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Can We Forecast the Implied Volatility Surface Dynamics of Equity Options? Predictability and Economic Value Tests

Alejandro Bernales and Massimo Guidolin[*]

Abstract

We examine whether the dynamics of the implied volatility surface of individual equity options contains exploitable predictability patterns. Predictability in implied volatilities is expected due to the learning behavior of agents in option markets. In particular, we explore the possibility that the dynamics of the implied volatility surface of individual equity options may be associated with movements in the volatility surface of S&P 500 index options. We present evidence of strong predictable features in the cross-section of equity options and of dynamic linkages between the implied volatility surfaces of equity options and S&P 500 index options. Moreover, time-variations in stock option volatility surfaces are best predicted by incorporating information from the dynamics in the implied volatility surface of S&P 500 index options. We analyze the economic value of such dynamic patterns using strategies that trade straddle and delta-hedged portfolios, and we find that before transaction costs such strategies produce abnormal risk-adjusted returns.

Keywords: Equity options; Index options; Implied volatility surface; Predictability; Trading strategies.

JEL Codes: C53, G13, G17.

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1. Introduction

Contrary to the constant volatility assumption of the Black and Scholes' (1973) model (henceforth BS model), the volatilities implicit in option contracts written on the same underlying asset differ across strike prices and time-to-maturities. This phenomenon is known as the implied volatility surface (henceforth, *IVS*).[1] In addition, there is abundant empirical evidence of predictable movements of the *IVS* (e.g., Dumas *et al.*, 1998, Cont and Fonseca, 2002, Gonçalves and Guidolin, 2006, and Fengler *et al.*, 2007). These studies show that the shape of the *IVS* in its two key dimensions, moneyness and time-to-maturity, would evolve over time in ways that can be forecasted using simple models. However, the financial literature has focused its attention mainly on the predictability of the *IVS* of index options, such as S&P 500 index options. As a result, the existence of similar dynamics involving the *IVS* of individual equity options has remained relatively under-researched. Moreover, the existence of potential dynamic relationships between the *IVS* of options written on equities and the *IVS* of index options has not been investigated, even though it may be of great practical importance. For instance, the dynamics in the *IVS* of index options could help traders and hedgers anticipate movements in the *IVS* of individual equity options, which may be highly valuable for the design of either speculative or hedging positions. The objective of this paper is therefore to fill these gaps by studying firstly the unexplored predictable dynamics in the *IVS* of equity options, and secondly, their relationships with movements in the volatility surface implicit in index options.

There are both strong academic and practical reasons to pursue a systematic investigation of the *IVS* dynamics in individual equity options. From an academic perspective, Gonçalves and Guidolin (2006) have analyzed how predictable the S&P 500 *IVS* has been over a 1992-1998 sample. They find that predictability of the S&P 500 *IVS* is strong, but fail to find compelling evidence that such predictable movements may easily translate in positive risk-adjusted profits net of sensible trading costs. Therefore, they conclude that their findings fail to represent first-order evidence that contradicts the efficient market hypothesis. On the one hand, this result provides a motivation to investigate whether alternative segments of the equity options market can be isolated in which *IVS* predictability may not only hold as a statistical fact, but also signal the existence of important pockets of market inefficiency. In fact, we would expect that such pockets of inefficiency may exist exactly with reference to options that are less liquid than S&P

[1] See, e.g., Rubinstein (1985), Campa and Chang (1995), and Das and Sundaram (1999).

500 index options. On the other hand, especially if the efficient market hypothesis is imposed so that any *IVS* predictability is traced back to either micro-structural imperfections or to unobserved and hard-to-estimate time-varying risk premia; then financial economists might have a lot to learn from a careful study of the cross-sectional distribution of predictability and/or economic value "scores" caused by *IVS* predictability.[2]

Understanding the *IVS* dynamics of equity options is not only crucial to participants in option markets such as market makers, option traders, or investors who aim at hedging equity option positions. Knowledge of the dynamic process of the *IVS* is also relevant for investment decisions in other markets, since option securities have been commonly used to obtain forward-looking market information. Forward-looking analyses based on option market information rely on the assumption that option prices should reveal agents' expectations about prospective economic scenarios, where the horizons of investors' forecasts correspond to the expiry dates of traded option contracts.[3] In practice, trading desks are often interested in estimating the dynamic process followed by the *IVS* of individual equity options, with the objective of taking strategic positions to hedge existing portfolios or other over-the-counter exotic derivatives offered to institutional customers. However, because trading volume may often be lumpy in individual equity option markets, it is at least doubtful that real-time updates of the entire equity option *IVS* may be feasible in practice. In fact, a non-negligible portion of all existing equity option contracts may be classified as infrequently traded securities. Therefore, given that investors are eager to learn any new information relevant to predict equity option *IVS* in real time, they are likely to be ready to avail themselves also of information revealed by transactions involving more liquid and related

[2] Examples of predictability "scores" are the root mean-squared prediction error or the mean absolute prediction error for *h*-step ahead BS implied volatilities. Examples of economic value "scores" are average trading profits or realized Sharpe ratios from trading strategies built on a given *IVS* dynamic model. Section 4 provides details on all the criteria used in our paper to measure predictability and its economic value.

[3] Option prices have been recently used in many occasions to capture forward-looking information on the dynamic process of asset returns (e.g., Xing *et al.*, 2010, and Bakshi *et al.*, 2011), their realized volatilities (e.g., Christensen and Prabhala, 1998, and Busch *et al.*, 2011), risk premiums (e.g., Duan and Zhang, 2010), betas (e.g., Siegel, 1995, and Chang *et al.*, 2009), correlation coefficients (e.g., Driessen *et al.*, 2009), and to solve forward-looking asset allocation problems (e.g., Kostakis *et al.*, 2011).

option contracts, such as those typically written on major market indices.[4] Consequently, in this paper we also endeavour to test whether there is any forecasting power in movements in the S&P 500 index options *IVS* for subsequent dynamics in the *IVS* of individual equity options. In this context, it is surprising that empirical research on derivatives has remained scarce when it comes to investigating similar relationships between the *IVS* of equity options and the *IVS* of market index options.[5] This may also be seen as an additional, novel academic contribution of our paper: in the same way that all students of finance apply the simple CAPM in their analyses in which the individual stock volatility moves proportionally with market volatility (e.g., as represented by the S&P 500 index), in our paper we test whether such relationship may also hold for the *IV* surfaces of equity options and index options.[6]

In our paper, we use daily data from individual equity and S&P 500 index options traded on the U.S. markets over the period 1996-2006. The choice of a sample that stops at the end of 2006 is also intended to provide evidence on the cross-sectional predictability dynamics in equity option *IVS* that is free from the effects of the recent U.S. financial turmoil of 2007-2009. Our modelling strategy is simple (one may argue, so simple that many trading desks may actually consider adopting it) and based on a two-stage econometric approach. First, we characterize the *IVS* of equity options and the *IVS* of S&P 500 index options by fitting on daily basis a straightforward deterministic *IVS* model. In this deterministic *IVS* model the dependent variable is the implied volatility (henceforth, also shortened as *IV*), and the explanatory variables are factors related to basic observable option contract features such as strike prices and time-to-maturities. Second, for each equity option we estimate a second-stage VARX predictive model in

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[4] In Section 2 we report market statistics concerning the trading activity levels on equity and index options. These statistics confirm, as one would expect, that index options are much more actively traded than even the most liquid individual equity options.

[5] Regarding the relationship of the *IVS* of equity options and the *IVS* of index options, it is important to mention the contributions of Dennis and Mayhew (2002) and Dennis *et al.* (2006), even though they do not explore directly the association of the shape characteristics between the equity option *IVS* and the market index option *IVS* as in our study. Dennis and Mayhew (2002) find that the skew of the risk neutral-neutral density implied in equity options is more negative when there is a high at-the-money implied volatility of S&P 500 index options. In addition, Dennis *et al.* (2006) use a similar relationship to the CAPM model with the implied volatilities (using at-the-money short-term option contracts) of equity option and the implied volatilities of S&P 500 index options; and thus to find the 'implied idiosyncratic volatility' present in equity options.

[6] Equity options and S&P 500 index options are also known as stock options and SPX options, respectively. In what follows, we will use any of these expressions/acronyms interchangeably, without any special or technical meaning.

which the endogenous variables are the time series coefficients estimated from the deterministic *IVS* models concerning each stock option from the first stage; while the exogenous variables are the time series coefficients estimated from deterministic *IVS* models for S&P 500 index options. In the following, we often refer to such VARX model as our 'dynamic equity-SPX *IVS* model'. Finally, the dynamic equity-SPX *IVS* model is used to recursively compute daily one-day-ahead forecasts for the *IVS* of individual equity options. The goal of our paper consists of assessing whether such a recursive, two-stage approach yields implied volatilities and option price forecasts that display any statistical accuracy (relative to benchmarks) and/or that may support valuable trading strategies.

We find evidence of strong cross-sectional relationships between the implied volatility surfaces of individual equity and S&P 500 index options. Moreover, we show that a remarkable amount of the variation in the *IVS* of stock options can be predicted using past dynamics in the *IVS* of S&P 500 index options. Firstly, we compare our VARX-type model (the dynamic equity-SPX *IVS* model) with a simpler VAR-type dynamic equity *IVS* model. This VAR-type dynamic equity *IVS* model follows a similar two-stage procedure as the dynamic equity-SPX *IVS* model describe above, but this benchmark model does not take into account the information from the *IVS* of S&P 500 index options. In particular, when we compare both models we find that the predictable dynamics in the *IVS* of stock options are better characterised by the VARX model that use the information in recent movements in the S&P 500 index *IVS*. The dynamic equity-SPX *IVS* model yields a superior one-day-ahead forecasting performance in comparison to the VAR-type framework that only includes information from past movements of the *IVS* of stock options. The intuition for this result comes from the slow updating process of the equity option *IVS* caused by the often modest trading frequency of a large fraction of stock option contracts. As a result, when such an updating is allowed to include information revealed by recent movements in the S&P 500 index *IVS*, the resulting forecasts out-perform the VAR-type model and other benchmarks, such as an *ad-hoc* 'strawman' random walk model for the first-stage deterministic *IVS* equity option coefficients (which is also used in Dumas *et al.*, 1998, and Christoffersen and Jacobs, 2004) and an option-GARCH model for American-style option contracts (see Duan and Simonato, 2001).

Furthermore, we also investigate the economic value of the predictable dynamics uncovered in the cross-section of the stock option *IVS*. We build a number of trading strategies that exploit the one-day-ahead forecasts of implied volatilities computed from the dynamic equity-SPX *IVS* model, and we compare their profits to those obtained by the benchmarks models discussed above. Of course, the idea of evaluating models under realistic economic loss functions typical of market traders—such as the profits derived from simple trading strategies—is not new in option markets (see, e.g., Day and Lewis, 1992, Harvey and Whaley, 1992, Bollen *et al*., 2000, Gonçalves and Guidolin, 2006, and Goyal and Saretto, 2009). However, such an effort becomes particularly crucial in the presence of complex back-testing exercises in which a relatively high number of parameters need to be recursively estimated, and hence an economic evaluation represents a natural and also interpretable way to guard against the dangers of over-fitting. Moreover, as already discussed, such trading strategies will allow us to ask whether any statistical evidence of predictable dynamics may represent a violation of the classical efficient market hypothesis. In this paper, we use straddle and delta-hedged strategies, which are free of risks caused by changes in the prices of the underlying stocks. We simulate daily $1,000 fixed-investment strategies that buy and sell straddles and delta-hedged option portfolios based on a simple principle: an option contract is purchased (sold) when a given model anticipates that the implied volatility for that option contract will increase (decrease) between t and $t+1$.[7] We find evidence of significant alphas using an asset pricing factor model that takes into account specific factors related to option securities, as in Coval and Shumway (2001).[8] However, most of this risk-adjusted profitability disappears when transaction costs are incorporated into the analysis, which is consistent with the efficiency of option markets, similarly to the results in Gonçalves and Guidolin (2006).

Our findings suggest that richer economic models such as those incorporating structural frameworks describing the investors' learning process might explain the predictable dynamic process on the equity option *IVS*. [9] For instance, in relation to GARCH type models commonly used to predict stock return volatilities (probably the most popular dynamic model used in financial economics), Engle (2001) writes that: "*Such an updating rule is a simple description of*

[7] This trading rule rests on the fact that option prices are positively related to implied volatilities.

[8] Goyal and Saretto (2009) use the same factor model to evaluate abnormal returns of option trading strategies based on differences between realized volatilities and at-the-money one-month implied volatilities.

[9] For instance, Timmermann (2001) shows in the stock market that the predictability patterns in stock returns can be explained by the learning process followed by investors. Although the literature regarding learning models that explain the predictable dynamics of option prices is limited, Ederington and Lee (1996) and Beber and Brandt (2006, 2009) present intuitive studies about the connection between learning and prices in option markets.

adaptive or learning behavior and can be thought of as Bayesian updating" (p. 160). In a similar way, our two-stage VARX-type models of the *IVS* dynamics may simply be stylized and yet powerful descriptive models hiding the way information is processed and spreads throughout a range of option markets.

The recent literature contains a number of studies about the *IVS* dynamics of index options; however the studies that have examined possible predictability patterns in the *IVS* of individual equity options are limited. Moreover, we currently have no knowledge of any links (simultaneous or predictive) between the *IVS* of stock options and the index (market) *IVS*. Nevertheless, a number of papers are related to our current efforts. Gonçalves and Guidolin (2006) find predictable dynamics in the *IVS* of S&P 500 index options using a two-stage approach in a similar fashion to our paper. In addition, a number of papers have explored the index *IVS* movements using principal component analysis (e.g., Skiadopoulos *et al.*, 1999, Cont and Fonseca, 2002, and Fengler *et al.*, 2003), semiparametric models (e.g., Fengler *et al.*, 2007), stochastic volatility models (e.g., Christoffersen *et al.*, 2009), and using a Kalman filter approach (e.g., Bedendo and Hodges, 2009).[10] Furthermore, recent contributions have examined the dynamics of higher order risk-neutral moments, but also in this case of index options (e.g., Panigirtzoglou and Skiadopoulos, 2004, and Neumann and Skiadopoulos, 2011). Finally, there are some studies that have explored a number of interesting features of individual equity options, although their focus is never on the *IVS* dynamics. For instance, Goyal and Saretto (2009) detect predictability patterns of equity options based on differences between historical realized volatilities and implied volatilities of at-the-money one-month option contracts. They report abnormal risk-adjusted returns from trading strategies. Additionally, Dennis and Mayhew (2000, 2002) analyze different factors that may explain the volatility smile and risk-neutral skewness for short-term equity option contracts, but their possible predictability is ignored.[11]

The paper is organized as follows. Section 2 describes the data. Section 3 introduces the deterministic *IVS* model used to characterise the *IVS* as well as the cross-sectional *IVS*

[10] In addition, some papers have investigated the predictability of the implied volatility of particular index option contracts, typically at-the-money short-term contracts (e.g., Harvey and Whaley, 1992, and Konstantinidi *et al.*, 2008).

[11] Moreover, recently Chalamandaris and Tsekrekos (2010) find predictable dynamics in the *IVS* of over-the-counter (OTC) currency options, which shows that predictability patterns are not unique to index options in accordance with the evidence presented in this paper.

relationships between equity and market index options. Section 4 presents the approach for modelling the joint dynamics of the *IVS*s of equity and index options; additionally this section reports the key results of statistical and economic measures to evaluate the predictability patterns in the *IVS* of equity options. Section 5 concludes.

2. The Data

We use data on daily equity and S&P 500 index option prices (American and European styles, respectively), spanning all calls and puts traded on the full set of option trading venues in the United States. This information is extracted from the OptionMetrics database covering the period between January 4, 1996 and December 29, 2006. The data include daily closing bid and ask quotes, volume, strike prices, expiration dates, underlying asset prices, dividends paid on each underlying asset, and the yield curve of riskless interest rates.[12] Reported option prices are bid-ask quote midpoints. We assume that dividend cash flows are perfectly anticipated by market participants as in Bakshi *et al.* (1997) and Dumas *et al.* (1998). In addition, we calculate the implied volatilities for American options using a binomial tree model under Cox *et al.'s* (1979) approach; while we numerically invert BS model to obtain implied volatilities in the case of European-style contracts.[13]

We apply four exclusionary criteria to filter out observations that are not likely to represent traded prices in well-functioning and liquid option markets. First, we eliminate all observations that violate basic no-arbitrage bounds, such as upper and lower bounds for call and put prices and call-put parity relationships (i.e., equalities in the case of European options and weaker bounds in the case of American options). Second, as argued in Dumas *et al.* (1998), we drop all option

[12] Battalio and Schultz (2006) have reported that the OptionMetrics database records option quotes and underlying stock prices with some minor time differences, which may represent a potential source of biases when arbitrage conditions are the main subject of investigation (e.g., the put-call parity). Using similar arguments to Goyal and Saretto (2009) about the irrelevance of this problem for their objectives, this feature of the data does not pose a problem for our research design because any residual non-synchronicity between option and stock prices would merely create spurious *in-sample* evidence of predictability, which is most likely to be punished by genuine recursive *out-of sample* strategies that are appropriately back-tested in recursive experiments, as we do in this paper.

[13] Of course, other approaches to extract implied volatilities from American options might have been used; however we consistently use the same model for all options and thus the small errors generated by Cox et al.'s (1979) approach should average out to zero.

contracts with less than six trading days or with more than one year to their expiration date, as their prices usually contain little information regarding the *IVS*. Third, similarly to Dumas *et al.* (1998) and Heston and Nandi (2000), we exclude contracts whose moneyness is either less than 0.9 or in excess of 1.1 because their prices are usually rather noisy, especially in the case of individual equity options of American style.[14] Fourth, following Bakshi *et al.* (1997) and Gonçalves and Guidolin (2006), we exclude contracts with price lower than $0.30 for equity options and $3/8 for S&P 500 index options, to avoid the effects of price discreteness on the *IVS* shape.[15]

We select the 150 equity options with the highest average daily trading volume over our sample period. Table 1 shows summary statistics for implied volatilities for these 150 equity options (Panel A, where we present averages across different individual equity contracts) and for S&P 500 index options (Panel B). This table presents statistics for data classified into a number of categories across moneyness and time-to-maturity. The moneyness categories are five (with break-points given by 0.94, 0.98, 1.02, and 1.06) and the maturity categories are three (short term options have a time to expiration between 7 and 120 days; medium term options have a time to expiration between 121 and 240 days; and long term options exceed 241 days to expiration). Besides reporting sample means and standard deviations for implied volatilities, we include a measure of trading frequency which is defined as the percentage of trading days in which we observe a non-zero trading volume for any of the option contracts in each of the categories defined in the table. Table 1 emphasizes the existence of remarkable differences in implied volatilities across moneyness and time-to-maturity for both individual equity options and S&P 500 index options. Therefore, this table shows the existence of an *IVS* for both types of options. In addition, even though in this paper we mostly focus on the sub-set of equity options with the highest trading volume, Table 1 reveals substantial differences in the average trading frequency of equity options vs. S&P 500 index options: the trading index for S&P 500 index options across all moneyness and maturity categories is at least 200% higher than it is for individual stock options. For instance, the mean trading frequencies for at-the-money equity options are 47.23% (short-term contracts), 38.48% (medium-term), and 8.52% (long-term). On the opposite, the

[14] We define the moneyness ratio as $Mon \equiv \frac{K}{S}$, where K and S are the strike price and the underlying asset price, respectively.

[15] This is due to the proximity of these prices to the minimum tick size: for equity options the minimum tick is $0.05 while for index options the minimum tick is $1/16.

trading frequency for S&P 500 index options are 100%, 97.69%, and 86.65%, respectively. The difference in mean trading frequency between the average long term stock option and SPX is indeed massive. In spite of these differences, following Goyal and Saretto (2009), we do not impose any constraints restricting option contracts to be traded for it to be included in our sample. This is because bid-ask quotes recorded on days without transactions still provide useful information that we want to capture through our modelling approach. For instance, it is true that investors will observe bid and ask prices in the market that are not supported by actual available transactions; however trading desk can use the information from bid and ask prices to forecast the *IVS* in following periods. Moreover, any usage in forecasting of stale information not supported by actual trades ought to be punished in subsequent, recursive out-of-sample trading experiments, which do represents the core of our research design.

[Insert Table 1 here]

The differences in trading activity reported in Table 1 suggest that changes in the *IVS* shape of S&P 500 ("market portfolio") index options might be more quickly incorporated into prices than they do for equity options. Therefore, if the *IV* surfaces of equity and of S&P 500 index options were dynamically related, investors could use the information obtained from the index *IVS* to predict changes in each individual equity *IVS*. The hypothesis by which the *IV* surfaces of equity options and that of the market portfolio are related, with the latter potentially predicting the former, is explored in depth in Section 4. However, Figure 1 provides preliminary suggestive evidence that such a link may actually be strong. Figure 1 displays the *IVS* of S&P 500 index options and the *IVS* of General Electric Co. options on two consecutive trading days (October 3, 2005 and October 4, 2005). For both the S&P 500 and General Electric Co., Figure 1 shows a pronounced smile shape in the *IVS* of short-term option contracts on October 3, 2005, which progressively weakens (i.e., the *IVS* "flattens") as time-to-maturity increase. Interestingly, on the next day (October 4, 2005) both *IVS*s fail to present a smile shape across moneyness; instead they take up a shape that is commonly called a "skew" (asymmetric smile). Figure 1 therefore shows one example supporting the hypothesis that the *IVS* of individual equity options and the *IVS* of S&P 500 index options could be related in the cross-section, moving over time in similar ways. In the following sections, we perform statistical and economic tests to document any significance of these dynamic relationships.

[Insert Figure 1 here]

3. Modelling the Implied Volatility Surface

A convenient and simple way to capture the key features of the shape of the *IVS* is by fitting a simple deterministic *IVS* model. This model consists of a linear regression in which the dependent variable is the implied volatility of each contract and the explanatory variables are variables related to moneyness and time-to-maturity. This type of representation is often called "deterministic" because all the explanatory variables are fully observable and correspond to simple transformations of basic contract parameters. Dumas *et al.* (1998), Peña *et al.* (1999), and Gonçalves and Guidolin (2006) present competing specifications within the general class of polynomial/spline deterministic *IVS* models. We adopt the functional form proposed and successfully applied by Gonçalves and Guidolin (2006), because in their empirical study they estimate a range of alternative model specifications and find that other competing representations yield a worse fit to option data.[16]

Suppose that the number of option contracts written on the same underlying asset observed on a given day is N and thus $\{\sigma_i\}_{i=1}^{N}$ is the full set of implied volatilities on the option contracts indexed by i. Then at one point in time, the deterministic linear function used in our paper can be written as:

$$\ln\sigma_i = \beta_0 + \beta_1 M_i + \beta_2 M_i^2 + \beta_3 \tau_i + \beta_4 (M_i \cdot \tau_i) + \varepsilon_i, \quad (1)$$

where the random error term (simply assumed to be a white noise) is represented by ε_i, τ_i is time-to-maturity, and M_i is time-adjusted moneyness (see, e.g., Tompkins, 2001, and Tompkins and D'Ecclesia, 2006), which is defined as:

$$M_i \equiv \frac{\ln\left(\frac{K_i}{\exp(r_i \tau_i) S - FVD_i}\right)}{\sqrt{\tau_i}}. \quad (2)$$

Here K_i is the strike price, S is the underlying asset price, r_i is the riskless nominal interest rate that depends on the option contract i through its time-to-maturity, and FVD_i is the forward

[16] In addition, and similarly to Gonçalves and Guidolin (2006), in unreported analyses, we experiment with alternative functional forms. We find that the strength of the *IVS* predictability captured by these alternative specifications is weaker and tends to yield lower economic value (trading profits). Detailed results are available upon request.

value until expiration date of all future dividends to be paid by the underlying asset (assumed to be perfectly anticipated by market participants).

In equation (1), β_0 is the intercept/level coefficient which in a Black and Scholes' (1973) world, where volatility is constant, should be equal to the common log-volatility implicit in all option contracts (i.e., $\beta_0 = \ln\sigma_1 = \cdots = \ln\sigma_N$ while $\beta_j = 0$ for $j = 1, \ldots, 4$). The moneyness (smile/skew) slope of the *IVS* is characterised by the coefficient β_1. β_2 captures the curvature of the *IVS* in the moneyness dimension, β_3 reflects the maturity (term-structure) slope, and β_4 describes possible interactions between the moneyness and the time-to-maturity dimensions. The coefficients in equation (1) are recursively estimated at daily frequency for each group of option contracts written on the same underlying asset, a procedure that is performed by generalized least squares (GLS), as recommended by Hentschel (2003).[17]

As a result of application of these methods, we obtain 151 daily sets of coefficients for the deterministic *IVS* model in equation (1), because in our sample we have a total of 150 "sets" of equity options (characterized by their underlying name) and one single set of S&P 500 index options. Table 2 reports summary statistics for the GLS coefficient estimates, the R^2, and the root mean squared error (RMSE) of the deterministic *IVS* model using equity options (Panel A) and S&P 500 index options (Panel B), obtained over time.[18] Table 2 shows that on average, the values of the R^2 and the F-statistics for equity options are 0.69 and 20.17, respectively; while for S&P index options we obtain an average R^2 of 0.78 and an average F-statistic of 382.85. Therefore, there is a sense in which, at least on average, our deterministic *IVS* model fits index options data better than it fits individual equity options, although the difference is far from massive. Although, in the daily time series for individual stock options not all estimated coefficients in equation (1) are individually significant, Table 2 emphasizes that the qualitative features of the *IVS* are common across index and stock options, with implied volatility declining

[17] Hentschel (2003) shows that linear models of option implicit volatilities cannot be estimated by simple ordinary least squares (OLS) because of the presence of pervasive measurement errors in implied volatilities (e.g., due to bid-ask spread bounce and/or minimum tick size rules) that may introduce heteroskedasticity and autocorrelation in standard OLS residuals. As a result, standard OLS estimates need to be presumed to be rather inefficient. For a detailed description of the implementation of the GLS method suggested by Hentschel (2003) to deterministic *IVS* models, see appendix B in Gonçalves and Guidolin (2006).

[18] In appendix A, we present the same summary statistics as in Table 2 but using ordinary least squares (OLS) as robustness check.

in moneyness, increasing in the square of moneyness, and decreasing as a function of the interaction between moneyness and time-to-maturity. This finding of implied volatilities declining in the level of moneyness and increasing in the square of moneyness yields an asymmetric smile shape that is what the literature has typically reported (see among the others Dumas *et al*., 1998, Neumann, and Skiadopoulos, 2011, and Xing *et al*., 2010). The only coefficient that carries a different estimated sign for individual equity options vs. the S&P 500 market index, is β_3: this implies that while the SPX *IVS* tends to be upward sloping as a function of maturity, on average the *IVS* of stock options slightly declines. Table 2 also presents *prima-facie* evidence of predictability patterns in the *IV* surfaces of equity options and S&P 500 index options: All the coefficients of the deterministic *IVS* model estimated with both option groups present on average significant serial correlation detected using the Ljung-Box test with one and three lags, LB(1) and LB(3), respectively.

[Insert Table 2 here]

[Insert Figure 2 here]

Additionally, as previously stated, one of the objectives of our paper is to explore possible relationships between the *IV* surfaces of equity and market index options. Consequently, Figure 2 shows the evolution of daily cross-sectional averages (over different underlying stocks) of the coefficients of the deterministic *IVS* model estimated with equity options, along with the coefficients for the *IVS* of S&P 500 options. Figure 2 shows some evidence of co-movements between each pair of coefficient time series, particularly visible with little effort in the case of the coefficients β_0 and β_3. In fact, we find significant linear correlation between the *IVS* coefficients characterizing individual equity options and the *IVS* of index options: Table 3 presents a correlation analysis applied to the individual *IVS* coefficients extracted from stock options data as well as from S&P 500 options. Table 3 shows that on average there are many significant correlations between the two sets of *IVS*s. In fact, some of the pair-wise correlations originate significant estimates in more than 90% of the cross-section of equity options. Therefore, the results presented in Table 3 provide some preliminary support to the hypothesis that the *IVS* of equity options may be related to the market index *IVS*.

[Insert Table 3 here]

4. Modelling the Joint Dynamics of Equity and Market Implied Volatility Surfaces

In this section we examine the time series as well as the cross-dynamics of the *IV* surfaces of both equity and index options. On the one hand, in Section 3 we have reported high levels of predictability as measured by the autocorrelations of the deterministic *IVS* model coefficients of equity options (see Table 2); these are also observable for the *IVS* of S&P 500 index options, consistently with the findings in Gonçalves and Guidolin (2006). On the other hand, we have shown evidence of cross-sectional linkages between individual equity options *IVS* coefficients and those for the market portfolio (see Figure 2 and Table 3). These findings suggest that the *IVS* of equity options could be characterised through a dynamic multivariate model that includes historical equity *IVS* movements—as measured by the time series of daily *IVS* coefficients obtained with equation (1)— as well as the dynamics of the SPX *IVS*. Therefore, the objective of this section is to investigate whether the predictability of implied volatilities of individual equities may benefit, both in a purely statistical perspective and through economic value tests, from the incorporation of information on historical dynamics in the S&P 500 *IVS*. To pursue this goal, we propose a simple vector time series model of VARX(p, q) type to be fitted to the time series of daily coefficients of the deterministic *IVS* models of equity and market S&P 500 index options:

$$\widehat{\boldsymbol{\beta}}_t^{Eq} = \boldsymbol{\gamma} + \sum_{j=1}^{p} \boldsymbol{\Phi}_j \widehat{\boldsymbol{\beta}}_{t-j}^{Eq} + \sum_{k=1}^{q} \boldsymbol{\Psi}_k \widehat{\boldsymbol{\beta}}_{t-k}^{SPX} + \mathbf{u}_t \qquad \mathbf{u}_t \sim IID\ \ N(0, \boldsymbol{\Omega}), \tag{3}$$

where $\widehat{\boldsymbol{\beta}}_t^{Eq} \equiv [\beta_{0t}^{Eq}\ \beta_{1t}^{Eq}\ \beta_{2t}^{Eq}\ \beta_{3t}^{Eq}\ \beta_{4t}^{Eq}]'$ is the 5×1 vector time series of the first-stage estimated coefficients specific to individual equity options obtained on a recursive daily basis from GLS estimations of the simple regression model in equation (1), and $\widehat{\boldsymbol{\beta}}_t^{SPX}$ is a similar 5×1 vector time series of estimated coefficients characterising the *IVS* of S&P 500 index options. We select the number of lags to be used in the model (p and q), via minimization of the Bayes-Schwarz criterion, after setting an arbitrary maximum value of three for both sets of parameters.[19] Consequently, the model introduced in equation (3) is a simple vector time series model, which

[19] The arbitrary choice of 3 as the maximum value for p and q is based on the analysis presented in Gonçalves and Guidolin (2006), where they show that parsimonious models with few lags tend to outperform richer models. Moreover, in preliminary analyses, we obtained worse statistical and economic measures of predictability with models using longer lag structures.

we use to forecast the *IVS* of equity options using recent co-movements in the *IVS*s from the equity options themselves and from market S&P 500 index options.[20]

For both testing and comparative purposes, besides the dynamic equity-SPX *IVS* model in equation (3) we also estimate and back-test three benchmark models. The first benchmark model nests equation (3) because it is derived by imposing the restrictions that $\boldsymbol{\Psi}_k \equiv \mathbf{O}$ for k = 1, ..., q, where $\mathbf{O}$ is a matrix of zeros. Therefore, the first benchmark model is a simple VAR(p) model where the information on past dynamics in the index *IVS* is disregarded:

$$\widehat{\boldsymbol{\beta}}_t^{Eq} = \boldsymbol{\delta} + \sum_{j=1}^{p} \boldsymbol{\Theta} \widehat{\boldsymbol{\beta}}_{t-j}^{Eq} + \mathbf{v}_t \qquad \mathbf{v}_t \sim IID \;\; N(0, \boldsymbol{\Xi}). \tag{4}$$

Also in this case, we select p by minimizing the Bayes-Schwarz criterion with a pre-selected maximum number of lags equal to three. The comparison of the model in equation (4) with the dynamic equity-SPX *IVS* model in equation (3) allows us to ask whether the index *IVS* dynamics may contain any useful and additional information regarding predictable movements in the cross-section of equity *IVS*s.

As a second benchmark, we entertain an *ad-hoc* 'strawman' model which has been used by Dumas *et al.* (1998) and Christoffersen and Jacobs (2004). This *ad-hoc* model is a simple random walk process for each of the coefficients of the deterministic *IVS* model for equity options. Under this naive benchmark, the best prediction for tomorrow's coefficients (hence, the forecast of the shape of the *IVS*) is simply given by today's values (i.e., $\widehat{\boldsymbol{\beta}}_t^{Eq} = \widehat{\boldsymbol{\beta}}_{t-1}^{Eq}$).

The third benchmark model is Duan and Simonato's (2001) American option GARCH model, which posits the following stochastic process for the underlying stock returns:[21]

$$r_{t+1} = r^f - (1/2)h_{t+1} + \sqrt{h_{t+1}} z_{t+1}^* \qquad h_{t+1} = \omega + \beta h_t + \gamma h_t (z_t^* - \delta - \psi)^2 \tag{5}$$

[20] The VARX model can be understood as a reduced form characterising the time variation in the equity option *IVS*, which may result from learning dynamics characterizing the behaviour of investors in option markets, (see e.g., Guidolin and Timmerman, 2003, or David and Veronesi, 2002).

[21] Notice that in American-style options such as stock options, volatility affects early exercise decisions because volatility enters in the calculation of the future value of the option. Therefore, it is important to consider all possible paths that the conditional volatility can follow in American-style options when GARCH-type models are used in option valuation. Duan and Simonato (2001) develop a numerical pricing method using Markov chains to deal with this issue which takes future volatility dynamics into account.

In recent years, a number of discrete-time single-factor GARCH models have been proposed in the applied econometrics literature, which when are applied to option pricing have shown performances often comparable to more complex frameworks, such as multi-factor structural models. For instance, using S&P 500 index options, Heston and Nandi (2000) report the superior performance of their NGARCH(1,1) model for European-style options over the *ad-hoc* 'strawman' model (our second benchmark model). The choice of our third American option GARCH benchmark aims at comparing the performance of the model in equation (3) with a different dynamic model in which the evolution of the quantity of interest—here volatility—is explicitly parameterized and estimated in one single step from option prices, instead of resorting to two steps, as in our baseline strategy. Implicitly, a Reader may consider the GARCH option pricing benchmark as an attempt to tease out from the data (especially in terms of economic value performances) whether and how our two-step estimation approach may capture any additional source of predictability in option prices, when standard time series models may have difficulty to take into account such dynamics. In practice, we use nonlinear least square (NLS) methods to recursively estimate on a daily basis the parameters of Duan and Simonato's (2001) American option GARCH model. In our NLS estimation program, we minimize the sum of the squared differences between the observed volatilities implicit in option contracts and the implied volatilities obtained by inverting the Duan and Simonato (2001) American option GARCH model. The main purpose of using estimators that are based on minimizing differences between market and implied quantities in the volatility space is to preserve full consistency with our dynamic equity-SPX *IVS* model, which is also estimated with reference to the implied volatility space.[22,23]

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[22] For an illustration of the use of NLS estimation in the implied volatility space, see Jackwerth (2000).

[23] We are grateful to Jin-Chuan Duan for sharing his codes implementing Duan and Simonato's (2001) American option GARCH model. To provide an idea of the type of estimates that a GARCH option pricing model yields, we obtain the following average estimates from our recursive exercise: $h_{t+1} = 1.14 \cdot 10^{-6} + 0.87h_t + 0.06h_t(z_t^* - 0.01 - 0.44)2$. This implies that on average there is a high persistence which is common for this kind of models (i.e., $0.87 + 0.06(1 + (0.01 + 0.44)^2) = 0.94$). In addition, Duan and Simonato's (2001) American option GARCH model leads to an average predictive RMSE of 0.045, which is indeed rather impressive predictive performance, given that this model has only five parameters.

4.1. Statistical Measures of Predictability

We use a recursive back-testing exercise to systematically evaluate the out-of-sample (one-day-ahead) performance of all models using three main statistical measures. We report the root mean squared forecast error (RMSE) and the mean absolute forecast error (MAE), calculated both in the implied volatility and in the option price spaces. In addition, we compute the mean correct prediction of direction of change (MCPDC). The MCPDC is defined as the percentage of predictions for which changes of the predicted variables have the same direction/sign as the realized movements followed by the same variable over the prediction horizon. Also in this case, we calculate MCPDC for both implied volatilities and option prices.

The recursive, out-of-sample nature of the exercise is structured in the following way. First, we estimate on a recursive daily basis all dynamic models, in which estimation is performed using six-month rolling windows of data (i.e., between day $t-(252/2)$ and day t). Second, we compute from all models one-day-ahead predictions of implied volatilities; and then we calculate prices for each option contract using the binomial tree method of Cox *et al.'s* (1979) approach. In the case of the dynamic equity-SPX *IVS* and VAR(p) models, we forecast one-day-ahead coefficients of the deterministic *IVS* function for equity options using equations (3) and (4), respectively. In addition, in the case of the benchmark *ad-hoc* 'strawman' model, the *IVS* coefficient forecasts are simply obtained from the random walk law of motion $\hat{\boldsymbol{\beta}}_t^{Eq}=\hat{\boldsymbol{\beta}}_{t-1}^{Eq}$. For these three models, we then obtain implied volatility predictions for all equity option contracts using equation (1) (i.e., plugging into the deterministic *IVS* function the predicted coefficients derived from any of the three dynamic frameworks). In the case of Duan and Simonato's (2001) American option GARCH model, implied volatilities are directly obtained from iterating the model one day forward. Nevertheless, we do not have predictions for one-day ahead stock prices and interest rates to calculate option price forecasts. Therefore, following Gonçalves and Guidolin (2006), we assume that the best one-day-ahead predictions for stock prices and interest rates are today's prices and rates, which seems to be consistent with the bulk of the literature on the efficient market hypothesis. The expected impact of this martingale assumption for stock prices and interest rates in terms of measurement error is to induce biases in the coefficients of the econometric tests which may make them drift away from significance; as a result, there is no reason to suspect that our findings can be mostly driven by these measurement errors. Moreover, any potential effect deriving from this assumption is mitigated by our use of trading strategies that are completely hedges against the effects of changes in the prices of the underlying stock, such as straddle and delta-hedged positions.[24]

We report in Table 4 our out-of-sample statistical indicators of predictive accuracy to assess the performance of the dynamic equity-SPX *IVS* model vs. the benchmark models. We also include in this table a 'pure' random walk model for implied volatilities, in which the best prediction of tomorrow's implied volatility for an option contract is today's level, similarly to Harvey and Whaley (1992). Table 4 shows that the dynamic equity-SPX *IVS* model outperforms all benchmark models in both the implied volatility and the option price spaces. It is interesting to observe that the dynamic equity-SPX *IVS VARX* model has a superior performance vs. the VAR model (i.e., a model does not take into account the dynamics of the market index *IVS*). This result starts providing some validation of our conjecture that the *IV* surfaces of equity and market index options are not only related in the cross-section, as reported in Table 3, but also dynamically. Therefore, movements in the S&P 500 index *IVS* provide additional and valuable information to anticipate the equity option *IVS* dynamics. In addition, it is important to emphasize the good out-of-sample results for the dynamic equity-SPX *IVS* model in relation to the measure evaluating the forecasting power for the direction of change (i.e., the MCPDC measure). Table 4 shows that the average MCPDC using the dynamic equity-SPX *IVS* model is 59.63% (54.03%) in the space of implied volatilities (options prices). This statistic is intrinsically related to the economic measures of predictability based on trading strategies which will be analyzed in the next section, because signals to buy or sell entirely depend on the direction of change of forecasts on implied volatilities.[25]

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[24] It is also easy to appreciate that the opposite assumption that tomorrow's stock prices and interest rates were known in advance would create a dangerous mixture between the predictive power of a model for the *IVS* and the assumed perfect foresight for stock prices and interest rates. Finally, assuming additional forecast models for stock prices and/or interest rates would make it difficult to distinguish between such models and *IVS* models as the main drivers on any realized out-of-sample results.

[25] In unreported in-sample statistical analysis we find that the dynamic equity-SPX *IVS* model yields the best in-sample fit among all models. For instance, the dynamic equity-SPX *IVS* model leads on average to an in-sample RMSE of 0.034 and 0.431 for implied volatilities and option prices, respectively; the VAR(p) model that includes only past information on *IVS* dynamics for equity options implies on average RMSEs of 0.041 and 0.584, respectively. The random walk *IVS* 'strawman' model and the 'pure' random walk implied volatility model yield in the in-sample analysis the same performance as the out-of-sample analysis presented in Table 4, because they do not require (by construction) additional parameter estimation or filtering. Therefore, the random walk *IVS* 'Strawman'

In Table 4, it is important to emphasize that the values of the MCPDC measure exceed 50% for all the models. In a no-predictability environment, we should expect values for the MCPDC measure around 50% for all modelling approaches; however the results of Table 4 suggest that we can forecast that the *IVS* of equity option even using simple models.

[Insert Table 4 here]

Table 5 shows the results of equal predictive accuracy tests for each of the four benchmark models against the dynamic equity-SPX *IVS* model, in which we use the methodology proposed by Diebold and Mariano (1995) applied to the one-day-ahead forecasts presented in Table 4. As a loss function to construct the test statistic, we use the differences between the squared forecast errors from the dynamic equity-SPX *IVS* model and the squared error from each of the benchmark models. A Newey and West (1987) heteroskedasticity and autocorrelation consistent (HAC) variance estimator is used to calculate the Diebold and Mariano (1995) test statistic. Table 5 shows that the average test statistic in the cross-section of stock options is negative and significant, which indicates that a VARX framework outperforms the simpler VAR model. Moreover, the out-of-sample performance of the dynamic equity-SPX *IVS* model is significantly superior in the vast majority of the pair-wise comparisons. Moreover, Table 5 shows that the null hypothesis of equal predictive accuracy for the dynamic equity-SPX *IVS* model and benchmark models is rejected for at least 73% of equity options in either the implied volatility space or the option price space. Such a 73% may be interpreted as a measure of how large is the fraction of the U.S. option markets containing pockets of illiquidity or other microstructural noise leading to evidence of statistically significant predictability in *IV*s.

[Insert Table 5 here]

In unreported results, we repeat the same analyses presented in Table 4 and Table 5 using forecast horizons of three and five days. Also in this case, we find a superior performance of our dynamic equity-SPX *IVS* model over the benchmark models. Therefore, the results of Table 4 and Table 5 have led to two key provisional conclusions concerning the *IVS* of equity options

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model yields in-sample RMSEs of 0.059 and 0.739 using implied volatilities and option prices, respectively. The 'pure' random walk implied volatility model leads to average RMSEs 0.049 and 0.641, respectively. Finally, Duan and Simonato's (2001) American option GARCH model leads to average in-sample RMSEs of 0.045 for implied volatilities and 0.674 for option prices. Complete results on recursively estimated coefficients and in-sample fit are available from the Authors upon request.

which we next proceed to check with further economic analyses: first, there is evidence of predictable dynamics in the *IVS* of equity options; and second, the movements of the *IVS* of index options provides useful information to forecast the *IVS* dynamics of equity option contracts.

4.2. The Economic Value of Predictability

The results from the statistical analysis in Subsection 4.1 show the existence of widespread predictability in the dynamics of the *IVS* of equity options; moreover, such predictability is greatly increased when we allow past movements in the S&P 500 *IVS* to predict subsequent shapes in the cross-section of stock option *IV* surfaces. However, it is reasonable to object that—although this may be encouraging—such empirical findings tell us little about whether any of such predictability might be actually exploited by investors in the option markets. Consequently, we evaluate the existence of any abnormal returns using two different and simple trading strategies, which exploit the one-day-ahead predictions generated by the dynamic equity-SPX *IVS* model and benchmark models in very intuitive ways. In fact, the trading strategies follow a straightforward rule: when a dynamic model forecasts that the implied volatility of a given option contact will increase (decrease) between (trading) day t and $t+1$, that option contract is purchased (sold) on day t to profit from potential option price movements. For this reason we have already emphasized how the previously reported MCPDC statistics may be crucial because they are highly correlated to the trading profits of our trading strategies. Notice that because of their extreme simplicity, the trading strategies pursued in this paper have to be interpreted as providing at best a *lower bound* on the actual trading profits that a sophisticated, real-world trading desk may eventually achieve using models such as ours.

We generate trading portfolios based on straddle and delta-hedged option strategies, because both are free of risks caused by changes in the prices of the underlying stocks. The first trading strategy consists of a portfolio composed of plain-vanilla straddle positions. A straddle strategy involves trading a combination of a call and a put option contracts with the same strike prices and expiration dates. A long straddle (in which options are purchased) is equivalent to a pure bet on a high(er) future volatility. A short straddle (in which options are sold) is equivalent to a pure bet on a low(er) future volatility. The second trading strategy consists of an even simpler portfolio that only contains delta-hedged positions. Delta-hedged positions are established by trading adequate volumes of the underlying stock on the basis of the option delta.[26] In practice, on every day in our back-testing period, we invest a fixed amount of $1,000 net in each straddle portfolio and an amount of $1,000 net in each delta-hedged portfolio. Both types of portfolios are re-balanced every day so that the initial $1,000 investment remains constant over time. Profits and losses are then recorded and used in the analyses that follow.

More specifically, in the case of a straddle portfolio, let Q_t be the number of option contracts written on the same underlying stock that should be traded following the trading rule introduced above. In addition, let $V_t^{Straddle}$ be the total value of all straddle positions in the portfolio on day t, which depends on Q_t. Given that the straddle portfolio involves buying and selling multiple calls and puts, we can write $V_t^{Straddle}$ as:

$$V_t^{Straddle} = \sum_{m \in Q_{t,+}} \left(C_{m,t} + P_{m,t}\right) - \sum_{m \in Q_{t,-}} \left(C_{m,t} + P_{m,t}\right) \tag{6}$$

where $Q_{t,+}$ ($Q_{t,-}$) is the sub-set of call and put contracts that should be purchased (sold), and $C_{m,t}$ ($P_{m,t}$) denotes the call (put) price of the option contracts in each sub-set. In the scenario that the net cost of the portfolio is positive (i.e., $V_t^{Straddle} > 0$), we purchase the quantity $X_t^{Straddle} = \$1{,}000/V_t^{Straddle}$ in units of the straddle portfolio, for a total cost of $1,000. As a result, the one-day net gain, $G_{t+1}^{Straddle}$, is:

$$\begin{aligned} G_{t+1}^{Straddle} &= X_t^{Straddle}\left[\sum_{m \in Q_{t,+}} \Big(\left(C_{m,t+1} + P_{m,t+1}\right) - \left(C_{m,t} + P_{m,t}\right)\Big)\right] \\ &+ X_t^{Straddle}\left[\sum_{m \in Q_{t,-}} \Big(-\left(C_{m,t+1} + P_{m,t+1}\right) + \left(C_{m,t} + P_{m,t}\right)\Big)\right]. \end{aligned} \tag{7}$$

However, under a scenario in which the net cost of the straddle portfolio is negative (i.e., $V_t^{Straddle} < 0$), we sell the quantity $X_t^{Straddle} = \$1{,}000/|V_t^{Straddle}|$ in units of the straddle portfolio, which yields a cash inflow of $1,000, and we invest the $1,000 generated in this way plus the $1,000 initially on hand at the riskless interest rate over one day. Therefore, in this scenario the net gain is $G_{t+1}^{Straddle} + \$2{,}000 \cdot (\exp(r_t/252) - 1)$, where $G_{t+1}^{Straddle}$ is calculated using equation (7).

[26] In the case of delta-hedged positions, implied deltas of equity option contracts are calculated using a binomial tree model following Cox *et al.'s* (1979) approach that accommodates the American style of individual options.

The same course of action is applied to delta-hedged portfolios. Let V_t^{D-H} be the total value of all delta-hedged positions on day t in a delta-hedged portfolio which also depends on Q_t; therefore we can write V_t^{D-H} as:

$$\begin{aligned} V_t^{D-H} = & \sum_{m \in Q_{t,+}^{call}} \left(C_{m,t} - S_t \Delta_{m,t}^C\right) + \sum_{m \in Q_{t,+}^{put}} \left(P_{m,t} + S_t \Delta_{m,t}^P\right) \\ & - \sum_{m \in Q_{t,-}^{call}} \left(C_{m,t} - S_t \Delta_{m,t}^C\right) - \sum_{m \in Q_{t,-}^{put}} \left(P_{m,t} + S_t \Delta_{m,t}^P\right) \end{aligned} \tag{8}$$

where $Q_{t,+}^{call}(Q_{t,-}^{call})$ is the sub-set of call contracts that have to be purchased (sold), while $Q_{t,+}^{put}(Q_{t,-}^{put})$ is the sub-set of put contracts that should also be purchased (sold), S_t is the price of the underlying stock, and $\Delta_{m,t}^C$ ($\Delta_{m,t}^P$) is the absolute value of the call (put) option delta. Similarly to straddle portfolios, in the case that the net value of the delta-hedged portfolio is positive (i.e., $V_t^{D-H} > 0$), we purchase the quantity $X_t^{D-H} = \$1{,}000/V_t^{D-H}$ in units of the delta-hedged portfolio, for a total cost of \$1,000. Consequently, the one-day net gain (G_{t+1}^{D-H}) is:

$$\begin{aligned} G_{t+1}^{D-H} = & X_t^{D-H} \left[\sum_{m \in Q_{t,+}^{call}} \left(\left(C_{m,t+1} - S_{t+1} \Delta_{m,t}^C\right) - \left(C_{m,t} - S_t \Delta_{m,t}^C\right) \right) \right] \\ & + X_t^{D-H} \left[\sum_{m \in Q_{t,+}^{put}} \left(\left(P_{m,t+1} + S_{t+1} \Delta_{m,t}^P\right) - \left(P_{m,t} + S_t \Delta_{m,t}^P\right) \right) \right] \\ & + X_t^{D-H} \left[\sum_{m \in Q_{t,-}^{call}} \left(-\left(C_{m,t+1} - S_{t+1} \Delta_{m,t}^C\right) + \left(C_{m,t} - S_t \Delta_{m,t}^C\right) \right) \right] \\ & + X_t^{D-H} \left[\sum_{m \in Q_{t,-}^{put}} \left(-\left(P_{m,t+1} + S_{t+1} \Delta_{m,t}^P\right) + \left(P_{m,t} + S_t \Delta_{m,t}^P\right) \right) \right]. \end{aligned} \tag{9}$$

However, when the net cost of the portfolio is negative (i.e., $V_t^{D-H} < 0$), we sell the quantity $X_t^{D-H} = \$1{,}000/|V_t^{D-H}|$ in units of the delta-hedged portfolio, which generates a cash inflow of \$1,000, and we invest the \$1,000 so generated together with the \$1,000 initially available at the riskless interest rate over one day. In this case, the net gain is $G_{t+1}^{D-H} + \$2{,}000 \cdot (\exp(r_t/252) - 1)$, where G_{t+1}^{D-H} is obtained from equation (9).[27]

Table 6 presents summary statistics for the average profits—over time and across equity options in the cross-section—obtained from trading straddle portfolios (Panel A) and delta-hedged portfolios (Panel B). Table 6 gives evidence on the economic value of the *IVS* predictability generated by the dynamic equity-SPX *IVS* model vs. the benchmark models. In addition, we include two further passive strategies (Panel C): the first passive benchmark follows a simple 'S&P 500 Buy and Hold' strategy (i.e., a daily investment of $1,000 in the S&P 500 index); and the second passive benchmark consists of an effortless investment of $1,000 at the riskless interest rate rolled over time, which only yields the time value of money (at least as a first approximation). Table 6 shows the superiority of the dynamic equity-SPX *IVS* model over all benchmark models under both the straddle-based and the delta-hedged strategies. The dynamic equity-SPX *IVS* model produces significant profits in more than 80% (59%) of the straddle (delta-hedged) portfolios with an average Sharpe ratio of 15.20% (5.67%). Of course, such daily Sharpe ratios are simply stunning, but we need to be reminded at this point that Table 6 does not take into account transaction costs and other frictions.[28] Because our trading strategies imply a need to potentially trade hundreds of options every day, this may be overly costly and expose an investor to massive risks (even under delta-hedging) that the Sharpe ratio may not fully take into account. Table 6 reports that delta-hedged portfolios are less profitable than straddle portfolios which is due to a one key reason: while straddle strategies take full advantage of predictability patterns in implied volatilities because they trade only equity option contracts, delta-hedged positions involve the need to invest in (or borrow) underlying shares stock, for which none of the models estimated in this paper is specifically designed to forecast. Although this may represent a reason to attach more weight to the straddle-based economic values than to delta-hedge based strategies, in our view it remains valuable to also report results for the latter as they truly represent a lower bound for the obtainable trading profits. In any event, the resulting Sharpe ratios are high and average mean profits statistically significant also in the case of simple, delta-hedged strategies.

[27] We invest only the $1,000 originally available at the riskless interest rate for one day in the (unlikely) case in which $V_t^{Straddle} = 0$ or $V_t^{D-H} = 0$.

[28] However the Sharpe ratios in Table 6 are reported in percentage terms. For instance, a 15.2% a day translates (using a simple square-root conversion) into a $0.152 \times (252)^{1/2} = 2.41$ annualized Sharpe ratio.

[Insert Table 6 here]

Many Readers may object that the brilliant daily performances reported in Table 6 are the consequence of an exposition to high risks that the simple Sharpe ratio fails to control for. Therefore, in Table 7 we supplement the Sharpe ratios in Table 6 with abnormal return calculations, which are obtained through an asset pricing model that includes specific factors that the literature has shown to capture risk exposures for option portfolios (see, e.g., Coval and Shumway, 2001). The factor model adopted in our analysis has the traditional functional form:

$$R_{port} = \alpha_{port} + \mathbf{B}_{port}^{'}\mathbf{F}_t + e_{port}, \quad (10)$$

where R_{port} is the excess return on either the straddle or delta-hedged trading strategies described above, $\mathbf{F}_t$ is a vector of risk factors, and e_{port} is a random error term that captures any idiosyncratic or unexplained risk. Therefore, a significant positive value of α_{port} can be interpreted as an abnormal return relative to the factor model in equation (10). In relation to the risk factors, we use the three Fama-French (1993) factors, the Carhart's (1997) momentum factor, and an option volatility factor as in Coval and Shumway (2001). The Coval and Shumway (2001) option volatility factor is based on the returns on one at-the-money short-term position on S&P 500 index options. In particular, in the case of straddle portfolios, the option volatility factor is the excess return of a straddle position which is zero-beta (ZbStrad – r_f), while in the case of delta-hedged portfolios this factor is calculated using the excess return of a delta-hedged position on a call option contract (DhCall – r_f).

Table 7 reports the average parameter estimates for the asset pricing factor model in equation (10) using the returns of straddle portfolios (Panel A) and the returns of delta-hedged portfolios (Panel B). Table 7 shows that the dynamic equity-SPX *IVS* model yields the highest average alpha amongst all the models, under both the straddle and the delta-hedged strategy. Such an alpha is 5% a day on average, and the alphas are statistically significant in almost 80% of the cross-section of stock options. This figure is indeed consistent with the idea that, at least before any frictions are taken into account, there may be pockets of unexploited value in the large majority of the U.S. stock options market. It is interesting to notice that on average, both portfolio strategies based on the equity-SPX *IVS* model imply a positive average loading on the option volatility factor. This means that abnormal returns are still present after a positive exposure to the Coval and Shumway (2001) option factor. The percentage of equity options with significant loadings on the market factor is also remarkable, which is higher for delta-hedged portfolios than for straddle portfolios. This is likely due to the fact that delta-hedged strategies have one of their component positions coming from trading shares of the underlying stocks.

[Insert Table 7 here]

In Table 8 we ask instead whether it may be that the exceptionally high trading profits reported in Table 6, and the positive abnormal performances listed in Table 7 may simply depend on the fact that up to this point in this paper we have failed to take transaction costs into account. Dynamic transaction costs are incorporated using the effective bid-ask spreads that are available in our data set, in which we buy (sell) option contracts and stocks at the ask (bid) price over time. However, the effective bid-ask spreads could be different from the quoted spreads. For instance, Battalio *et al.* (2004) show that the effective spread in equity options is around 0.8 times the quoted spread. Therefore, Table 8 presents the profits generated by the trading strategies after netting transaction costs out using a conservative effective bid-ask spread equal to 0.5 times the quoted spread. Table 8 shows that straddle and delta-hedged trading strategies built on the *IVS* forecasts derived from all models under consideration imply large negative average profits and Sharpe ratios in the cross-section. Furthermore, although at least 80% (even in the best case) of the equity options imply statistically significant negative returns in the cross-section, it must be emphasized that we obtain negative profits from both strategies for all equity options (which is not directly reported in Table 8).[29] In addition, Table 8 highlights that delta-hedged portfolios give less negative returns than straddle portfolios. These differences are explained by the low level of transaction costs for stocks in relation to options (i.e., stocks tend to display on average narrower relative bid-ask spreads than option contracts).

[Insert Table 8 here]

Nevertheless, in spite of the impact of transaction costs on the economic profits of our trading strategies, we emphasize that we have anyway reported the existence of clear predictability patterns in the *IVS* of equity options, and this holds both in a statistical and in an economic value perspective. Moreover, these results support our conjecture that the information captured in the movements of the *IVS* of index options can help forecast subsequent dynamics in the *IVS* of equity options. These predictable features of the equity option *IVS* are obviously relevant to

[29] We have also experimented with a different level for the effective bid-ask spread in unreported results. In this case, we assume that the effective bid-ask spread is equal to 1.0 times the quoted spread. As one would expect, the results show even more negative profits than those presented in Table 8.

operators in derivatives markets, as well as to all investors that may want to use option prices and implied volatility to extract forward-looking information on the state of the economy.

Similarly to Goncalves and Guidolin (2006), these findings by which trading strategies have a hard time producing positive returns after transaction costs induce two key implications.[30] First, although dynamic predictability in the *IVS* is statistically strong, only investors (trading desks) that can economize on transaction costs by trading inside the bid-ask spread may actually turn such predictability into effective, realized profits. Second, our earlier evidence of predictability in the *IVS* does not necessarily imply that option markets may fail to be efficient, at least in a weak-form sense. However, one must be careful before concluding that as a result of market efficiency, the past dynamics of the *IVS* carries no useful information for market operators interested in estimating the dynamic process followed by the *IVS* of individual equity options: because trading volume may often be lumpy in individual equity option markets, trading desks are likely to be ready to avail themselves of information revealed by transactions involving more liquid index option contracts also for their hedging and general forecasting goals related to portfolio management, for instance connecting the shape and dynamics of individual equity *IVS* to prediction of expected stock returns (see e.g., Xing et al., 2010).

4.3. Additional Trading Strategies

In unreported results we have examined two additional trading rules that are applied to straddle and delta-hedged portfolios to mitigate the large negative effects of transaction costs on profits. Nevertheless, these trading rules also produce negative profits on straddle and delta-hedged portfolios, under forecasts produced by all the *IVS* models pursued in our paper. First, we select only one option contract for the straddle strategy and one option contract for the delta-hedged strategy per each of the 150 sets of option contracts written on the same underlying stock. One contract is picked daily per each option set which produces the highest expected (ex-ante) trading profit *after* transaction costs using straddle positions, and the other contract is selected in the same way according to ex-ante expected utility profit maximization under the delta-hedged strategy. The expected transaction costs are calculated according to a round-trip logic as today's transaction costs—as measured by 0.5 times bid-ask spread—multiplied by two. Subsequently,

[30] This remark applies also to the additional results in Section 4.3, when the trading strategies are set up to limit the amount of contracts effectively traded.

we invest $1,000 daily in the straddle position and $1,000 in the delta-hedged position following the rules set out in Section 4.2. The key intuition for this trading rule is to decrease transaction costs caused by the need of trading multiple contracts under the strategies used so far in our study.

Second, following Harvey and Whaley (1992), we use strategies that are constrained to only purchase/sell contracts that are at-the-money and short-term; and so we generate a single straddle position and a single delta-hedged position on a daily basis. Obviously, this is a neat way to reduce the overall amount of transaction costs charged on the trading investor. However, also these constrained trading system ends up producing negative returns after netting transaction costs out under all *IVS* forecast models.

5. Conclusion

In this paper we have studied the predictability patterns in the *IVS* of individual equity options. In addition, we explored the existence of dynamic linkages between the *IV* surfaces of equity and S&P 500 index options. We use a simple two-stage modelling approach. In the first stage, we characterise the daily shape of the *IVS* of equity and index options by fitting a simple deterministic *IVS* model. In the second stage, we estimate a VARX-type model to forecast the equity option *IVS*. This VARX model uses the historical coefficients of the deterministic *IVS* model estimated in the first stage, which describe the recent dynamics of the *IVS*s of equity and index options.

We find that there are strong cross-sectional and dynamic relationships between the *IVS* of equity options and the *IVS* of index options. In addition, we show that the two-stage procedure not only generates an accurate forecasting that outperforms in a statistical sense the predictions produced by competing models of common use in the literature; it also produces abnormal returns when trading strategies are back-tested in a recursive out-of-sample exercise. However, the trading profits disappear when we take into account transaction costs, which is consistent with the hypothesis of efficient option markets. In spite of the effects of transaction cost on the profits of our trading strategies, it is important to indicate that in any case we show evidence that there are predictability pattern in the *IVS* of equity options; and thus these predictability features of equity option can be used by diverse agents in option markets and other markets given that option contracts are usually used to obtain forward-looking information.

Finally, the two stage modelling approach presented is simple and intuitive; nevertheless the results motivate the exploration of future research endeavours. For example, a complete economic learning model to explain the sources and structure of the predictability patterns in the implied volatility surface is beyond the scope of this study. Although the mapping between our two-stage approach and the optimizing behaviour of a representative investor who learns the process of the underlying asset is not straightforward, our results suggest the presence of a strong Markov structure in the *IVS*. Additionally, a learning process followed by option market participants could provide an explanation for the existence of a precisely estimable dynamic relationship between the *IVS* of equity options and the *IVS* of market index options. These relationships could be understood by using models of agents' cognitive mechanisms after changes of global fundamental variables or economic news which affect option pricing and the *IVS*s for all option securities. Moreover, it would be interesting to analyze a possible relationship between the *IVS* shape dynamics of equity options and equity features (e.g. leverage, liquidity, betas, among others), while it may also prove useful to study the dynamic relationships among the *IV* surfaces of options written on different equities which are in the same industry or in other economically relevant sub-groups.

Appendix A

In this appendix we report summary statistics for the deterministic *IVS* model (equation (1)) recursively estimated by OLS. This represents a robustness check of the GLS estimates for the same model presented in Table 2. Table A.1 shows the OLS coefficients, the R^2 coefficients, and the RMSE statistics of the deterministic *IVS* model estimated using equity options (Panel A) and index S&P 500 index options (Panel B). Table A.1 shows that on average in the cross-section of stock options, OLS coefficients are similar to those estimated by GLS as in Table 2. The goodness of fit measures show that GLS estimation yields R^2 and RMSE statistics that are marginally lower than OLS estimates. In addition, the similar values of the LB(1) and LB(3) statistics in Table A.1 and Table 2 suggest that predictability patterns of the *IVS*s discussed in the main text are independent of the estimation approach.

[Insert Table A.1 here]

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Figures

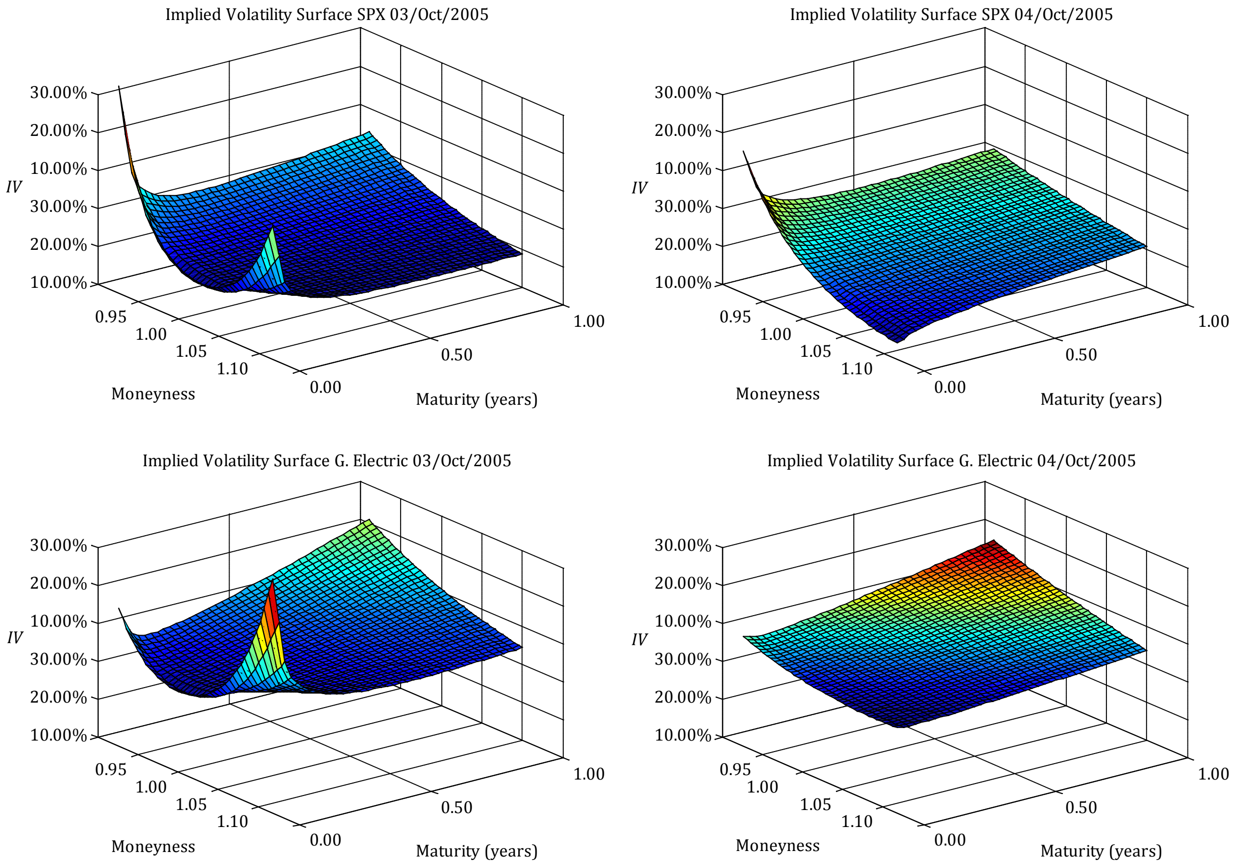

Figure 1. Changes in the implied volatility surface between two consecutive trading days for S&P 500 index options and for General Electric Co. options. The figure shows the *IVS* of S&P 500 index options (two upper windows) and the *IVS* of General Electric Co. options (two lower windows) on two consecutive trading days: October 3, 2005 and October 4, 2005.

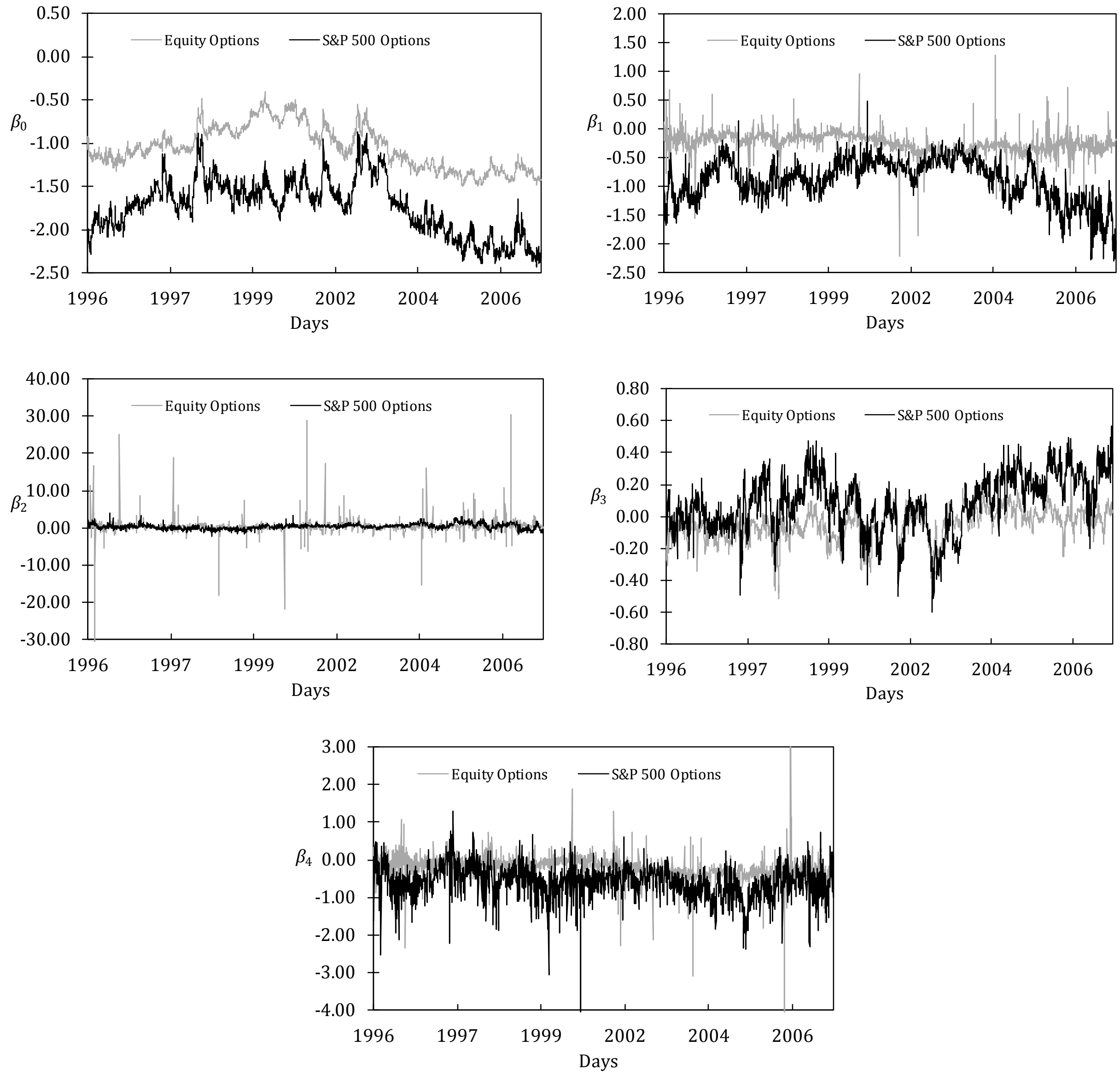

Figure 2. Evolution of coefficients of the deterministic implied volatility surface model estimated by GLS for equity options and for S&P 500 index options. The figure shows the time variation of daily cross-sectional averages of the coefficients of the deterministic *IVS* model in equation (1) estimated with equity options along with the estimated coefficients that describe the *IVS* of S&P 500 index options using the same model. The data cover the period between January 4, 1996 and December 29, 2006.

Tables

Table 1

Summary Statistics of Implied Volatilities across Moneyness and Time-to-Maturity for Equity Options and for S&P 500 Index Options.

	Short-Term (6<Calend. Days≤120)			Medium-Term (120<Calend. Days≤240)			Long Term (240<Calend. Days)		
	Average Trading Freq.	Mean *IV*	Std. Dev. *IV*	Average Trading Freq.	Mean *IV*	Std. Dev. *IV*	Average Trading Freq.	Mean *IV*	Std. Dev. *IV*
	Panel A: Equity Options								
$K/S \leq 0.94$	45.94%	42.27%	17.34%	35.20%	37.63%	14.87%	8.07%	36.56%	14.10%
$0.94<K/S \leq 0.98$	46.87%	39.33%	17.11%	36.63%	36.48%	14.77%	8.36%	35.72%	14.14%
$0.98<K/S \leq 1.02$	47.23%	37.84%	17.23%	38.48%	35.67%	14.88%	8.52%	35.32%	14.30%
$1.02<K/S \leq 1.06$	46.11%	37.70%	17.11%	38.53%	35.28%	14.81%	8.53%	34.92%	14.48%
$1.06<K/S$	42.86%	39.23%	17.28%	36.66%	35.20%	15.03%	8.20%	34.46%	14.30%
	Panel B: S&P 500 Options								
$K/S \leq 0.94$	100.00%	23.45%	6.73%	92.55%	20.86%	5.08%	68.35%	20.38%	4.89%
$0.94<K/S \leq 0.98$	100.00%	19.47%	6.06%	95.30%	19.37%	5.01%	75.66%	19.16%	4.78%
$0.98<K/S \leq 1.02$	100.00%	16.57%	5.85%	97.69%	18.00%	4.95%	86.65%	18.21%	4.76%
$1.02<K/S \leq 1.06$	100.00%	15.83%	5.92%	93.49%	17.19%	5.03%	72.22%	17.85%	4.90%
$1.06<K/S$	99.10%	17.47%	6.49%	88.07%	16.66%	5.05%	66.33%	16.87%	4.88%

Notes: The table contains summary statistics for implied volatilities across moneyness (K/S) and time-to-maturity (calendar days to the expiration). Panel A (Panel B) reports statistics for equity options (market S&P 500 index options). IV is the implied volatility, K is the strike price, and S is the underlying asset price. The table presents trading frequencies, means, and standard deviations. The trading frequency is defined as the percentage of trading days in which we observe at least one trade for an option contract with specific characteristics (given by the moneyness and the time-to-maturity). The data cover the period between January 4, 1996 and December 29, 2006.

Table 2

Summary Statistics of Deterministic *IVS* Model Coefficients Estimated by GLS for Equity Options and for Market S&P 500 Index Options

Coefficients Statistics	Mean	Std. Dev.	Skew	Exc. Kurt.	Min.	Max.	t-test	F-test	LB(1)	LB(3)
				Panel A: Equity Options						
β_0	-1.01	0.30	0.28	0.69	-1.78	-0.03	-51.49		779.23	3022.05
							(97.33)		(100.00)	(100.00)
β_1	-0.22	1.03	0.79	260.71	-15.45	16.03	-2.01		21.80	30.12
							(48.75)		(51.33)	(71.33)
β_2	0.41	9.94	0.76	372.00	-151.97	172.70	0.63		17.03	50.41
							(29.11)		(42.66)	(70.66)
β_3	-0.05	0.21	-0.59	40.80	-1.80	1.58	-1.37		206.36	985.35
							(60.78)		(97.33)	(98.66)
β_4	-0.23	1.93	-1.49	237.12	-30.76	26.65	-0.61		20.48	86.81
							(20.83)		(74.00)	(80.00)
R^2	0.69	0.02	-1.60	2.85	0.03	0.96		20.75	35.65	133.52
								(79.93)	(92.66)	(95.33)
RMSE	0.01	0.01	8.38	148.47	0.00	0.33			28.07	58.68
									(83.33)	(94.00)
				Panel B: S&P 500 Options						
β_0	-1.73	0.32	-0.03	-0.56	-2.43	-0.87	-320.88		980.62	6858.89
							(100.00)		(100.00)	(100.00)
β_1	-0.89	0.36	-0.72	0.52	-2.30	0.49	-16.22		95.99	273.88
							(93.92)		(100.00)	(100.00)
β_2	0.37	0.66	0.45	1.23	-2.12	3.97	2.42		59.23	247.25
							(71.08)		(100.00)	(100.00)
β_3	0.08	0.17	-0.28	-0.03	-0.60	0.56	4.93		309.26	1786.68
							(85.49)		(100.00)	(100.00)
β_4	-0.60	0.43	-1.56	18.37	-6.95	1.31	-3.09		27.52	106.58
							(65.34)		(100.00)	(100.00)
R^2	0.78	0.20	-1.85	3.46	0.16	0.98		382.85	39.60	129.31
								(100.00)	(100.00)	(100.00)
RMSE	0.01	0.01	22.67	661.59	0.00	0.25			13.70	26.55
									(100.00)	(100.00)

Notes: The table shows average summary statistics for daily GLS coefficient estimates, the R^2, and the root mean squared error (RMSE) of the model introduced in equation (1). Panel A concerns average estimates and regression statistics across days in the sample and in the cross-section of stock options; panel B concerns average estimates across days for S&P 500 index options. LB(1) and LB(3) are the values of the Ljung-Box test statistics using one and three lags, respectively. The data cover the period between January 4, 1996 and December 29, 2006. The percentage of statistics with a significant value (using a standard 10% size) for each of the diagnostic tests is reported in parentheses The values in parentheses for the LB(1) and LB(3) statistics are percentages of significant values (at 10%) based on time series computed on each set of individual option contracts.

Table 3

Cross-Sectional Relationships of *IVS* Features Characterised by the Deterministic *IVS* Model Estimated on Equity Options and on S&P 500 Index Options

	$\beta_{0,Equities}$	$\beta_{1,Equities}$	$\beta_{2,Equities}$	$\beta_{3,Equities}$	$\beta_{4,Equities}$	$\beta_{0,SPX}$	$\beta_{1,SPX}$	$\beta_{2,SPX}$	$\beta_{3,SPX}$	$\beta_{4,SPX}$
	Correlations									
$\beta_{0,Equities}$	1.00									
	(100.00)									
$\beta_{1,Equities}$	-0.17	1.00								
	(91.33)	(100.00)								
$\beta_{2,Equities}$	-0.09	-0.21	1.00							
	(74.00)	(90.66)	(100.00)							
$\beta_{3,Equities}$	-0.54	-0.13	0.07	1.00						
	(99.33)	(86.66)	(82.66)	(100.00)						
$\beta_{4,Equities}$	0.01	-0.66	0.02	0.15	1.00					
	(77.33)	(98.66)	(88.66)	(78.00)	(100.00)					
$\beta_{0,SPX}$	0.68	-0.04	-0.03	-0.28	0.06	1.00				
	(98.00)	(52.66)	(49.33)	(96.00)	(62.00)	(100.00)				
$\beta_{1,SPX}$	-0.29	0.02	-0.02	-0.14	0.03	-0.34	1.00			
	(97.33)	(52.00)	(44.00)	(84.66)	(48.66)	(100.00)	(100.00)			
$\beta_{2,SPX}$	-0.19	-0.04	0.05	0.05	-0.02	-0.25	-0.19	1.00		
	(95.33)	(46.00)	(54.00)	(67.33)	(41.33)	(100.00)	(100.00)	(100.00)		
$\beta_{3,SPX}$	-0.49	-0.02	0.01	0.31	-0.05	-0.75	-0.59	0.06	1.00	
	(96.00)	(38.00)	(46.66)	(92.00)	(57.33)	(100.00)	(100.00)	(100.00)	(100.00)	
$\beta_{4,SPX}$	0.08	0.02	0.02	0.03	0.03	0.11	-0.37	-0.48	-0.01	1.00
	(84.00)	(32.66)	(29.33)	(59.33)	(42.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)

Notes: The table contains the average value of a correlation analysis of time series coefficients of the deterministic *IVS* model for market S&P 500 index options and for each individual set of equity options written on the same underlying stock. Daily coefficients from the deterministic *IVS* model are estimated by GLS. The data cover the period between January 4, 1996 and December 29, 2006. The percentage of correlations with significant estimated correlations is reported in parentheses (using a 10% test size); therefore the values in parentheses report percentages across the number of individual equity option time series (i.e., in total 150 different time series, each correlated with the coefficients characterizing the S&P 500 *IVS*).

Table 4

Statistical Measures of Predictability to Evaluate the Forecasting Performance of the Dynamic Equity-SPX *IVS* Model vs. Benchmark Models

	RMSE	MAE	MCPDC (%)	RMSE	MAE	MCPDC (%)
	Implied Volatilities			Option Prices		
VARX (Equity and SPX *IVS* Dynamics)	0.039	0.028	59.63%	0.483	0.392	54.03%
VAR (only Equity *IVS* Dynamics)	0.046	0.037	56.68%	0.619	0.484	52.90%
Random Walk *IVS* ('Strawman')	0.059	0.049	53.23%	0.739	0.649	51.08%
Option GARCH(1,1)	0.053	0.044	54.79%	0.712	0.569	52.45%
Random Walk *IV*	0.049	0.041	NA	0.641	0.503	NA

Notes: The table contains average out-of-sample statistical measures of predictability to evaluate the forecasting properties of the dynamic equity-SPX *IVS* model (equation 3) and benchmark models. The statistical measures are calculated in the implied volatility and in the option price spaces. The four benchmark models are: (i) a VAR(p) model that takes into account only the past dynamics in the *IVS* of individual equity options written on the same underlying stock (equation (4)); (ii) a simple random walk model for the coefficients of the deterministic *IVS* function; (iii) the Duan and Simonato's (2001) American option GARCH model; and (iv) a 'pure' random walk model for implied volatilities. In the table, RMSE is the root mean squared forecast error, MAE is the mean absolute forecast error, and MCPDC is the mean correct prediction of direction of change statistic. The MCPDC cannot be computed for the 'pure' random walk model because this model, by construction, forecasts no change in implied volatilities between time t and any future date.

Table 5
Equal Predictive Accuracy Tests of the Dynamic Equity-SPX *IVS* Model against Benchmark Models

	VAR Only Equity *IVS* Dynamics	Random Walk *IVS* 'Strawman'	Option GARCH(1,1)	Random Walk *IV*
	Panle A: Implied Volatilities			
Comparative Accuracy Test	-4.89	-11.34	-7.39	-5.98
	(73.33)	(88.00)	(81.33)	(78.66)
	Panel B: Option Prices			
Comparative Accuracy Test	-5.04	-12.81	-8.93	-6.17
	(75.33)	(89.33)	(83.33)	(77.33)

Notes: The table shows average cross-sectional Diebold and Mariano's (1995) test statistics computed from a function based on the difference between the RMSEs from the dynamic equity-SPX *IVS* model (equation (3)) and benchmark models. The test statistics are computed both with reference to the implied volatility (Panel A) and the option price (Panel B) spaces. Benchmark models are described in Table 4. The Newey and West (1987) heteroskedasticity and autocorrelation consistent (HAC) variance estimator is used to calculate the Diebold and Mariano (1995) test. The percentages of test statistics that in the cross-section of stock options lead to rejection of the null hypothesis in tests at a 10% size are reported in the parentheses.

Table 6
Economic Value of *IVS* Predictability-Based Trading Strategies (before Transaction Costs)

	Mean Profit (%)	Std. Dev. Profit (%)	t-test	Sharpe Ratio (%)
	Panel A: Straddle Portfolios			
VARX (Equity and SPX *IVS* Dynamics)	5.07%	32.89%	7.91	15.20%
			(80.66)	
VAR (only Equity *IVS* Dynamics)	4.27%	34.04%	6.53	12.64%
			(75.33)	
Random Walk *IVS* ('Strawman')	2.37%	33.30%	3.49	6.62%
			(60.00)	
Option GARCH(1,1)	3.36%	39.00%	4.43	8.71%
			(65.33)	
	Panel B: Delta-Hedged Portfolios			
VARX (Equity and SPX *IVS* Dynamics)	1.82%	30.17%	2.89	5.67%
			(59.33)	
VAR (only Equity *IVS* Dynamics)	1.44%	28.41%	2.46	4.88%
			(56.00)	
Random Walk *IVS* ('Strawman')	0.71%	32.97%	1.09	2.02%
			(19.33)	
Option GARCH(1,1)	1.06%	40.43%	1.33	2.49%
			(35.33)	
	Panel C: Benchmark Portfolios			
S&P Buy and Hold	0.04%	1.11%	1.27	2.22%
			(0.00)	
T-Bill Portfolio	0.02%	0.01%	73.62	0.00%
			(100.00)	

Notes: The table shows summary statistics for recursive out-of-sample daily measures of economic value to evaluate the forecasting power of the dynamic equity-SPX *IVS* model (equation (3)) and of benchmark models. Benchmark models are described in Table 4. The economic measures of predictability are based on profits from straddle portfolios (Panel A) and delta-hedged portfolios (Panel B), before transaction costs. We invest $1,000 net on straddle portfolios and $1,000 net on delta-hedged portfolios on each day in the sample. We re-balance every day so that the $1,000 investment remains constant over time. Straddle portfolios include only straddle positions following the rule in equation (6); while delta-hedged portfolios follow the rule described in equation (8). The percentage of profitability measures that in the cross-section of options are significant using a test size of 10% are in parentheses.

Table 7

Risk-Adjusted Returns from Option Trading Strategies (before Transaction Costs)

	VARX Equity and SPX *IVS* Dynam.	VAR Only Equity *IVS* Dynam.	Random Walk *IVS* 'Strawman'	Option GARCH(1,1)
	Panel A: Straddle Portfolios			
Alpha	0.05	0.04	0.02	0.03
	(79.33)	(71.33)	(53.33)	(59.33)
$MKT-r_f$	-0.80	-0.52	-0.62	0.59
	(35.33)	(27.33)	(26.00)	(30.66)
SMB	-0.42	-0.44	-0.55	-0.49
	(9.33)	(12.66)	(17.33)	(14.00)
HML	-0.92	-0.79	-0.82	-0.63
	(13.33)	(16.00)	(19.33)	(11.33)
MOM	0.35	0.39	0.42	0.34
	(12.00)	(8.67)	(7.33)	(13.33)
$ZbSTrad-r_f$	0.12	0.13	0.10	0.12
	(40.66)	(35.33)	(36.00)	(43.33)
R^2	0.09	0.07	0.09	0.08
	(20.00)	(19.33)	(16.66)	(17.33)
	Panel B: Delta-Hedged Portfolios			
Alpha	0.02	0.01	0.01	0.01
	(55.33)	(47.33)	(15.33)	(32.00)
$MKT-r_f$	1.90	1.47	2.33	1.52
	(63.33)	(59.33)	(68.00)	(62.66)
SMB	0.08	0.06	0.03	0.06
	(14.66)	(11.33)	(7.33)	(8.66)
HML	-0.32	-0.24	-0.18	-0.28
	(12.00)	(10.66)	(14.66)	(13.33)
MOM	0.09	0.08	0.07	0.08
	(14.66)	(13.33)	(9.33)	(10.66)
$DhCall-r_f$	0.25	0.23	0.279	0.383
	(38.00)	(34.66)	(30.66)	(39.33)
R^2	0.06	0.04	0.04	0.05
	(24.66)	(21.33)	(19.33)	(22.66)

Notes: The table contains average parameter estimates for the asset pricing factor model in equation (10) estimated on excess returns of straddle portfolios (Panel A) and delta-hedged portfolios (Panel B). Straddle and delta-hedged portfolios are formed as in Table 6 and they are based on forecasts of the dynamic equity-SPX *IVS* model (equation (3)) and of benchmark models. The asset pricing factor model includes the Fama and French (1993) three factors (i.e., excess market returns, size-sorted returns, and HML returns), the Carhart's (1997) momentum factor (MOM), and the Coval and Shumway's (2001) option volatility factor built from one at-the-money short-term position on S&P 500 index option contracts. The option volatility factor in the case of straddle portfolios is the excess return of a straddle position which is zero-beta (ZbStrad-rf); while the option volatility factor in the case of delta-hedged portfolios is calculated using the excess return of a delta-hedged position on a call option contract ($DhCall-r_f$). The percentage of the cross-section of the 150 equity options with significant parameters using a 10% test size is reported in parentheses; while the percentage of asset pricing models across the equity options with a significant F-statistic using a 10% test size is also reported in parentheses below the R^2.

Table 8

Economic Value of *IVS* Predictability-Based Trading Strategies (after Transaction Costs)

	Mean Profit (%)	Std. Dev. Profit (%)	t-test	Sharpe Ratio (%)
		Panel A: Straddle Portfolios		
VARX (Equity and SPX *IVS* Dynamics)	-9.88%	45.03%	-11.05 (91.33)	-22.17%
VAR (only Equity *IVS* Dynamics)	-11.64%	48.13%	-12.96 (92.66)	-25.99%
Random Walk *IVS* ('Strawman')	-13.79%	46.83%	-15.09 (96.66)	-30.15%
Option GARCH(1,1)	-12.79%	52.84%	-12.82 (94.00)	-24.31%
		Panel B: Delta-Hedged Portfolios		
VARX (Equity and SPX *IVS* Dynamics)	-5.73%	38.30%	-7.74 (81.33)	-15.45%
VAR (only Equity *IVS* Dynamics)	-6.52%	39.76%	-8.87 (84.66)	-16.32%
Random Walk *IVS* ('Strawman')	-7.42%	46.12%	-8.42 (87.33)	-16.73%
Option GARCH(1,1)	-7.08%	42.43%	-9.18 (88.66)	-17.07%

Notes: The table shows out-of-sample economic measures of predictability of the dynamic equity-SPX *IVS* model (equation (3)) and of benchmark models. Benchmark models are described in Table 4. The measures of profitability are based on the profits from straddle (Panel A) and delta-hedged portfolios (Panel B) after transaction costs. Straddle and delta-hedged portfolios are formed as in Table 6. Transaction costs are incorporated by setting them to equal the effective bid-ask spread. We use a conservative effective bid-ask spread that is 0.5 times the quoted spread. The percentage of t-test statistics across the 150 sets of equity option contracts that lead to a rejection of the null hypothesis of zero mean profits using a 10% size test is reported in parenthesis.

Table A.1

Summary Statistics of Deterministic *IVS* Model Coefficients Estimated by OLS for Equity Options and for Market S&P 500 Index Options

Coefficients Statistics	Mean	Std. Dev.	Skew	Exc. Kurt.	Min.	Max.	t-test	F-test	LB(1)	LB(3)
				Panel A: Equity Options						
β_0	-1.01	0.30	0.26	0.61	-1.81	-0.01	-54.43		867.26	3971.06
							(99.26)		(100.00)	(100.00)
β_1	-0.19	1.06	0.36	257.28	-15.68	16.40	-2.04		23.27	34.83
							(50.04)		(52.66)	(74.00)
β_2	0.33	9.30	1.27	373.44	-144.64	160.84	0.84		19.81	51.79
							(38.90)		(44.66)	(72.00)
β_3	-0.05	0.23	-0.40	35.82	-1.81	1.63	-1.11		238.56	1147.01
							(57.21)		(99.33)	(99.33)
β_4	-0.30	2.06	-1.43	215.50	-31.55	27.01	-0.65		27.49	99.69
							(22.32)		(75.33)	(80.66)
R^2	0.75	0.21	-1.20	1.10	0.04	0.99		28.68	39.61	150.87
								(84.88)	(100.00)	(100.00)
RMSE	0.00	0.01	11.85	283.21	0.00	0.31			31.21	60.19
									(85.33)	(88.00)
				Panel B: S&P 500 Options						
β_0	-1.75	0.32	0.01	-0.53	-2.44	-0.85	-263.25		1064.92	7786.29
							(100.00)		(100.00)	(100.00)
β_1	-0.80	0.41	-0.39	0.23	-2.17	0.48	-17.18		107.97	333.21
							(96.16)		(100.00)	(100.00)
β_2	0.72	0.85	1.53	3.84	-1.21	6.07	3.90		65.45	263.62
							(72.27)		(100.00)	(100.00)
β_3	0.11	0.20	-0.28	-0.10	-0.60	0.81	5.38		323.40	2416.80
							(87.33)		(100.00)	(100.00)
β_4	-0.89	0.83	-1.51	4.72	-6.94	1.30	-3.22		30.89	108.68
							(69.99)		(100.00)	(100.00)
R^2	0.85	0.14	-1.73	3.22	0.17	0.99		518.54	43.04	136.56
								(100.00)	(100.00)	(100.00)
RMSE	0.00	0.01	22.20	814.67	0.00	0.18			43.81	54.34
									(100.00)	(100.00)

Notes: The table shows average summary statistics for daily OLS coefficient estimates, the R^2, and the root mean squared error (RMSE) of the model introduced in equation (1). Panel A concerns average estimates and regression statistics across days in the sample and stock options in the cross-section; panel B concerns average estimates across days for S&P 500 index options. LB(1) and LB(3) are the values of the Ljung-Box test statistics using one and three lags, respectively. The data cover the period between January 4, 1996 and December 29, 2006. The percentage of statistics with a significant value (using a standard 10% size) for each of the diagnostic tests is reported in parentheses The values in parentheses for the LB(1) and LB(3) statistics are percentages of significant values (at 10%) based on time series computed on each set of individual option contracts.