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A Theory of Corporate Boards and Forced CEO Turnover

ABSTRACT

We develop a theory of corporate boards and their role in forcing CEO turnover. We consider a firm with an incumbent CEO of uncertain management ability and a board consisting of a number of directors whose role is to evaluate the CEO and fire her if a better replacement can be found. Each board member receives an independent private signal about the CEO's ability, after which board members vote on firing the CEO (or not). If the CEO is fired, the board hires a new CEO from the pool of candidates available. The true ability of the firm's CEO is revealed in the long run; the firm's long-run share price is determined by this ability. Each board member owns some equity in the firm, and thus prefers to fire a CEO of poor ability. However, if a board member votes to fire the incumbent CEO but the number of other board members also voting to fire her is not enough to successfully oust her, the CEO can impose significant costs of dissent on him. In this setting, we show that the board faces a coordination problem, leading it to retain an incompetent CEO even when a majority of board members receive private signals indicating that she is of poor quality. We solve for the optimal board size, and show that it depends on various board and firm characteristics: one size does not fit all firms. We develop extensions to our basic model to analyze the optimal composition of the board between firm insiders and outsiders and the effect of board members observing imprecise public signals in addition to their private signals on board decision-making. Finally, we develop a dynamic extension to our basic model to analyze why many boards do not fire CEOs even when they preside over a significant, publicly observable, reduction in shareholder wealth over a long period of time. We use this dynamic model to distinguish between the characteristics of such boards from those that fire bad CEOs proactively, before significant shareholder wealth reductions take place.

1 Introduction

If you shoot at a king, you must kill him.

Ralph Waldo Emerson

The seeming inability of corporate boards to monitor the performance of firm management proactively, and thus prevent corporate crises, has become the topic of debate among both practitioners and academics over the last few years in the wake of various corporate scandals. Citing several case studies where boards allowed CEOs to destroy shareholder value over a number of years before pressuring the CEOs to resign, Monks and Minow (1995) ask: "Why does it take boards so long to respond to deep-seated competitive problems? And, if one of the leading responsibilities of directors is to evaluate the performance of the CEO, why do boards wait too long for proof of managerial incompetence before making a move?" A number of related questions also arise in the above context. First, are there situations where boards do not fire CEOs, even when a majority of board members are individually convinced that the CEO is of poor quality? In particular, what are the characteristics of boards that fire poor quality CEOs proactively versus waiting and firing such CEOs only after considerable destruction in shareholder value has occurred? Second, what is the relationship between board size and the effectiveness of board decision-making in terms of monitoring the CEO? Third, what is the relationship between board composition (the proportion of outside directors) and the effectiveness of board decision-making and forced CEO turnover? Finally, how will the various policy proposals for corporate governance and board reform that have been suggested by academics, practitioners, and policy-makers affect the ability of the board to appropriately monitor the CEO and fire him or her as necessary?

While there has been little research on the dynamics of boards' CEO firing decisions, there has been a large empirical literature in finance partially addressing some of the

other questions we have raised above.[1] However, the above evidence has been mixed. For example, while some papers argue that smaller boards of directors are more effective and therefore maximize shareholder value (e.g., Yermack (1996)), others challenge this notion, documenting that, depending on various firm characteristics, either large or small board sizes may be appropriate from the point of view of enhancing shareholder value (see, e.g., Coles, Daniel, and Naveen (2008)). Similarly, the evidence is also inconclusive on the question of whether a larger proportion of inside directors enhances shareholder value: see, e.g., Linck, Netter, and Yang (2008), Boone, Field, Karpoff, and Raheja (2007), Coles, Daniel, and Naveen (2008). While theoretical models of corporate boards (see, e.g., Hermalin and Weisbach (1998), Harris and Raviv (2008), Adams and Ferreira (2007)) have helped to resolve some of the above ambiguities by providing guidance for empirical research, a number of interesting questions remain unanswered. The objective of this paper is to provide answers to such questions by developing a theoretical analysis of corporate boards and their role in forcing CEO turnover based on trade-offs that have not been studied before in the literature.

We consider a firm with an incumbent CEO, whose quality is uncertain, and a board consisting of a number of directors whose role is to evaluate the CEO and decide whether to fire her or to retain her. Each board member receives an independent private signal about the CEO's quality, after which board members vote regarding whether or not to fire the CEO. While we consider other voting rules, the bulk of our analysis is carried out under the assumption that a majority of votes is required to fire the CEO. If the CEO is fired, the board hires a new CEO from the pool of candidates available. The quality of the firm's CEO is publicly revealed in the long run, and the firm's long-run share price is determined by this revealed quality of its CEO. We assume that each board member owns

[1] An exception is Ertugrul and Krishnan (2011), who empirically study the characteristics of boards that fire CEOs proactively, before the firm sustains a significant loss in shareholder value.

some equity in the firm, so that they have an incentive to fire a bad CEO and hire a good one (or retain a good CEO), based on their assessment of the CEO's quality. However, if a board member votes to fire the incumbent CEO but fails to oust him, the CEO can impose significant costs of dissent on him (by not re-nominating that director to the board or otherwise imposing costs on him even if he continues on the board).[2]

We first study the benchmark equilibrium where the board members' dissent-costs are zero. We show that, in the absence of dissent-costs, each board member votes informatively (i.e., they vote to fire the CEO if they get a bad signal and vote to retain her if they get a good signal). Further, in the absence of dissent-costs and if the board's voting rule is optimal, we show that the board's firing decision efficiently aggregates all board members' private signals, in the sense that the board fires the CEO only if, in the opinion of a social planner who has access to all board members' signals, the incumbent CEO is worse (in terms of expected quality) than a potential replacement; the board retains the CEO if the reverse is true. We also show that, under reasonable conditions, a simple majority voting rule is optimal in the above sense.

We then analyze our basic model with dissent-costs. We show that, once such dissent-costs are significant, coordination problems arise between board members, so that the board may become suboptimally passive. In particular, the board may choose to retain the CEO even when a majority of board members are privately convinced that the CEO is of poor quality. This is because, even when each board member has a bad signal about the CEO's quality, his probability assessment that enough other board members will vote against the CEO (and thus will be able to successfully fire the CEO and avoid incurring

[2]Mace (1971) discusses anecdotal evidence of CEOs exercising authority in selecting candidates for the board, in effect hand-picking nominees. Similarly, Lorsch and MacIver (1994) report survey evidence indicating that CEOs wield major influence in selecting new board members. Tejada (1997) presents a news account of an outside director of a prominent company being denied nomination for reelection after criticizing management.

dissent-costs) is not large enough, so that the board member is better off voting to retain the CEO.[3] In this case, we show that whether each board member votes informatively or not depends on how well he is incentivized: if each board member's equity holdings in the firm is large enough, so that his dissent-cost to equity loss ratio is low, then he votes informatively; he votes to retain the CEO even if he receives a bad signal about the CEO's ability if this ratio is high.

We then analyze the effect of board size on the quality of the board's decision-making. On the one hand, since each board member's signal about the CEO's quality is independent, a larger number of board members increases the amount of information that is potentially available (collectively) to the board. On the other hand, a larger board worsens the coordination problems across board members, since in a larger board, a larger number of votes is required to fire the CEO. We show that the optimal board size emerges from the above trade-off. Further, this optimal board size depends on the precision of each board member's signal and his dissent-cost to equity loss ratio. Our results contradict the hypothesis of Lipton and Lorsch (1992) and Jensen (1993) that larger boards are always less effective, and sheds light on the findings of the recent empirical literature, which documents that while smaller board sizes are value maximizing for simpler firms, larger board sizes seem to be optimal for more complex firms (with greater information requirements for evaluating the CEO's performance): see, e.g., Coles, Daniel, and Naveen (2008).

In section 5, we extend our basic model to analyze issues of optimal board composition. Here we distinguish between insider board members and independent outsiders: while outsiders are similar to the board members in our basic model, insiders are assumed to receive more precise signals but have higher dissent-costs than outsiders. The advantage

[3] In the basic model, we assume that each board member cannot communicate his private signal to other board members prior to voting. However, in section 8, we show that even if such credible communication of private signals across board members at a cost is introduced into our basic model, our results go through essentially unchanged as long as the communication costs involved are significant.

of insider dominated boards is that insiders have better information about the quality of the CEO; the disadvantage is that, since insiders have high dissent costs, they are less likely to vote according to their private signals. By the same token, the advantage of outsider dominated boards is that they can vote truthfully; the disadvantage is that their information is less precise. We show that there is an optimal composition of the board. Boards should include as many insiders as possible to take advantage of their superior signal quality but have enough outsiders so that all board members vote according to their private signals. Our results thus contradict the conventional wisdom that outsider dominated boards are always value-maximizing. Further, they explain the recent empirical evidence documenting that, while a larger number of outsiders is value-maximizing for complex firms with greater informational requirements in evaluating and monitoring the CEO (see Coles, Daniel, and Naveen (2008)), this is not necessarily the case for simpler firms and those where firm-specific knowledge (likely to be available to insiders) is more important (see Linck, Netter, and Yang (2008) and Coles, Daniel, and Naveen (2008)).

In section 6, we extend the basic model in a different direction: we study the effect of a public signal on the coordination problem among directors in the basic model. One might at first think that publicly observable information regarding the CEO's quality (for example, the accounting performance of the firm under the CEO's management) is a natural coordination mechanism. In order to examine this question, we assume in this section that, in addition to their own private signals, all directors observe a noisy public signal of the CEO's quality, before voting.[4] In this setting, we show that the effect of the signal on board decision-making is asymmetric. On the one hand, it is indeed the case that a negative public signal improves board decision-making, in the sense that board

[4]Our assumption here is that the public signal is imprecise, so that it does not make the board members' private signals redundant. Of course, if the public signal is entirely precise, all board members will ignore their private signals and will vote to fire the CEO after they observe a negative public signal (for example).

members vote informatively for ranges of the dissent-cost to equity loss ratio where, in the absence of such a signal, they would have voted to retain the CEO (ignoring their private signals). On the other hand (and more interestingly), a positive public signal worsens board decision-making, in the sense that board members vote to retain the CEO in the presence of such a signal for values of the dissent-cost to equity loss ratio for which they would have chosen to vote informatively (i.e., according to their private signals) otherwise. The intuition here is that, when a board member receives a negative private signal but a positive public signal, he may choose not to vote against the CEO, since, given the positive public signal, he assesses a greater probability that many other board members vote to retain the CEO, thus allowing the CEO to stay in power and impose dissent costs on any director who voted against her. In summary, we show that having noisy public measures of CEO performance available to the board will not always improve board decision-making, since such signals can sometimes make the board members' coordination problem worse rather than better.

Finally, in section 7 we develop a dynamic version of our basic model to address the question we raised at the beginning of this paper, namely, why many corporate boards wait too long in the face of shareholder value destruction before firing the CEOs involved, and the difference between the characteristics of such boards and those that fire CEOs proactively (before significant destruction in shareholder value takes place). In this dynamic model, we introduce a second round of voting on the incumbent CEO (if she was not fired as a result of the first round of voting), with a highly informative public signal (such as whether significant destruction in shareholder value has taken place or not) between the two rounds of voting. We also assume that the equity loss of the board is significantly higher if a low quality CEO is fired only after board members receive the public signal (consistent with the higher loss in the value of their equity holdings that board members would sustain

compared to the situation where they replace the low quality CEO earlier).

In the above dynamic setting, we show that the coordination problem faced by board members may explain the board's reluctance to act to remove CEOs until considerable value-destruction has taken place. The dynamic trade-off faced by board members is the following. On the one hand, as time goes by, the firm's share price goes further and further down, thus increasing the board members' equity loss. On the other hand, as more and more evidence of the CEO's incompetence arrives (publicly) over time, each board members' assessment of a sufficient number of other members also voting against the CEO to oust her increases, thus reducing his own probability of incurring a dissent-cost if he votes against her. The timing of the CEO's dismissal emerges from this dynamic trade-off between expected equity loss and expected dissent-cost. We show that boards that are better incentivized (in terms of dissent-cost to equity loss ratio) will fire CEOs earlier, before significant shareholder value-destruction has taken place. The predictions of our dynamic model explain the empirical findings of Ertugrul and Krishnan (2011), who document that boards that dismiss CEOs early (i.e., in the absence of significant negative prior stock returns) are those that are characterized by higher equity ownership by board members, higher board equity compensation as a fraction of total compensation, higher institutional ownership, and better firm corporate governance characteristics.

Even though our model is set in the context of a board deciding whether to force a CEO turnover, the intuition behind our model generalizes to other binary corporate decisions made by a board as well. For example, consider the case of a CEO proposing a takeover of another firm, and the board voting on the CEO's proposal, with each board member having a private signal about the desirability of the takeover from the point of view of shareholder value maximization. While, unlike in the turnover setting, rejecting the CEO's preferred policy will not rid the firm of her, the basic trade-offs of our model will go through even in

a non-turnover setting as long as we make an additional assumption that the costs imposed by the CEO on a dissenting board member will be lower if the majority of directors went against the CEO's proposal. Here again the relationship between dissent-costs and equity-loss will determine the equilibrium. Larger boards are less likely to sustain informative voting if directors' signals are relatively imprecise and there emerges an optimal board size even in this case. Further, there will also be an optimal combination of insiders and outsiders on the board in equilibrium, based on the trade-offs we discussed earlier. The role of publicly available information will also be similar to that in a CEO turnover setting in the sense that having a public signal may not always improve the quality of decision-making.

The rest of the paper is organized as follows. Section 2 relates our paper to the existing literature. Section 3 describes the set-up of our basic model. Section 4 characterizes its equilibrium and analyzes issues related to optimal board size. Section 5 extends our basic model to analyze optimal board composition. Section 6 extends our basic model to study the effect of an imprecise public signal on the effectiveness of board decision-making. Section 7 presents a dynamic extension of our basic model to study the timing of forced CEO turnover. In section 8 we show that the main results of our paper are robust to introducing finite communication costs. Section 9 discusses the empirical and policy implications of our model. Section 10 concludes. The proofs of all propositions are confined to the appendix.

2 Relation to the Existing Literature

There have been several theoretical models of corporate boards in the existing literature, driven by trade-offs different from the ones we analyze here. The first theoretical analysis in this literature was by Hermalin and Weisbach (1998). In their model, board structure is the outcome of negotiation between the CEO and outside directors: CEOs, who generate

a surplus for their firms (for whom good substitutes are unavailable), wield considerable influence over their outside directors and use this influence to capture some of the surplus they generate by placing insiders in open board positions. Adams and Ferreira (2007) develop a model in which the CEO's preferred projects, which yield him control benefits, differ from those of shareholders. The CEO faces a trade-off in disclosing information to the board: if he reveals his information, he receives better advice from it, but may lose control benefits, since an informed board will also monitor him more intensively. They thus show that management-friendly boards, which can pre-commit not to use the information provided by the CEO to monitor him, may be value-maximizing for the firm.[5]

Harris and Raviv (2008) develop a model of a corporate board consisting of both insiders and outsiders and which can profitably use the information held by both insiders and outsiders to make optimal project choices. Insiders have private information relevant to this choice, but have private benefits that lead their incentives to be misaligned with those of shareholders; outsiders, whose interests are perfectly aligned with those of shareholders, are initially uninformed but can engage in producing information relevant to project choice at a cost. Control of the board entitles the controlling party (insiders or outsiders) either to make decisions themselves or to delegate decisions to the other party. In this setting, they characterize the conditions under which there will be insider versus outsider control of the board, when each party will delegate decision-making to the other party, the extent of communication between the two parties, and the number of outside directors.

Raheja (2005) also models a board consisting of both insiders and outsiders in charge of project selection, where insiders have decision-relevant private information, have private benefits that distort their incentives, and where outsiders can engage in costly production of information relevant to project choice. Outsiders' information costs are reduced if insid-

[5]See also Ferreira, Ferreira, and Raposo (2011), who develop a model demonstrating that stock price informativeness affects the structure of corporate boards, and test its implications.

ers reveal their private information. She argues that, since insiders are the source of future CEOs in her model, outsiders can use their CEO succession votes to motivate insiders to reveal their private information. She solves for the combination of insiders and outsiders on the board that leads to optimal project selection. Gillette, Noe, and Rebello (2003) show both theoretically and experimentally that when agency problems are especially severe, having uninformed outsiders in control of a board can prevent inefficient outcomes. Hirshleifer and Thakor (1994) model the interaction between internal (corporate boards) and external (acquisitions) governance mechanisms in the maintenance of managerial quality.

While some of the above models also address two of the central issues regarding corporate boards that we study here, namely, board size and optimal board composition, the above models address these questions based on trade-offs quite different from that in our model.[6] Further, none of these models study issues related to the effect of public signals on board decision-making, and the dynamics (timing) of CEO firing decision that we study here.[7] Our paper is also related to the large empirical literature on corporate boards and the relation between boards and firm characteristics, which we discuss in section 9 in the context of the empirical and policy implications of our model: see Adams, Hermalin, and Weisbach (2010) for an excellent review of the literature on corporate boards.[8,9]

[6] In a subsequent paper, Malenko (2011) analyzes a setting with biased board members to study issues of pre-vote communication among board members.

[7] Warther (1998) develops a simple model of a two-member board in which the CEO can eject hostile board members. He shows that this results in board members being reluctant to oppose management and votes being often unanimous in favor of management. However, given that he exogenously assumes a two-member board, he is unable to study any of the issues of coordination among board members that we analyze here; neither does he analyze optimal board size, board composition, the effect of public signals, or the dynamics of CEO turnover. Almazan and Suarez (2003) show that passive (weak) boards may be optimal in a setting where severance pay and weak boards are substitutes for costly incentive compensation paid to the CEO.

[8] Our paper is also related to the literature on voting in committees: see, e.g, Crawford and Sobel (1982), Coughlan (2000), Doraszelski, Gerardi, and Squintani (2003) and Austen-Smith and Feddersen (2005), who allow for pre-vote communication and study a cheap-talk setting. Our setting differs from the above papers in the sense that we do not allow pre-vote communication (except in our section 8, where we show robustness to pre-vote communication) and do not focus on a cheap-talk setting.

[9] Our paper is also related to the broader literature on the optimal amount of dissent: see, e.g, Acharya,

3 The model

Our basic model consists of five dates. It starts at time 0 with an incumbent CEO in place. The CEO can be of either good or bad quality (denoted G for good and B for bad). The board consists of n directors and does not include the CEO (we will use the terms board member and director interchangeably throughout the paper).[10] At time 1 each of n board members obtains a signal, s_i, $i = 1, \ldots, n$, about the CEO's quality. At time 2, board members meet before the vote and discuss the CEO (we call it the discussion round). Board members decide simultaneously whether to report their private signals about the CEO's quality truthfully or to lie. With some probability, the CEO will learn what signal each director revealed during the discussion round. We denote this probability by ρ. At time 3, based on the signals they received at time 1 and discussion at time 2, board members vote whether to keep the current CEO or to replace her.[11] If the board fires the CEO, it must hire a replacement from the pool of available candidates. The quality of this pool is indexed by γ: γ is the ex-ante probability of hiring a high-quality replacement. At time 4 the true quality of the then-incumbent CEO is revealed (either of the CEO existing at time 0 if she was not fired or the new CEO hired at time 3). The game ends and all payoffs are distributed. The timeline of the model is depicted in figure 1. All agents are risk-neutral, the risk-free rate of return is zero.

Myers, and Rajan (2011), Che and Kartik (2009), Landier, Sraer, and Thesmar (2009), in the sense that we model how the presence of costs of dissent potentially imposed on corporate board members can stifle dissent, thus leading to inefficient CEO turnover and other decisions by boards. Note, however, that the focus of the above models and the trade-offs driving them are quite unrelated to those in our model.

[10] This assumption is introduced for ease of exposition and is innocuous if one assumes that the CEO always votes to retain herself. Adding the CEO to the board in this case will simply decrease the effective board size to $n - 1$.

[11] For expositional simplicity, we assume open ballot. However, all of our qualitative results will remain unchanged even if board members vote by secret ballot, as long as there exists a high enough probability that the board member who voted against the CEO will suffer a dissent-cost (described below). One way this can happen is as follows. If the CEO cannot learn who voted against her, she can attempt to replace the entire board.

The signal that each board member receives can be either good, denoted g, or bad, denoted b. The signals are assumed to be conditionally independent across directors and informative. In particular, $p(s_i = g|G) = p(s_i = b|B) = \alpha > \frac{1}{2}$. Thus, α gives the precision of each board member's signal about the CEO's quality. We assume that all board members have signals of identical precision. For expositional simplicity we assume that the prior probability assessment of a board member that the incumbent CEO is good is γ, the same as the probability of hiring a high-quality replacement. The assumption of informative signals guarantees that $p(B|b) > 1 - \gamma > p(B|g)$; that is, after observing private signal b, a board member's private assessment of the incumbent CEO's quality is lower than of a potential replacement, and after observing private signal g, a board member's private assessment of the incumbent CEO's quality is higher than of a potential replacement

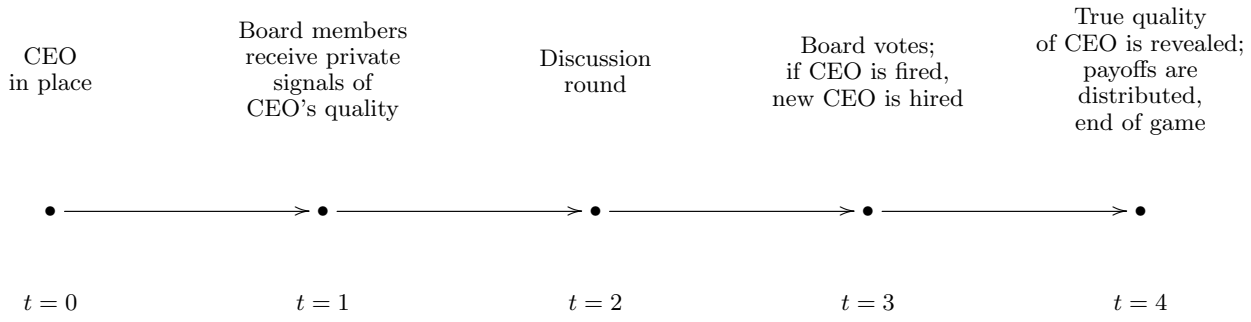

Figure 1: Timeline of the basic model.

3.1 Board members' voting rule

Using Bayes' rule, each board member's updated probability assessment of the CEO being of bad quality conditional on his private signal, denoted $p(B|s_i)$, is given by:

$$p(B|g) \equiv p(B|s_i = g) = \frac{(1-\gamma)(1-\alpha)}{(1-\gamma)(1-\alpha)+\gamma\alpha}, \tag{1}$$

$$p(B|b) \equiv p(B|s_i = b) = \frac{(1-\gamma)\alpha}{(1-\gamma)\alpha+\gamma(1-\alpha)}. \tag{2}$$

Following the existing voting literature (see, e.g., Austen-Smith and Banks (1996)), we define:

Definition 1. *Voting is informative if $v_i = 1$ iff $s_i = b$.*

Definition 2. *Voting rule, k, is the minimum number of votes required to fire the CEO.*

Definition 3. *A board member is pivotal if his vote determines the outcome of the vote.*

Informative voting implies that the board member votes in accordance with his information. In particular, if the outcome of the discussion round is uninformative, then the board member will vote in the direction of his private signal about the CEO's quality. If the outcome of the discussion round is informative, the board member will condition his vote on the information he received during the discussion round. We will assume throughout that the board votes according to simple majority. That is, there must be $\frac{n+1}{2}$ votes against the CEO to fire her.

3.2 Board members' objective

The objective of each board member in making his voting decision is to retain the CEO if she is of good quality and fire her if she is of bad quality (and hire a good quality replacement if the incumbent CEO is fired by the board) but only to the extent that

this affects his own costs and benefits, as described below.[12] We assume without loss of generality that the maximum payoff a director can receive is 0.[13] If it is revealed at time 4 that the CEO chosen by the board is of bad quality, each board member suffers a cost, $q, q > 0$. q is the loss in value to directors from making a bad decision that arises from the reduction in the value of the firm's equity held by individual board members (due to the mismanagement of the firm by a low quality CEO). We will refer to q from now on the the directors' "equity loss." Under this interpretation, one can think of the equity loss as being proportional to the difference in the long-run value of the firms's equity under a good quality CEO versus a bad quality CEO. Thus, one way to increase a board member's equity loss would be to increase the equity fraction in his compensation. q can also be thought of as reputational damage or a direct cost imposed by shareholders on the members of a board which retains a bad CEO.[14]

In addition to the above cost of choosing a low quality CEO, each board member may suffer a cost of dissent, denoted $c, c > 0$. In the discussion round, if a board member reports a signal that will later be interpreted as negative, he suffers the dissent-cost if the CEO is not subsequently fired and learns about the signals reported before the vote (with probability ρ).[15] Also, during voting, if a board member votes against the CEO but fails

[12] If good quality CEOs can generate higher long-run cash flows for the firm than bad quality CEOs, this translates into the assumption that the board wishes to maximize the firm's long-run value net of personal costs incurred.

[13] This is a normalization. If we assumed a fixed amount here (e.g., representing the board member's compensation that is independent of firm value), our results would go through qualitatively unchanged.

[14] We treat the value of q as being exogenous to the problem we study for several reasons. First, equity holdings that align interests of all directors may have to be prohibitively large. In particular, new, competent directors may be in short supply, and insisting on their holding large equity positions in the firm may have significant negative effects on their own portfolios, imposing large costs on them. Alternatively, the firms can compensate directors with equity, but in this case, awarding equity compensation to board members large enough to induce efficient voting may impose huge costs on the firm.

[15] There can exist an equilibrium in non-monotone strategies when during the discussion round board members with bad signals say that they have good signals while board members with good signals say that they have bad signals. At the voting stage, all board members will know this and the good signals reported before the vote will be correctly interpreted as bad signals about the CEO's quality and vice versa. We assume that in this case it is directors who reported good signals will suffer dissent costs because

to receive enough support from fellow board members, he incurs the dissent-cost.[16] This "dissent-cost" can be thought of as potential punishment imposed by the CEO (such as the CEO's reluctance to nominate the director in the next election cycle) or as the reputational damage from being perceived as a "troublemaker" by other directors on this board or by other boards who refuse to grant this director a seat. What matters to individual directors is the magnitude of c relative to q and the probability with which the CEO can learn that a director reported a negative signal during the discussion round, ρ. We will refer to $\frac{\rho c}{q}$ as the directors' expected dissent-cost to equity loss ratio.

Let μ_i denote the signal reported by board member i to other board members in the discussion round. We say that $\mu_i = 0$ if during the discussion round board member i says that he has a good signal and $\mu_i = 1$ if he says that he has a bad signal about the CEO's quality. Board member i's vote is denoted by v_i. We say that $v_i = 0$ if board member i votes to retain the CEO and $v_i = 1$ if he votes to fire the CEO. Given the above, director i's payoff after making his report during the discussion round and subsequent voting decision is given by:[17]

$$\begin{aligned}\pi_i = &- \Big\{ I\big(\sum_{j=1}^{n} v_j \geq \frac{n+1}{2}\big) q(1-\gamma) + I\big(\sum_{j=1}^{n} v_j < \frac{n+1}{2}\big) qp(B|s_i) \Big\} \\ &- I\big(\sum_{j=1}^{n} v_j < \frac{n+1}{2}\big) cv_i - I\big(\sum_{j=1}^{n} v_j < \frac{n+1}{2}\big) \rho c \mu_i,\end{aligned} \tag{3}$$

those signals are (correctly) interpreted as negative signals about the CEO's quality. However, this is for simplification only. Without this assumption, an additional restriction will have to be put on the magnitude of dissent-costs for informative voting to be feasible and none of our qualitative conclusions will change.

[16] The assumption that dissent-costs are incurred independently in the voting and the discussion round is for simplicity only. It significantly simplifies the algebraic exposition because in the objective function expected dissent-costs from the discussion round can be separated from dissent-costs of the voting stage. Assuming that a director who both speaks against the CEO and votes against her suffers only the dissent costs at the voting stage requires that the payoff be conditioned on every possible combination of board members' action in both rounds, which does not change the qualitative nature of the model but adds considerable expositional complexity.

[17] Notice that there is no discounting since the risk-free rate is assumed to be zero and all agents are risk-neutral.

where I denotes an indicator function in each term (it takes the value of 1 if the relation in parenthesis is satisfied and 0 otherwise), and $p(B|s_i)$ denotes the probability that the current CEO is of low quality given director i's signal. The terms in curly brackets in (3) represent the cost of choosing a bad CEO. The first term is the expected cost of choosing a bad CEO when the current CEO is fired: the corresponding indicator function, $I\big(\sum_{j=1}^{n} v_j \geq \frac{n+1}{2}\big)$, takes on the value of 1 when there are enough votes to oust the CEO ($\sum_{j=1}^{n} v_j \geq \frac{n+1}{2}$). In that case, the probability that the replacement is of low quality is given by $1-\gamma$, and hence the expected equity loss is $-q(1-\gamma)$. The second term in curly brackets is the expected equity loss from retaining the incumbent: the corresponding indicator function takes on the value of 1 when there are not enough votes to oust the CEO ($\sum_{j=1}^{n} v_j < \frac{n+1}{2}$). In that case, the probability that the board member will suffer equity loss is given by $p(B|s_i)$ so that the expected equity loss is $-qp(B|s_i)$. The third term in (3), outside the curly brackets, represents the expected dissent-cost from voting. A board member incurs it when two conditions are satisfied: (*i*) he votes to fire the CEO ($v_i = 1$), and (*ii*) there are not enough votes to oust the incumbent ($\sum_{j=1}^{n} v_j < \frac{n+1}{2}$, so that the corresponding indicator function takes on the value of 1). The last term in (3) represents the expected equity loss from reporting a bad signal during the discussion round. A board member incurs it when the CEO is retained and subsequently learns who reported bad signals, with probability ρ. For example, if board member i reports a bad signal and then votes to fire the CEO but fails to attain enough support from other board members, his expected payoff will be $-qp(B|s_i)-(1+\rho)c$. If, on the other hand, board member i reports a bad signal but then votes to retain the CEO and the CEO stays, his expected payoff will be $-qp(B|s_i)-\rho c$.

4 Equilibrium of the basic model

In this section we characterize the equilibrium of our basic model. The equilibrium concept we use is pure Bayesian Nash equilibrium, in which no player plays a (weakly) dominated strategy. An equilibrium consists of: (*i*) a strategy profile by each director given his signal (whether to report the signal truthfully before the vote and then whether to vote to fire or to keep the current CEO); (*ii*) each director's conjecture about the signals received by the other directors. Each director's strategy profile maximizes his expected payoff given his beliefs about the distribution of signals among the other board members and their strategies.

Our setup is different from the standard voting literature, which usually considers only the pivotal voter. In the standard literature, votes of non-pivotal votes do not affect their payoffs. In our setting, on the other hand, even if a board member is not pivotal, his vote can affect his payoff: if he votes to fire the CEO but there are not enough votes to fire her, he will suffer the dissent-cost. Analogously, in the discussion round, what a board member reports to other board members can affect his payoffs even if he is not pivotal (via dissent-costs). Hence, board members will calculate their payoffs not just conditional on being pivotal but also conditional on the probability of being on a particular side of the vote given how other board members vote and what signals they report.

4.1 The benchmark equilibrium with no dissent-costs

Before we solve for the equilibrium of our basic model and its enriched versions (in later sections), we consider the benchmark case in which each director's dissent-cost, c, is zero, i.e., the case when the only cost imposed on board members is the equity loss. The following proposition describes the equilibrium in this case.

Proposition 1. *(Benchmark Equilibrium).*

(*i*) *In the benchmark case described above there exists an equilibrium, in which each board member reports his signal about the CEO's quality truthfully in the discussion round and then votes informatively.*

(*ii*) *In this case, the board efficiently aggregates all board members' information in the absence of dissent-costs, in the sense that if the board decides to retain the CEO, her expected quality is better than that of a potential replacement; if the board decides to fire the CEO, her expected quality is worse than that of a potential replacement.*

Since all directors vote informatively, the outcome of this vote is equivalent to aggregating all directors' private information in making the final decision of the board regarding whether to fire the incumbent CEO or to retain her. In this case, since the board effectively aggregates private information, it is clearly optimal to have as many independent signals is possible. This logic is made precise in the following proposition.

Proposition 2. *(Optimal board size without dissent-costs). The quality of the board's decision is increasing in the number of board members, n.*

Proposition 2 says that when board members are willing to reveal their private valuation it is better to have as many of them as possible, since this increases the amount of information collectively available to the board.

4.2 Equilibrium of the basic model with dissent-costs

We now characterize the equilibrium of our model in the presence of dissent-costs, which leads to imperfect coordination among board members.

Proposition 3. *(Equilibrium of the basic model). Index board members by i and let* $e_i = 1$ *if the* i^{th} *board member observes a bad signal* ($s_i = b$); $e_i = 0$ *if the signal is good* ($s_i = g$). *Then:*

(*i*) *If the expected dissent-cost to equity loss ratio* $\frac{\rho c}{q} \leq \Upsilon(n, \alpha, \gamma|b)$*, there exists an equilibrium in which each board member reports his signal about the CEO's quality truthfully in the discussion round then votes informatively;*

(*ii*) *If* $\frac{\rho c}{q} > \Upsilon(n, \alpha, \gamma|b)$*, each board member reports a good signal about the CEO's quality and subsequently votes to retain the CEO, regardless of his private signal, where* $\Upsilon(n, \alpha, \gamma|b) = \frac{p\left(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\middle|b\right)\left(p(B|b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\middle|b\right)}$ *and all conditional probabilities represent beliefs of director i given his signal.*

The intuition for Proposition 3 is as follows. When board members report their signals about the CEO's quality truthfully, information can be aggregated at the voting stage and the CEO will be fired if the majority of observed signals are bad. In that case, no director ever suffers dissent-costs at the voting stage since all of them have the same information about the CEO's quality and vote in a coordinated manner. However, board members who reported bad signals in the discussion round will suffer partial dissent-costs (ρc) if the CEO is not ultimately fired. If they report their signals truthfully, they reduce the probability of suffering the equity loss q because a bad CEO is more likely to be fired. However, if they report their signals truthfully and fail to attain enough support they will suffer ρc, the cost of dissent scaled by the probability that the CEO learns about the outcome of the discussion round, regardless of whether the CEO is good or bad. Hence, directors with bad signals face the trade-off between equity loss and this expected dissent-cost. When ρc is high relative to q, board members will choose to disregard their signals and vote to keep the current CEO. Hence, it is the ratio $\frac{\rho c}{q}$ that will determine whether board members vote informatively or not. The threshold value of $\frac{\rho c}{q}$ above which informative voting is impossible is given by

$$\Upsilon(n, \alpha, \gamma|b) = \frac{p\left(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\middle|b\right)\left(p(B|b) - (1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\middle|b\right)}. \tag{4}$$

Notice that (4) makes intuitive sense. $\Upsilon(n, \alpha, \gamma|b)$ is positively related to the probability that enough directors observe bad signals, given that director i observes b. It is also positively related to the difference between the quality of outside replacement and the current CEO: when $p(B|b) - (1 - \gamma)$ is large, the current CEO is perceived to be much worse by board members with bad signals than the outside replacement and is more likely to be fired. On the other hand, $\Upsilon(n, \alpha, \gamma|b)$ is inversely related to the conjectured number of positive signals observed by board members (given that director i's own signal is bad): if director i believes that only a small number of other board members obtained a bad signal, he understands that it is unlikely they will have enough votes to oust the current CEO, so that he will not disclose his signal in order to avoid the expected dissent-cost ρc. It is natural to consider the ratio $\frac{\rho c}{q}$ to measure the board's incentives.

Definition 4. *Consider two boards, A and B, and denote the corresponding expected dissent-costs and equity loss by $\rho_A c_A$, q_A, $\rho_B c_B$, and q_B, respectively. We say that board A is better incentivized than board B if $\frac{\rho_A c_A}{q_A} < \frac{\rho_B c_B}{q_B}$.*

It is immediate from Proposition 3 that better incentivized boards are more likely to sustain informative voting. Our analysis emphasizes two sources of influence that a director faces: the CEO (and potentially other directors) via expected dissent-costs ρc, and the shareholders and directors' own shareholdings through the equity loss q. Providing better incentives involves changes to either or all of those parameters.

Proposition 4. *(Suboptimally passive boards). In the presence of dissent-costs, boards are likely to keep the incumbent CEO more often than is socially optimal.*

In particular, if dissent-costs are sufficiently large relative to the equity loss the board will not fire the CEO even if the majority of directors observe a bad signal of CEO's quality. This may explain the criticism of current boards in the financial press: in general, they

are less likely to fire bad CEOs than is desirable. As long as some boards suffer from coordination problems, the average turnover decision in the economy will be suboptimal.

Proposition 5. *(Large vs. small boards). When simple majority is the voting rule and n is odd:*

(*i*) *if* $\gamma < \alpha$*, then adding directors to the board increases the threshold value of* $\frac{c}{q}$ *above which informative voting is unsustainable,*

(*ii*) *if* $\gamma > \alpha$*, then adding directors to the board decreases the threshold value of* $\frac{c}{q}$ *above which informative voting is unsustainable,*

(*iii*) *if* $\gamma = \alpha$*, then adding directors to the board doesn't change the threshold value of* $\frac{c}{q}$ *above which informative voting is unsustainable.*

Proposition 5 implies, in particular, that when $\alpha < \gamma$, then larger board are less likely to sustain informative voting by having directors report their signals truthfully in the discussion round. Adding directors to the board improves chances that signals are reported truthfully if board members who observe bad signals expect a majority of new directors to observe a bad signal. What matters in this case is the probability of a board member who privately observes a bad signal that an additional board member observes a bad signal. If that probability is less than $\frac{1}{2}$, such a board member expects that the majority of additional directors observe a good signal, and is therefore less willing to report his private signal truthfully and subsequently vote informatively. This happens when the precision of the signal is low relative to the prior about the CEO's quality ($\alpha < \gamma$). The opposite happens when the signal is precise ($\alpha > \gamma$). In that case, a director with bad signal expects that the majority of new board members observe a bad signal and will be more willing to report their private signals truthfully in the discussion round and vote informatively after that.

Proposition 6. *(Optimal board size).*

(*i*) *If* $\alpha < \gamma$*, then there exists an optimal board size,* n^**.*

(*ii*) *If* $\alpha > \gamma$*, there will be a corner solution where a larger board is always better.*[18]

The trade-off driving part (*i*) of the preceding proposition is as follows. On the one hand, a larger board brings more information into the decision making process since a larger board collectively has access to more information. On the other hand, bigger boards increase the difficulty of coordinating across board members. Whether a larger board fosters or inhibits efficient voting depends on whether the first effect dominates the second, or vice versa. When the signal available to an individual board member is not very precise ($\gamma > \alpha$), then the second effect dominates and as the board's size grows, the likelihood that voting will be informative decreases. In general, boards must be large enough to obtain a reasonably precise signal but small enough to enable informative voting. More directors bring additional information into the board's decision making. However, this will occur only they report their signals about the CEO's quality truthfully in the discussion round and then vote informatively. Otherwise, additional board members only exacerbate the coordination problem among board members. On the other hand, if the signal very precise ($\alpha > \gamma$), then the first effect dominates the second, and informative voting becomes more likely when the board becomes larger. If the private signals obtained by individual board members are very precise (as postulated in part (*ii*) of the above proposition), a larger board will always make better decisions, since, under these conditions, a larger board will have more information, collectively, available to it. In that case, there will be no interior optimum in our model.

[18] Other considerations, such as compensating board members or the availability of good board members would limit the board size.

5 Board Composition

One of the recurring topics in the literature on corporate boards is the relationship between insiders and outsiders on the board and the optimal board composition from the point of vie of shareholder value maximization. In this section we explore the role that these two types of directors play in the CEO turnover decision. As before, we assume that there are no public signals and that each director observes a private signal of the incumbent CEO's quality.

We introduce two types of board members: insiders and outsiders. We assume that there are h insiders and $n-h$ outsiders on the board. We define insiders as board members employed by the firm. Since insiders work alongside the CEO, they are likely to have a better ability to judge her quality. At the same time, they are likely to suffer a greater cost of dissent if they vote against the CEO. To capture this idea we introduce two differences between insiders and outsiders in this section. First, on the positive side, insiders' signals are more precise: $p(s_i = g|G) = p(s_i = b|B) = \beta > \alpha > \frac{1}{2}$. On the negative side, however, insiders suffer larger dissent costs if they vote against the CEO but fail to oust her.[19] We denote insiders' dissent costs by $b, b > c$. Outsiders have the same characteristics as board members in our basic model.

We now characterize the optimal composition of the board.

Proposition 7. *(Optimal board composition). Index board members by i and let $e_i = 1$ if the i^{th} board member observes a bad signal ($s_i = b$); $e_i = 0$ if the signal is good ($s_i = g$). Also, denote insider board members by the superscript "ins" and outsider board members by the superscript "$outs$". Then,*

(i) If the expected dissent-cost to equity loss ratio for insiders $\frac{\rho b}{q} \leq \Upsilon^{ins}(n, \alpha, \gamma|b)$, then

[19] One justification for this assumption that inside directors have larger dissent costs compared to outside directors is that insiders' careers may be tied closely to that of the CEO: see, e.g., Weisbach (1988).

the board should consist only of insiders.

(*ii*) *If there exists a value of* h *for which* $\Upsilon^{ins}(n,\alpha,\gamma|b) < \frac{\rho b}{q} \leq \Theta^{ins}(n,\alpha,\gamma,h)$ *and* $\frac{\rho c}{q} \leq \Theta^{outs}(n,\alpha,\gamma,h)$*, then the board should include both insiders and outsiders.*[20]

(*iii*) *If* $\frac{\rho c}{q} \leq \Upsilon^{outs}(n,\alpha,\gamma|b)$ *but there exists no value of* h *for which* $\Upsilon^{ins}(n,\alpha,\gamma|b) < \frac{\rho b}{q} \leq \Theta^{ins}(n,q,\gamma,\gamma,h)$ *and* $\frac{\rho c}{q} \leq \Theta^{outs}(n,\alpha,\gamma,h)$*, then the board should consist only of outsiders, where*

$$\Upsilon^{ins}(n,\alpha,\gamma|b) = \frac{p\left(\sum_{j=1}^{n} e_j^{ins} \geq \frac{n+1}{2} \middle| s_i^{ins}=b\right)\left(p(B|s_i^{ins}=b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j^{ins} < \frac{n+1}{2} \middle| s_i^{ins}=b\right)},$$

$$\Upsilon^{outs}(n,\alpha,\gamma|b) = \frac{p\left(\sum_{j=1}^{n} e_j^{outs} \geq \frac{n+1}{2} \middle| s_i^{outs}=b\right)\left(p(B|s_i^{outs}=b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j^{outs} < \frac{n+1}{2} \middle| s_i^{outs}=b\right)},$$

$$\Theta^{ins}(n,\alpha,\gamma,h) = \frac{p\left(\sum_{j=1}^{h} e_j^{ins} + \sum_{l=1}^{n-h} e_l^{outs} \geq \frac{n+1}{2} \middle| s_i^{ins}=b\right)\left(p(B|s_i^{ins}=b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{h} e_j^{ins} + \sum_{l=1}^{n-h} e_l^{outs} < \frac{n+1}{2} \middle| s_i^{ins}=b\right)},\ \textit{and}$$

$$\Theta^{outs}(n,\alpha,\gamma,h) = \frac{p\left(\sum_{j=1}^{h} e_j^{ins} + \sum_{l=1}^{n-h} e_l^{outs} \geq \frac{n+1}{2} \middle| s_i^{outs}=b\right)\left(p(B|s_i^{outs}=b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{h} e_j^{ins} + \sum_{l=1}^{n-h} e_l^{outs} < \frac{n+1}{2} \middle| s_i^{outs}=b\right)}.$$

The intuition behind Proposition 7 is as follows. When insiders' dissent costs are not very high, so that an insiders-only board reports signals truthfully before the vote and then votes informatively, the board should consist only of insiders, since the quality of their information is higher (i.e., their signals are more precise). This case is described in part (*i*) of the above proposition. It is when insiders alone would not vote informatively that adding outsiders improves board decision-making (part (*ii*) of the above proposition). In this case, replacing some insiders with outsiders can make the remaining insiders more willing to report their private signals truthfully (since insiders believe that outsiders are likely to report their signals truthfully due to their lower dissent-costs), thus alleviating the coordination problem. As a result, informative voting will be more likely.

The trade-off driving the optimal board composition is as follows. On the one hand, it is desirable to have many insiders because the quality of information available to them

[20] For example, let a board consist of eleven board members, with the following parameters: $\gamma = 0.7, \gamma = 0.7, \alpha = 0.6, \beta = 0.65, \frac{\rho c}{q} = 0.1, \frac{\rho b}{q} = 0.2$. Then the board should consist of nine insiders and two outsiders. The relevant numerical simulations can be obtained from the authors upon request.

is high. At the same time, they face large dissent costs and may be unwilling to vote informatively. Outsiders, on the other hand, possess less precise information about the CEO's quality but are more willing to vote according to this information because of their lower dissent-costs. The optimal board composition is such that it retains as many insiders as possible but has enough outsiders to induce truthful reporting of private signals in the discussion round and subsequent informative voting. However, under certain parameter values (described in part (*iii*) of the above proposition) it is optimal to have only outsiders on the board. This happens when insiders' dissent costs are so high that the benefit from their having more precise signals is swamped by their higher dissent-costs. In other words, when the dissent-costs of insiders are so high that they never report their private signals truthfully, only outsiders should sit on the board.

6 Public signals

So far we have been assuming that directors were able to obtain only private information about the CEO's quality. We have shown that in this case lack of coordination may lead to inefficient decision-making. In this section we investigate what implications adding a public signal brings to our model. We argue that publicly observable information is a natural coordination mechanism. Simple intuition would suggest that having such a mechanism should improve decision-making. Somewhat surprisingly, however, we show that it is not always the case.

We assume that, in addition to their own private signals, all directors observe a public signal of CEO's quality. The public signal can be either H (high) or L (low). As in our basic model, we assume that directors are homogeneous (there's no distinction between insiders

and outsiders). The informativeness of the public signal is determined by parameter ϕ:[21]

$$\begin{aligned} &p(H|G) = p(L|B) = \phi; \\ &p(L|G) = p(H|B) = 1 - \phi; \\ &\phi > \frac{1}{2}. \end{aligned} \tag{5}$$

We also assume that public and private signals are conditionally independent. That is, $p(s_{private}, S_{public}|Q) = p(s_{private}|Q)p(S_{public}|Q)$ for $s_{private} \in \{g, b\}, S_{public} \in \{H, L\}$, and $Q \in \{G, B\}$.

6.1 Equilibria

If the public signal is very precise compared to private signals, directors should disregard their private information and fire the CEO upon observing public signal L but retain her upon observing public signal H. This happens when $p(B|b, H) < 1 - \gamma < p(B|g, L)$. In order to keep matters interesting, we assume that the precision of the public signal does not allow directors to disregard their private signals. This is a reasonable assumption, since otherwise the board of directors brings no value into the turnover decision and it would be optimal to simply dismiss the CEO after observing negative public information (such as a fall in the stock price or in operating performance).

To capture the above ideas, in this section only we impose the following restriction:[22]

$$p(B|g, H) < p(B|g, L) < 1 - \gamma < p(B|b, H) < p(B|b, L). \tag{6}$$

Proposition 8. *(Equilibrium with a public signal). Index board members by i and let*

[21] For simplicity of modeling, we use only one parameter to describe the public signal's precision.

[22] This assumption ensures that the private signal continues to be informative to board members even in the presence of a public signal (i.e., the public signal is not too precise).

$e_i = 1$ if the i^{th} board member observes a bad private signal ($s_i = b$); $e_i = 0$ if the private signal is good ($s_i = g$). Then, in the presence of a public signal:

(*i*) *After observing public signal H:*

(*a*) *If the expected dissent-cost to equity loss ratio $\frac{\rho c}{q} \leq \Upsilon(n, \alpha, \gamma | b, H)$, there exists an equilibrium in which each board member reports his signal about the CEO's quality truthfully in the discussion round then votes informatively;*

(*b*) *If $\frac{\rho c}{q} > \Upsilon(n, \alpha, \gamma | b, H)$, each board member reports a good signal about the CEO's quality and subsequently votes to retain the CEO, regardless of his private signal, where*

$$\Upsilon(n, \alpha, \gamma | b, H) = \frac{p\Big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} \Big| b,H\Big)\Big(p(B|b,H) - (1-\gamma)\Big)}{p\Big(\sum_{j=1}^{n} e_j < \frac{n+1}{2} \Big| b,H\Big)}.$$

(*ii*) *After observing public signal L:*

(*a*) *If the expected dissent-cost to equity loss ratio $\frac{\rho c}{q} \leq \Upsilon(n, \alpha, \gamma | b, L)$, there exists an equilibrium in which each board member reports his signal about the CEO's quality truthfully in the discussion round then votes informatively;*

(*b*) *If $\frac{\rho c}{q} > \Upsilon(n, \alpha, \gamma | b, L)$, each board member reports a good signal about the CEO's quality and subsequently votes to retain the CEO, regardless of his private signal, where*

$$\Upsilon(n, \alpha, \gamma | b, L) = \frac{p\Big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} \Big| b,L\Big)\Big(p(B|b,L) - (1-\gamma)\Big)}{p\Big(\sum_{j=1}^{n} e_j < \frac{n+1}{2} \Big| b,L\Big)}.$$

Notice that unlike in the basic model there are two thresholds in this case, depending on the type of the public signal that board members observe. This is because, after observing a public signal each director reassesses the validity of his own private information, conditional on the public information. The quality of decision-making by the board depends on the relation between $\frac{\rho c}{q}$ and these conditional thresholds.

Proposition 9. *(Quality of decision making with a public signal).*

(*i*) *If* $\frac{\rho c}{q} \leq \Upsilon(n, \alpha, \gamma | b, H)$*, directors report their private signals truthfully and vote informatively, regardless of the public signal, i.e., the public signal is irrelevant.*

(*ii*) *If* $\Upsilon(n, \alpha, \gamma | b, H) < \frac{\rho c}{q} \leq \Upsilon(n, \alpha, \gamma | b)$*, directors report their private signals truthfully and vote informatively only if there is no public signal or if the public signal is L, i.e., the public signal worsens the board's decision-making on average.*

(*iii*) *If* $\Upsilon(n, \alpha, \gamma | b) < \frac{\rho c}{q} \leq \Upsilon(n, \alpha, \gamma | b, L)$*, directors report their private signals truthfully and vote informatively only when there is a public signal and it is L, i.e., the public signal improves the board's decision-making on average.*

(*iv*) *If* $\Upsilon(n, \alpha, \gamma | b, L) < \frac{\rho c}{q}$*, directors and always vote to retain the CEO, i.e., the public signal is irrelevant again, where*

$$\Upsilon(n, \alpha, \gamma | b, H) = \frac{p\Big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} \Big| b, H\Big)\Big(p(B|b,H) - (1-\gamma)\Big)}{p\Big(\sum_{j=1}^{n} e_j < \frac{n+1}{2} \Big| b, H\Big)},$$

$$\Upsilon(n, \alpha, \gamma | b) = \frac{p\Big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} \Big| b\Big)\Big(p(B|b) - (1-\gamma)\Big)}{p\Big(\sum_{j=1}^{n} e_j < \frac{n+1}{2} \Big| b\Big)},$$

$$\Upsilon(n, \alpha, \gamma | b, L) = \frac{p\Big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} \Big| b, L\Big)\Big(p(B|b,L) - (1-\gamma)\Big)}{p\Big(\sum_{j=1}^{n} e_j < \frac{n+1}{2} \Big| b, L\Big)}.$$

Part (*i*) of Proposition 9 states that when the expected dissent-cost to equity loss ratio is low, the public signal is irrelevant. Since $\frac{\rho c}{q}$ is low, the directors would have reported their private signals truthfully and voted informatively even in our basic model. On the other hand, when this ratio goes up but remains below $\Upsilon(n, \alpha, \gamma | b)$ of our basic model, as in part (*ii*) of the above proposition, the presence of a public signal may worsen decision-making. This happens because conditional on observing a low public signal or no public signal at all, directors vote informatively. Observing a high public signal, however, may prevent board members with bad private signals from reporting them truthfully, since they may assess that many other directors may vote for retaining the CEO, given the high public

signal. A favorable public signal may thus make it less likely that there will be enough directors to overturn the incumbent CEO.

Part (*iii*) of Proposition 9 implies that when the expected dissent-cost to equity loss ratio is higher than $\Upsilon(n, \alpha, \gamma|b)$ of our basic model but is below $\Upsilon(n, \alpha, \gamma|b, H)$, the presence of a public signal is beneficial. This happens because in this case the only way for directors to report their private signals truthfully and vote informatively is to observe a low public signal. Directors with bad private signals would have sided with the CEO in our basic model. Upon learning negative public information about her quality, however, there is a higher probability that enough other directors will also want to oust her. This higher chance of success facilitates informative voting. When $\frac{\rho c}{q}$ is very high, as in part (*iv*) of Proposition 9, even a low public signal will not make board members vote informatively. In this case, the public signal is irrelevant.

7 The dynamic model: Proactive boards and early firing decisions

We now turn to the dynamic aspect of the board's decision-making. In particular, we ask the following question: which boards are likely to fire their CEO sooner rather than later? The financial press has been long asking the question why some boards seem to keep their CEOs until the damage is so obvious that the stock price is in a free fall. This subject, however, has seen little attention in the academic literature, with Ertugrul and Krishnan (2011) being the sole empirical paper on the topic.

In this section we modify our initial game to allow two rounds of voting: one after directors observe their private signals and another after some public information about the current CEO is revealed. We also assume the board is homogeneous (there's no difference

between insiders and outsiders). To make the notion of early versus late firing decision operational, we say that the CEO is fired early if the board made this decision prior to the revelation of a negative public signal. The CEO is said to be fired late if the decision came only after a negative public signal. The negative public signal we have in mind is a drop in share price (loss of shareholder value) under the current CEO.

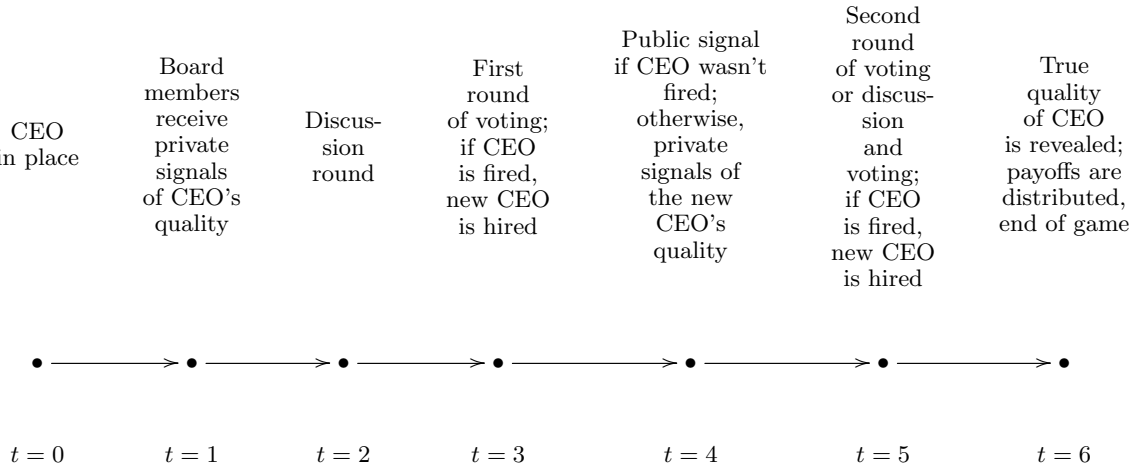

Figure 2: Timeline of the dynamic model.

The timeline of the extended model is as follows. At time 1 each director observes a private signal about the CEO's quality. At time 2 board members discuss the CEO's quality (discussion round). At time 3 the board votes on whether or not to fire the CEO. If the CEO is retained, at date 4 the public signal of her quality is revealed. If a new CEO is hired, at time 4 directors observe private signals about her ability, and there will be no public signal at time 4 if a new CEO is hired at time 3. At time 5 the second round of voting takes place and at time 5 the true quality of the then-incumbent CEO is revealed. If a new CEO was hired at time 3, then at time 5 board members first discuss the new CEO and then vote whether to fire or retain her. Unlike in section 6, we assume that the public signal is highly

informative, that is, $p(B|g,H) < p(B|b,H) < 1-\gamma < p(B|g,L) < p(B|b,L)$. Given this, directors should disregard their private signals upon observing the public signal. Apart from modeling simplicity, we introduce this assumption because it represents the trade-off we intend to study in this section. One may view this public signal as the result of a series of revelations, which are compressed into a single publicly observed sufficient statistic (H for high or L for low). As in the previous section, the precision of the public signal is determined by the parameter ϕ.

In addition, we impose a penalty for keeping a bad CEO for too long. We increase the amount of potential equity loss to reflect the idea that some damage is being done to the firm during a bad CEO's tenure.[23] Specifically, if a low public signal is observed at time 4, each board member suffers an equity loss q: this loss is in addition to any equity loss they will suffer if it is revealed that the quality of the CEO at time 6 is bad (e.g., if the board fired the incumbent CEO after observing a low public signal but the replacement CEO was of bad quality, the total equity loss by each board member will be $2q$). Also, if a bad CEO was retained from time 0 until time 5, each board member's dissent cost rises to $2q$. The potential equity loss can be as high as $3q$ in the following scenario: a bad incumbent is retained after both rounds of voting even though the public signal revealed at time 3 was L. This never happens in equilibrium, however, since board members will disregard their private signals and vote in the direction of the more precise public signal. If a new CEO was hired at time 3 (i.e., if the incumbent CEO was fired early, before the realization of the public signal), there is no increase in the equity loss.[24]

Proposition 10. *(Firing in the dynamic game). In the two-round voting game described*

[23] Without such an increase it would always be optimal to wait until the public signal is revealed, since this would allow board members to always avoid the dissent-cost without suffering any additional equity loss.

[24] If a new CEO was hired after the first round of voting, the game after time 2 is equivalent to the original game.

above, there exists a threshold, $\Psi(\gamma, \alpha, \phi, n)$*, such that if* $\frac{\rho c}{q} > \Psi(\gamma, \alpha, \phi, n)$*, the board never fires the incumbent CEO after the first round of voting, but waits for the second round of voting. If* $\frac{\rho c}{q} \leq \Psi(\gamma, \alpha, \phi, n)$*, the board will sometimes fire the incumbent CEO after the first round of voting.*

The trade-off faced by board members is as follows. As in the basic model, they face an equity loss if it is revealed that they chose a bad CEO and the dissent-cost if they attempt to fire the incumbent but fail. However, in this dynamic setting, they have the additional choice of whether to fire the CEO in the first round by reporting their private signals truthfully and voting according, or to wait until some public information is observed. Since we assume that the public signal is more precise than directors' private signals, waiting is beneficial. There is some residual uncertainty, however, so that even after observing a positive public signal the CEO's quality may be revealed as bad at time 5. In that case, the board members' equity loss will be significantly greater than it would have been had the board fired the CEO in the first round of voting (at time 3). The equity loss will also be higher if a low public signal is revealed, even if the board fires the CEO upon observing this public signal. If the expected dissent-cost to equity loss ratio is high, board members would rather risk a greater equity loss than face the higher probability of suffering dissent-costs. When this ratio is low, directors will report their private signals truthfully and vote informatively in the first round since the potential of an increased equity loss outweighs the benefits of avoiding the cost of dissent.

To summarize, as time goes by and more evidence of poor firm performance by the CEO becomes available to board members, their equity loss becomes greater; however, the coordination problem faced by board members is also reduced over time, since, once each board member becomes aware that others are also receiving more and more such evidence of the CEO's incompetence (over time), their probability assessment of a sufficient number

of board members voting against the CEO goes up (and correspondingly, their probability assessment of having to incur the dissent-cost goes down). Proposition 10 implies that better incentivized boards (smaller $\frac{\rho c}{q}$) are more likely to fire the CEO earlier, before significant damage is done to shareholder wealth.

8 Empirical and policy implications

Our model has several testable and policy implications. We highlight some of these below.

(i) Passive boards, suboptimal firing decisions, and the effects of shareholder activism: Our model develops a new rationale for why many boards are suboptimally passive (i.e., they choose to retain the existing CEO for too long). We showed that when the dissent-cost to equity loss ratio is large (for example, when the CEO has the ability to dictate the composition of the board without any uncertainty), coordination problems between board members prevent the board from firing the CEO even when collectively, the board has enough information to know that it is optimal for shareholders to fire the existing CEO and hire a new one from outside. Further, our model implies that if the board's assessment of the quality of the existing CEO relative to the pool of outside CEOs is low, then the board is more likely to fire the existing CEO and hire a new CEO from the outside.

The above has several testable predictions. First, more entrenched CEOs with greater ability to influence the payoff of board members are less likely to be fired. Second, there will be a positive relationship between a CEO being fired and the pool of outside candidates for the CEO's job and a negative relationship between industry-adjusted firm performance and the CEO being fired. Third, firms in industries where board members are able to evaluate CEOs more precisely (i.e., where board members receive more precise signals of CEO's quality) are more likely to make better CEO firing decisions. Thus, boards of firms in more homogeneous industries are likely to make better firing decisions than those in

more heterogeneous industries (where it is harder to benchmark the CEO's performance relative to other firms). Further, boards in which members have, on average, larger stock ownership (larger q in our model) and are institutional or otherwise activist shareholders (lower c) are more likely to make better decisions in terms of firing the CEO.

Evidence consistent with the first prediction above is provided by Allen (1981), who documents that CEOs having greater power over directors have longer tenures. Strong evidence supporting the second and third predictions above is provided by Parrino (1997). First, he documents a strong negative relationship between industry-adjusted firm performance and the likelihood that an outsider is appointed CEO. Second, he documents that the likelihood of forced CEO turnover and outside succession are both greater in homogeneous industries (that consist of similar firms) than in heterogeneous industries.

(ii) Optimal board size: The optimal board size in our model is determined by the trade-off between the information requirements of the board in evaluating the CEO, which favors a larger board (the larger the board is, the greater is the amount of information available to the board as a whole), and the greater coordination problems that arise with larger boards. Thus, our model predicts that, in order to maximize firm value, firms will have the smallest board size consistent with the information requirements of the board in evaluating and monitoring the performance of the CEO. Thus, our model predicts that larger, multi-divisional, and more complex firms (in the sense that the board requires more information in evaluating the CEO's performance) will have larger boards than simple firms.[25] In the latter case, our model predicts that the board members will (optimally) be better incentivized (smaller $\frac{c}{q}$) either through larger holdings in the firm's equity (larger

[25] As we demonstrated in Proposition 5(a), if board members' signals are relatively precise, a larger board makes better decisions. In large and complex firms, each board member can specialize in evaluating the CEO's performance in a particular division, so that each board member will have a signal of higher precision compared to the case where each board member has to evaluate the entire firm. This, in turn, implies that larger boards are optimal for larger and more complex firms.

equity loss q) or through appointing board members (like activist shareholders) who suffer from smaller dissent-costs.

Several empirical papers provide evidence consistent with the first prediction above of our model. Yermack (1996) shows that, controlling for firm size and other firm characteristics, there is an inverse relationship between market value (Tobin's Q) and the size of the board of directors. He also shows that measures of operating efficiency and profitability are negatively related over time to board size within firms, and that smaller boards are more likely to dismiss CEOs following periods of poor performance. Related evidence is provided by Kini, Kracaw, and Mian (1995), who document that board size shrinks after successful tender offers of underperforming firms. Evidence supporting the second prediction of our model above is provided by Mikkelson, Partch, and Shah (1997) and Boone, Field, Karpoff, and Raheja (2007), who document that firms start out with smaller boards at IPO, with board size increasing significantly after the firm has become seasoned. The latter paper also documents that board size is related to measures of the scope and complexity of the firm's operations such as firm size, firm age, and the number of business segments the firm operates in. Finally, Coles, Daniel, and Naveen (2008) document that complex firms, which have greater advising requirements than simple firms, have larger boards with more outside directors. They also document that the relation between a firm's market value (Tobin's Q) and board size is U-shaped: Tobin's Q increases (decreases) in board size for complex (simple) firms.

(iii) Optimal board composition: Conventional wisdom among practitioners seems to be that a greater level of board independence (a larger fraction of outside directors) unambiguously increases firm performance. For example, TIAA-CREF, one of the largest pension funds in the world, has stated that it will invest only in companies that have a majority of outside directors on its board; similarly, CALPERS, another large pension

fund, recommends that the CEO should be the only inside director on a firm's board. The empirical evidence, however, has been mixed: For example, Coles, Daniel, and Naveen (2008) present evidence challenging the notion that restrictions on board size and insider representation necessarily enhance firm value (see also Bhagat and Black (2001)).

Our model contributes significantly to the above debate about the optimal proportion of insiders versus outsiders on corporate boards. Our analysis suggests that there may be an optimal mix of insiders and outsiders on corporate boards: the advantage of having insiders arises from the more precise information they possess, while the advantage of having outsiders arises from their lower dissent-costs. Our model therefore suggests that replacing insiders with outsiders will be value-enhancing mainly when outsiders have significantly lower dissent-cost to equity loss ratios compared to firm insiders, so that adding more outsiders alleviates the coordination problem faced by the board. Consequently, our model predicts that a greater proportion of independent directors (outsiders) is not necessarily value improving in all situations. For example, our model suggests that firms where the CEO has greater influence (as measured by the CEO's share ownership and job tenure), generating higher dissent-costs for board members (both insiders and outsiders) will be associated with a smaller fraction of independent outsiders. Evidence supporting this prediction is provided by Boone, Field, Karpoff, and Raheja (2007), who document such a relationship. On the other hand, our model predicts that more complex firms that have larger boards should have a greater proportion of outsiders so as to mitigate the coordination problem (which is larger in larger boards); further, in such boards, the outside board members should have greater equity holdings, so as to reduce their dissent-cost to equity loss ratio. Evidence supporting the former prediction is provided by Coles, Daniel, and Naveen (2008), who document that more complex firms, with greater advising requirements than simpler firms, have larger boards with more outside directors. Evidence supporting

the latter prediction that the proportion of independent outside directors will be greater in firms where outsiders have lower dissent-cost to equity loss ratio is provided by Boone, Field, Karpoff, and Raheja (2007), who document that the proportion of independent outsiders on a firm's board is positively related to the equity ownership of outside directors. Finally, our model predicts that a board with a greater proportion of (appropriately incentivized) outsiders is more likely to replace a poorly performing CEO. Evidence supporting this prediction is supported by Weisbach (1988) and Borokhovich, Parrino, and Trapani (1996).

(iv) The ambiguous effect of low-precision public signals on board performance: Our model demonstrates that, contrary to common intuition, low-precision public signals may in fact worsen the board's decision-making instead of improving it (relative to the case of no public signal). Thus, short-term improvements in accounting performance by a firm may induce a board member to vote to retain the current CEO, even though he is privately convinced that the firm is pursuing a wrong long-term strategy under the current CEO. This arises from the fact that such a public signal worsens coordination problems among board members by increasing each member's fear that other members may vote to retain the CEO, thus increasing his likelihood of having to incur dissent-costs arising from voting against the CEO but failing to oust her.

(v) The dynamics of forced CEO turnover and its relation to board characteristics: Both practitioners and academics have bemoaned the tendency of boards to allow CEOs to pursue wrong-headed policies for a number of years, and fire them only after millions (if not billions) of dollars of shareholder value has been destroyed. For example, Monks and Minow (1995) document a number of cases where CEOs were able to destroy shareholder value for a number of years, and raise the following questions: "Why does it take boards so long to respond to deep-seated competitive problems? And, if one of the leading responsibilities

of directors is to evaluate the performance of the CEO, why do boards wait too long for proof of managerial incompetence before making a move?" Jensen (1993) makes a similar point and calls this a failure of corporate internal control systems, and comments: "They seldom respond in the absence of a crisis."[26]

The dynamic version of our model provides some answers to the above questions raised by Monks and Minow (1995), regarding why, even when board members are individually aware of the wrong-headedness of the CEO's policies, they do not immediately vote to fire the CEO. Our dynamic analysis indicates that the timing of a board's firing of a CEO who is destroying shareholder value is driven by a trade-off between the equity loss incurred by board members versus the probability of incurring a dissent-cost. On the one hand, as time goes by and more evidence of poor firm performance becomes available to board members, their equity loss becomes greater; on the other hand, the coordination problem faced by board members is also reduced over time, since, once each board member becomes aware that others are also receiving more and more such evidence (over time), their probability assessment of of having to incur a dissent-cost also goes down. Thus, our dynamic analysis has two predictions. First, CEOs who are entrenched and have greater influence over the board (e.g., as measured by longer tenure) are likely to be allowed to pursue policies that reduce share price for a longer time period before being asked to resign by the board. Second, boards that are better incentivized (smaller dissent-cost to equity loss ratio) are more likely to fire CEOs proactively, before considerable destruction of shareholder value

[26]Monks and Minow (1995) give the case studies of the CEOs of General Motors, Westinghouse, American Express, IBM, Eastman Kodak, Scott Paper, and Borden, who were pressured to resign in the face of their companies' long-term underperformance. They comment that, while the above moves by their companies' boards were heralded in the media as breakthroughs in boardroom activism, in all these instances the board took the necessary drastic action years too late. Jensen (1993) points out that "The [...] GM board revolt [...] which resulted in the firing of CEO Robert Stempel exemplifies the failure, not the success, of GM's governance system. General Motors, one of the world's high-cost producers in a market with substantial excess capacity, avoided making major changes in its strategy for over a decade. The revolt came too late: the board acted to remove the CEO only in 1992, after the company had reported losses of $6.5 billion in 1990 and 1991."

has taken place. Evidence consistent with the latter prediction of our model is provided by Ertugrul and Krishnan (2011), who document that boards that dismiss CEOs early (i.e., in the absence of significant negative prior stock returns) are those that are characterized by higher equity ownership by board members, higher board equity compensation as a fraction of total compensation, higher institutional ownership, and better firm corporate governance characteristics.[27]

(vi) Policy implications for corporate governance and board reform: Our model suggests several ways in which corporate governance, especially corporate boards, can be reformed to improve firm performance. Our analysis suggests that reforms that reduce the dissent-costs of board members or increase their equity loss will significantly increase the likelihood of boards firing incompetent CEOs. One proposal that would reduce dissent-costs would be to grant security of tenure (i.e., guarantee their presence on the board for some length of time) for current board members, thus reducing their dissent-costs in the event they vote against the current CEO but end up being unable to oust him or her. Another reform proposal to reduce dissent-costs would make it easier for dissidents to acquire a seat on the board: due to the cost of proxy fights, few dissidents bother to make board challenges.[28] One such proposal currently being considered by the SEC would let owners of at least 1% of a company with an equity value of $700 million or more (3% for smaller companies) include information about their board nominees in corporate proxy materials. Another such proposal would reimburse successful dissident board candidates for all their campaign expenses (and partially reimburse candidates who obtained at least 40% of the

[27] Ertugrul and Krishnan (2011) document that firms that dismiss CEOs early do not experience poor operating performance after the CEO dismissal relative to a control sample, suggesting that such dismissals are not cases of value-reducing mistakes by the board.

[28] It can cost hundreds of thousands of dollars or more for an outside contender to run against a director. For example, a campaign for a board seat may involve repeated mailings to every investor; directors endorsed by the firm management, on the other hand, can make use of the company coffers to finance their candidacies. Thus, RiskMetrics, a proxy-advisory firm, points out that there were just 75 shareholder contests in 2009 (as of October of that year); challengers won one seat each in 58 of those fights.

votes cast).[29] Another such proposal that would reduce dissent-costs would be to reduce the involvement of CEOs in the selection of board members and have boards choose directors through nominating committees composed only of independent members of the board (see, e.g., The Working Group on Corporate Governance (1991)).[30]

A second set of board reform proposals suggested by our analysis would increase the equity loss of board members. One such proposal would be to increase the proportion of board members' compensation that depends directly on the firm's long-run share value. Another proposal (perhaps harder to implement) would require board members to invest a significant amount of their own wealth in the firm's equity.

9 Conclusion

In this paper, we have developed a theory of corporate boards and their role in forcing CEO turnover. We considered a firm with an incumbent CEO of uncertain management ability and a board consisting of a number of directors whose role is to evaluate the CEO and fire her if a better replacement can be found. In our setting, each board member receives an independent private signal about the CEO's ability, after which board members vote on firing the CEO (or not). If the CEO is fired, the board hires a new CEO from the pool of candidates available. The true ability of the firm's CEO is revealed in the long run; the firm's long-run share price is determined by this ability. Each board member owns some equity in the firm, and thus prefers to fire a CEO of poor ability. However, if a board

[29] Such a proposal was adopted by HealthSouth Corporation in October 2009. For details of this and other proposals being considered by the SEC, see Joann S. Lublin, *Reimbursements Aim for a Fairer Proxy Fight.* The Wall Street Journal, page A22, October 27, 2009.

[30] Shivdasani and Yermack (1999) document that when the CEO is involved in the selection of a new director, firms appoint fewer independent outside directors and more "grey" outsiders. They define a CEO as "involved" in such selection (*i*) if the board has a separate nominating committee and the CEO serves as a member or (*ii*) if such a committee does not exist and directors are selected by the entire board including the CEO.

member votes to fire the incumbent CEO but the number of other board members also voting to fire her is not enough to successfully oust her, the CEO can impose significant costs of dissent on him. In this setting, we show that the board faces a coordination problem, leading it to retain an incompetent CEO even when a majority of board members' private signals indicate that she is of poor quality. We solved for the optimal board size, and show that this depends on various board and firm characteristics: one size does not fit all firms. We developed extensions to our basic model to analyze the optimal composition of the board between insiders and outsiders and the effect of board members observing imprecise public signals (such as the firm's short-term operating performance) in addition to their private signals on the effectiveness of board decision-making. Finally, we developed a dynamic extension to our basic model to analyze why many boards do not fire CEOs even when they preside over a significant, publicly observable, reduction in shareholder wealth over a long period of time, and used this dynamic model to distinguish between the characteristics of such boards from those that fire bad CEOs proactively, before significant wealth reductions take place.

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A Appendix A: Proofs of Propositions 1 through 8

Proof of Proposition 1

Part (i). With no dissent-costs, the directors' objective, given by (3), simplifies to

$$\pi_i^{benchmark} = -I\big(\sum_{j=1}^{n} v_j \geq \frac{n+1}{2}\big)q(1-\gamma) - I\big(\sum_{j=1}^{n} v_j < \frac{n+1}{2}\big)qp(B|s_i). \tag{A.1}$$

We will show that the conjectured equilibrium exists. Let u denote the number of bad signals reported in the discussion round (before the vote). First consider the voting stage. All board members observe u and since signals were reported truthfully, all board members infer that exactly u bad private signals were observed and $n-u$ good private signals were observed. All board members, therefore, have the same assessment of the CEO's quality. It is easy to see that if $u \geq \frac{n+1}{2}$, then $P(B|u) > 1-\gamma$ (the CEO's quality is worse than that of a potential replacement):

$$\begin{aligned} p(B|u \geq \frac{n+1}{2}) &\geq \frac{(1-\gamma)\alpha^{\frac{n+1}{2}}(1-\alpha)^{\frac{n-1}{2}}}{(1-\gamma)\alpha^{\frac{n+1}{2}}(1-\alpha)^{\frac{n-1}{2}} + \gamma(1-\alpha)^{\frac{n+1}{2}}\alpha^{\frac{n-1}{2}}} \\ &= \frac{1-\gamma}{1-\gamma+\gamma\frac{1-\alpha}{\alpha}} > 1-\gamma \text{ (since } \alpha > \tfrac{1}{2}) \ . \end{aligned} \tag{A.2}$$

In the conjectured equilibrium, all board members vote informatively, i.e., each board member votes to fire the CEO when $u \geq \frac{n+1}{2}$ and votes to retain her otherwise. If a board member i votes to fire the CEO when $u \geq \frac{n+1}{2}$, his payoff (conditional on other board members playing the conjectured equilibrium) is given by:

$$\pi_i^{benchmark}(v_i = 1|u \geq \frac{n+1}{2}) = -q(1-\gamma). \tag{A.3}$$

If a board member i votes to retain the CEO when $u \geq \frac{n+1}{2}$, his payoff (conditional on other board members playing the conjectured equilibrium) is given by:

$$\pi_i^{benchmark}(v_i = 0|u \geq \frac{n+1}{2}) = -qp(B|u \geq \frac{n+1}{2}). \tag{A.4}$$

Since $P(B|u \geq \frac{n+1}{2}) > 1-\gamma$, it follows that $\pi_i^{benchmark}(v_i = 1|u \geq \frac{n+1}{2}) > \pi_i^{benchmark}(v_i = 0|u \geq \frac{n+1}{2})$. Hence, each board member votes to fire the CEO when $u \geq \frac{n+1}{2}$. Analogously, each board member votes to fire the CEO when $u < \frac{n+1}{2}$.

Now consider the discussion round. Index board members by i and let $e_i = 1$ if the i^{th} board member observes a bad signal ($s_i = b$); $e_i = 0$ if the signal is good ($s_i = g$). Given the outcome of the voting round, the expected payoff from reporting a bad signal for a board member who privately observes a bad signal (conditional on other board members playing the conjectured equilibrium) is given by $-q(1-\gamma)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big) - qp(B|b)p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)$. The expected payoff from reporting a good signal for a board member who privately observes a bad signal (conditional on other board members playing the conjectured equilibrium) is given by $-q(1-\gamma)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+3}{2}\big|b\big) - qp(B|b)p\big(\sum_{j=1}^{n} e_j < \frac{n+3}{2}\big|b\big)$. The probabilities changed in the latter case because if this board member reports a good signal, the total number of bad private signals observed by all board

needs to be at least $\frac{n+3}{2}$ in order for the CEO to be fired (since this board member's signal will be reported as good). Since $p(B|b) > 1-\gamma$ and $p\big(\sum_{j=1}^{n} e_j < \frac{n+3}{2}\big|b\big) > p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)$, this board member's payoff is lower if he reports a good signal. Hence, he will report his signal truthfully. Analogously, a board member with a good private signal will also report it truthfully in the discussion round.

Part (ii). Assume that the number of board members who observe bad private signals about the CEO's quality equals r. Then, the updated assessment that the CEO is of bad quality, given all board members private information, can be written as

$$p(B|r) = \frac{(1-\gamma)\alpha^r(1-\alpha)^{n-r}}{(1-\gamma)\alpha^r(1-\alpha)^{n-r} + \gamma(1-\alpha)^r\alpha^{n-r}}. \tag{A.5}$$

The CEO should be fired if and only if $p(B|r) > 1-\gamma$, which with some algebra simplifies into

$$\begin{aligned} &\alpha^r(1-\alpha)^{n-r} > (1-\alpha)^r\alpha^{n-r} \;\Leftrightarrow \\ &2r > n \text{ (since } \alpha > \tfrac{1}{2}) \text{ .} \end{aligned} \tag{A.6}$$

We assumed that n is odd. Hence, the CEO should be fired when there is at least a majority of bad signals about her quality and be retained otherwise, which is exactly the outcome of part (i).

Proof of Proposition 2

Since n is odd, let $n = 2m+1$. Let $P(k;n)$ be the probability that there are at least k votes out of n for the correct outcome (to fire the CEO if she is of poor quality and to keep the CEO if she is of high quality). We will show that $P(m+1;2m+1) > P(m;2m-1)$. That is, it is more likely that the majority of votes (since $m+1 = \frac{n+1}{2}$ if $n = 2m+1$) will be in the right direction when the number of directors grows from $2m-1$ to $2m+1$ (we increase the number of directors by 2 to keep it odd).

There are three possible combinations of the signals observed by the added directors (after $2m-1$ initial signals):

1. their signal is split (one observes a bad signal and the other observes a good signal), which happens with probability $2\alpha(1-\alpha)$;
2. both of them observe the right signal, with probability α^2;
3. both of them observe the wrong signal, with probability $(1-\alpha)^2$.

Notice that

$$\begin{aligned} P(m+1,2m+1) = 2\alpha(1-\alpha)P(m;2m-1) + \alpha^2 P(m-1;2m-1) \\ + (1-\alpha)^2 P(m+1;2m-1). \end{aligned} \tag{A.7}$$

(A.7) follows by the following logic. The left-hand side is the probability that at least $m+1$ directors observe the correct signal. If the signal of the new board members is split, which happens with probability $2\alpha(1-\alpha)$, then to have at least $m+1$ correct signals it must be the case that there were at least m correct signals among the initial $2m-1$ directors, which happens with probability $P(m,2m-1)$. If both of the added board members observe correct signals, $m-1$ correct signals among the initial $2m-1$ directors suffices to bring the total to $m+1$. If both of the added board members observe wrong signals, there must already be $m+1$ correct signals.

Observe that

$$\begin{aligned} P(m-1,2m-1) &= P(m,2m-1) + \binom{2m-1}{m-1}\alpha^{m-1}(1-\alpha)^m, \\ P(m+1,2m-1) &= P(m,2m-1) - \binom{2m-1}{m}\alpha^{m}(1-\alpha)^{m-1}. \end{aligned} \tag{A.8}$$

Hence,

$$\begin{aligned} &P(m+1;2m+1) - P(m;2m-1) = \\ &\alpha^2\binom{2m-1}{m-1}\alpha^{m-1}(1-\alpha)^m - (1-\alpha)^2\binom{2m-1}{m}\alpha^m(1-\alpha)^{m-1} = \\ &\binom{2m-1}{m-1}\alpha^m(1-\alpha)^m(2\alpha-1) > 0, \end{aligned} \tag{A.9}$$

since $\binom{2m-1}{m-1} = \binom{2m-1}{m}$ and $\alpha > \frac{1}{2}$.

Proof of Proposition 3

Part (i). Our setup is different from the standard voting literature, which usually considers only the pivotal voter. In the standard literature, votes of non-pivotal votes do not affect their payoffs. In our setting, on the other hand, even if a board member is not pivotal, his vote can affect his payoff: if he votes to fire the CEO but there are not enough votes to fire her, he will suffer the dissent-cost. Analogously, in the discussion round, what a board member reports to other board members can affect his payoffs even if he is not pivotal (via dissent-costs). Hence, board members will calculate their payoffs not just conditional on being pivotal but also conditional on the probability of being on a particular side of the vote given how other board members vote and what signals they report.

Let u denote the number of bad signals reported in the discussion round (before the vote). First consider the voting stage. All board members observe u and if signals were reported truthfully, all board members infer that exactly u bad private signals were observed and $n-u$ good private signals were observed. All board members, therefore, have the same assessment of the CEO's quality: if $u \geq \frac{n+1}{2}$, then $P(B|u) \geq 1-\gamma$ (the CEO's quality is worse than that of a potential replacement). If all board members vote to fire the CEO when $u \geq \frac{n+1}{2}$ and vote to retain her otherwise, no board member is ever pivotal and nobody incurs the dissent-costs at the voting stage (however, some board members will expect to incur dissent-costs in the discussion round). A board member who deviates from the conjectured equilibrium by changing his vote can only lower his payoff: if all other board members vote to retain the CEO, a board member voting to fire her will incur additional dissent costs. Now consider the discussion round. If all board members report their true signal, a director with a good private signals never incurs a dissent-cost, and has no incentive to deviate from the equilibrium. A director with a bad private signal incurs dissent-costs in the discussion round if he communicates a bad signal but not enough other directors have bad private signals to obtain majority. Given the voting strategy at the voting stage in the conjectured equilibrium, if board members with bad signals report their true signals, the CEO will be fired with the probability $p\big(\sum_{j=1}^n e_j \geq \frac{n+1}{2}\big|b\big)$. If board members with bad signals report good signals, the CEO is never fired. Hence, for a director with a bad private signal, the expected payoff from reporting his true signal is given by $-\big(\rho c + qp(B|b)\big)p\big(\sum_{j=1}^n e_j < \frac{n+1}{2}\big|b\big) - q(1-\gamma)p\big(\sum_{j=1}^n e_j \geq \frac{n+1}{2}\big|b\big)$. The

payoff from deviating, assuming that board members with good signals do not deviate, is given by $-qp(B|b)$. Board members with bad private signals have no incentive to deviate if and only if

$$-qp(B|b) \leq -\big(\rho c + qp(B|b)\big)p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big) - q(1-\gamma)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big). \tag{A.10}$$

With some algebra, we can transform (A.12) into

$$\frac{\rho c}{q} \leq \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)\big(p(B|b) - (1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)}. \tag{A.11}$$

Hence, the conjectured equilibrium exists when $\frac{\rho c}{q} \leq \frac{p\left(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\right)\left(p(B|b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\right)}$.

Part (ii). We prove by contraction. First, observe that a board member with a good private signal always (weakly) prefers to reveal his true signal in the communication stage. This way he avoids the cost of dissent (in the discussion round) and increases the chances that a good CEO will stay when his private assessment of the CEO is higher than that of a potential replacement since $p(B|g) < 1 - \gamma$.

Consider the case when board members with bad signals report them truthfully in the discussion round. Start with the voting round. Since directors with good signals always report good signals, the number of bad signals reported in the discussion round equals to the number of bad signals observed by all board members. All board members, therefore, have the same assessment of the CEO's quality and we can treat them as having the same type. The CEO is fired when the assessment of his quality by all board member is lower than that of a potential replacement: if $u \geq \frac{n+1}{2}$, then $P(B|u) \geq 1 - \gamma$. No board member suffers dissent-costs at the voting stage. Now consider the discussion round. Given the strategy of the voting stage, the CEO is fired with the probability that $u \geq \frac{n+1}{2}$ bad signals are reported. For a board member who privately observes a bad signal, the CEO will be fired with the probability $p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)$. If board members with bad signals report good signals, the CEO is never fired. Hence, for a director with a bad private signal, the expected payoff from reporting his signal truthfully is given by $-\big(\rho c + qp(B|b)\big)p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big) - q(1-\gamma)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)$. The payoff from deviating, assuming that board members with good signals do not deviate, is given by $-qp(B|b)$. Board members with bad private signals have no incentive to deviate if and only if

$$-qp(B|b) \leq -\big(\rho c + qp(B|b)\big)p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big) - q(1-\gamma)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big). \tag{A.12}$$

With some algebra, we can transform (A.12) into

$$\frac{\rho c}{q} \leq \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)\big(p(B|b) - (1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)}. \tag{A.13}$$

We assumed that $\frac{\rho c}{q} > \frac{p\left(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\right)\left(p(B|b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\right)}$. Contradiction. Hence, board members with bad signals always deviate and report good signals.

Now consider the case when board members with bad private signals report good signals at the communication stage. In this case, the communication stage is totally uninformative and doesn't change the information available to the players. At the voting stage, therefore, a board member with good private signals (weakly) prefers to vote to retain the CEO: he avoids the dissent-cost and votes for the alternative he prefers if he is pivotal. The only equilibrium in which the CEO can be fired is when board members with bad private signals vote to fire her. If board members with bad signals vote to retain the CEO, the CEO is never fired. Hence, for a director with a bad private signal, the expected payoff from voting informatively is given by $-\big(c+qp(B|b)\big)p\big(\sum_{j=1}^{n}e_j<\frac{n+1}{2}\big|b\big)-q(1-\gamma)p\big(\sum_{j=1}^{n}e_j\geq\frac{n+1}{2}\big|b\big)$ (instead of ρc, his dissent-cost is now given by c). The payoff from deviating, assuming that board members with good signals do not deviate, is given by $-qp(B|b)$. Board members with bad private signals have no incentive to deviate if and only if

$$-qp(B|b)\leq-\big(c+qp(B|b)\big)p\big(\sum_{j=1}^{n}e_j<\frac{n+1}{2}\big|b\big)-q(1-\gamma)p\big(\sum_{j=1}^{n}e_j\geq\frac{n+1}{2}\big|b\big). \tag{A.14}$$

With some algebra, we can transform (A.14) into

$$\frac{c}{q}\leq\frac{p\big(\sum_{j=1}^{n}e_j\geq\frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n}e_j<\frac{n+1}{2}\big|b\big)}. \tag{A.15}$$

We assumed that $\frac{\rho c}{q}>\frac{p\big(\sum_{j=1}^{n}e_j\geq\frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n}e_j<\frac{n+1}{2}\big|b\big)}$ and since $\rho<1$, it follows that $\frac{c}{q}>\frac{p\big(\sum_{j=1}^{n}e_j\geq\frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n}e_j<\frac{n+1}{2}\big|b\big)}$. Contradiction. Hence, board members with bad signals always deviate and vote to retain the CEO.

Proof of Proposition 4

Compare equilibrium outcomes in Proposition 1 and Proposition 3. It is immediately clear that they coincide except when $\frac{\rho c}{q}>\frac{p\big(\sum_{j=1}^{n}e_j\geq\frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n}e_j<\frac{n+1}{2}\big|b\big)}$. In that case, had there been no dissent-cost, the CEO would have been fired with a positive probability. In the presence of dissent-costs the board will never fire the CEO. Since the equilibrium in Proposition 1 efficiently aggregates all board members' information, any deviation from this equilibrium is suboptimal.

Proof of Proposition 5

Let $P(k,n)\equiv p\big(\sum_{j=1}^{n}e_j\geq k\big|b\big)$. It suffices to show that $P(m,2m-1)\lessgtr P(m+1,2m+1)$ when $\gamma\lessgtr\alpha$. Let $\lambda\equiv p(e_j=1|b)=\frac{\gamma-2\gamma\alpha+\alpha^2}{\gamma-2\gamma\alpha+\alpha}$ (the probability that a board member j observes a bad signal, conditional on board member i observing a bad signal. The proof parallels that of Proposition 2, with λ replacing α. If $\lambda\gtrless\frac{1}{2}$, then $P(m+1,2m+1)\gtrless P(m,2m-1)$. This happens when

$$\frac{\gamma-2\gamma\alpha+\alpha^2}{\gamma-2\gamma\alpha+\alpha}\gtrless\frac{1}{2}\Leftrightarrow(\gamma-\alpha)(1-2\alpha)\gtrless0\Leftrightarrow\alpha\gtrless\gamma,\text{ since }\alpha>\frac{1}{2}. \tag{A.16}$$

Proof of Proposition 6

Part (i). By Proposition 5, if $\alpha < \gamma$, when n increases, the value of $\frac{p\left(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} \middle| b\right)\left(p(B|b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < \frac{n+1}{2} \middle| b\right)}$ decreases. Hence, for any given value of $\frac{\rho c}{q}$, there exists a maximum value of n so that (A.13) holds. Denote it n^*. We will show that n^* dominates any $n > n^*$ and any $n < n^*$. First, consider some $n' > n^*$. It follows from Proposition 5 that a board consisting of n' members will always retain the incumbent, regardless of directors' signals, while at n^* a bad CEO is fired with a positive probability. Hence, n^* dominates n'. Now, consider some $n'' < n^*$. The board votes informatively. Hence, the outcome of the vote coincides with the social planner's choice. By Proposition 2, however, n^* produces more efficient outcomes. Hence, n^* dominates n'' as well.

Part (ii). By Proposition 5, adding more board members increases the value of $\frac{p\left(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} \middle| b\right)\left(p(B|b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < \frac{n+1}{2} \middle| b\right)}$ above which informative voting is unsustainable. Therefore, adding more board members never decreases the probability of informative voting and at the same time increases the amount of information available to the board. In particular, if $\frac{\rho c}{q}$ is below the threshold of informative voting when the number of board members is N, it will remain under the threshold for all $n \geq N$, while the number of independent signals about the CEO's quality will be larger when the board is larger. If $\frac{\rho c}{q}$ is above the threshold of informative voting when the number of board members is N, it is possible that at some $n > N$ the threshold will increase sufficiently for informative voting to be possible. Hence, adding board members beyond N never makes the voting outcome less efficient and sometimes makes it more efficient.

Proof of Proposition 7

Part (i). Since insiders have more precise signals, it is optimal to have as many of them as possible as long as they report their signals truthfully and vote accordingly. If a board consists only of insiders, the game is analogous to the game in our basic model with dissent-costs, with b replacing c and β replacing α (the equilibrium in this case is analogous to the equilibrium in the basic model with dissent-costs, described in Proposition 3). When $\frac{\rho b}{q} \leq \frac{p\left(\sum_{j=1}^{n} e_j^{ins} \geq \frac{n+1}{2} \middle| s_i^{ins}=b\right)\left(p(B|s_i^{ins}=b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j^{ins} < \frac{n+1}{2} \middle| s_i^{ins}=b\right)}$, an insiders-only board will vote informatively and is therefore optimal in this case.

Part (ii). When $\frac{\rho b}{q} > \frac{p\left(\sum_{j=1}^{n} e_j^{ins} \geq \frac{n+1}{2} \middle| s_i^{ins}=b\right)\left(p(B|s_i^{ins}=b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j^{ins} < \frac{n+1}{2} \middle| s_i^{ins}=b\right)}$, an insiders-only board does not report signals truthfully and therefore cannot sustain informative voting. Assume that there are h insiders on the board. Insiders who privately receive a bad signal now have to assess how many other insiders observe a bad signal as well as how many outsiders observe a bad signal. Analogously, outsiders who receive a bad private signal have to assess how many other outsiders observe a bad signal as well as how many insiders observe a bad signal. The game for insiders is now analogous to our basic model with dissent-costs, the only difference being that the probability of suffering an equity loss or dissent-costs is affected by votes of outsiders. Similarly, the game for outsiders is now analogous to our basic model with dissent-costs, the only difference being that the probability of suffering an equity loss or dissent-costs has is affected by votes of insiders.

By the logic analogous to Proposition 3, both insiders and outsiders report their private signals truthfully and vote informatively after that when, simultaneously,

$$\frac{\rho b}{q} \leq \frac{p\left(\sum_{j=1}^{h} e_j^{ins} + \sum_{l=1}^{n-h} e_l^{outs} \geq \frac{n+1}{2} \middle| s_i^{ins} = b\right)\left(p(B|s_i^{ins} = b) - (1-\gamma)\right)}{p\left(\sum_{j=1}^{h} e_j^{ins} + \sum_{l=1}^{n-h} e_l^{outs} < \frac{n+1}{2} \middle| s_i^{ins} = b\right)} \tag{A.17}$$

and

$$\frac{\rho c}{q} \leq \frac{p\big(\sum_{j=1}^{h} e_j^{ins} + \sum_{l=1}^{n-h} e_l^{outs} \geq \frac{n+1}{2} \big| s_i^{outs} = b\big)\big(p(B|s_i^{outs} = b) - (1-\gamma)\big)}{p\big(\sum_{j=1}^{h} e_j^{ins} + \sum_{l=1}^{n-h} e_l^{outs} < \frac{n+1}{2} \big| s_i^{outs} = b\big)}. \tag{A.18}$$

Hence, if there exists a value h for which (A.17) and (A.18) hold simultaneously, such a board will share information. Since insiders have more precise signals, the highest value of h for which informative voting is possible is the optimal number of insiders. For example, let a board consist of eleven board members, with the following parameters: $\gamma = 0.7, \gamma = 0.7, \alpha = 0.6, \beta = 0.65$, $\frac{\rho c}{q} = 0.1$, $\frac{\rho b}{q} = 0.2$. Then, the board should consist of nine insiders and two outsiders.

Part (iii). If there exists no value h for which (A.17) and (A.18) hold simultaneously, insiders will always vote to retain the CEO, regardless of their private signals. In this case, having insiders on the board does not improve decision-making. On the other hand, similar to our basic model with dissent-costs, if $\frac{\rho c}{q} \leq \frac{p\big(\sum_{j=1}^{n} e_j^{ins} \geq \frac{n+1}{2} \big| s_i^{outs} = b\big)\big(p(B|s_i^{outs} = b) - (1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j^{ins} < \frac{n+1}{2} \big| s_i^{outs} = b\big)}$, then an outsiders-only board will vote informatively. Consequently, only outsiders should be on the board.

Proof of Proposition 8

Conditional on observing public signal H, the game with public signals is equivalent to the game in our basic model with dissent costs, with $p(B|g, H)$ replacing $p(B|g)$ and $p(B|b, H)$ replacing $p(B|b)$. Hence, by Proposition 3, the board always retains the CEO unless

$$\frac{\rho c}{q} \leq \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} | b, H\big)\big(p(B|b, H) - (1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2} | b, H\big)}. \tag{A.19}$$

Analogously, conditional on observing L, directors will share their private information and vote accordingly if and only if

$$\frac{\rho c}{q} \leq \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2} | b, L\big)\big(p(B|b, L) - (1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2} | b, L\big)}. \tag{A.20}$$

The proofs of Proposition 9 and Proposition 10 are lengthy and are therefore confined to a separate online appendix, not to be published.

B Appendix B: Proofs of Propositions 9 and 10 (not to be published)

Proof of Proposition 9

First we calculate updated probabilities conditional on observing a particular public signal.

$$\begin{aligned} p(B|g,H) &= \frac{p(B)p(g,H|B)}{p(B)p(g,H|B)+p(G)p(g,H|G)} \\ &= \frac{p(B)p(g|B)p(H|B)}{p(B)p(g|B)p(H|B)+p(G)p(g|G)p(H|G)} \\ &= \frac{(1-\gamma)(1-\alpha)(1-\phi)}{(1-\gamma)(1-\alpha)(1-\phi)+\gamma\alpha\phi}, \end{aligned} \tag{B.1}$$

where the second line follows from the assumption of conditional independence between public and private signals. It is easy to see that $p(B|g,H) < p(B|g)$:

$$\begin{aligned} &\frac{(1-\gamma)(1-\alpha)(1-\phi)}{(1-\gamma)(1-\alpha)(1-\phi)+\gamma\alpha\phi} < \frac{(1-\gamma)(1-\alpha)}{(1-\gamma)(1-\alpha)+\gamma\alpha} \Leftrightarrow \\ &\frac{\gamma\alpha}{(1-\gamma)(1-\alpha)} < \frac{\gamma\alpha\phi}{(1-\gamma)(1-\alpha)(1-\phi)} \Leftrightarrow \\ &\phi > \frac{1}{2}. \end{aligned} \tag{B.2}$$

Similarly,

$$p(B|g,L) = \frac{(1-\gamma)(1-\alpha)\phi}{(1-\gamma)(1-\alpha)\phi+\gamma\alpha(1-\phi)} > p(B|g), \tag{B.3}$$

$$p(B|b,H) = \frac{(1-\gamma)\alpha(1-\phi)}{(1-\gamma)\alpha(1-\phi)+\gamma(1-\alpha)\phi} < p(B|b), \tag{B.4}$$

$$p(B|b,L) = \frac{(1-\gamma)\alpha\phi}{(1-\gamma)\alpha\phi+\gamma(1-\alpha)(1-\phi)} > p(B|b). \tag{B.5}$$

The probability of director j observing private signal b, given director i's signal, can be rewritten as

$$p(s_j = b|s_i) = (1-\alpha) + (2\alpha-1)p(B|s_i). \tag{B.6}$$

(B.6) is increasing in $p(B|s_i)$. We showed above that $p(B|b,H) < p(B|b) < p(B|b,L)$. Hence, $p(s_j = b|b,H) < p(s_j = b|b) < p(s_j = b|b,L)$. It follows from the properties of the Binomial distribution that $p(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b,H) < p(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b) < p(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b,L)$ and

$p(\sum_{j=1}^{n} e_j < \frac{n+1}{2}|b,H) > p(\sum_{j=1}^{n} e_j < \frac{n+1}{2}|b) > p(\sum_{j=1}^{n} e_j < \frac{n+1}{2}|b,L)$. Therefore,

$$\begin{aligned}
&\frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b,H\big)\big(p(B|b,H)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b,H\big)} < \\
&\frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)} < \\
&\frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b,L\big)\big(p(B|b,L)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b,L\big)}.
\end{aligned} \tag{B.7}$$

We let $\Upsilon(n,\alpha,\gamma|b,H) \equiv \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b,H\big)\big(p(B|b,H)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b,H\big)}$,
$\Upsilon(n,\alpha,\gamma|b) \equiv \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)}$, and
$\Upsilon(n,\alpha,\gamma|b,L) \equiv \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b,L\big)\big(p(B|b,L)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b,L\big)}$.

With this notation, (B.7) can be rewritten as $\Upsilon(n,\alpha,\gamma|b,H) < \Upsilon(n,\alpha,\gamma|b) < \Upsilon(n,\alpha,\gamma|b,L)$. The rest of this proposition follows from the equilibrium described in Proposition 8 and the relation between $\Upsilon(n,\alpha,\gamma|b,H)$, $\Upsilon(n,\alpha,\gamma|b)$, and $\Upsilon(n,\alpha,\gamma|b,L)$ that we have just derived.

Proof of Proposition 10

We solve starting with the outcome of the second round of voting. The CEO is never fired in the first round if $\frac{\rho c}{q} > \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)}$. We prove by contradiction. Assume the CEO was fired in the first round of voting (at time 3). In this case, the board hired a replacement and at time 5 (the second round of communication and subsequent voting) the game is equivalent to the initial game. Given that $\frac{\rho c}{q} > \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)}$, Proposition 3 implies that the board doesn't fire the CEO in the second round (at time 5): directors simply keep the replacement they hired after the first round of voting. The true quality of this replacement will be revealed at time 5. Going back to time 3, the game is now equivalent to the initial game because each director knows that if a replacement is hired, she will stay in the second round and therefore they will learn her true quality at date 5. However, since $\frac{\rho c}{q} > \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)}$, they can never fire the CEO in the initial game, yielding a contradiction.

Now let $\frac{\rho c}{q} \leq \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\big|b\big)\big(p(B|b)-(1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\big|b\big)}$. If the CEO was retained in the first round of voting, then the outcome of the second round of voting is completely determined by the public signal observed at time 4. This follows because $p(B|g,H) < p(B|b,H) < 1-\gamma < p(B|g,L) < p(B|b,L)$ by assumption of informative private signals and a very precise public signal. Given this, we can calculate the expected equity loss from having the CEO retained in the first round of voting

conditional on director i's signal:

$$\begin{aligned}
Q(retained|s_i) =& -qp(G|s_i)p(L|G) - q(1-\gamma)p(G|s_i)p(L|G) \\
& - qp(B|s_i)p(L|B) - q(1-\gamma)p(B|s_i)p(L|B) - 2qp(B|s_i)p(H|B) \\
=& -q(2-\gamma)p(G|s_i)(1-\phi) - 2qp(B|s_i)(1-\phi) - q(2-\gamma)p(B|s_i)\phi \\
=& -q(2-\gamma)(1-\phi) - qp(B|s_i)(1+(2\phi-1)(1-\gamma)) \\
\equiv& \, qA(\gamma,\phi,\alpha,n|s_i).
\end{aligned} \tag{B.8}$$

It is easy to see that $qA(\gamma,\phi,\alpha,n|g) > qA(\gamma,\phi,\alpha,n|b)$, since $1+(2\phi-1)(1-\gamma) > 0$ and $p(B|b) > p(B|g)$. Hence, the value of waiting is greatest for directors who already think that the CEO is good with a high probability.

If the CEO was fired in the first round of voting, the game starting at t=4 is equivalent to the game in our basic model with dissent-costs. We now calculate the expected payoff of that game to each board member before he observes his private signal (this is the payoff he expects to receive conditional on having the CEO fired at t=3). Since $\frac{\rho c}{q} \leq \frac{p\left(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}\middle|b\right)\left(p(B|b)-(1-\gamma)\right)}{p\left(\sum_{j=1}^{n} e_j < \frac{n+1}{2}\middle|b\right)}$, board members will report their signals truthfully and vote accordingly in the second round, by Proposition 3. Hence, the CEO will be fired with the probability that $\frac{n+1}{2}$ or more directors observe a bad signal, and will be retained otherwise. Let V denote the number of directors who observe a bad signal in the second round. V is binomially distributed, with the distribution parameter $1-\alpha$ if the true quality of the replacement hired after the first round of voting is good, and α otherwise. Thus, the time t=3 payoff from having the CEO fired in the first round is given by:

$$\begin{aligned}
Q(fired) =& -\gamma p(b|G)\rho c p(V < \frac{n+1}{2}|G) - \gamma p(b|G)q(1-\gamma)p(V \geq \frac{n+1}{2}|G) \\
& - \gamma p(g|G)q(1-\gamma)p(V \geq \frac{n+1}{2}|G) - (1-\gamma)p(b|B)(\rho c+q)p(V < \frac{n+1}{2}|B) \\
& - (1-\gamma)p(b|B)q(1-\gamma)p(V \geq \frac{n+1}{2}|B) - (1-\gamma)p(g|B)qp(V < \frac{n+1}{2}|B) \\
& - (1-\gamma)p(g|B)q(1-\gamma)p(V \geq \frac{n+1}{2}|B) \\
=& -\rho c\gamma(1-\alpha)p(V < \frac{n+1}{2}|G) - \rho c(1-\gamma)\alpha p(V < \frac{n+1}{2}|B) \\
& - q\gamma(1-\alpha)(1-\gamma)p(V \geq \frac{n+1}{2}|G) - q\gamma\alpha(1-\gamma)p(V \geq \frac{n+1}{2}|G) \\
& - q(1-\gamma)\alpha p(V < \frac{n+1}{2}|B) - q(1-\gamma)\alpha(1-\gamma)p(V \geq \frac{n+1}{2}|B) \\
& - q(1-\gamma)(1-\alpha)p(V < \frac{n+1}{2}|B) - q(1-\gamma)(1-\alpha)(1-\gamma)p(V \geq \frac{n+1}{2}|B) \\
\equiv& \, \rho c B(\gamma,\alpha,n) + qC(\gamma,\alpha,n).
\end{aligned} \tag{B.9}$$

If $qA(\gamma,\phi,\alpha,n|g) > qA(\gamma,\phi,\alpha,n|b) > cB(\gamma,\alpha,n) + qC(\gamma,\alpha,n)$, it is always optimal to retain the incumbent CEO, regardless of the signal. If, on the other hand, $cB(\gamma,\alpha,n) + qC(\gamma,\alpha,n) > qA(\gamma,\phi,\alpha,n|g) > qA(\gamma,\phi,\alpha,n|b)$, then it is always optimal to fire the incumbent. We therefore focus on the interesting case when $qA(\gamma,\phi,\alpha,n|g) > cB(\gamma,\alpha,n) + qC(\gamma,\alpha,n) > qA(\gamma,\phi,\alpha,n|b)$. Directors with signal g always report that they observe a good signal, so that it is the directors

with signal b who determine the equilibrium. Considering the cost of dissent, they choose to report their private signals truthfully if and only if:

$$\begin{aligned} qA(\gamma,\phi,\alpha,n|b) \leq & -\big(\rho c - qA(\gamma,\phi,\alpha,n|b)\big)p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}|b\big) \\ & + \big[\rho c B(\gamma,\alpha,n) + qC(\gamma,\alpha,n)\big]p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b\big). \end{aligned} \tag{B.10}$$

Otherwise, all board members report good signals and vote to retain the CEO. (B.10) can be rewritten as

$$\frac{\rho c}{q} \leq \frac{\Big(- A(\gamma,\phi,\alpha,n|b) + C(\gamma,\alpha,n)\Big)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}|b\big) - B(\gamma,\alpha,n)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b\big)}. \tag{B.11}$$

Define

$$\begin{aligned} \Psi(\gamma,\alpha,\phi,n) \equiv \min\Bigg\{ & \frac{\Big(- A(\gamma,\phi,\alpha,n|b) + C(\gamma,\alpha,n)\Big)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}|b\big) - B(\gamma,\alpha,n)p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b\big)}, \\ & \frac{p\big(\sum_{j=1}^{n} e_j \geq \frac{n+1}{2}|b\big)\big(p(B|b) - (1-\gamma)\big)}{p\big(\sum_{j=1}^{n} e_j < \frac{n+1}{2}|b\big)} \Bigg\}. \end{aligned} \tag{B.12}$$

Then, if $\frac{\rho c}{q} > \Psi(\gamma,\alpha,\phi,n)$, the CEO is never fired in the first round and if $\frac{\rho c}{q} \leq \Psi(\gamma,\alpha,\phi,n)$, the CEO is fired in the first round with a positive probability (equal to the probability that $\frac{n+1}{2}$ directors observe a bad signal, given the CEO's true quality).