_t driving mortality at all ages. This common factor, usually known as the unobservable mortality index, evolves over time as an autoregressive process and the favorite Carter-Lee specification makes is a unit-root process by setting $c_1 = 1$. Identification is achieved by imposing the restrictions $\sum_t k_t = 0$ and $\sum_x b_x = 1$, so that the unobserved mortality index k_t is estimated through Singular Value Decomposition[4]. Identification and estimation of the system allows to perform stochastic simulations to obtain projections for $\ln[m_{x,t+i}]$ and its associated confidence intervals.

3.1 The performance of the Lee-Carter model on Italian data

Before using the Lee-Carter model to project future mortality rates we have evaluated its ex-post performance by fitting the model to Italian mortality rates for the period 1965-1999 and generating forecasts for the subsequent nine years, up to 2008. Cohorts ranging from 20 to 110 years have been considered. In fact, this is the age-range needed to project pension profiles up to 2050 as the latest cohort to retire in our projection period is made by individuals in their twenties today. Figures 2.1 and 2.2 report the estimates of the coefficients a_x and b_x, with their associated 95 per confidence intervals. The constant term a_x, monotonically increasing in x, reflects the heterogeneity in mortality for different age groups, while b_x pins down the response of mortality rates at different age to the common (stochastic) trend in mortality k_t (note that this coefficient decreases roughly uniformly from age 70 onwards). The cross-sectional variability of the estimates b_x is a typical feature of the Lee-Carter model, reflecting the higher volatility of observed mortality rates in the right tail of the population distribution. Figure 2.3 shows the goodness of fit in the

[4]See Appendix 1 for a full description of the adopted identification and estimation strategy.

cross section of ages by reporting the R^2 of each of the ninety equations estimated for mortality rates in the age range 20 to 100 over the sample 1965-1999. The model performs very well in explaining mortality from 40 to 100 years, while its power significantly drops as the observations on mortality rates become more volatile at very old ages. Still, the overall within-sample fit of the model is remarkable. Figure 2.4 illustrates the point by showing the performance of the model on the 65-years cohort. Out-of-sample projections are obtained by simulating the models for mortality rates at all ages jointly with the following autoregressive model for k_t (standard deviation of coefficients in parentheses).

$$k_t = \underset{(0.0148)}{1.0174} k_{t-1} - \underset{(0.28062)}{1.6165} + e_t \tag{4}$$

In practice, upper and lower bounds for projected k are derived by first bootstrapping the above model, allowing for parameters uncertainty, to obtain 1000 simulated path for k_t based on drawing with replacement from the empirical distribution of the estimated residuals. Projected mortality rates and their associated uncertainty are then derived by combining the simulated paths for k_t with the estimated coefficients a_x and b_x and their associated uncertainty.The evidence reported in Figure 3 allows pseudo-out-of-sample model simulation by assessing the projected mortality, survivor and frequency of death over the period 2000-2008 against those observed ex-post. The overall performance of the model is overall satisfactory, as the realized ex-post data almost never violate the 95 per cent confidence bounds.

3.2 From Mortality Projections to Retired Population Projections

Projections for retired populations and its age structure are immediately obtained by applying the projected mortality rates to the observed population pyramid in 2011. Figures 4.1-4.4 show our mean and lower-bound projections for the Italian retired population's mortality, specifically of age 65, 75, 85 and 95 years old, from 2012 to 2050 and compare them to a benchmark offered by a constant-mortality scenario, i.e. the projections given by a random walk (without drift) model for mortality rates at all ages. Projected mortality for 65-years old people in 2050 is equal to 0.17% (while it stands at 0.94% in 2008) and converges to zero in the lower-bound case. Similarly, mortality rates of 75-, 85- and 95-years old people are expected to decline from 2.57%, 8.70% and 27.90% in 2008 to 0.49%, 2.01% and 10.78% in 2050, respectively. Figure 4.5 reports the expected residual life at 65 consistent with the path of mortality projections at ages of 65 and over. Expected life at 65 increases from 20 years in

2009 (fully consistent with the projections made by the Italian Institute of Statistics, ISTAT) to roughly 30 years in 2050 in the mean-mortality case, and to 36 years in the lower bound mortality (upper bound life expectancy) case. Figure 5.1 summarizes the results of projections of mortality at all ages by reporting the resulting over-65 population structure over the period 2012-2050. The projected retired population grows at an average rate of 1.65% per year from 2012 to 2050, almost doubling from an initial level of around 12.5 millions to 23.5 millions in 2050. Interestingly, projections based on constant mortality rates at all ages would on average underestimate the retired population by as much as 7.5 millions, and by 14 millions if the upper bound for longevity is considered.

4 Projecting the Future Payments of the Italian Pension System

Mapping the future structure of retired population in future pension payments requires a representation of the institutional design of the pension system. Several pension reforms have progressively changed the Italian Pension system over the last thirty years. Three main reforms, namely the Amato (1992) reform, the Dini (1995) reform and the Sacconi-Tremonti(2010), completed by the Fornero-Monti(2012) reforms, that have so far affected the system have determined a progressive shift from the retributive to the contributive pension system. We therefore identify three periods:

- Pre-Amato reform period (pre-1992). This period was characterized by a retributive method for calculating pensions. The initial annual pension paid to the retiree was 2% of the average income of the last five years before retirement, multiplied by the number of years of the individual's contribution. Pensions were then revaluated with a perfect indexation to inflation.

- Amato reform (1992). The Amato reform was aimed at lowering pension expense by making the annual pension function of the income earned during the entire working life rather than that earned in the last years before retirement. The calculation method still was retributive, and the pension was calculated as 2% of the average income of all the contributing life, times the number of years of contribution. Pensions were revaluated at inflation plus 1%.

- Dini reform (1995). This reform shifted the Italian pension system from retributive to contributive. Under the contributive regime workers contribute during their working life with a share (33%) of their income to the formation of a capital. This capital is revalued each year at the five-year moving average of the nominal GDP growth. Upon retirement, an annual pension is then calculated to equate the present value of the total contributed capital at retirement to the resent value of totale pension payments. As annual pension payments in real terms are constant, the equalization is obtained by multiplying the accumulated capital by a transformation coefficient, that depends on life expectancy and a long-run discount rate. In theory, life expectancy in the transformation coefficient can be automatically indexed to the evolution of mortality over time, in practice this parameter has been kept constant from 1996 to 2007, it was then changed on a one-off basis in 2007, while automatic indexation has been postponed until the 2010-2012 reform. The minimum age for retirement under this method was 57 years, the maximum one 65, and the reform fully applied for those who started working in 1996.

- The 2010-2012 reform. This reform has moved the Italian system to a fully contributive and it has introduced the indexation of the pension system to the evolution of mortality after retirement. Two aspects of this reforms are particularly important to us. The first one is the indexation of the pension payments to the expected length of life at retirement, the second one is the indexation of retirement age to expected life at retirement.

To take into account the effect over time of these reforms, the pension of a retiree as of the beginning of 2012 is computed as a function of his working history with one of the three following alternative computation methods: contributive, mixed or retributive.For all agents retiring before 1992, the Pre-Amato regime applies. This regime implies that

$$
\begin{aligned}
E\left(PD_{65+(2012+j-r),2012+j} \mid \Omega_{2011}\right) &= y_r \prod_{k=1}^{2012+j-r} \left(1+\pi_{r+k,r+k-1}+D_{1993}0.01\right) \\
1967 &\leq r \leq 1992 \qquad (5) \\
y_r &= \beta \overline{I}_{r-5,r} C_r \\
j &= 0, ..., 45-(2012-r)
\end{aligned}
$$

where r is the year of retirement, $\beta = 2\%$, C_r is the number of years for which he has contributed at retirement and $\overline{I}_{r-5,r}$ is the average labour income for the last five years before retirement. π is inflation and

D_{1993} is a dummy taking value of 1 from 1993 onward and zero otherwise. This dummy captures the change in mechanism of indexation of 1992 that adjusted annual pension from 1993 onward with a mark-up of one per cent on the annual realized inflation rate.

For all individuals retiring between 1993 and 1995 the Amato reform applies and we have:

$$\begin{aligned} E\left(PD_{65+(2012+j-r),2012+j} \mid \Omega_{2011}\right) &= y_r \prod_{k=1}^{2012+j-r} \left(1.01 + \pi_{r+k,r+k-1}\right) \\ 1993 &\leq ret \leq 1995 \\ y_r &= \beta\left(\overline{I}_{r-5,r}\left(C_r - (r-1993)\right) + \overline{I}_{r-10,r}\left(r-1993\right)\right) \\ j &= 0, ..., 45 - (2012 - r) \end{aligned} \tag{6}$$

where $\overline{I}_{ret-10,ret}$ is the average labour income for the last 10 years before retirement and all the other variables have the same meaning as in Equation 5.

For individuals retiring from 1996 onwards the Dini reform's framework applies, with the necessary adjustments for the 2010-2012 reforms

$$\begin{aligned} E\left(PD_{65+(2012+j-r),2012+j} \mid \Omega_{2011}\right) &= y_r \prod_{k=1}^{2012+j-r} \left(1.01 + \pi_{r+k,r+k-1}\right) \\ 1996 &\leq r \leq 2050 \\ y_r &= \beta\left(\overline{I}_{r-5,r}\left(C_r - (r-1993)\right) + \overline{I}_{r-10,r}\left(r-1993\right)\right) \\ if\ C_{1995} &\geq 18 \\ y_r &= \beta\left(\overline{I}_{r-5,r}\left(C_r - (r-1993)\right) + \overline{I}_{r-10,r}\left(1996-1993\right)\right) \\ &+ \gamma_{r,2012+j} \sum_{t=1996}^{r} \delta I_t (1+gr)^{r-t} \\ if\ 0 < C_{1995} &< 18 \\ y_r &= \gamma_{r,2012+j} \sum_{t=r-C_r}^{r} \delta I_t (1+gr)^{r-t} \\ if\ C_{1995} &\leq 0 \\ \gamma_{r,2012+j} &= \left(\sum_{\tau=1}^{E(L|\Omega_r)} \frac{1}{(1+r_z)^\tau}\right)^{-1} \\ j &= 0, ..., 45 - (2012 - r) \end{aligned} \tag{7}$$

where $\delta = 33\%$, I_t is income earned in contribution year t, gr is the five-year moving average of nominal GDP growth and $\gamma_{ret,2012+j}$ is a coefficient, calculated by ISTAT, based on life expectancy at retirement,

$E(L)$, conditional on the information set on life expectancy available at retirement, Ω_r. This coefficient has been kept constant from 1996 to 2007, it has been then changed on a one-off basis in 2007 while automatic indexation (every three-years) has been introduced by the Sacconi reform in 2010. We report in Table 1 the values of the contributive coefficients for different ages as of the beginning of 2012 (assuming a long-run discount rate $r_z = 1.5\%$). Baseline simulations will be performed by including this aspect of the 2010-2012 reforms in our analysis but not its second feature, which is the indexation of the retirement age. A specific section will be then devoted to the evaluation of the effect of this important modification of our baseline scenario.

4.1 Old-Age Pension Expense Projections

Within the institutional framework discussed in the previous section projections of total pension expenditure over the period 2012-2050 requires as input projections for future labour income and inflation. We adopt very simple models for these macroeconomic variables, as the main objective of our exercise is the evaluation the impact of longevity risk on the pension system. As we expect a low correlation between macroeconomic and longevity risk, the model adopted for macroeconomic projections should be very little relevant for the estimation of the effect of longevity risk. Historical data on labour income in Italy are taken from the annual Survey on Households Income ("Indagini sui Bilanci delle Famiglie Italiane") made available by the Bank of Italy through its website for the annual sample 1965-2008[5]. These data are based on annual surveys conducted by the Bank of Italy on income, real wealth, financial assets' diffusion, use of different means of payment, housing market, use of public services and quality of life. Given the availability of these data we construct a time series of average labour income from employment and self-employment, by excluding other sources of income that do not enter the computation of pensions (such as, for example, interest income). These computations make available a consistent time series for five types of categories: self-employed (other than entrepreneurs), entrepreneurs, employees (other than office workers), office workers and managers[6].We then combine the historical time-series with forecasts for nominal labour income over the period 2012-2050. These forecasts are based on the prediction of a time-series model for real labour income and a scenario for inflation. Future real labour income growth is projected to move one-to-one with future real GDP growth. Real GDP growth is modelled, using data available up to 2011, as an AR(1) process. The following equation

[5] http://www.bancaditalia.it/statistiche/indcamp/bilfait

[6] Please refer to Appendix B for the analysis of the surveys and their elaboration.

has been estimated by OLS:

$$r_{GDP,t} = \alpha + \beta r_{GDP,t-1} + \eta_t \tag{8}$$

Using the estimates of α and β, respectively equal to 0.0152 and 0.252, we project forward the AR model that converges to a long-run equilibrium real GDP growth of 2.03%.[7] The generated projections for the level of average real labour income imply that real labour income will more than double in the next forty years, going from the current value of around € 16,000 to a value of around € 35,000 in 2050. Given these forecasts for annual real labour income, we generate the corresponding nominal labour income by making the assumption that CPI inflation will be in the future determined by the ECB's target of 2 percent. The resulting nominal incomes' projections range from € 15,831 in 2009 to € 36,175 in 2050. The projections of the future numerosity of each cohort of retired individuals are then combined with the macroeconomic scenario and the institutional framework to generate total old-age pension expenditure. In particular, the computation of pensions with the retributive method is based on the assumption of an age at retirement equal to 65 years and 40 years of contribution. The retributive system is relevant for those who have retired between 1969 and 2012 (as the Fornero-Monti(2012) reform does not allow for retirement with the retirbutive method after 2011). The mixed system applies instead to those individuals who had less than 18 years of contribution at 31.12.1995, and in this case the retributive part of the pension is computed for the years of contribution before 1996, and the contributive part applies to the income earned after 1996, which is observable up to 2008 and projected by the AR model from 2009 onwards. The annual payment of the contributive part of the pension is then computed taking the transformation coefficient δ to be equal to 0.33 and using the method of computing the $\gamma(E(L))$ coefficients adopted by ISTAT - whose values are shown in Table 1 for the 2010-2013 period. Under simulation the coefficient $\gamma(E(L))$ is modified every three years to reflect mortality improvements and the change in life expectancy at 65. Finally, for those retiring from 2036 onwards, only the contributive method applies.By combining the information on pension payments to retired individuals of different cohorts with the projections on the population at different ages obtained with the Lee-Carter mortality model, we have derive mean estimate for future pension expenditure as a fraction of real GDP, and its associated upper

[7] We have also experimented with specifications allowing the long-run to be function of the demographic structure of the population. As the results obtained were not stastically strong and unstable over-time, we have preferred the simple AR model.

bound [8]. The difference between the mean and the upper bound can be taken as a measure of the impact of longevity risk on Italian Pension payments. By adding up pensions paid to each cohort, we have generated an estimate of total old-age pension expenditure ranging from € 197 billion in 2012 to € 1.09 trillion in 2050. Figure 5.2 reports total old-age pension expense as percentage of nominal GDP. Our results indicate that at constant (2012) mortality rates over the next 40 years, the ratio will pass from 11.70% in 2012 to 13.70% in 2040. The peak is reached when the cohort of those who are in their 35-50 years today retire. These constant-mortality projections are drastically different from those taking the model-generated pattern of mortality over the next 50 years. According to our mean mortality estimates, old-age pensions over nominal GDP will reach a peak of 15.58% in the middle of the 2040's and will stay well above 15% until the end the period. Neglecting longevity improvements has an impact of 2.7% of GDP in 2050, when the difference between base and constant-mortality projections is the widest. Longevity risk for the Italian Pension System is represented by higher-than-expected longevity resulting into an unexpected increase in total old-age pension expense. Longevity risk can be measured by the difference between the upper bound of the total old-age pension expense and its mean estimate, ranging between 0.06% in 2012 and 4.35% in 2050. Total longevity risk over the period 2040-2050 is as high as 40 per cent of total GDP over the same period.

4.2 The Effects of the Indexation of Life Expectancy and Retirement Age

The results derived so far have been obtained in simulations where retirement age has been kept constant at 65. In this section we investigate the impact of introducing changes in retirement age over time, as established by the 2010-2012 reforms. In particular we present the results of an experiment made by adjusting retirement age to generate a constant expected pension period of 20 years over time. In this simulation retirement age is progressively adjusted to reflect mortality improvements in such a way that the pension period is kept constant at 20 years (retire-

[8] In our computation we have also taken into account the constraint posed by the minum pension and its evolution over time. We obtained the data relative to the minimum monthly pension in Italian £ (from 1994 to 2001) and in Euros (from 2002 to 2011) from INPS' website. From 2012 onwards, we let the minimum pension of 2011 grow at the constant ECB inflation rate of 2% already discussed when forecasting of average salaries, consistently with the assumption that revisions of minimum pensions are made to preserve retirees' purchasing power. Moreover, future pensions have been revaluated using projected inflation

ment age is set to the age in which expected residual life is 20 years[9]). In this scenario, indexation occurs every three years and it affects both the coefficient $\gamma(E(L))$ (as in the baseline model) and retirement age. Figure 6 reports the upper and lower bounds of the pension payments to GDP ratio and the projected expected age of retirement in our simulation. The results shows that the indexation of retirement age has a very sizeable impact on longevity risk, which is reduced from four per cent per year over the period 2040-2050 in the baseline scenario to about one per cent per year over the same period in the alternative after-2010/12 reform scenario. The driver of this reduction in the impact of longevity risk on the Italian Pension System is the increase in expected retirement age that is sizably increased form 65 year in 2012 to 74 years in 2050[10].

5 Conclusions

This paper has shown that pension systems in which the retirement age is not indexed to expected life at retirement have a sizeable exposure to longevity risk. This exposure is sizeably reduced by the indexation of retirement age to expected life at the cost of a sizeable increase of the retirement age over time. The stochastic simulation of a mortality model to project future population over 65 illustrates that longevity risk might generate incremental pension payments as high as 4 per cent of GDP per year in the period 2040-2050, without indexation of retirement age. This exposure is reduced to one per cent of GDP over the same period when indexation of retirement age is introduced. The longevity risk that affects the pension system is not idiosyncratic and therefore not diversifiable. There are two main conclusions that bear important policy implications that we would like to highlight. First, the diversification of longevity risk requires specific instruments, such as longevity bonds or longevity swaps, which are not yet widely available. The introduction of these instrumetns would be particularly important if an indexation of retirement age is not introduced or it is difficult to sustain after its introduction. Our results show that sovereign states, being liable for pension payments, should not increase their exposure towards longevity by issuing longevity bonds. However, exposure to longevity might help the diversification of a private portfolio. If this is the case, than the private sector could take the role of issuer of longevity bonds. In a

[9]In formulae, we have that retirement age in Equation 7 is adjusted by a term h reflecting the difference between expected life at 65 from 2012 to 2050 and expected life at 65 in 2009 (which is roughly equal to 20)

[10]Note that the coefficient $\gamma(E(L))$ is always automatically adjusted to the actual "projected" mortality while the indexation of retirement age depends on the mean expected life in all simulations.

companion paper Bisetti et al. (2012) offer some evidence on this issue by showing that including a longevity-linked security helps diversification of a portfolio of equity and bonds and it has a diversification effect that varies with the investment horizon. Second, our results also hint at the potential importance of the redistributive effect of longevity. If the exposition to the common trend of mortality improvement is different across high- and low-income groups of the population, then the relevance of longevity risk poses serious (and most likely regressive) redistributive issues in pension payments which are not currently sufficiently debated. The same argument makes questionable the equity of the application of a unique indexation of retirement age to the entire population of retirees.

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7 Figures and Tables

Figure 1: Longevity for over-65 in Italy

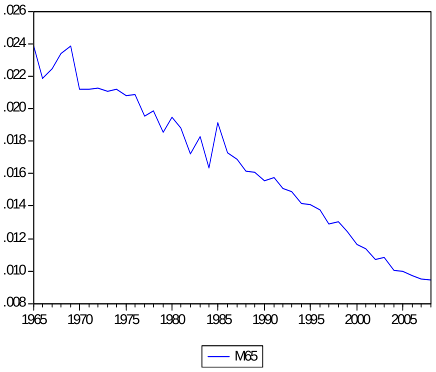


Figure 1.1: Mortality rates

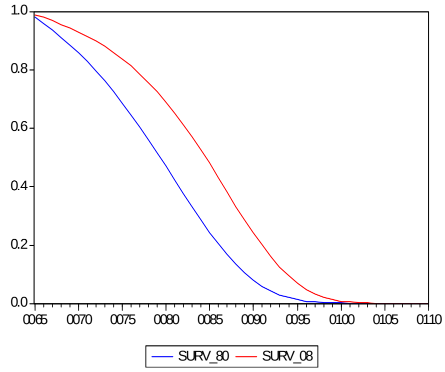


Figure 1.2: Survival probabilities

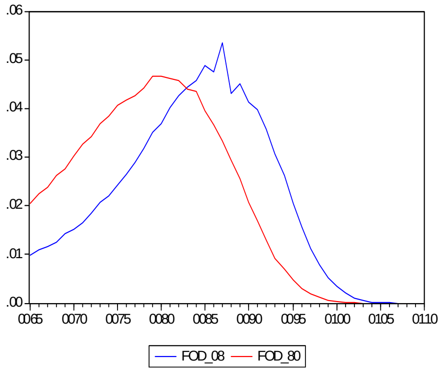


Figure 1.3: Frequencies of death

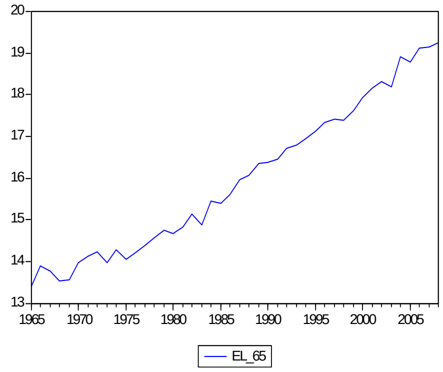


Figure 1.4: Expected life at 65 years

Figure 2: Within-sample performance of the Lee-Carter model for Italy

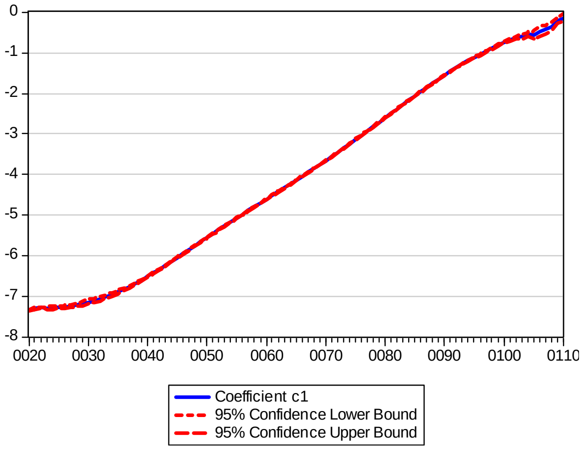


Figure 2.1: Estimates of a_x

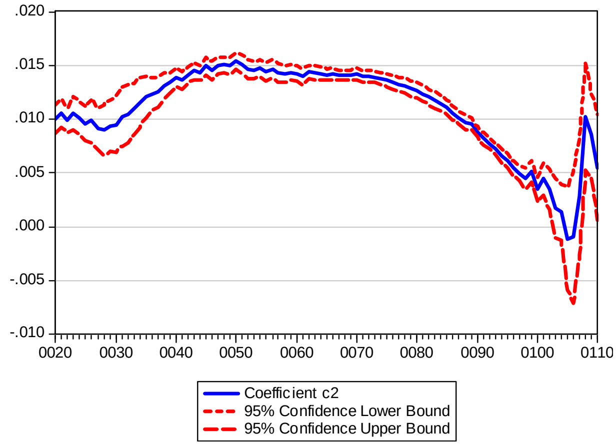


Figure 2.2: Estimates of b_x

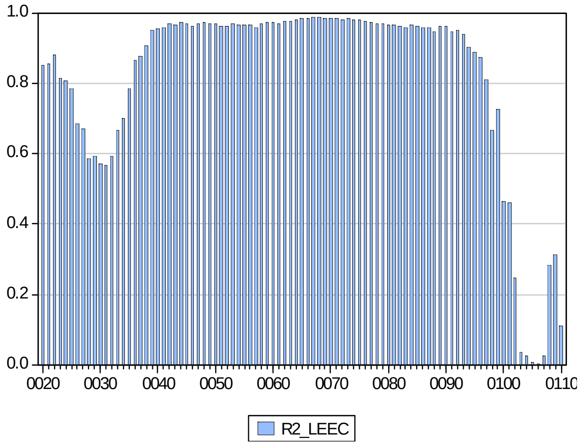


Figure 2.3: Cross-sectional R^2

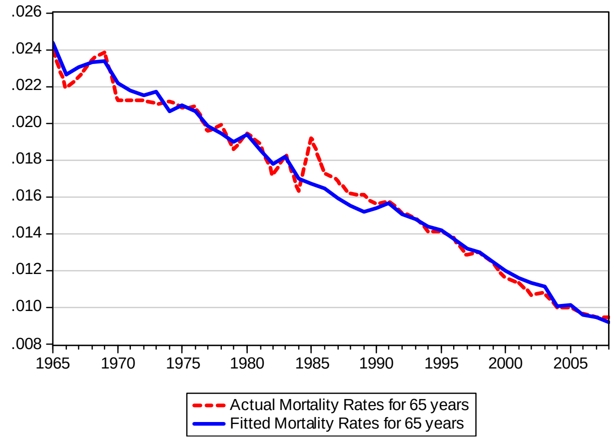


Figure 2.4: Mortality rates

Figure 3: Pseudo out-of-sample (1999-2008) projections

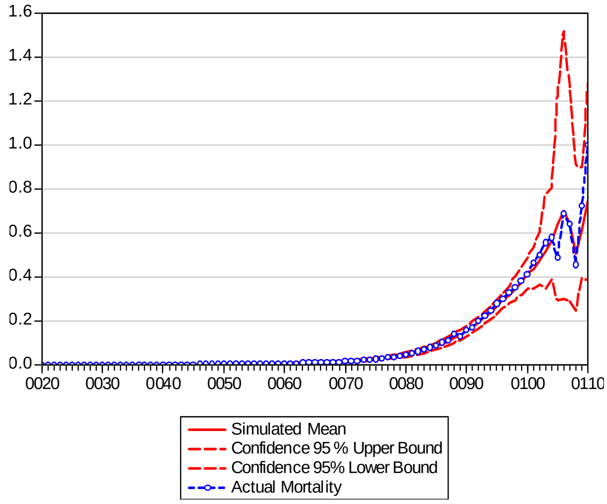


Figure 3.1: Mortality rates in 2008

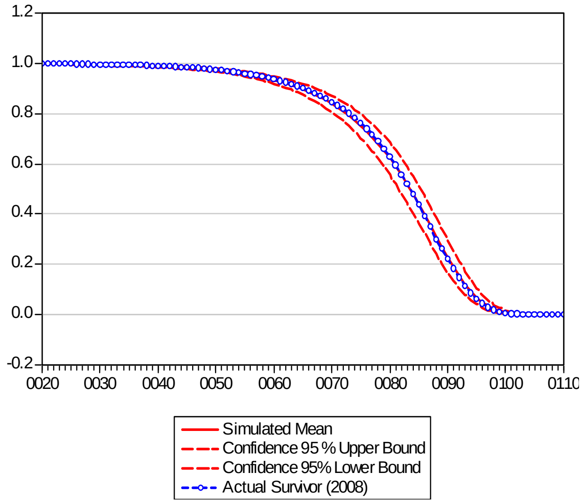


Figure 3.2:Survival probabilities in 2008

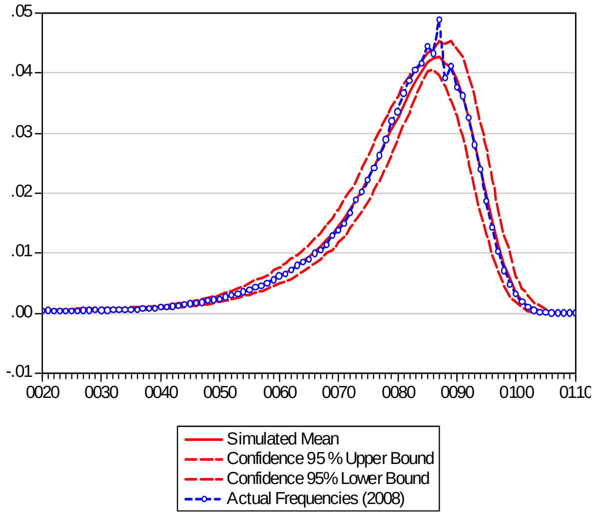


Figure 3.3: Frequencies of death in 2008

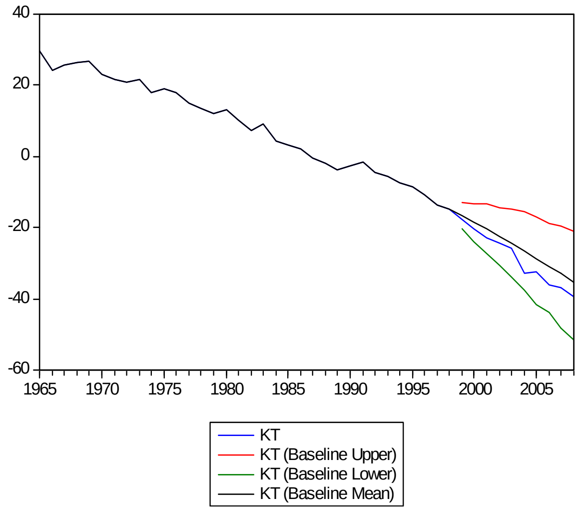


Figure 3.4: Mortality Index k_t

Figure 4: Out-of-sample (2009-2050) projections

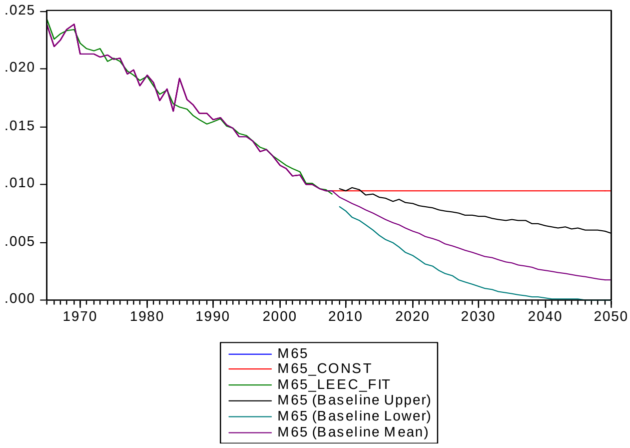


Figure 4.1: Mortality at 65-years

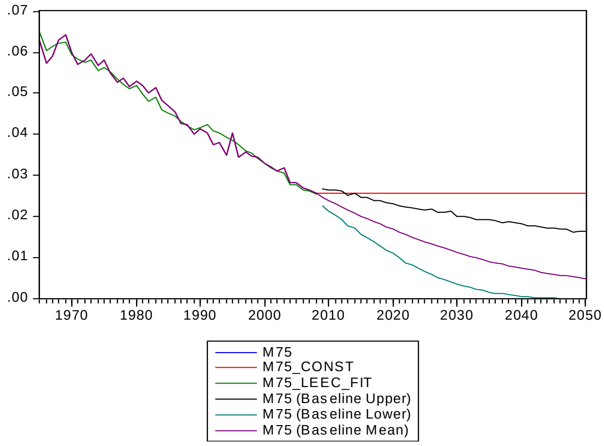


Figure 4.2: Mortality at 75-years

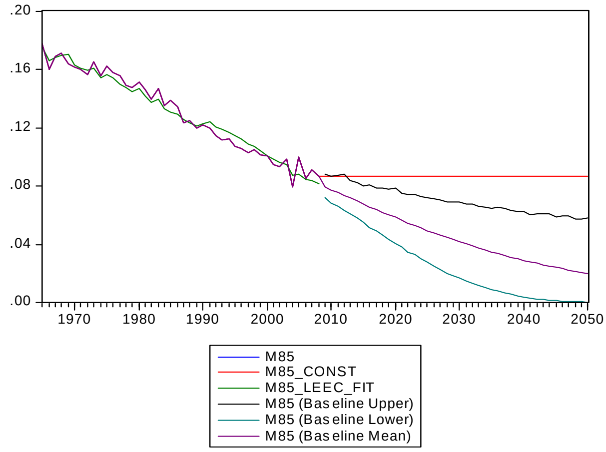


Figure 4.3: Mortality at 85-years

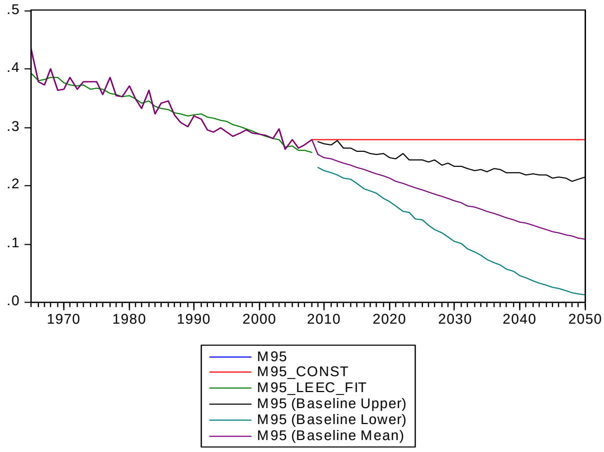


Figure 4.4: Mortality at 95-years

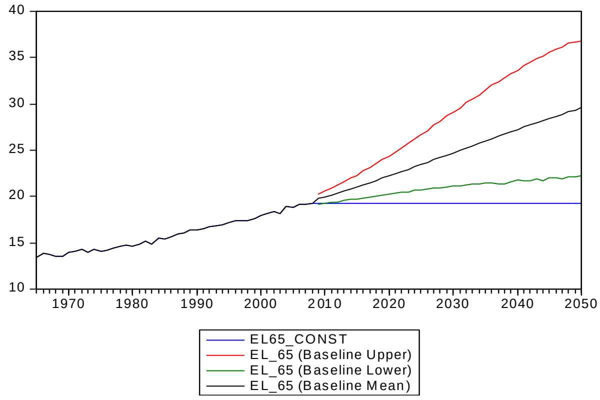


Figure 4.5: Life-Expectancy at 65

Figure 5: Retired population and old-age pension predictions (2012-2050)

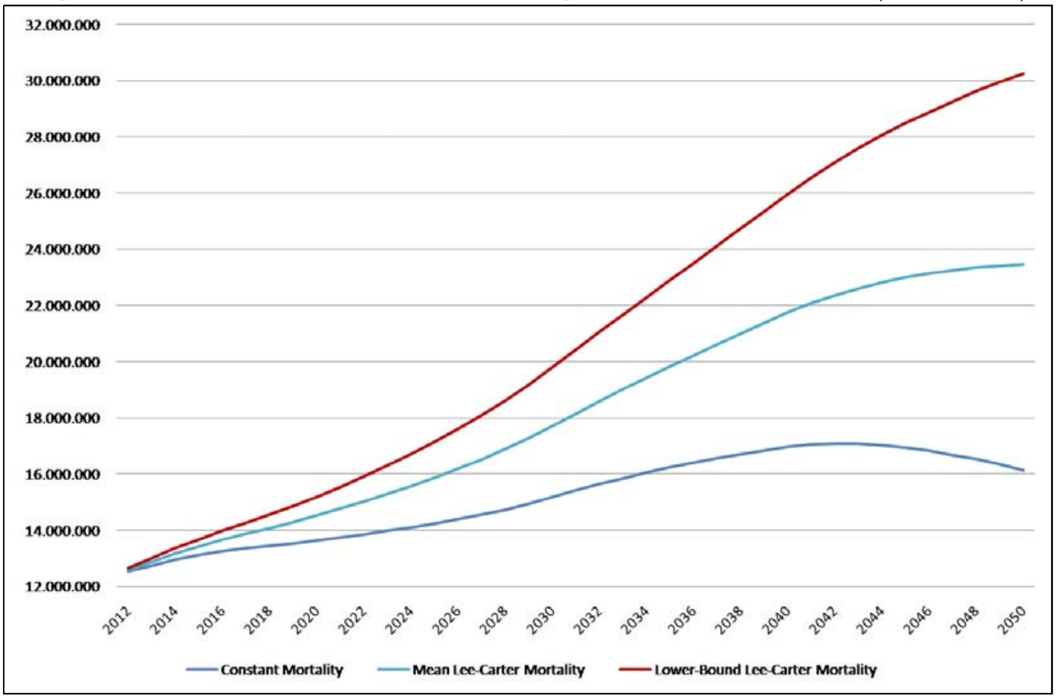


Figure 5.1: Retired population projections

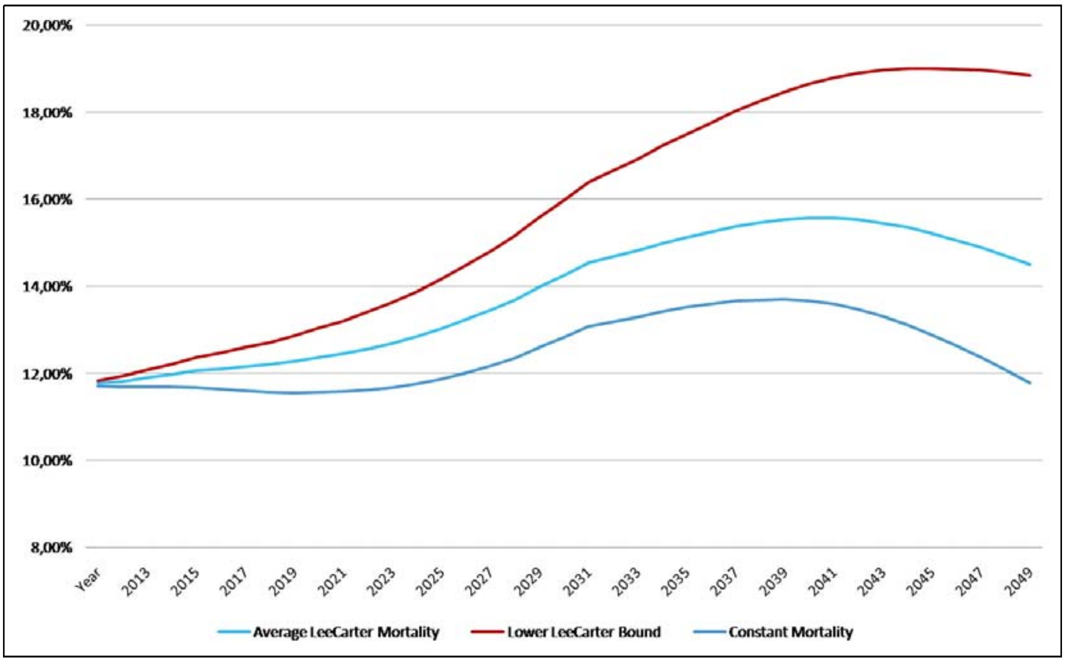
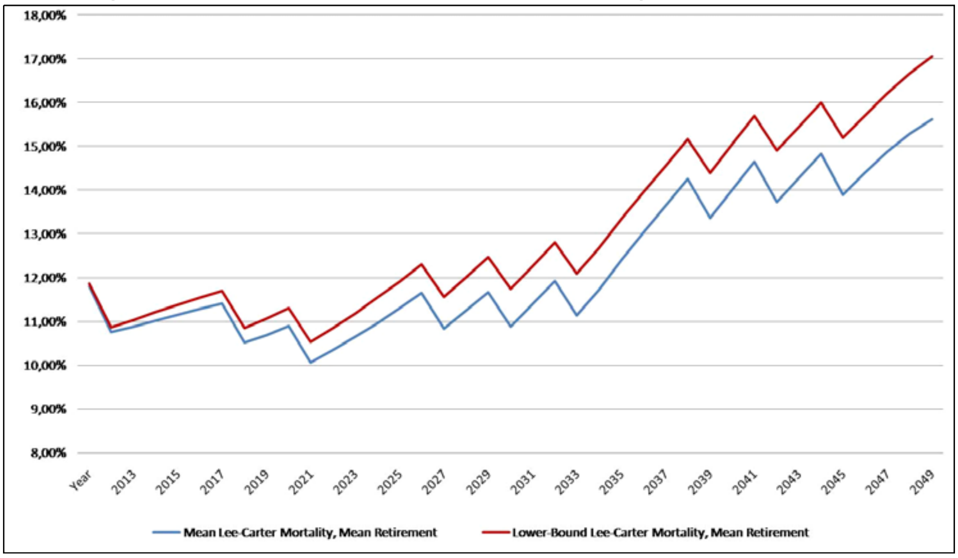
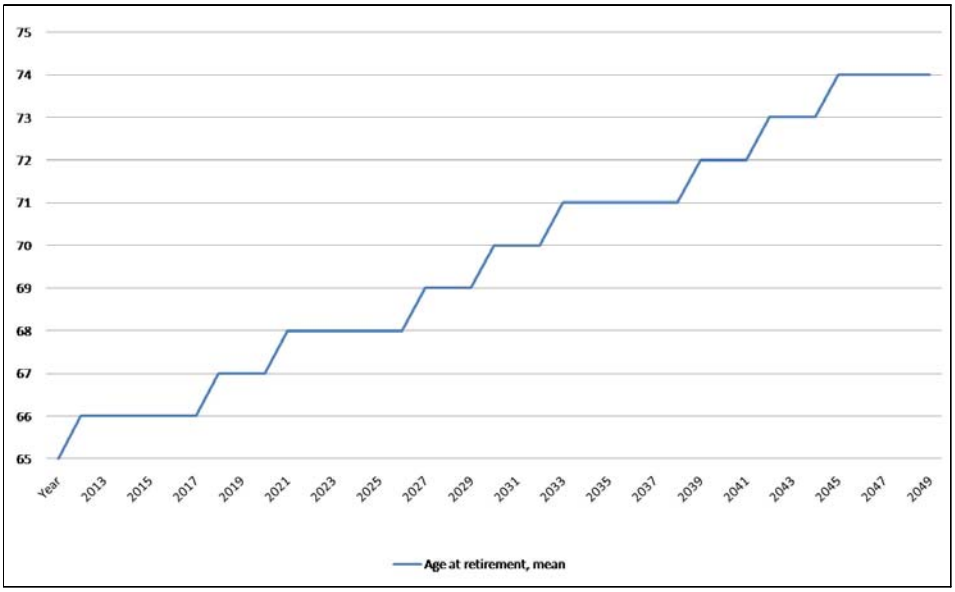


Figure 5.2: Old-age pensions, no adjustment in retirement age

Figure 6: The indexation of retirement age to expected life



Longevity risk when retirement age is indexed to the increase in life expectancy to deliver a constant expected retirement period of 20 years



Age at retirement when indexation applies

Age at Retirement	Contributive Coefficient	Expected Residual Life
57	4.903%	24.5
58	5.049%	23.5
59	5.204%	23
60	5.368%	22
61	5.542%	21
62	5.727%	20.5
63	5.925%	20
64	6.136%	19
65	6.361%	18

Table 1: Contributive Coefficient as Function of Age at Retirement in 2012. Source: www.inps.it

8 Appendix 1:Identification and Estimation of the Lee-Carter Mortality Model

The Lee-Carter (1992) consists of a system of equations for logarithms of mortality rates for age cohort x at time t, $\ln[m_{x,t}]$, and a time-series equation for an unobservable time-varying mortality index k_t:

$$\ln[m_{x,t}] = a_x + b_x k_t + \epsilon_{x,t} \tag{9}$$

$$k_t = c_0 + c_1 k_{t-1} + e_t$$

$$\epsilon_{x,t} \sim NID\left(0, \sigma_\epsilon^2\right) \tag{10}$$

$$e_t \sim NID\left(0, \sigma_e^2\right) \tag{11}$$

where a_x and b_x are age-specific constants. The error term $\epsilon_{x,t}$ captures cross-sectional errors in the model based prediction for mortality of different cohorts, while the error term e_t captures random fluctuations in the time series of the common factor k_t driving mortality at all ages.. This common factor, usually known as the unobservable mortality index evolves over time as an autoregressive process and the favourite Carter-Lee specification makes is a unit-root process by setting $c_1 = 1$. Identification is achieved by imposing the restrictions $\sum_t k_t = 0$ and $\sum_x b_x = 1$, so that the unobserved mortality index k_t is estimated through Singular Value Decomposition. SVD is a technique based on a theorem of linear algebra stating that a $(m \times n)$ rectangular matrix M can be broken down into the product of three matrices - an $(m \times m)$ orthogonal matrix U, a diagonal $(m \times n)$ matrix S, and the transpose of an orthogonal $(n \times n)$ matrix V. The SVD of the matrix M will be therefore be given by $M = USV'$ where $U'U = I$ and $V'V = I$. The columns of U are orthonormal eigenvectors of AA' , the columns of V are orthonormal eigenvectors of $A'A$, and S is a diagonal matrix whose elements are the square roots of eigenvalues from U or V in descending order. The restriction $\sum_t k_t = 0$ implies that a_x is the average across time of $\ln[m_{x,t}]$, and Equation 9 can be rewritten in terms of the mean-centered log-mortality rate as

$$\ln[m_{x,t}] - \overline{\ln[m_{x,t}]} \equiv \tilde{m}_{x,t} = b_x k_t + \epsilon_{x,t}$$

Grouping all the $\tilde{m}_{x,t}$ in an unique $(X \times T)$ matrix $\tilde{m}$ (where the columns are mortality rates at time-t ordered by age groups and the rows are mortality rates through time for a specific age-group x), leads naturally to use SVD to obtain estimates of b_x and k_t. In particular, if $\tilde{m}$ can be decomposed as $\tilde{m} = USV'$, $b = [b_0, b_1, \ldots, b_X]$ is represented by the normalized first column of U, $u_1 = [u_{0,1}, u_{1,1}, \ldots, u_{X,1}]$, such that

$$b = \frac{u_1}{\sum_{x=0}^{X} u_{x,1}}$$

On the other hand the mortality index vector $k = [k_1, k_2, \ldots, k_T]$ is given by

$$k = \lambda_1 (\sum_{x=0}^{X} u_{x,1}) \nu_1$$

where $\nu_1 = [\nu_{1,1}, \nu_{1,2}, \ldots, \nu_{1,T}]'$ is the first column of the V matrix and λ_1 is the highest eigenvalue of the matrix S (see Girosi and King (2007) and Giacometti et al.(2010)). The values of mortality rates obtained with this method will not, in general, be equal to the actual number of deaths. The authors hence re-estimated k_t in a second step, taking the values of a_x and b_x as given from the first-step SVD estimate and using actual mortality rates. The new values of k were obtained such that, for each year, the actual death rates would have been equal to the implied ones. This two-step procedure allows to take into account the population age distribution, providing a very good fit for 13 of the 19 age groups in the authors' sample, where more than 95% of the variance over time was explained. For seven of these, more than 98% of the variance was explained.

9 Appendix 2: The Data on Italian Labour Income

- Data in Italian £
  - 1965-1969: data are available for monthly income in the various sectors and for the average monthly income. We consid-

ered 13 months of income per year to get the yearly average income.

- 1970-1971: data available for average monthly income, at which the various employment sectors are indexed (e.g. If the average monthly income is 130 and the index for managers is 120, then the monthly income for managers is (130*100)/120). Figures are then translated into yearly values by multiplying by 13 working months.
- 1973-1974: data available on total yearly income for each employment sector, on labour and mixed income (work into the unincorporated sector, valid for the computation of the pension) as percentage of total income and on employment sectors as percentages of total workforce. Overall labour income is then an average of the labour income of the various employment sectors, weighted by the relative amount of that sector on the total workforce.
- 1975-1983;1986: data available for average yearly income, at which the various employment sectors are indexed (e.g. If the average yearly income is 130 and the index for managers is 120, then the monthly income for managers is (130*100)/120).
- 1987;1989: figures on yearly labour income directly available from reports.
- 1991;1993;1995;1998: data are separately available for the total income from subordinated work, the total income from self-employment and the percentages of these two sources of income relative to total income. Average labour income is calculated accordingly as a weighted average between income from self-employment and income from subordinated labour.
- 1972; 1984-1985;1988;1990;1992;1994;1996;1999: no surveys available for these years.

• Data in Euros

- 2000;2002;2004;2006;2008: data are separately available for the total income from subordinated work, the total income from self-employment and the percentages of these two sources of income relative to total income. Average labour income is calculated accordingly as a weighted average between income from self-employment and income from subordinated labour.
- 2001;2003;2005;2007: no surveys available for these years.

Once obtained our nominal estimates of average labour income during the years between 1965 and 2008 for which surveys were available, first of all we converted all the data into the same currency by using the fixed exchange rate of 1936,27 ITL/EUR. We then interpolated nominal labour income for the years for which no surveys were conducted. Finally, using the CPI Index[11], we deflated average nominal labour income with which we subsequently produced our forecasts.

[11] Obtained from the WorldBank's website, http://databank.worldbank.org/ddp/home.do