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Loans from the Government, Overinvestment by Households, and Asset Bubbles

Viktar Fedaseyeu[1] and Vitaliy Strohush[2]

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Abstract

We investigate the role of government-provided loans on market outcomes. First, we show that government-provided financing can lead to asset bubbles when enough households have adaptive expectations and determine the minimum share of households with adaptive expectation that is sufficient for bubbles to arise. Second, we show that in addition to causing bubbles government-provided loans can generate a propagation mechanism behind them. Third, we show that bubbles can be avoided if financing is provided over a sufficiently large number of periods rather than all at once, even when households have adaptive expectations.

Key words:
Asset bubbles, government-provided loans

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1. Introduction

Government-provided financing plays a significant role in some markets.[3] In the student loans market, whose size was $904 billion in March of 2012, federal student loans constitute more than 70% of all outstanding student debt.[4] The government's role in the mortgage market is also significant, with close to 50% of all loans outstanding in 2011 being owned or guaranteed by government agencies and government-sponsored enterprises.[5] In this paper, we investigate whether the government's role as provider of financing can generate inefficient allocations that lead to asset bubbles. We also investigate how such bubbles can be prevented.

Our analysis is motivated by the 2008 financial crisis and the simple observation that any financial transaction is a deal between at least two parties. Therefore, a financial crisis cannot arise if at least one of the parties involved in the underlying financial transactions refuses to participate. When households acquire goods on

[3]Examples include housing and education markets as well as loans to "green energy" and auto companies.

[4]This number is an approximation because the Department of Education does not publish data on the amount of loans outstanding. We arrived at this estimate in the following way. Based on the President's Budget Report for 2012 (available at `http://www.whitehouse.gov/sites/default/files/omb/budget/fy2012/assets/edu.html`), the amount of federal student aid outstanding was $610 billion in 2010. According to the Quarterly Report on Household Debt and Credit from the Federal Reserve Bank of New York (available at `http://www.newyorkfed.org/research/national_economy/householdcredit/DistrictReport_Q32011.pdf`), the total amount of student debt outstanding was $845 billion in the second quarter of 2011 (the earliest for which adjusted data are available). $610 billion is 72% of $845 billion. This estimate is roughly in line with what student debt analysts report. See, for example, `http://www.fastweb.com/financial-aid/articles/2589-total-college-debt-now-exceeds-total-credit-card-debt`, where the proportion of federal loans is estimated at 80% of the market ($665 billion of $830 billion).

[5]Source: Flow of Funds Accounts of the United States, June 7, 2012, available at `http://www.federalreserve.gov/releases/z1`. This market share has not been stable. It reached a high of 57% in 2003 and fell as low as 33% in 2007.

credit, two things must happen. First, somebody must be willing to provide loans, and second, households must believe that the benefits they will receive will outweigh the costs they have to incur.

Our analysis is based on two main premises. First, we assume that not all households have rational expectations.[6] Second, we assume that the government lacks information that would enable it to determine the efficient amount of financing. It is the combination of the two that will create conditions necessary for a bubble to arise. If the government is not restricting access to financing for the households who are not rational, their irrationality cannot be eradicated by the market and they will be unable to learn until it is too late. Besides the straightforward conclusion that deviation from rational expectations leads to asset bubbles, our model enables us to explore three novel dimensions. First, we show that government-provided financing, in addition to causing the bubble, also generates an endogenous propagation mechanism behind it. Second, we show that timing of financing can prevent asset bubbles, even if households have adaptive expectations and the government is unable to determine the efficient amount of financing. And third, we determine the proportion of households with adaptive expectations that is sufficient for bubbles to arise, even when the remaining households have rational expectations.

[6]The assumption that not all households are rational is supported by large experimental evidence, which shows that people do not behave rationally in relevant economic situations. The literature on behavioral biases and bounded rationality is too vast to provide a comprehensive overview here. See, for example, Tversky and Kahneman (1981), Thaler, Tversky, Kahneman, and Schwartz (1997). See also Barberis and Thaler (2003) for a survey of behavioral finance. Some recent research indicates that human behaviors that deviate from the notion of rationality can have physiological origins and may be innate rather than learned (De Martino, Kumaran, Seymour, and Dolan (2006), Chen, Lakshminarayanan, and Santos (2006)).

We consider partial equilibrium in an economy with households and the government. There exists an investment good. Households decide whether to acquire the investment good or not. If acquired, the investment good generates an income stream that depends on the total number of households who acquired the investment good. We assume that the income stream generated by the investment good exhibits diminishing marginal returns: the larger the number of households who acquired the investment good, the lower the income stream.[7] If households want to acquire the investment good, they have to obtain financing from the government because their endowment is insufficient to cover the associated costs. Those costs are different across households. We say that households have different ability levels in acquiring the investment good and that households with higher abilities have lower costs.[8] The government determines the total amount of financing it will provide. We assume that the government provides loans to households in the most efficient way possible, in the

[7]Consider mortgages. When a household acquires a home, it benefits from house price appreciation and also from the flow of services that the house provides. To see clearly that housing purchases exhibit diminishing marginal returns consider two identical households that buy two identical homes at different times, and assume that the hosing market is in expansion (if house prices are expected to fall, nobody will buy them). Even though both households will gain from house price appreciation, the household that bought its house first will gain more (since it bought it at a lower price and the flow of housing services is the same for both households). Hence, households that make their purchases later receive lower returns.

Education is another example. When too many people go to college, the wage premium associated with the college degree falls, and the salaries of relatively less skilled workers increase due to their scarcity. As an example, according to the May 2010 wage estimates by the U.S. Bureau of Labor Statistics, electricians (an occupation that does not require advanced training) made on average $51,810 per year, which is roughly the same or more than what people earned in some occupations that require a bachelor's or even a master's degree (such as tax examiners and collectors, or substance abuse councelors). In addition, electricians do not have the debt burden they would have accumulated had they gone to college.

[8]The ability we have in mind here is intellectual ability in the case of education and creditworthiness in the case of mortgages.

sense that households who benefit most from government-provided financing receive such financing first. Inefficiencies arise when the government cannot determine the efficient level of total financing. If too much financing is provided (in the sense that it is enough to generate a bubble, as will become apparent below), we say that there is excess financing. For simplicity, there are no financial markets in our model.[9]

In the above setting, we derive the following results:

1. households that have adaptive expectations overinvest and this behavior gives rise to asset bubbles when the government provides excess financing;

2. in addition to causing the bubble, excess financing generates an endogenous propagation mechanism behind it;

3. even when households have adaptive expectations and the government is willing to provide excess financing, the degree of overinvestment can be reduced if financing is provided over a sufficiently large number of periods, rather than all at once;

4. bubbles can arise even when only a certain proportion of households have adaptive expectations.

The first result follows in a straightforward manner. When households have adaptive expectations, they decide whether they wish to acquire the investment good by observing the income it generates at the beginning of the period, when they haven't acquired it yet. Hence, such households will overestimate the benefits of

[9]Introducing short-sellers will not affect our results. See section 5 for a discussion.

acquiring the investment good since it exhibits diminishing marginal returns (and will therefore generate a lower income once they acquire it). As a result, more households will be willing to acquire the investment good than is efficient. If the government had perfect information, it would provide financing to households only as long as acquiring the investment good is beneficial to those households. If, on the other hand, the government is providing too much financing, then too many households acquire the investment good. When the amount of financing is large enough, a bubble emerges, and some households who obtained loans in order to acquire the investment good will default on those loans with certainty.

Further, excess financing creates a propagation mechanism behind the bubble. As the government provides financing to more and more households, the income stream from the investment good keeps decreasing for all households, including those that acquired it previously. At this point, some households who previously acquired the investment good also default since their income stream is no longer sufficient to cover their loan payments. Hence, the ensuing defaults affect a larger number of households than just the households who received excess financing.[10]

However, even when households have adaptive expectations and the government is willing to provide too much financing, the damage can be minimized if that financing is spread over a number of periods. This happens because with adaptive expectations households will adjust their estimates of the benefits provided by the investment good as more households gradually acquire the investment good. Hence, because the

[10]The subprime mortgage crisis illustrates this point very clearly. As households who obtained subprime loans started to default (because ultimately their income could not cover their mortgage payments), house prices began to fall. This, in turn, affected prime homeowners.

investment good exhibits diminishing marginal returns, with passage of time fewer and fewer households will be willing to obtain the investment good. This insight may explain why the system of mortgage financing that had worked fine for decades broke down so suddenly. It was not just the sheer volume of financing in the U.S. housing market before the crisis that was extraordinary but also the speed with which it was provided.[11]

Even though it is easy to see that bubbles are inevitable when households have adaptive expectations, our fourth result demonstrates that not all households need to have adaptive expectations in order for a bubble to arise. Whether high ability households have rational or adaptive expectations is irrelevant since they would be acquiring the investment good anyway. It suffices that households below the minimum level of ability at which acquiring the investment good is efficient have adaptive expectations. In that case, adaptive expectations influence decisions of precisely the households who should not be acquiring the investment good. Casual observation suggests that this may indeed be the case in the real world: lower ability households are more likely to have cognitive difficulties in rationally assessing their options.

Our paper has clear policy implications. First, we show that in general government-provided loans are not innocuous unless households have perfect information: when government provides too much financing too fast asset bubbles ensue. Second, our model suggests that there are two ways to address the cause of government-induced

[11]Consider the following statistics. Home ownership rate in the United States between 1960 and 1994 increased from 62.1% to 64.0% (with a brief peak of 65.6% in the early 1980s). In comparison, between 1994 and 2005, the homeownership rate increased from 64.0% to 68.9% (in percentage points, more than 2.5 times the increase in the previous three decades). Source: `http://www.census.gov/hhes/www/housing/hvs/annual09/ann09t14.xls`

asset bubbles: self-restraint by the government or self-restraint by households (or both). In theory, bubbles can be avoided and social policies implemented as long as the government is able to perfectly allocate financing by correctly estimating the optimal number of households who will benefit from those policies. However, we think that such a scenario is unlikely. A more feasible way for the government to promote social policies without generating inefficiencies and asset bubbles is to require that people who take advantage of those policies are made aware of potential risks associated with them. Consumer education and full and clear disclosure by all market participants is one way to prevent financial collapse stemming from government overspending.

Third, we show that it is not just the amount of financing the government is willing to provide that matters but also the speed with which it does so. Even when the government is willing to provide too much financing, spreading that financing over a sufficient number of periods will enable households to adjust their expectations. This suggests that government-provided financing and fiscal policy stimuli in general may be most efficient if they are released gradually.

We focus on the government as the source of excess financing. Of course, this role can in principle be performed by other agents. However, we find it plausible that the government (which spends taxpayer money) is more likely to provide excess financing than private investors (who pledge their own money).[12]

[12]In the recent crisis, foreign governments and quasi-government entities may have contributed to the extraordinary amount of financing in the U.S. housing markets. Those investors were often driven by non-market motives. On the other hand, private bondholders in U.S. financial institutions enjoyed the implicit guarantees of the U.S. government, which were eventually honored. Hence, they as well may have been prompted to invest because it was ultimately the government that stood

The rest of the paper is organized as follows. In section 2 we relate our model to the existing literature. Section 3 describes the model and its implications. In section 4 we provide a numerical example. Section 5 discusses our assumptions and the policy implications of our results. Section 6 concludes. All proofs are confined to the appendix.

2. Relation to the existing literature

This paper supports the view that government-provided financing and expansionary fiscal policy more generally can lead to financial crises. As Charles Calomiris noted in an April 2012 interview, had the government's aggressive encouragement of high-risk mortgage lending not occurred, that would have been enough to prevent the 2008 financial crisis from happening (Epstein (2012)).[13] This view was also expressed by Peter Wallison in his dissenting statement in the Financial Crisis Inquiry Report and also by Raghuram Rajan in his recent book, "Fault Lines: How Hidden Fractures Still Threaten the World Economy". We also emphasize that household behavior is critical to generating a bubble, and hence U.S. households cannot be entirely absolved of the blame for the recent financial collapse.

Our paper is also related to the literature on banking crises and the literature that emerged in the aftermath of the 2008 financial crisis (Allen and Gale (2000), Allen, Babus, and Carletti (2009), Gennaioli, Shleifer, and Vishny (2010)). However,

behind. See Rajan (2010) for a discussion.

[13]Charles Calomiris also added in the same interview that it was the combination of government subsidies and lack of prudential regulation that created the crisis. In this paper, we show that government subsidies alone can in principle lead to financial collapse.

the focus of those literatures is on the role of financial intermediaries in generating and propagating asset bubbles. We, on the other hand, show the government can generate and propagate asset bubbles by acting on its own. We do not suggest that financial intermediaries played no role in the recent financial collapse. However, their role in that collapse may have to be reassessed. The degree to which it was the government that created the bubble and not financial intermediaries is an empirical issue, which needs to be investigated in further research.

Our paper is also related to the large literature on the effects of fiscal policy. A significant part of that literature focuses on the crowding-out effect of fiscal policy and its influence on market outcomes. Buiter (1977) classified different types of crowding out of private economic activity with public spending. Blinder and Solow (1973), Feldstein (1982), Kormendi (1983), Aschauer (1985; 1989), and, more recently, Farmer (2010) and Farmer and Plotnikov (2012) focus on whether there is a crowding out between public expenditures and private expenditures and how effective is fiscal policy in restoring full employment. Another strand of that literature, more closely related to our paper, focuses on the efficiency of government subsidies in various markets. For example, Spencer and Brander (1983) develop a theoretical framework for government intervention in the form of R&D subsidies to help domestic companies acquire higher market shares internationally. Johnson (1984), Fernandez and Rogerson (1995), Blankenau (1999), Hanushek, Leung, and Yilmaz (2003), Caucutt and Kumar (2003), Akyol and Athreya (2005) study the effects of government spending on public education. For example, Hanushek, Leung, and Yilmaz (2003) find that subsidies raise output and create a more equal income distribution. Fi-

nally, Benabou (2002), Seshadri and Yuki (2004) study the effects of redistribution policies in a dynamic context. In our paper, we consider another dimension of how government policy can affect market outcomes. We show that government-provided loans can create misallocations and cause asset bubbles.

In addition, this paper is broadly related to the general literature on asset bubbles (Tirole (1985), Santos and Woodford (1997), Farhi and Tirole (2011)). However, both our purpose and the approach we use are significantly different from that literature. Our objective is to show how asset bubbles can arise in a non-rational expectations setting and what policy response can prevent them. We describe a simple static economy, and the above literature generally concludes that rational asset bubbles are impossible in a static economy (Kreps (1977), Tirole (1982)). Tirole (1982), in particular, provides a set of assumptions that need to be relaxed in order to generate a bubble in a static economy. We relax the assumption that all agents have rational expectations and investigate the precise mechanism by which it leads to asset bubbles. We investigate the minimum amount of irrationality that is needed to generate asset bubbles. Another key difference of our setup from the previous literature is the nature of the asset we are considering here. Our paper is concerned with the assets that exhibit diminishing marginal returns with respect to the number of households who acquire those assets. Education and housing are examples of such assets.[14]

[14]See footnote 7 for an explanation.

3. The model

We consider partial equilibrium in an economy with mass 1 of households who live for two dates (we will add multiple dates in a later section): households are born at date 1 and live until date 2. There is an investment good (such as housing or education) in unlimited supply, which households can acquire at date 1. Each household can acquire only one unit of the investment good.

Households differ in their ability to acquire the investment good. Let θ denote this ability (θ can be viewed as intellectual ability in the case of education or credit worthiness in the case of housing). We assume that θ is uniformly distributed across households, from 0 to μ. In order to acquire the investment good, households need to pay a cost, determined by the non-negative function $cost(\theta)$, with $cost'(\theta) < 0$ (the higher the intellectual ability or creditworthiness, the lower the cost). We assume that households are born with zero endowment and have to borrow in order to acquire the investment good. If a household with ability level θ_0 decides to acquire the investment good, it has to borrow the full amount of $cost(\theta_0)$. The loan will then have to be fully repaid at the date 2. We set the interest rate to zero.

If acquired, the investment good generates a payment, and the amount of that payment depends on how many other households also acquired the investment good. Formally, the investment good generates a payment according to the non-negative continuously differentiable function $s(y)$, with $s'(y) < 0$, where y is the total mass of households who acquired the investment good.[15] Households that do not acquire

[15]For example, when households with abilities from θ_0 to θ_1 acquired the investment good, $y = \frac{\theta_1 - \theta_0}{\mu}$.

the investment good receive a payment determined by the non-negative continuously differentiable function $u(x)$, with $u'(x) < 0$, where x is the total mass of households who do not acquire the investment good. Since households face a binary choice whether to acquire the investment good or not, $y + x = 1$. For simplicity, we assume that $s(y)$ and $u(x)$ depend only on the total mass of households that acquired the investment good and that didn't acquire it, respectively. It means that the payment received from the investment good is the same for all household who acquired it, regardless of their ability, and all differences between them are reflected in their cost of acquiring the investment good.

We can think of education as one example of the investment good. Households acquire education to increase their human capital, which will supposedly generate benefits that outweigh the costs of acquiring education. In this case, the income generated by the investment good can be viewed as the salary that skilled workers receive. Housing is another example. There, the utility from owning a house and the appreciation of housing assets over time represent the income stream that the investment good generates. Hence, functions $s(y)$ and $u(x)$ can be viewed, for example, as wage rates of skilled vis-a-vis unskilled labor or utility from owning a home relative to renting.

Households maximize their final consumption by choosing whether to acquire the investment good or not. At the end of their lives (at date 2), households consume their entire net income in excess of any loan payment they have to make. We abstract from households' labor-leisure choice and from their savings decision since these are not central to the problem we are studying. Let $i(\theta)$ denote households' choice: if

household with ability θ_0 chooses to acquire the investment good we set $i(\theta_0) = 1$, and we set $i(\theta_0) = 0$ otherwise.

Households solve the following problem:

$$\max_{i \in \{0,1\}} \bar{u}_\theta(1 - i) + \bar{s}_\theta i - cost(\theta)i, \tag{1}$$

where $\bar{u}_\theta$ denotes the payment that a household with ability level θ expects to receive at date 2 if it does not acquire the investment good and $\bar{s}_\theta$ denotes the payment this household expects to receive if it acquires the investment good.

When households have rational expectations, they correctly estimate $\bar{u}_\theta$ and $\bar{s}_\theta$ by taking into account the government's behavior (described below). When households have adaptive expectations, they observe the payment that the investment good generates at date 1 and assume that the same payment will be generated at date 2.[16]

We impose the following restrictions on $s(y)$, $u(x)$, and $cost(\theta)$:

$$s(1) - cost(0) < 0, \tag{2}$$

$$s(0) - cost(0) > u(1). \tag{3}$$

Restriction (2) ensures that when all households acquire the investment good some of them default. $s(1)$ is the income that the investment good generates when all households choose to acquire it, while $cost(0)$ is the cost of acquiring the investment

[16]Since nobody possesses the investment good at date 1, this means that households with adaptive expectations set $\bar{s}_\theta = s(0)$ and $\bar{u}_\theta = u(1)$.

good for the lowest ability/creditworthiness (highest cost) household. This restriction precludes a situation when even the household with the highest cost of acquiring the investment good does not default on its debt after acquiring the investment good. Since $u(x)$ is a non-negative function, restriction (2) also implies that the highest possible payoff from not acquiring the investment good must be greater than the lowest possible payoff from acquiring the investment good. Otherwise, the acquisition of the investment good is so beneficial that it is always optimal to make all households acquire it. Hence, this restriction is necessary to make inefficient acquisitions of the investment good possible in principle.[17]

Restriction (3), on the other hand, ensures that some households with adaptive expectations find it beneficial to acquire the investment good. $s(0)$ is the income that the investment good generates when no household chooses to acquire it, $cost(0)$ is the cost of acquiring the investment good for the lowest ability/creditworthiness (highest cost) household, and $u(1)$ is the income households receive when all households choose not to acquire the investment good.[18]

The second agent of our economy is the government, which at date 1 can pro-

[17]It is likely that there are some social policies that do not satisfy restriction (2). One example could be the provision of cheap cell phones to farmers in Africa, so that they could communicate with potential buyers and better react to market conditions. In those cases, no matter how large the acquisition of the investment good is, it cannot be suboptimal. We think, however, that many of the markets the government is involved in, such as education and the mortgage market, are likely to be characterized by restriction (2), in the sense that there can exist suboptimal outcomes when too much of the investment good is acquired by households.

[18]This restriction is stronger than what we need, and we impose it because it simplifies exposition. All of our results go through as long as a weaker restriction is satisfied, namely, $s(0) - cost(\theta^e) > u(1)$, where $\theta^e < \theta^d$, and θ^d is defined as the value of θ that satisfies the following equality: $s(y(\theta)) - cost(\theta) = 0$, where $y(\theta)$ denotes that all households with abilities from θ to μ acquired the investment good. See footnote 26 in the appendix.

vide financing to households in order for them to acquire the investment good. The government is benevolent: if it has enough information, it provides financing only to households that will benefit from acquiring the investment good. There are no private financial markets and consumers have to borrow from the government if they wish to acquire the investment good (because they have zero endowment).[19] The government, should it choose to intervene, determines the total amount of financing it is going to provide, which we denote by L. If the government decides to provide financing to some households, it loans them precisely the amount needed to acquire the investment good. For example, if the government provides financing to households with abilities between θ_0 and θ_1, then $L = \int_{\theta_0}^{\theta_1} cost(\theta)d\theta$.

We assume that the government provides financing in a sequential manner, starting with the highest ability (lowest cost) households first. In particular, if two households, with abilities θ_1 and θ_2 such that $\theta_1 > \theta_2$, want to acquire the investment good, the government will provide financing to the household with ability θ_1 first and will finance the other household only if the remaining funds can cover that household's acquisition of the investment good. Formally, the government adopts the following algorithm when it provides financing to households.[20]

Algorithm 1. *The government determines the total amount of financing, L, and*

[19]The assumption that there are no private financial markets is for simplicity only. Relaxing it will not change the qualitative nature of our results because if the government steps in after some involvement of private financial markets, it will probably have to subsidize relatively less solvent households, making a bubble more likely. In essence, the government plays the role of financial intermediaries by providing households with loans. The only difference is that the government has no profit motive and provides loans until it allocates all the funds it is willing to provide. The total amount of financing is a policy choice, determined exogenously.

[20]Notice that any allocation of financing different from Algorithm 1 will make bubbles more likely.

provides it to households in a sequential manner, starting from households with the highest ability. It loans funds equal to the cost of acquiring the investment good to households with lower and lower abilities until the total amount L is reached. Hence, $L = \int_{\theta^g}^{\mu} cost(\theta)d\theta$*, where* θ^g *represents the lowest ability level of households who obtain financing from the government.*

A household acquires the investment good if and only if both of the following conditions are satisfied. First, this household wishes to acquire the investment good. Second, the amount of financing provided by the government is enough to cover this household's costs of acquiring the investment good as well as the costs of all households with abilities higher than this household's ability. Because of Algorithm 1, households must acquire the investment good continuously, starting with households of ability μ and until some threshold level of ability θ_0 is reached.

The problem we have in mind can be described as follows. There exists a set of households, and a decision must be made whether some of them need to acquire the investment good or not. We proceed in a sequential manner, starting with the lowest cost (highest ability) households first. A decision is made whether those households are better off by acquiring the investment good or not. If they acquire the investment good, then households with the second lowest level of cost are considered, and so on.

3.1. Preliminaries

In this section we develop some preliminaries in order to be able to characterize how efficient is the outcome of households' investment decisions. We will also define what we mean by an asset bubble here. We start by showing that, under the re-

strictions that we imposed above, there exists a maximum mass of households who should acquire the investment good.

Lemma 1. *There exists θ^*, such that $\mu > \theta^* > 0$, and:*

(*i*) *If households with abilities from θ^* to μ acquire the investment good, then all households are better off than if no household acquired the investment good.*

(*ii*) *If households with abilities from θ' to μ acquire the investment good, where $\theta' < \theta^*$, all households are worse off than if only households with abilities from θ^* to μ acquired the investment good.*

The intuition behind θ^* is as follows. Start with a situation when no household acquires the investment good. It is clear that households with the highest level of ability (and consequently lowest costs) will find it beneficial to acquire the investment good, assuming nobody else acquires it (because of restriction (3)). The income generated by the investment good, which is determined by $s(y)$, will be very high since y is very small. One these households acquire the investment good, the mass of households who did not acquire the investment good goes down. As a result, their income also rises, even if they decide not to acquire the investment good (their income stream, determined by $u(x)$, rises as x, the mass of households who do not acquire the investment good, goes down). This process repeats until the marginal household is indifferent between acquiring and not acquiring the investment good, which happens exactly at θ^*. For any household with ability below θ^*, acquiring the investment good makes it strictly worse off. Moreover, it makes some households who previously acquired the investment good worse off as well since their incomes,

determined by $s(y)$, fall when y, the mass of households who acquire the investment good, increases.

Think of education. If very few people go to college, the marginal product of skilled labor is extremely high. The marginal product of unskilled labor also rises as more people become skilled because unskilled workers now become relatively more rare. As more an more people become skilled, however, the marginal benefit of going to college diminishes. Hence, it must be the case that at some point further education will bring negative benefits to the people who acquire it. A case in point is the scarcity of manufacturing workers in the United States. Consider the following example. "An aspiring machinist – a popular factory job – can start training at 18 and then do a one- or two-year manufacturing apprenticeship. In five years, he or she could be making more than $50,000. In 10 years, that could double to $100,000."[21] This is more than many college graduates can expect to earn when they turn 28. On top of that, this aspiring machinist won't have the significant debt burden he or she would have accumulated while in college: in 2010, the average amount of student debt in the U.S. stood at $25,250.[22]

A similar story applies to the housing market. When there are very few homeowners, housing prices are likely to be very low. Hence, acquiring a house can be a good financial investment. It also brings utility to homeowners from the flow of housing services. However, when more and more people start to buy houses, house prices increase and have less room to climb further. At some point they reach a value

[21]Source: `http://finance.yahoo.com/news/100-000-factory-job-whats-145600750.html`
[22]Source: `http://projectonstudentdebt.org/files/pub/classof2010.pdf`.

where further price appreciation is impossible. Households who acquire housing after that point are bound to be making a negative net present value investment.

However, not all households with abilities below θ^* will default if they acquire the investment good. Let ϕ be the total mass of households that acquired the investment good. Then, a household with ability θ is better off by not acquiring the investment good if $u(1-\phi) > s(\phi) - cost(\theta)$. However, even if $u(1-\phi) > s(\phi) - cost(\theta)$, it may still be the case that $s(\phi) > cost(\theta)$ and the payment that the investment good generates is sufficient to cover the cost of acquiring it. In this case, the household with ability θ does not default after it acquires the investment good.

There exists the ability level θ^d, $\theta^d < \theta^*$, such that as long as only households with abilities above θ^d acquire the investment good, there are no defaults. As soon as households with abilities below θ^d start acquiring the investment good, defaults ensue.

Lemma 2. *There exists θ^d, such that $\theta^* > \theta^d > 0$, and:*

(*i*) *If only households with abilities from θ^d to μ acquire the investment good, no household defaults on its debt.*

(*ii*) *If households with abilities from θ'' to μ acquire the investment good, where $\theta'' < \theta^d$, some households default on their debt.*

We will now define a bubble in our setting. Let $y(\theta) \equiv \frac{\mu-\theta}{\mu}$. Given that the government provides loans in a sequential manner, the smallest mass of households that acquire the investment good if household with ability θ acquires the investment good is given by $y(\theta)$. Hence, the highest possible payment that a household of

ability θ can receive after acquiring the investment good is given by $s(y(\theta))$. The price this household has to pay for the investment good is equal to this household's cost of acquiring it, $cost(\theta)$. We say that there is a bubble if for some households who acquire the investment good $cost(\theta) > s(y(\theta))$. It follows from this definition and Lemma 2 that there is a bubble if households with abilities below θ^d acquire the investment good.

3.2. Equilibrium when households have rational expectations

Proposition 1. *Assume that the government has no knowledge of $s(y)$, $u(x)$, but knows $cost(\theta)$ and the distribution of θ, and provides unlimited financing to households, so that it will extend a loan to buy the investment good to any household that wishes to acquire it. Also assume that households have rational expectations and perfect knowledge of $s(y)$, $u(x)$, $cost(\theta)$, and the distribution of θ. Then, in equilibrium, only households with abilities from θ^* to μ acquire the investment good.*

When perfectly rational and fully informed households face the choice of acquiring the investment good, they will do so only when their estimate of future income from acquiring the investment good, net of loan repayment, is higher than their estimate of the income without the investment good. In order to produce those estimates, they will condition their acquisition of the investment good on the fact that all households with abilities higher than theirs will also acquire the investment good. Hence, even if the government is willing to provide unlimited financing to households, they will use it up only to the point where they are indifferent between acquiring the investment good and not acquiring the investment good.

3.3. Households with adaptive expectations

We will now assume that households have adaptive expectations. For expositional simplicity, we start by assuming that all households have adaptive expectation. We will relax this assumption later.

Households with adaptive expectations observe the income stream that the investment good generates at date 1 and choose whether to acquire it or not. Their actual payoff, however, is realized at date 2, after some households acquired the investment good. Unlike households with rational expectations, households with adaptive expectations do not condition their estimate of the income generated by the investment good on the fact that all households with abilities higher than theirs will also acquire the investment good.

Proposition 2. *Assume that households have adaptive expectations. Also assume that the government has no knowledge of $s(y)$, $u(x)$, but knows $cost(\theta)$ and the distribution of θ, and that the amount of government-provided financing, L, satisfies $L = \int_{\theta^g}^{\mu} cost(\theta)d\theta$. Then, households with abilities from θ^g to μ acquire the investment good.*

Households with adaptive expectations overestimate returns from acquiring the investment good. Therefore, they are willing to acquire the investment good as long as they are able to obtain financing from the government. Hence, it is the amount of financing provided by the government that determines the outcome in this case.

In the next proposition we will show that a bubble is generated when the government provides too much financing (too much financing means that households with

abilities below θ^d are given funds to acquire the investment good).[23]

Proposition 3. *Assume that households have adaptive expectations. Also assume that the amount of government-provided financing, L, satisfies $L = \int_{\theta^g}^{\mu} cost(\theta)d\theta > \int_{\theta^d}^{\mu} cost(\theta)d\theta$. Then,*

(*i*) *There is an asset bubble.*

(*ii*) *There exists $\theta^{gg} > \theta^d$ so that all households with abilities between θ^g and θ^{gg} default on their debts.*

Part (*i*) of Proposition 3 states that asset bubbles arise when households have adaptive expectations and the government provides too much financing. We say that there is excess financing if the total amount of government-provided loans is large enough so that households with abilities below θ^d can obtain financing. Part (*ii*) of Proposition 3 shows that excess financing, in addition to causing the bubble, creates an endogenous propagation mechanism behind it. Once the amount of government-provided loans reaches a level at which households with abilities below θ^d are able to obtain financing, it is not just the households with abilities below θ^d that default. Some households with abilities above θ^d also default on their loans. This is what we call an endogenous propagation mechanism for asset bubbles induced by government overspending. It occurs because as more and more households acquire the investment

[23]When households are willing to acquire the investment good as long as the government provides them with financing, then the amount of this financing uniquely determines the mass of households that acquire the investment good. A close analogy is the decision to go to college in the United States, where students receive federally provided financial aid if they are admitted at an institution of higher learning. In that case, the federal government is willing to provide financing to anyone able to pass entrance tests, regardless of their expected future payoffs.

good, the income stream that it generates decreases. It is clear (from Lemma 2) that all households with abilities lower than θ^d default if they acquire the investment good. However, some households with abilities above θ^d were just about breaking even after acquiring the investment good. Once the income stream generated by the investment good goes down, these households will also default.[24]

Table 1: Correlation coefficients between delinquency and home ownership rates

	Delinquency rate on all mortgages, t
Home ownership rate, t	.1489
	(.0872)
Home ownership rate, $t-1$	.2946
	(.0007)
Home ownership rate, $t-2$	.4317
	(.0000)
Home ownership rate, $t-3$	.5580
	(.0000)
Home ownership rate, $t-4$	.6638
	(.0000)

Delinquency rate is defined as the number of mortgage loans 30 or more days past due divided by the total number of mortgage loans outstanding in a particular quarter. Correlation coefficients are constructed for the period 1979-2011 using quarterly data, p-values are in parentheses.

Proposition 3 is indirectly supported by empirical evidence from Table 1. Table 1 shows correlation coefficients between delinquency rates and homeownership rate in the U.S. for the period between 1972 and 2011, using quarterly data. While it is natural to expect that the number of delinquencies should rise as homeownership

[24]The subprime mortgage crisis illustrates the point of Proposition 3 very clearly. As households who obtained subprime loans started to default (because ultimately their income could not cover their mortgage payments), house prices began to fall. This, in turn, affected prime homeowners.

increases, it is not obvious that delinquency *rates* should rise as well. However, this is precisely what Table 1 demonstrates. As more households receive loans to obtain a home, a larger share of them default, consistent with the logic of Proposition 3.

3.4. Timing of financing

It is not only the amount of financing that is important but also its timing. In particular, if financing is spread over a sufficiently large number of dates, then the outcome can be close to efficient even if households have adaptive expectations and the government is willing to provide too much financing. To show this we extend our model to include a number of periods.

At date 1, the government decides how much financing to provide, L, and also over how many dates, T. In particular, the government will provide financing at date 1, date 2, and so on until date T or until there are no more households that wish to acquire the investment good. At each of those points in time the government is willing to allocate $\frac{L}{T}$. As before, the government allocates loans according to Algorithm 1. Unlike before, however, at each date the government stops its financing when the total amount $\frac{L}{T}$ has been allocated or if no more households wish to acquire the investment good.

Households are born at date 1 and live infinitely. At each date, they observe the income stream generated by the investment good and also the income stream from not acquiring the investment good. Based on this observation, they decide whether they wish to acquire the investment good or not. A household acquires the investment good at a particular date if and only if both of the following conditions are satisfied. First, this household wishes to acquire the investment good. Second,

the amount of financing provided by the government at that date is enough to cover this household's costs of acquiring the investment good as well as the costs of all households with abilities higher than this household's ability and who have not yet acquired the investment good.

Proposition 4. *Assume that households have adaptive expectations. Also assume that the amount of government-provided financing, L, satisfies $L > \int_{\theta^*}^{\mu} cost(\theta)d\theta$ and is provided over T dates. Then,*

(*i*) *There exists θ^{gm}, such that $\theta^* \geq \theta^{gm} > 0$ and households with abilities from θ^{gm} to μ acquire the investment good.*

(*ii*) *The maximum difference between θ^* and θ^{gm} is decreasing with T and increasing with L.*

Proposition 4 shows that the difference between the efficient outcome and the outcome actually achieved can be minimized if T is sufficiently large, even when the government is willing to provide financing to people with ability below θ^* and households have adaptive expectations. The intuition behind this proposition is as follows. After each successive round of financing, households observe the income stream that the investment good generates. Since the income stream exhibits diminishing marginal returns, fewer households will find it worthwhile to acquire the investment good as time goes by (because the number of households that acquired the investment good increases and hence the income stream that it generates decreases). This adjustment mechanism insures that even if households have adaptive

expectations, fewer of them will make suboptimal decisions. If the length of time over which financing is provided is large enough, the outcome can be close to efficient.

3.5. What proportion of households needs to have adaptive expectations for bubbles to be possible?

So far we have been assuming that all households have adaptive expectations. It simplified exposition and helped us convey the main intuition of our results more clearly. We will now relax this assumption and show that bubbles are possible even if some households have rational expectations. For tractability, we will assume that there is a range of abilities between $\underline{\theta}$ and $\bar{\theta}$, $\underline{\theta} < \bar{\theta}$, in which all households have adaptive expectations. This precludes situations when households with adaptive expectations are scattered across various ability levels.

Proposition 5. *Assume that households with abilities between $\underline{\theta}$ and $\bar{\theta}$, $\underline{\theta} < \bar{\theta}$, have adaptive expectations while all other households have rational expectations. Also assume that the amount of government-provided financing, L, satisfies $L > \int_{\theta^d}^{\mu} cost(\theta)d\theta$. Then,*

(*i*) *if $\underline{\theta} < \theta^d$ and $\bar{\theta} \geq \theta^*$, there is an asset bubble;*

(*ii*) *if $\underline{\theta} > \theta^d$ or $\bar{\theta} < \theta^*$, there is no asset bubble.*

The existence of a bubble depends on the proportion of households that have adaptive expectations and on precisely which households have adaptive expectations, relative to θ^d (the ability level below which households start to default) and θ^* (the ability level below which acquiring the investment good is inefficient). If households

with adaptive expectations cover a range of abilities that includes both θ^d and θ^*, bubbles will ensue. Since households with abilities right below θ^* have adaptive expectations, they will keep acquiring the investment good, even though it makes them worse off. In addition, the share of households with adaptive expectations must be large enough to cover some ability levels below θ^d because otherwise no household defaults and there is no bubble.

If all households that have adaptive expectations have ability levels above θ^d, there will be no asset bubbles: households with abilities below θ^d all have rational expectations and hence they never acquire the investment good. The same applies when the highest level of ability at which households have adaptive expectations happens to be below θ^*. In that case, financing will never reach those households since the government provides financing sequentially and households with rational expectations that have ability right below θ^* don't acquire the investment good.

It follows from Proposition 5 that in order for a bubble to be possible at least $\frac{\theta^* - \theta^d}{\mu}$ mass of households needs to have adaptive expectations. It is clear that $\frac{\theta^* - \theta^d}{\mu} < 1$ and hence not all households need to have adaptive expectations for bubbles to occur.

4. A numerical example: the decision to go to college

We describe an economy characterized by a Cobb-Douglas production function. There are two inputs: skilled labor and unskilled labor. Households are born unskilled and can decide whether to remain unskilled or to become skilled by acquiring education (going to college). The wage to each input is determined as its marginal product. Each worker is described by his/her ability level at birth, θ; θ is uniformly

distributed in the population. We assume that the costs of going to college are quadratic, $cost(\theta) = (\mu - \theta)^2$, where μ is the highest level of ability in the population.

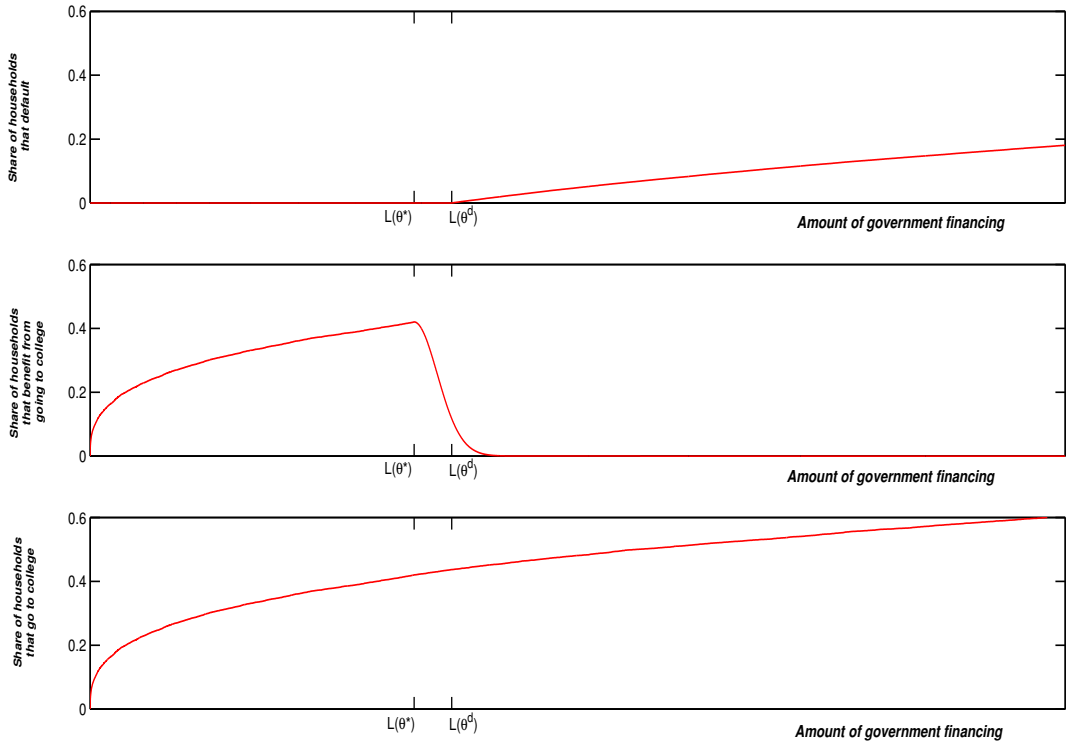
Our simulation is meant for illustrative purposes only, it is not a calibration exercise. We choose the following parameter values: $\alpha = 0.9$ (the share of income paid to skilled labor) and $\mu = 2.2$. We simulate using 10,000 draws from a uniform distribution. We assume, as in Proposition 3, that workers (as well as the government) are fully aware of their abilities and of the distribution of ability in the population. They are also aware of the direct costs of acquiring education. We assume that workers have adaptive expectations. They observe the income stream that the investment good generates at the beginning of each period and based on this observation decide whether they wish to acquire it or not. In Figure 1 we depict what happens as the government increases the amount of financing it provides to households.

Figure 1 consists of three panels.[25] The horizontal axis in all panels shows the amount of financing provided by the government. The bottom panel shows the share of population that goes to college as a function of the amount of financing provided by the government. Unsurprisingly, as the amount of financing grows, so does the number of people who go to college.

The middle panel shows the share of population that is better off after going to college. To calculate this share we start by comparing, for each individual worker, the

[25]For clarity, Figure 1 shows what happens to the first 60% of workers only. Depicting it for the entire population will not change any of the conclusions but will make the graph less readable as all effects will be concentrated in the far left part of the graph. That is why we chose to magnify the relevant part of the graph.

Figure 1: Amount of financing and college attainment



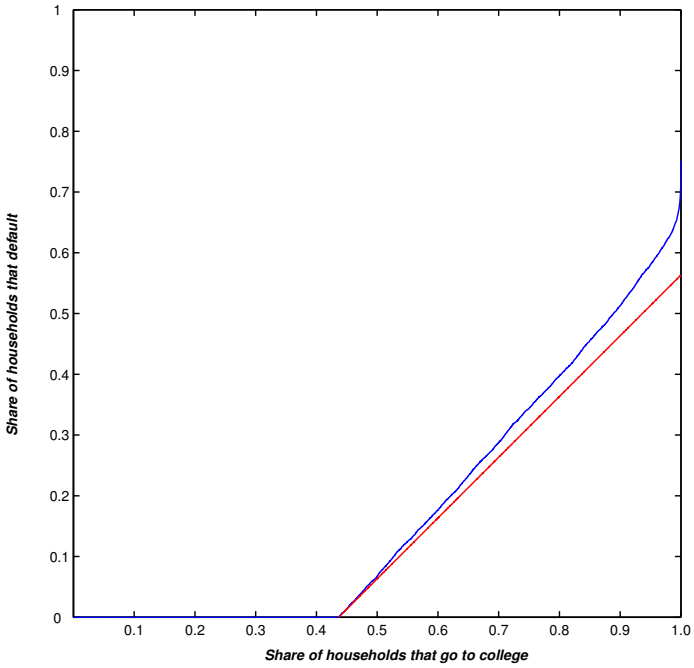
income (net of costs) he/she receives after acquiring education to the income he/she would have received if he/she didn't acquire education. We then divide the number of people for whom the income (net of costs) after going to college is greater than their income had they remained unskilled by the total population (10,000 workers in our case). Initially, this share grows as the number of people who go to college increases. However, after the amount of financing hits $L(\theta^*)$, this share starts to fall. This happens because for all people with abilities below θ^* going to college is

dominated by not going to college. Those workers receive less from being skilled, after costs are taken into account, than the wage rate they would have received had they remained unskilled. Still, initially those workers do not default on their loans, as the top panel of Figure 1 demonstrates (being worse off does not necessarily lead to default).

The top panel of Figure 1 depicts the share of households who default on their loans as a function of the total amount of financing provided by the government. It is the share of households whose income after acquiring education is below their costs of acquiring education. Households start to default only when the amount of government-provided financing reaches $L(\theta^d)$, which is higher than $L(\theta^*)$.

In Figure 2 we show the propagation mechanism that government-provided financing produces. This figure depicts the share of households who default on their loans as a function of the share of households who go to college. Initially, this share is zero: up until a certain point, as long as the amount of government-provided liquidity is below $L(\theta^d)$, the income received by skilled workers is enough to cover their costs of acquiring education. Once households with abilities below θ^d start to acquire education, defaults ensue. What is more important, however, is that the share of people who default grows faster than the share of people who go to college (the relevant line is always above the 45 degree line and is diverging upwards away from it). It means that every additional worker who acquires education causes not only his/her own default but also cases defaults of some people who would not have defaulted had this worker not acquired education. This happens because the marginal product of skilled labor goes down as the number of skilled workers increases. Hence, the wage

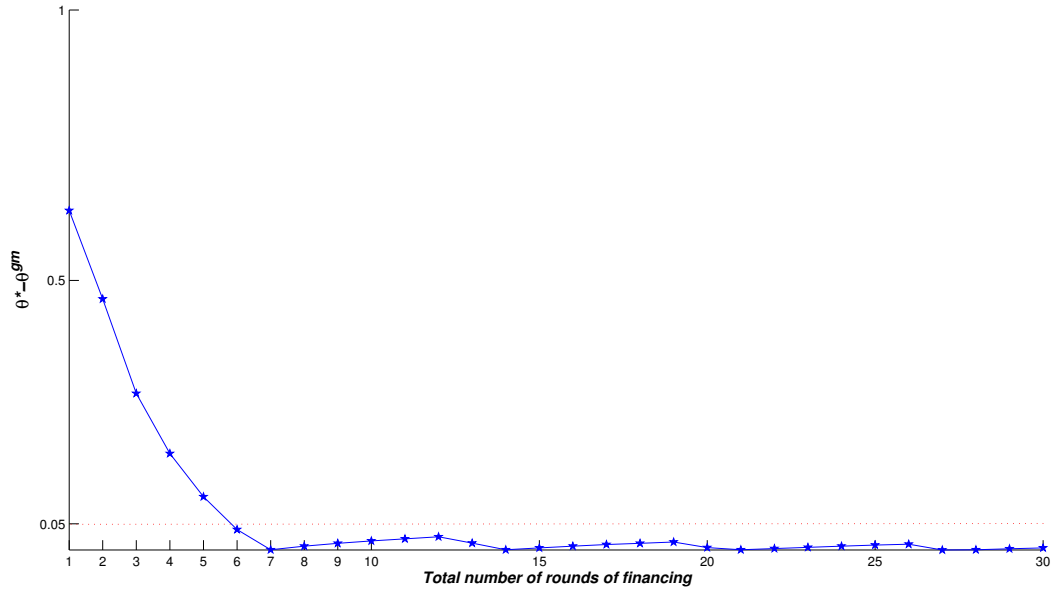
Figure 2: Propagation



rate of skilled labor also goes down and some workers who were previously able to cover their loan payments will no longer be able to do so.

Figure 3 illustrates that the outcome of government-provided financing can be close to efficient if this financing is provided over a sufficient number of periods. On the vertical axis we depict the difference between θ^* and the ability of the marginal household that acquires education (θ^{gm}. On the horizontal axis we depict the number of rounds over which financing is provided. We fix L, the total amount of financing that the government provides, at 50% of the level that would be sufficient for all

Figure 3: Financing over a number of periods



households to go to college. We then change the number of periods over which this amount of financing is provided. As the number of periods grows, less financing is provided in each of them and hence fewer households are able to go to college each period. As a result, fewer households are overestimating the benefits of going to college. In our numerical example, six periods are enough to ensure that the ability of the marginal household that goes to college is close to θ^*.

5. Discussion and policy implications

We made several simplifying assumptions to maintain a simple and clear setting. However, our qualitative results will be unaffected if one relaxes those assumptions.

We assumed no private financial market. As a result, there are no short-sellers

in our model. However, adding informed agents who can sell the investment good short will only exacerbate things. Short-sellers will benefit from the eventual price drop after households default. They will not, however, preclude defaults. To see this notice that short-selling will depress the price of the investment good, which will temporarily inflate households' returns from acquiring the investment good. Hence, even more households will be willing to obtain the investment good. In addition, assuming away private financial markets leaves us with the government as the only provider of financing to households. However, it is natural to assume that private investors (who pledge their own money) are better incentivized to screen borrowers than the government (which spends taxpayer money). In that case, the government will be providing financing to the relatively higher risk individuals, and inefficiencies and asset bubbles will be more likely to occur than in our setting.

The logic of this paper shows that the 2008 subprime mortgage collapse could not have happened without consumers willingly obtaining mortgages they could not possibly repay, most likely due to their misunderstanding of the costs and benefits associated with home ownership. It also could not have happened without the excess financing that stemmed from the U.S. government's desire to increase home ownership in the United States. A similar situation may now be taking place in the U.S. system of higher education. There, the government provides loans to students once they get accepted to an accredited institution of higher learning. Those loans are provided irrespective of students' ability to pay them back after graduation. Unless college applicants perfectly understand their prospects after graduation and are able to assess the costs and benefits associated with obtaining student loans, they are

prone to take out loans they cannot possibly repay.

The general regulatory response to the recent crisis has been to increase oversight of financial intermediaries and put additional regulatory burden on them. However, our paper suggests that this does not address the core problem behind government-induced asset bubbles. As long as the government provides excess financing, no amount of regulatory oversight will prevent eventual financial collapse. Malevolent intent by financial intermediary is in no way necessary to generate bubbles, although it can make them more likely.

Our paper has clear policy implications. First, we show that in general government-provided loans are not innocuous unless households have perfect information: when government provides too much financing too fast asset bubbles ensue. Second, our model suggests that there are two ways to address the cause of government-induced asset bubbles: self-restraint by the government or self-restraint by households (or both). In theory, bubbles can be avoided and social policies implemented as long as the government is able to perfectly allocate financing by correctly estimating the optimal number of households who will benefit from those policies. However, we think that such a scenario is unlikely. A more feasible way for the government to promote social policies without generating inefficiencies and asset bubbles is to require that people who take advantage of those policies are made aware of potential risks associated with them. Consumer education and full and clear disclosure by all market participants is one way to prevent financial collapse stemming from government overspending.

Third, we show that it is not just the amount of financing the government is

willing to provide that matters but also the speed with which it does so. Even when the government is willing to provide too much financing, spreading that financing over a sufficient number of periods will enable households to adjust their expectations. This suggests that government subsidies and fiscal policy stimuli in general may be most efficient if they are released gradually.

6. Conclusion

This paper describes a general mechanism by which government overspending can lead to asset bubbles. In general, when the government chooses to provide financing to households based on some simple criterion (such as admission to an institution of higher education or compliance with simple mortgage standards), it is only by chance that it will provide the optimal amount of financing. If too much financing is provided, a bubble ensues and households start to default. In addition to causing bubbles, government-provided financing creates an endogenous propagation mechanism behind them. However, if financing is spread over a sufficient number of periods, households will have time to adjust their expectations and the negative consequences of excess financing can be mitigated.

Avoiding welfare loss and asset bubbles is possible if households clearly understand the terms of financing and the associated risks and can calculate the costs and benefits of taking advantage of the government's policy. Full and clear disclosure by all market participants is one way to avoid financial collapse induced by government overspending. In addition, providing financing over a sufficiently long period of time may mitigate its potential negative consequences.

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A. Appendix: Proofs of propositions

We adopt the following notation that we will use in all the proofs. Let $y(\theta) = \frac{\mu-\theta}{\mu}$ denote the mass of households who acquired the investment good when all households with abilities from θ to μ acquired the investment good. In particular, $y(0) = 1$ and $y(\mu) = 0$. Similarly, $x(\theta) = \frac{\theta}{\mu}$ is the mass of households who did not acquire the investment good when all households with abilities from θ to μ acquired the investment good. Since households make a binary choice, $y(\theta) + x(\theta) = 1$.

Lemma 1. *There exists θ^*, such that $\mu > \theta^* > 0$, and:*

(*i*) *If households with abilities from θ^* to μ acquire the investment good, then all households are better off than if no household acquired the investment good.*

(*ii*) *If households with abilities from θ' to μ acquire the investment good, where $\theta' < \theta^*$, all households are worse off than if only households with abilities from θ^* to μ acquired the investment good.*

Proof. Part (*i*). Define θ^* such that for a household with ability θ^* the value of its consumption if it acquires the investment good is equal to the value of its consumption if it does not acquire the investment good, conditional on all households with abilities above θ^* acquiring the investment good:

$$s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*)). \tag{A.1}$$

It is clear that $\mu > \theta^* > 0$. First notice that the left-hand side of (A.1) is monotonically increasing in θ^* while the right-hand side of (A.1) is monotonically decreasing in θ^*. Then notice that if we let $\theta^* = 0$, then the left-hand side of equation (A.1) is smaller than its right-hand side by restriction (2) and the fact that $u(x)$ is a non-negative function. If we let $\theta^* = \mu$, then the left-hand side of equation (A.1) is larger than its right-hand side by restriction (3) because $cost(\mu) < cost(0)$. Hence, (A.1) must hold with equality for some $\theta^* \in (0, \mu)$.

Now consider any household with ability θ_0, such that $\theta_0 > \theta^*$. The value of this household's consumption, conditional on mass $y(\theta^*)$ households acquiring investment

good, is given by $s(y(\theta^*)) - cost(\theta_0)$. The value of this household's consumption, conditional on no household acquiring the investment good, is given by $u(1)$. Since $1 - y(\theta^*) < 1$, it follows that $u(1 - y(\theta^*)) > u(1)$. Moreover, since $\theta_0 > \theta^*$, we have that $cost(\theta_0) < cost(\theta^*)$, and it follows from equation (A.1) that $s(y(\theta^*)) - cost(\theta_0) > u(1 - y(\theta^*)) > u(1)$. Hence, for this household acquiring the investment good, conditional on mass $y(\theta^*)$ households acquiring the investment good, dominates not acquiring the investment good, conditional on no household acquiring the investment good.

Lastly, consider any θ_1, such that $\theta_1 < \theta^*$. Observe that $u(1 - y(\theta^*)) > u(1)$, and hence this household's consumption is higher when households with abilities from θ^* to μ acquire the investment good than when no household acquires the investment good.

Part (ii). Pick any household with ability θ_0, such that $\theta_0 \geq \theta^*$. We will show that this household is worse off when all households with abilities between θ' and μ acquire the investment good than when only households with abilities between θ^* and μ acquire the investment good. The value of this household's consumption, if all households with abilities between θ' and μ acquire the investment good, is given by $s(y(\theta')) - cost(\theta_0)$. The value of this household's consumption, if only households with abilities between θ^* and μ acquire the investment good, is given by $s(y(\theta^*)) - cost(\theta_0)$. Since $\theta^* > \theta'$, we have that $s(y(\theta')) < s(y(\theta^*))$, and it follows that $s(y(\theta')) - cost(\theta_0) < s(y(\theta^*)) - cost(\theta_0)$.

Pick any household with ability θ_1, such that $\theta^* > \theta_1 \geq \theta'$. We will show that this household is also worse off when all households with abilities between θ' and μ acquire the investment good than when only households with abilities between θ^* and μ acquire the investment good. The value of this household's consumption, if all households with abilities between θ' and μ acquire the investment good, is given by $s(y(\theta')) - cost(\theta_1)$. The value of this household's consumption, if only households with abilities between θ^* and μ acquire the investment good, is given by $u(1 - y(\theta^*))$. Since $\theta^* > \theta_1 \geq \theta'$, we have that $cost(\theta_1) > cost(\theta^*)$, $s(y(\theta')) < s(y(\theta^*))$, and it follows from equation (A.1) that $s(y(\theta')) - cost(\theta_1) < s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*))$.

Pick any household with ability θ_2, such that $\theta' > \theta_2$. We will show that this

household is also worse off when all households with abilities between θ' and μ acquire the investment good than when only households with abilities between θ^* and μ acquire the investment good. The value of this household's consumption, if all households with abilities between θ' and μ acquire the investment good, is given by $u(1 - y(\theta'))$. The value of this household's consumption, if only households with abilities between θ^* and μ acquire the investment good, is given by $u(1 - y(\theta^*))$. Since $\theta^* > \theta'$, it follows that $u(1 - y(\theta')) < u(1 - y(\theta^*))$. □

Lemma 2. *There exists θ^d, such that $\theta^* > \theta^d > 0$, and:*

(*i*) *If only households with abilities from θ^d to μ acquire the investment good, no household defaults on its debt.*

(*ii*) *If households with abilities from θ'' to μ acquire the investment good, where $\theta'' < \theta^d$, some households default on their debt.*

Proof. Part (*i*). Define θ^d such that the for a household with ability θ^d the value of its consumption, conditional on the mass $y(\theta^d)$ of households acquiring the investment good is equal to 0:

$$s(y(\theta^d)) - cost(\theta^d) = 0. \tag{A.2}$$

It is clear that $\theta^* > \theta^d > 0$. First notice that the left-hand side of (A.2) is monotonically increasing in θ^d. Then notice that if we let $\theta^d = 0$, then the left-hand side of equation (A.2) is smaller than zero (its right-hand side) by restriction (2). If we let $\theta^d = \theta^*$, then the left-hand side of equation (A.2) is greater than zero (its right-hand side) by equation (A.1) and the fact that $u(1 - y(\theta^*)) > 0$ since $\theta^* < \mu$. Hence, (A.2) must hold with equality for some $\theta^* \in (0, \theta^*)$.

Pick any household with ability θ_0, such that $\theta_0 > \theta^d$. The value of this household's consumption when households with abilities from θ^d to μ acquire the investment good is equal to $s(y(\theta^d)) - cost(\theta^0)$. Since $\theta_0 > \theta^d$, we have that $cost(\theta_0) < cost(\theta^d)$ and $s(y(\theta^d)) - cost(\theta^0) > 0$. Hence, this household doesn't default.

Now pick any household with ability θ_1, such that $\theta_1 < \theta^d$. The value of this household's consumption when households with abilities from θ^d to μ acquire the

investment good is equal to $u(1 - y(\theta^d))$. Since $u(x)$ is a nonnegative function, this household doesn't default.

Part (ii). Pick a household with ability θ_0 such that $\theta'' \leq \theta_0 < \theta^d$. The value of this household's consumption when households with abilities from θ'' to μ acquire the investment good is equal to $s(y(\theta'')) - cost(\theta_0)$. Since $\theta_0 < \theta^d$, we have that $cost(\theta_0) > cost(\theta^d)$, and $s(y(\theta^d)) - cost(\theta_0) < s(y(\theta^d)) - cost(\theta^d) = 0$ by equation (A.2). Hence, this household defaults. □

Proposition 1. *Assume that the government has no knowledge of $s(y)$, $u(x)$, but knows $cost(\theta)$ and the distribution of θ, and provides unlimited financing to households, so that it will extend a loan to buy the investment good to any household that wishes to acquire it. Also assume that households have rational expectations and perfect knowledge of $s(y)$, $u(x)$, $cost(\theta)$, and the distribution of θ. Then, in equilibrium, only households with abilities from θ^* to μ acquire the investment good.*

Proof. Households have perfect knowledge of $s(y)$, $u(x)$, $cost(\theta)$, and the distribution of θ. Since the government distributes its financing continuously (via Algorithm 1), each household conditions its payoff on the fact that all households with higher abilities also have to acquire the investment good if it acquires the investment good. Formally, this means that in the maximization problem (1) such households set $\bar{u}_\theta = u(1 - y(\theta))$ and $\bar{s}_\theta = s(y(\theta))$.

Consider any household with ability θ_0, such that $\theta^* \leq \theta_0 < \mu$. By (1), the value of consumption of a household with ability θ_0 from acquiring investment good, conditional on mass $y(\theta_0)$ of households acquiring the investment good, is equal to $s(y(\theta_0)) - cost(\theta_0)$. Notice that $s(y(\theta_0)) - cost(\theta_0) > s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*)) \geq u(1 - y(\theta_0))$, where the last term is the value of consumption of a household with ability θ_0 if it does not acquire the investment good, conditional on mass $y(\theta_0)$ of households acquiring the investment good. Hence, this household is better off acquiring the investment good. It follows that all households with abilities between θ^* and μ acquire the investment good. Analogously, all households with abilities from 0 to θ^* are worse off by acquiring the investment good than by not acquiring the investment good. Consider any θ_1, such that $\theta_1 < \theta^*$. Observe that $s(y(\theta_1)) -$

$cost(\theta_1) < s(y(\theta^*)) - cost(\theta^*) = u(1 - y(\theta^*)) < u(1 - y(\theta_1))$. Thus, only households with abilities from θ^* to μ acquire the investment good. □

Proposition 2. *Assume that households have adaptive expectations. Also assume that the government has no knowledge of $s(y)$, $u(x)$, but knows $cost(\theta)$ and the distribution of θ, and that the amount of government-provided financing, L, satisfies $L = \int_{\theta^g}^{\mu} cost(\theta)d\theta$. Then, households with abilities from θ^g to μ acquire the investment good.*

Proof. Consider any household with ability θ_0. Since this household has adaptive expectations and at date 1 nobody acquired the investment good, it calculates the payoff from acquiring the investment good as $s(0) - cost(\theta_0)$. It follows from restriction (3) and the fact that $cost(\theta_0) < cost(0)$ that $s(0) - cost(\theta_0) > u(1)$. Hence, this household wishes to acquire the investment good. As θ_0 was chosen arbitrarily, it follows that all households wish to acquire the investment good.

The government uses Algorithm 1 to provide loans to household. Hence, the government provides financing to all households with abilities between θ^g and μ, where θ^g is such that $L(\theta^g) = \int_{\theta^g}^{\mu} cost(\theta)d\theta$. As we just observed, all of those households wish to acquire the investment good. Hence, they acquire it. □

Proposition 3. *Assume that households have adaptive expectations. Also assume that the amount of government-provided financing, L, satisfies $L = \int_{\theta^g}^{\mu} cost(\theta)d\theta > \int_{\theta^d}^{\mu} cost(\theta)d\theta$. Then,*

(*i*) *There is an asset bubble.*

(*ii*) *There exists $\theta^{gg} > \theta^d$ so that all households with abilities between θ^g and θ^{gg} default on their debts.*

Proof. Part (*i*). By Proposition 2, all households with abilities between θ^g and μ acquire the investment good. Since $L(\theta^g) > \int_{\theta^d}^{\mu} cost(\theta)d\theta$, it follows that $\theta^g < \theta^d$. Thus, by Lemma 2 (part (*i*) states that $\theta^d < \theta^*$), there is an asset bubble.[26]

[26] As we stated above, restriction (3) is stronger than what we need. Consider the following

Part (ii). Pick a household with ability θ^g. The value of this household's consumption when households with abilities from θ^g to μ acquire the investment good is equal to $s(y(\theta^g)) - cost(\theta^g)$. Since $\theta^g < \theta^d$, we have that $cost(\theta^g) > cost(\theta^d)$, $s(y(\theta^g)) < s(y(\theta^d))$, and it follows that $s(y(\theta^d)) - cost(\theta^g) < s(y(\theta^d)) - cost(\theta^d) = 0$ by equation (A.2). Hence, this household defaults.

Define θ^{gg} such that

$$s(y(\theta^g)) - cost(\theta^{gg}) = 0. \tag{A.3}$$

It is clear that $\theta^{gg} > \theta^d$. First notice that the left-hand side of (A.3) is monotonically increasing in θ^{gg}. Also notice that $\theta^g < \theta^d$ by assumption and hence $s(y(\theta^g)) < s(y(\theta^d))$. Thus, it follows from equation (A.2) that if we let $\theta^{gg} = \theta^d$, then the left-hand side of equation (A.3) is smaller than zero (its right-hand side). If $s(y(\theta^g)) - cost(\mu) \leq 0$, then all households with abilities between θ^g and μ default, and hence $\theta^{gg} = \mu$. If $s(y(\theta^g)) - cost(\mu) > 0$, then the left-hand side of equation (A.3) is greater than zero (its right-hand side) if we let $\theta^{gg} = \mu$. Hence, (A.3) must hold with equality for some $\theta^{gg} \in (\theta^d, \mu)$, and all households with abilities between θ^g and θ^{gg} default. □

Proposition 4. *Assume that households have adaptive expectations. Also assume that the amount of government-provided financing, L, satisfies $L > \int_{\theta^*}^{\mu} cost(\theta)d\theta$ and is provided over T dates. Then,*

- (i) *There exists θ^{gm}, such that $\theta^* \geq \theta^{gm} > 0$ and households with abilities from θ^{gm} to μ acquire the investment good.*
- (ii) *The maximum difference between θ^* and θ^{gm} is decreasing with T and increasing with L.*

Proof. First, we introduce some new notation. Let θ_i denote the lowest ability household that the government is willing to finance at date i. Formally, $\int_{\theta_i}^{\theta_{i-1}} cost(\theta)d\theta = L/T$, where $i = \overline{1,T}$ and $\theta_0 = \mu$.

restriction instead: $s(0) - cost(\theta^e) > u(1)$, for some $\theta^e < \theta^d$. Then, all households with abilities from θ^e to μ wish to acquire the investment good, which is still inefficient since $\theta^e < \theta^d < \theta^*$.

Part (i). Let us consider date j such that $\theta_{j-1} > \theta^* \geq \theta_j$. This date exists since we assumed that $L > \int_{\theta^*}^{\mu} cost(\theta)d\theta$. If $\theta^* = \theta_j$ then we set $\theta^{gm} = \theta^*$. As we showed in Proposition 1, all households with abilities higher than θ^* are better off by acquiring the investment good. Now consider an arbitrary household with ability θ' such that $\theta' < \theta_j$ we have that $s(y(\theta^*)) - cost(\theta') < u(1 - y(\theta^*))$. It follows from equation (A.1) and the fact that since $\theta' < \theta^*$, $cost(\theta') > cost(\theta^*)$. This household decides whether it wishes to acquire investment good or not based on the return from acquiring the investment good at date j. This household does not acquire investment good, because it estimates the return from acquiring the investment good as $s(y(\theta^*)) - cost(\theta')$, which is smaller than the return from not acquiring investment good, $u(1 - y(\theta^*))$. Since θ' was chosen arbitrarily, it follows that no household with ability below θ_j acquires the investment good and the government stops provision of financing at date j.

If $\theta^* > \theta_j$, there are two possibilities:

1. For all θ'' such that $\theta^* \geq \theta'' \geq \theta_j$ it holds that $s(y(\theta_{j-1})) - cost(\theta'') > u(1 - y(\theta_{j-1}))$. In this case, we set $\theta^{gm} = \theta_j$. Consider an arbitrary household with ability θ' such that $\theta' < \theta_j$. It follows from equation (A.1) and from the fact that $cost(\theta') > cost(\theta^*)$ (because $\theta' < \theta^*$) that $s(y(\theta^*)) - cost(\theta') < u(1 - y(\theta^*))$. Since $s(y(\theta^*)) > s(y(\theta_j))$ and $u(1 - y(\theta^*)) < u(1 - y(\theta_j))$, it implies that $s(y(\theta_j)) - cost(\theta') < u(1 - y(\theta_j))$. Since this household decides whether to acquire investment good based on the return from acquiring the investment good at date j, this household does not acquire investment good. It follows because the return from acquiring the investment good is $s(y(\theta_j)) - cost(\theta')$, which is smaller than the return from not acquiring investment good, $u(1 - y(\theta_j))$. Similarly, all households with abilities above θ^{gm} acquire the investment good.

2. There exists θ''' such that $\theta^* \geq \theta''' \geq \theta_j$ and:

$$s(y(\theta_{j-1})) - cost(\theta''') = u(1 - y(\theta_{j-1})). \tag{A.4}$$

In this case, we set $\theta^{gm} = \theta'''$. For any household with ability θ^{iv} such that $\theta^{iv} < \theta'''$ we have that $cost(\theta^{iv}) > cost(\theta''')$. From equation (A.4) it follows that $s(y(\theta_{j-1})) - cost(\theta^{iv}) < u(1 - y(\theta_{j-1}))$ and hence this household does not acquire the investment good. Similarly, all households with abilities above θ''' acquire the investment good.

Part (ii). We proved that $\theta^* \geq \theta^{gm}$. If $\theta^* = \theta^{gm}$, then $\theta^* - \theta^{gm} = 0$. Now consider the non-trivial case when $\theta_{j-1} > \theta^* > \theta^{gm} \geq \theta_j$. We know that $\int_{\theta_i}^{\theta_{i-1}} cost(\theta) d\theta = L/T$, which implies that $cost(\theta^*) + cost(\theta^{gm}) < L/T$. Hence, $cost(\theta^{gm}) < L/T - cost(\theta^*)$. Increasing L would increase maximum possible value for $cost(\theta^{gm})$ and hence decrease the minimum possible value for θ^{gm}, thus increasing the distance between θ^* and θ^{gm}. Similarly, increasing T would decrease maximum possible value for $cost(\theta^{gm})$ and hence increase the minimum possible value for θ^{gm}, thus decreasing the distance between θ^* and θ^{gm} . □

Proposition 5. *Assume that households with abilities between $\underline{\theta}$ and $\bar{\theta}$, $\underline{\theta} < \bar{\theta}$, have adaptive expectations while all other households have rational expectations. Also assume that the amount of government-provided financing, L, satisfies $L > \int_{\theta^d}^{\mu} cost(\theta) d\theta$. Then,*

(i) *if $\underline{\theta} < \theta^d$ and $\bar{\theta} \geq \theta^*$, there is an asset bubble;*

(ii) *if $\underline{\theta} > \theta^d$ or $\bar{\theta} < \theta^*$, there is no asset bubble.*

Proof. Part (i). Households with abilities above $\bar{\theta}$ have rational expectations. Since $\bar{\theta} > \theta^*$, all of those households acquire the investment good by Proposition 1. All households with abilities between $\underline{\theta}$ and $\bar{\theta}$ have adaptive expectations and therefore wish acquire the investment good. Hence, all households with abilities between $\underline{\theta}$ and μ wish to acquire the investment good. As before, let θ^g be such that $L(\theta^g) = \int_{\theta^g}^{\mu} cost(\theta) d\theta$. It follows that households with abilities between $\min\{\theta^g, \underline{\theta}\}$ and μ acquire the investment good. Since $L(\theta^g) > \int_{\theta^d}^{\mu} cost(\theta) d\theta$, it follows that $\theta^g < \theta^d$, and hence $\min\{\theta^g, \underline{\theta}\} < \theta^d$. By Lemma 2, there is an asset bubble.

Part (ii). If $\underline{\theta} > \theta^d$, then all households with abilities below θ^d have rational expectations and hence they don't acquire the investment good (by Proposition 1). If $\bar{\theta} < \theta^*$, then households with abilities between $\bar{\theta}$ and θ^* have rational expectations and hence they don't wish to acquire the investment good. As the government provides financing continuously (according to Algorithm 1), it follows that households with abilities below θ^* do not acquire the investment good. □