INNOCENZO GASPARINI INSTITUTE FOR ECONOMIC RESEARCH

Institutional Members: CEPR, NBER and Università Bocconi

WORKING PAPER SERIES

Tick Size Regulation and Sub-Penny Trading

Sabrina Buti, Barbara Rindi, Yuanji Wen, Ingrid M. Werner

Working Paper n. 492

This Version: September 9, 2013

IGIER – Università Bocconi, Via Guglielmo Röntgen 1, 20136 Milano –Italy
http://www.igier.unibocconi.it

Tick Size Regulation and Sub-Penny Trading*

Sabrina Buti† Barbara Rindi‡
Yuanji Wen § Ingrid M. Werner¶

September 9, 2013

*With thanks to Thierry Foucault, Mark Van Achter, Thomas McInish, Elvira Sojili and Travis Box for thoughtful comments and suggestions. We also thank participants at FMA2011, EFMA2011, AFA2012, EFA2013 and the Central Bank Workshop on the Microstructure of Financial Markets 2012 in Stavanger for valuable comments and discussions. We thank the Center for Applied Research in Finance (Carefin, Bocconi University) for financial support. Yuanji Wen acknowledges support from the Fondazione Cariplo. The usual disclaimer applies.

†University of Toronto, Rotman School of Management, Sabrina.Buti@rotman.utoronto.ca
‡Bocconi University, IGIER and Carefin, barbara.rindi@unibocconi.it
§Bocconi University, yuanji.wen@phd.unibocconi.it
¶Fisher College of Business, Ohio State University, werner_47@fisher.osu.edu

Tick Size Regulation and Sub-Penny Trading

We show that following a tick size reduction in a decimal public limit order book (PLB) market quality and welfare fall for illiquid but increase for liquid stocks. If a Sub-Penny Venue (SPV) starts competing with a penny-quoting PLB, market quality deteriorates for illiquid, low priced stocks, while it improves for liquid, high priced stocks. As all traders can demand liquidity on the SPV, traders' welfare increases. If the PLB facing competition from a SPV lowers its tick size, PLB spread and depth decline and total volume and welfare increase irrespective of stock liquidity.

1 Introduction

The minimum price variation - the tick size - is one of the most important factors affecting liquidity of securities traded on public limit order books (PLBs). As a consequence, it has been at the top of the regulatory agenda over the past decade (e.g., SEC 2010 and SEC 2012). Decimalization, i.e., the transition to trading and quoting securities in one penny increments, started in 2001 in the U.S. and in 2004 in Europe and had profound consequences both for the level of liquidity, and for the business model of those institutions which support liquidity. The anticipated and unanticipated consequences of tick-size reductions on the quality of the markets and on the welfare of market participants have been debated extensively.[1] The rationale for reducing the tick-size to decimals was to encourage trading activity by reducing transaction costs; the unanticipated consequence was that the increased competition for the provision of liquidity reduced the incentive for market participants to supply liquidity. Several empirical studies indeed found that the quoted spread declined following a tick size reduction, but so did displayed depth, resulting in potentially higher costs for institutional traders.

However, it is challenging to draw inferences from historical tick-size reductions for our current markets. The reason is that today lit markets compete in tick size for the provision of liquidity with both transparent and dark markets, some of which quote in smaller price increments. The Securities and Exchange Commission (SEC) regulates the tick size for all transparent venues in the U.S., but more than one-third of U.S. consolidated equity volume (SEC, 2010) executes in non-transparent venues that permit limit orders in sub-penny increments, i.e., at fractions of the minimum regulated tick size on transparent markets. In Europe, even transparent markets may use different tick sizes. Hence, European exchanges trading the same security compete on tick size. Further, the Markets in Financial Instruments Directive (MiFID) opened up European markets to competition from transparent and opaque venues that quote in sub-penny. Examples of venues generating sub-penny executions are internalization pools where brokers execute their own customers' buy orders against their own customers' sell orders and dark pools where subscribers submit orders to dark trading platforms. There are of course also dark venues that mix internalized order flow with orders from

[1] See Sections 2 and 3.

other brokers. Since we are primarily focusing on the tick-size aspect of these competing venues, we collectively label any venue that accepts sub-penny orders as a Sub-Penny Venue (SPV), and for modeling purposes allow the SPV to be either transparent or opaque.

The tick size is a policy instrument that regulators and exchanges can use to affect the willingness of market participants to provide liquidity to a PLB. However, by modifying the tick size, regulators and exchanges also influence the incentive for traders to undercut the existing liquidity on the top of the PLB, and trade in sub-penny increment in the SPV, thus diverting liquidity away from the PLB. Since the amount of sub-penny trading in the U.S. markets is now substantial (Buti, Consonni, Rindi and Werner, 2013), the interaction between tick size variations, liquidity provision and sub-penny trading is a key policy issue.[2] This interaction is complex and subtle as our model will show.

To help guide the debate about tick-size regulation in today's fragmented market structure, we start by analyzing the effects of a tick size variation on order flow, market quality and welfare within the context of a single PLB that works like a double auction trading platform. This discussion sets the stage for a more sophisticated analysis of a tick size variation in a more realistic setting in which the same PLB competes for the provision of liquidity with a market characterized by a smaller tick size. More specifically, in this dual market model the platform that competes with the PLB is a SPV as defined above.

Our model contributes to the current regulatory debate on tick size regulation and on sub-penny trading in three ways. First, we show how regulators and empirical researchers should evaluate the effects of a tick size change when the market operates as a transparent limit order book. Second, we discuss the effects that sub-penny trading has on the liquidity of the lit markets, the factors that drive trading in sub-penny, and the costs and benefits of sub-penny trading for market participants. Finally, we provide guidelines for regulators and exchanges on how to set the tick size when there is inter-market competition for the provision of liquidity and they wish to limit sub-penny trading. We show that the critical regulatory issue is in fact to adjust the tick size for

[2] Buti, Consonni, Rindi and Werner (2013) study sub-penny trading for a sample of 180 NASDAQ and NYSE stocks stratified by price and market capitalization. They find that over a sample period of 42 days in 2010, the daily average of sub-penny trading is equal to 12.07% and 11.17% for the NASDAQ and NYSE low priced stocks respectively; and to 8.75% and 8.04% for high priced stocks.

each stock to optimally balance the effect it has on sub-penny trading with the effect it has on liquidity provision, which in turn depends on the initial level of liquidity and on the price of the stock (Goldstein and Kavajecz, 2000).

To build our theoretical model, we extend Parlour's (1998) framework to include more price levels, and we add competition from a second trading platform. More price levels are necessary to study how the aggressiveness of the liquidity provision changes after a variation in the tick size. Inter-market competition is necessary to embed the most salient feature of today's financial markets. Within our three-period framework, we are able to discuss results for stocks with different characteristics. To this end, we use the opening state of the limit order book as a proxy for the liquidity of a stock and solve the model for different values of the asset price.

Starting with a single market model, we show that the effects of setting a different tick size depend both on the liquidity of the stock, and on its price. For liquid stocks, a smaller tick size increases competition among liquidity suppliers and hence improves both market quality and traders' welfare. For illiquid stocks, instead, a smaller tick size discourages liquidity provision and worsens both market quality and traders' welfare. Moreover, the effects of introducing a smaller tick size are more significant for low priced stocks. When the price of the stock is low, the costs that traders face if the order is not executed (non-execution costs) are lower, all else equal. The result is that traders use limit orders more intensively. Therefore, the provision of liquidity plays a major role by amplifying the effect of a smaller tick size. Based on these findings, which are all consistent with the empirical results of Goldstein and Kavajecz (2000), we argue that when setting the minimum price improvement, regulators and exchanges need to consider both the asset price level and the liquidity of the stock.

We extend the framework to a dual market model in which a group of Broker-Dealers (BDs) choose between executing their customers' orders on a PLB or on a SPV. We model the SPV as a limit order book with a finer price grid than the PLB, that can be either transparent or opaque. While only BDs can post limit orders on the SPV, all traders can take advantage of the liquidity offered by both trading platforms. This assumption is consistent with a fast market in which a smart order routing (SOR) technology allows all investors to simultaneously access multiple sources of liquidity (Butler, 2010).[3] Specifically, Regular Traders (RTs) can access the liquidity posted on the SPV through SORs, which compare the liquidity in the two markets and send the order to the market that offers the best execution.

In reality, the more sophisticated the SOR technology, the better the traders' inference on the state of the SPV is. Hence, to consider different regimes of SPV pre-trade transparency, we first assume perfect inference on the state of the SPV, and then extend the model to include partial inference and Bayesian learning which is what would happen in a dark SPV. With transparency RTs can observe the state of the SPV; with opacity they can only make inference about the state of the SPV based on publicly available information.

The dual market model allows us to investigate the consequences of sub-penny trading in a setting in which the main PLB trades in penny increments. It is also an ideal framework for us to investigate the effects of a tick size reduction. This setup captures the essence of today's stock trading environment where market venues such as NYSE-Euronext and Nasdaq-OMX compete with internalizers and dark pools.

Our results show that the SPV attracts both limit and market orders away from the PLB. The reduction of limit orders causes a reduction in the provision of liquidity on the PLB and hence has a detrimental effect on both market depth and the inside spread. Conversely, the reduction of market orders leads to a decrease in the demand for liquidity. This preserves depth and spread on the PLB, generating a positive effect on liquidity. We find that the overall effect on the quality of the PLB is dictated by the net result of these two forces, which in turn crucially depends on the initial state of the PLB, i.e., whether the stock is liquid or illiquid.

When the stock is liquid, the introduction of the SPV induces BDs to trade intensively on the SPV in order to undercut the orders sitting on the PLB, thus reducing liquidity supply on the PLB. However, aggressive market orders are intercepted by the SPV so that also liquidity demand decreases on the PLB, thus preserving liquidity. We show that in liquid books the positive effect of the reduced liquidity demand dominates. The reason is that, all else equal, when the stock is liquid competition for the provision of liquidity is intense and the execution probability of limit orders on

[3] Examples are ITG Dark Aggregator and Smartrade Liquidity Aggregator.

the PLB is low. As a result, traders rely more heavily on market orders and these are intercepted by the SPV. This effect is even stronger for high priced stocks: when the stock price is higher, non-execution costs are also higher and traders opt even more intensively for market orders.

On the other hand, for illiquid stocks the existence of a SPV is detrimental for the level of liquidity on the PLB. When traders perceive the competition from the SPV, they are afraid of being undercut both on the PLB and on the SPV. Hence they reduce their provision of liquidity in the PLB to such a degree that both market depth and the inside spread worsen. This time the effect is weaker for high priced stocks. The greater role played by liquidity demand when the price of the stock is high makes the positive effect of the reduced demand that follows the introduction of a SPV stronger, thus attenuating the negative effect of the reduced liquidity supply.

Our results also show that overall the opacity of the SPV has a negative effect on the quality of the PLB, especially for illiquid stocks. When RTs cannot condition their order choice on the state of the SPV, they cannot observe whether liquidity is available on the SPV and therefore market orders bounce back to the PLB more frequently. When the initial book is liquid, the market orders bouncing back and executing on the PLB reduce the positive effect of the SPV competition on market quality. When the initial book is illiquid, uncertainty on the state of the SPV generates an additional effect: it decreases the execution probability of limit orders so that RTs rely more heavily on market orders. The resulting reduction in liquidity provision has a substantial further negative effect on market quality.

Moreover, our results show that even though competition from a SPV has mixed effects on the quality of the PLB, it makes all traders better off, be they BDs who supply liquidity, or RTs who can access this extra liquidity via SORs. The results also show that gains from trade are higher for low priced stocks, which explains why our model predicts that BDs should be more active in these stocks. A word of caution is warranted, though. The welfare of RTs increases with sub-penny trading provided that SORs allow them to benefit from the liquidity posted on the SPV. Hence, unsophisticated retail traders, who are likely to be unable to take advantage of this optional liquidity, could be harmed when the quality of the PLB deteriorates, as we observe for illiquid stocks.

The final step is to use the extended framework with inter-market competition between a PLB and a SPV to investigate the effect of a tick size reduction within this more realistic setting. We find that when the PLB tick size is smaller, competition from the SPV is more beneficial. The negative effect on the market quality of illiquid stocks is tempered and traders use the SPV less intensively. Likewise, the positive effect on the market quality of liquid stocks is magnified and traders supply more liquidity to the SPV. As we discuss in the conclusions, this result leads to a straight forward policy prescription on the optimal regulation of the tick size.

Our model provides new predictions on the effects on market quality and welfare of tick size changes in a PLB. Specifically, we show that the effects depend on the liquidity and price of the stock being traded. We also provide several new empirical predictions regarding the impact of tick size changes when a regular exchange faces competition from SPVs summarized above. The model shows that it is necessary to study the dynamics of order flow to understand how PLBs and SPVs interact and thereby affect both the provision of and the demand for liquidity.

This paper is related to three strands of the existing theoretical literature, that is to inter-market competition, to the optimal tick size, and to the internalization of order flows by BDs (Battalio and Holden, 2001).[4] To the best of our knowledge, it is the first model that allows researchers to investigate the tick size rule within a framework that takes into account both the asset value and the liquidity of the stock. It is also the first paper that investigates the regulation of the tick size within the context of inter-market competition. It also departs from the existing theoretical works as it embeds sub-penny trading through modeling a SPV.

The remaining part of this paper is structured as follows: in Section 2, we discuss the regulatory debate on tick size, in Section 3 we overview the related literature. In Section 4 and 5, we focus on the single market model, whereas Section 6 and 7 contains the model with a SPV. In Section 8, we discuss the effects of tick size changes on the welfare of market participants. We present the empirical implications in Section 9, and we draw policy conclusions in Section 10. All the proofs appear in the Appendix.[5]

[4] See, for example, Chowdhry and Nanda (1991), Foucault and Menkveld (2008), and Parlour and Seppi (2003) on intermarket competition, and Anshuman and Kalay (1998), Cordella and Foucault (1999), Foucault et al. (2005), Goettler et al. (2005), Kadan (2006), and Seppi (1997) on optimal tick size.

[5] An extended version of the proofs is available in the Internet Appendix.

2 Regulatory Debate

As a vast body of empirical literature has shown, when the tick size is reduced spread decreases but depth at the top of the book deteriorates.[6] For this reason, regulators are concerned about trading strategies that exploit the possibility to submit orders at fractions of the minimum tick size. In 2005 the SEC introduced the Sub-Penny Rule [adopted Rule 612 under Regulation National Market System (NMS)]. The rule is aimed at protecting displayed limit orders from being undercut by trivial amounts. It prohibits market participants from displaying, ranking, or accepting quotations in NMS stocks that are priced at smaller increments than the allowed minimum price variation.

In the years following the introduction of Rule 612, however, the development of SPVs deeply affected inter-market competition, and made the rule ineffective in protecting displayed limit orders. In particular, two features of the rule paved the way for sub-penny trading. First, Rule 612 prohibits market participants from quoting prices in sub-penny, but in the belief that sub-penny trading would not be as detrimental as sub-penny quoting, it expressly allows BDs to provide price improvement to a customer order that results in a sub-penny execution, thus allowing sub-penny trading. Second, the Rule 612 prohibition of sub-penny quoting does not apply to dark markets; this means that BDs can exploit dark SPVs to jump the queue by a fraction of a penny and so preempt the National Best Bid Offer (NBBO).

Another important factor that facilitates sub-penny trading is the growing importance of fast trading facilities. Using programs to generate algorithmic replications of trading strategies, BDs trading large volumes can make significant profits even though they sacrifice a fraction of a penny in order to step ahead of the PLB.[7] As a result, the proportion of sub-penny trading (and in particular that of trades submitted at price increment smaller than half a tick, i.e., queue-jumping) has dramatically increased over the past 10 years as shown in Figure 1.

[**Insert Figure 1 here**]

The SEC (2010) has recently proposed a Trade-At Rule that would prohibit "any trading center

[6] See Section 3.

[7] Jarnecic and Snape (2010) suggest that high frequency trading is negatively related to the tick size.

from executing a trade at the price of the NBBO unless the trading center was displaying that price at the time it received the incoming contra-side order." This rule would effectively prohibit dark venues from trading at the NBBO, which would significantly affect inter-market competition. On October 12, 2012 the Investment Industry Regulatory Organization of Canada introduced a Trade-At Rule that now gives lit orders priority over dark orders in the same venue. In particular small dark orders under 5,000 shares or C$100,000 dollars in value must offer at least half a tick in price improvement for stocks that have a one tick spread, and a full tick of price improvement for stocks with higher spreads. Following the implementation of this rule, the Canadian dark share of volume dropped by more than 50% (Rosenblatt Securities Inc., February 2013). Even the Australian Securities and Investments Commission on May 26, 2013 adopted a new regulation for dark venues aimed at containing dark trading for transactions of size smaller than blocks. The key component of the new regulatory regime is the adoption of a minimum size threshold for dark orders.

By contrast, in 2009 BATS proposed to reduce the minimum price increment of publicly displayed market centers to sub-pennies, in order to level the playing field. Our model shows that this approach would indeed be helpful in reducing the adverse consequences on market quality that may arise because of intra-market competition from a venue that trades in sub-pennies.

Finally, in April 2012, the U.S. Congress passed the Jumpstart Our Business Startup (JOBS) Act which instructed the SEC to study the impact of decimalization on liquidity for small and medium capitalization companies. According to the JOBS Act, if needed, the SEC is allowed to increase the minimum trading increment of emerging growth companies. However, the conclusions of the SEC Report to Congress on Decimalization (2012) discouraged the Commission from proceeding with the rulemaking required to increase the tick size. Our model shows that an increase in the tick size for smaller stocks may be ineffective in today's trading environment. We explain why in Section 10.

3 Literature Review

There is extensive research on the relationship between the reduction of the tick size and market quality.

Theoretical models have been developed to study the effect of a tick size variation in different market structures. Seppi (1997) investigates the optimal tick size in a market in which a specialist competes for liquidity provision against a competitive limit order book and finds that large traders may prefer a larger tick size than small traders. Cordella and Foucault (1999) study competition between dealers who arrive at the market sequentially and whose bidding strategy depends of the value of the tick size. They show that a larger tick size can increase the speed at which dealers adjust their quotes towards the competitive price, especially when monitoring costs are high. Hence transaction costs can ultimately decrease following an increase in the tick size. Similarly, competition among dealers for the provision of liquidity to an incoming market order drives the results obtained by Kadan (2006). He shows that when the number of dealers is small, a small tick size is associated with higher liquidity since this prevents dealers from exploiting their market power. When the number of dealers is instead large, a small tick size may also improve liquidity. The reason is that a small tick size allows dealers to post quotes as close as possible to their reservation value, thus transferring welfare to liquidity demanders.

Our model departs from all these protocols by considering a pure order driven market in which liquidity provision is endogenously created by market participants who choose limit as opposed to market orders. To evaluate the effects of a tick size variation in pure limit order markets, one has to consider the competitive interaction of both patient traders who supply liquidity via limit orders, and impatient traders who demand liquidity via market orders. Our framework shares this feature with the dynamic model of Foucault et al. (2005) who show that a reduction of the tick size may harm market resiliency and have adverse effects on transaction costs. In their model, however, traders cannot refrain from trading and when submitting limit orders they must provide a price improvement. Hence, because patient traders cannot join the queue at the existing best bid or offer, a larger tick size has the effect of making their orders more rather than less aggressive, resulting in an increased resiliency and a narrowed spread. By contrast, in our model traders are free to submit market and limit orders at any level on the price grid, as well as to refrain from trading.

Our protocol is closer to Goettler et al. (2005) who consider an infinite horizon version of

Parlour (1998) and model a limit order book as a stochastic sequential game with rational traders submitting orders at, above or below the existing best quotes. Goettler et al. (2005) show that, by reducing the tick size, regulators increase the total surplus of investors. We show instead that the effects of a tick size variation depend on the liquidity of the limit order book and on the price of the stock considered. Differently from Goettler et al. (2005), we also study the effects of competition from a SPV.

Empirical studies from various markets around the world have found that a tick size reduction is associated with a decline in both the spread and depth, and that the spread is not equally affected across stocks.[8] These findings are confirmed by a more recent pilot program implemented by the major European platforms aimed at investigating the effect of a reduction of the tick size, and are consistent with the early predictions of Angel (1997) and Harris (1994).[9]

More recent empirical studies have focused on sub-penny trading. Hatheway, Kwan and Zheng (2013) suggest that sub-penny trading attracts uninformed traders away from lit markets and hence reduces adverse selection costs. Kwan, Masulis and McInish (2013) conclude that the ability to circumvent time priority of displayed limit orders increases the fragmentation of U.S. equity markets. Buti, Consonni, Rindi and Werner (2013) investigate two related issues. First, they study under which conditions and in which type of stocks BDs trade more intensively at sub-penny increments. They separate queue-jumping from mid-crossing and find that queue-jumping is negatively related to the price of the stock and positively related to liquidity and volatility.[10] Second, they investigate the effect of sub-penny trading on market quality, measured by relative spread and inside depth, and find no evidence that it harms the quality of the market for liquid

[8] See for example Ahn et al. (1996), Bacidore (1997), Bourghelle and Declerck (2004), Goldstein and Kavajecz (2000), Griffiths et al. (1998), and Ronen and Weaver (2001).

[9] In December 2008, BATS Europe, in conjunction with Chi-X, Nasdaq OMX Europe and Turquoise, developed a proposal to standardize the tick size of the pan European trading platforms. Starting June 1, 2009, Chi-X, followed by Turquoise, BATS Europe, and finally the LSE and Nasdaq OMX Europe, reduced the tick size for a number of stocks. This pilot program, aimed at studying the effect of a change in the tick size based on actual market data, showed that following the reduction of the tick size, effective spread, inside spread, inside depth and average trade size decreased (BATS, 2009).

[10] By definition, sub-penny trading takes place at price increments that are smaller than one tick. However, it is important to separate queue-jumping from mid-crossing. Queue-jumping is the practice that competes for the provision of liquidity at the top of lit markets and worries regulators. Technically, mid-crossing is still trading at sub-penny increments, but a significant fraction of it derives either from dark pools that execute at the spread mid-quote according to a derivative pricing rule, or from lit markets that allow traders to use hidden midpoint-pegged orders.

stocks.

Our paper is also related to the literature on inter-market competition that documents an improvement in market efficiency when competing venues enter a market.[11] Chowdhry and Nanda (1991) extend Kyle (1985) model to accommodate multi-market trading and show that markets with the lowest transaction costs attract liquidity. Closer to our framework, Hendershott and Mendelson (2000) discuss costs and benefits of order flow fragmentation by modeling the interaction between a dealer market and a crossing network. Degryse et al. (2009) also focus on a dealer market competing with a crossing network, and show that overall welfare is not necessarily enhanced by the introduction of a crossing network. Our setup substantially differs from theirs as we consider a PLB instead of a dealer market and a SPV instead of a crossing network. Our model also departs from Buti, Rindi and Werner (2013) who model competition between a PLB and a dark pool by focusing squarely on the tick size. Furthermore, the SPV that we model has its own discriminatory pricing rule and a thinner price grid than the PLB, whereas the dark pool in Buti, Rindi and Werner (2013) is based on a derivative pricing rule and on midpoint execution.

Finally, this paper is related to the literature on BDs' internalization and payment for order flow.[12] Chordia and Subrahmanyam (1995) and Kandel and Marx (1999) show that these practices arise from the existence of the tick size. By contrast, Battalio and Holden (2001) show that when the tick size is set equal to zero, brokers still internalize their clients' orders as they make profits by exploiting their direct relationships with customers. This prediction is consistent with the related empirical works.[13]

4 Single Market Model

In this Section we introduce the single market framework and in the next one we solve the model and compare the results for two different values of the tick size. In Section 6 we add competition

[11] See, for example, Barclay et al. (2003), Bessembinder and Kaufman (1997), Biais et al. (2010), Foucault and Menkveld (2008), and Goldstein et al. (2008).

[12] Internalization is either the direction of order flows by a BD to an affiliated specialist, or the execution of order flows by that BD acting as a market maker.

[13] See Chung et al. (2004), Hansch et al. (1999), He et al. (2006), Hendershott and Jones (2005), and Porter and Weaver (1997).

from a SPV where BDs can post quotes at sub-penny increments.

4.1 The Market

A market for a security is run over a trading day divided into 3 periods, $t = t_1, t_2, t_3$. At each period t a trader arrives. For simplicity we assume that the size of his order is unitary. Following Parlour (1998), traders are risk neutral and have the following linear preferences:

$$U(C_1, C_2; \beta) = C_1 + \beta C_2 \tag{1}$$

where C_1 is the cash inflow from selling or buying the security on day 1, while C_2 is the cash inflow from the asset payment on day 2 and is equal to $+v$ $(-v)$ in case of a buy (sell) order. β is a patience indicator drawn from the uniform distribution $U(\underline{\beta}, \overline{\beta})$, with $0 \leq \underline{\beta} < 1 < \overline{\beta}$. Traders have a personal trade-off, which depends on β, between consumption in the two days. A patient trader has a β close to 1 while an eager one has values of β close to either $\underline{\beta}$ or $\overline{\beta}$. For all of our numerical simulation and the analytical solution to the model we assume that the support of the β distribution is $U(0, 2)$.

Upon arrival in period t, the trader observes the state of the book, which is characterized by the number of shares available at each level of the price grid. To investigate how changes in the tick size affect traders' aggressiveness in the provision of liquidity as well as depth at different levels of the book, we need at least two price levels on each side of the PLB. Therefore we extend Parlour's (1998) model to include two prices on the ask ($A_1 < A_2$) and two prices on the bid side of the market ($B_1 > B_2$), symmetrically distributed around the asset value v. The difference between two adjacent prices -the minimum price increment- is the tick size, τ, which also corresponds to the minimum inside spread. Thus the possible prices are equal to $A_1 = v + \frac{\tau}{2}$, $A_2 = v + \frac{3\tau}{2}$, $B_1 = v - \frac{\tau}{2}$, and $B_2 = v - \frac{3\tau}{2}$. The state of the book that specifies the number of shares Q_t available at each price level is defined as $S_t = [Q_t^{A_2}, Q_t^{A_1}, Q_t^{B_1}, Q_t^{B_2}]$. As in Seppi (1997) and Parlour (1998) we assume that a trading crowd provides liquidity at the highest levels of the limit order book and prevents traders from quoting prices that are too far away from the top of the book. In addition, traders are allowed to submit limit orders queuing in front of the trading crowd.

In this market, which enforces price and time priority, traders can choose between two types of orders that cannot be modified or cancelled after submission: limit orders that we indicate by $+1$, and market orders that we indicate by -1. We define H_t a trader's strategy at time t. Traders can submit limit orders to sell or to buy one share respectively at the two levels of the ask and of the bid side of the PLB, $H_t = \{+1^{A_k}, +1^{B_k}\}$, where $k = 1, 2$. These orders stay on the PLB until they are executed against a market order of opposite sign. Alternatively, traders can submit market orders which hit the best bid or ask prices available on the PLB and are hence executed immediately, $H_t = \{-1^{B_{k'}}, -1^{A_{k'}}\}$, where $k' = 1, 2$ refers to the best price. Finally, traders can decide not to trade, $H_t = \{0\}$. The trader's strategy space is therefore $H_t = \{-1^{A_{k'}}, +1^{A_k}, 0, +1^{B_k}, -1^{B_{k'}}\}$. The change in the limit order book induced by the trader's strategy H_t is indicated by h_t and defined as:

$$h_t = [h_t^{A_2}, h_t^{A_1}, h_t^{B_1}, h_t^{B_2}] = \begin{cases} [\pm 1, 0, 0, 0] & \text{if } H_t = \pm 1^{A_2} \\ [0, \pm 1, 0, 0] & \text{if } H_t = \pm 1^{A_1} \\ [0, 0, 0, 0] & \text{if } H_t = 0 \\ [0, 0, \pm 1, 0] & \text{if } H_t = \pm 1^{B_1} \\ [0, 0, 0, \pm 1] & \text{if } H_t = \pm 1^{B_2} \end{cases} \tag{2}$$

The state of the book is hence characterized by the following dynamics:

$$S_t = S_{t-1} + h_t \tag{3}$$

where S_{t-1} is the state of the book that the trader arriving at t observes before he submits his order, and S_t is the state of the book after his order submission. We assume that when a trader arrives at t_1 he observes the initial state of the book denoted by S_0, which is exogenous.

4.2 Order Submission Decision

To select his order submission strategy, a trader needs to choose an order type and a price. His goal is to maximize utility, which in this risk neutral setting is equivalent to maximize his payoff, considering all the available strategies. Market orders guarantee immediate executions, while limit orders enable traders to obtain a better price. When traders choose a limit rather than a market order, they increase their non-execution costs as, to obtain a better price, they forgo execution certainty. At the same time, however, they reduce their price opportunity cost, which is the cost associated with an execution at a less favorable price. Hence in this market traders face the trade-off between non-execution costs and price opportunity costs. The payoffs of the different strategies available to traders, $U(\cdot)$, are listed in Table 1.

[**Insert Table 1 here**]

Market orders are always executed at the best ask or bid price and their payoff depends on the traders' degree of patience, β_t, which can also be interpreted as their personal valuation of the asset. Limit orders' payoff also depends on the execution probability that we indicate by $p_t(A_k|S_t)$ and $p_t(B_k|S_t)$ for a limit sell and for a limit buy order respectively submitted at the ask price A_k, or at the bid price B_k. Note that we relax Parlour's (1998) assumption that nature exogenously selects buyers and sellers at the beginning of each trading period. In our model, a trader's choice to buy or sell the asset only depends on his own β_t. So if he comes to the market with a high personal valuation, he will certainly buy the asset. This differs from Parlour's model where the same trader would refrain from trading had nature selected him as a seller.

Figure 2 shows the extensive form of the game when the book opens empty at t_1, $S_0 = [0, 0, 0, 0]$. Assume that a trader arriving at the market at t_1 decides to submit a limit sell order at A_2, $H_{t_1} = +1^{A_2}$. The execution probability of this order depends both on the initial state of the book and on the future orders submitted by the other market participants. Because before submission no shares were standing at A_1 and A_2, this order is at the top of the queue on the sell side and would be executed as soon as a market buy order arrives in period t_2 or t_3. However, this order will not be executed if a trader arrives in period t_2 and gains price priority by submitting a limit sell order at A_1, $H_{t_2} = +1^{A_1}$. Figure 2 shows that a limit sell order at A_1 is one of the possible strategies of a trader arriving at t_2. If this order is actually chosen by the incoming trader, then at t_3 the book will open with one share at both the first and the second level of the ask side, $S_{t_2} = [1, 1, 0, 0]$, and only the order posted at A_1 will have positive execution probability. Recall that only one trader arrives at each trading round.

Figure 2 focuses on the ask side, but the order execution probability also depends on the state of the other side of the book: for instance, a deep book on the bid side increases the incentive for a seller to post limit orders as he knows that incoming buyers will be more inclined to post a market rather than a limit order, due to the long queue on the bid side.

[**Insert Figure 2 here**]

4.3 Market Equilibrium

Traders use information from the state of the limit order book to rationally compute the execution probabilities of different orders. Having done so, they are then able to choose the optimal strategy consistently with their own β, by simply comparing the expected payoff from each order.

We solve the model by backward induction. At time t_3, the execution probability of limit orders is zero, and traders can only either submit market orders or decide not to trade. It is easy to show that traders' equilibrium strategies are:

$$H^*_{t_3}(\beta|S_{t_2}) = \begin{cases} -1^{B_{k'}} & \text{if } \beta \in [0, \frac{B_{k'}}{v}) \\ 0 & \text{if } \beta \in [\frac{B_{k'}}{v}, \frac{A_{k'}}{v}) \\ -1^{A_{k'}} & \text{if } \beta \in [\frac{A_{k'}}{v}, 2] \end{cases} \tag{4}$$

By using these equilibrium strategies together with the distribution of β, we calculate the equilibrium execution probabilities of limit orders submitted at t_2:

$$p^*_{t_2}(A_k|S_{t_2}) = \begin{cases} \int\limits_{\beta \in \{\beta : H^*_{t_3} = -1^{A_{k'}}\}} \frac{1}{2} d\beta = 1 - \frac{A_{k'}}{2v} & \text{if } A_k = A_{k'} \text{ and } Q^{A_k}_{t_1} = 0 \\ 0 & otherwise \end{cases} \tag{5}$$

$$p^*_{t_2}(B_k|S_{t_2}) = \begin{cases} \int\limits_{\beta \in \{\beta : H^*_{t_3} = -1^{B_{k'}}\}} \frac{1}{2} d\beta = \frac{B_{k'}}{2v} & \text{if } B_k = B_{k'} \text{ and } Q^{B_k}_{t_1} = 0 \\ 0 & otherwise \end{cases} \tag{6}$$

These execution probabilities are the dynamic link between period t_3 and t_2. Because there is only one period left in the trading game, the execution probability of a limit order submitted at t_2

is positive only if the order is posted at the best ask ($A_{k'}$) or bid price ($B_{k'}$), and if there are no other orders already standing in the book at that price.

A trader arriving at t_2 can choose between a market and a limit order. The equilibrium strategies for t_2 depend on the state of the book. As an example, here we discuss the equilibrium strategies for the book opening with room for limit orders on both sides of the market, i.e., $p_t^*(A_k|S_t) \neq 0$ & $p_t^*(B_k|S_t) \neq 0$.[14] The trader's optimal strategies are:

$$H_{t_2}^*(\beta|S_{t_1}) = \begin{cases} -1^{B_{k'}} & \text{if } \beta \in [0, \beta_{-1^{B_{k'}},+1^{A_k},t_2|S_{t_1}}) \\ +1^{A_k} & \text{if } \beta \in [\beta_{-1^{B_{k'}},+1^{A_k},t_2|S_{t_1}}, \beta_{+1^{A_k},+1^{B_k},t_2|S_{t_1}}) \\ +1^{B_k} & \text{if } \beta \in [\beta_{+1^{A_k},+1^{B_k},t_2|S_{t_1}}, \beta_{+1^{B_k},-1^{A_{k'}},t_2|S_{t_1}}) \\ -1^{A_{k'}} & \text{if } \beta \in [\beta_{+1^{B_k},-1^{A_{k'}},t_2|S_{t_1}}, 2] \end{cases} \quad (7)$$

where $\beta_{-1^{B_{k'}},+1^{A_k},t_2|S_{t_1}} = \frac{B_{k'}}{v} - \frac{p_{t_2}^*(A_k|S_{t_2})}{1-p_{t_2}^*(A_k|S_{t_2})} \cdot \frac{A_k - B_{k'}}{v}$, $\beta_{+1^{A_k},+1^{B_k},t_2|S_{t_1}} = \frac{p_{t_2}^*(A_k|S_{t_2})A_k + p_{t_2}^*(B_k|S_{t_2})B_k}{p_{t_2}^*(A_k|S_{t_2}) + p_{t_2}^*(B_k|S_{t_2})} \cdot \frac{1}{v}$, and $\beta_{+1^{B_k},-1^{A_{k'}},t_2|S_{t_1}} = \frac{A_{k'}}{v} + \frac{p_{t_2}^*(B_k|S_{t_2})}{1-p_{t_2}^*(B_k|S_{t_2})} \cdot \frac{A_{k'} - B_k}{v}$.

These thresholds are derived by taking into account that the trader arriving at the beginning of period t_2 observes the state of the book S_{t_1}. For instance $\beta_{-1^{B_{k'}},+1^{A_k},t_2|S_{t_1}}$ denotes the threshold between a market sell order hitting the best bid price, $B_{k'}$, and a limit sell order posted at the ask price A_k, and it is derived by equating the payoffs of the two orders. Note that the greater the limit order execution probability, $p_{t_2}^*(A_k|S_{t_2})$, the smaller this threshold and the higher will the probability that traders choose limit rather than market orders be.

More generally, if the execution probability at time t is high enough for non-execution costs to be lower than price opportunity costs, the trader will submit a limit order. If instead the execution probability is low, he will choose a market order. As explained in Section 5, this trade-off crucially depends on the value of the tick size, τ, and on the price of the stock, v.

When a trader chooses a limit order, he also has to decide how aggressively to submit this order at better prices next to v. The optimal price at which a trader submits a limit order is the result of the trade-off between non-execution costs and price opportunity costs: a more aggressive price implies a higher execution probability due to both the lower risk of being undercut by incoming

[14] We refer to the Appendix for a more detailed discussion.

traders and the fact that the order becomes more attractive for traders on the opposite side of the market. However, this is obtained at the cost of lower revenue once the order is executed.

From the equilibrium strategies at t_2, we can derive the execution probabilities for limit orders submitted at t_1 and the corresponding equilibrium strategies. Due to the recursive structure of the game and because traders are indifferent between orders with a zero execution probability, a unique equilibrium always exists and is defined as follows:

Definition 1 *Given an initial book S_0, an equilibrium is a set of order submission decisions $\{H_t^*\}$ and states of the limit order book $\{S_t\}$ such that at each period the trader maximizes his payoff $U(\cdot)$ according to his Bayesian belief over the execution probabilities $p^*(\cdot)$, i.e.,*

$$\{H_t^* := \arg\max \ U(\cdot | \, S_{t-1})\}$$
$$\{S_t := S_{t-1} + h_t^*\}$$
where h_t^* is defined by (2)

In our analysis we use two different starting books, S_0, to proxy stocks with different degrees of liquidity; and, without loss of generality, in our numerical simulations we assume that $\tau = 0.1$ and $v = \{1, 5, 10, 50\}$. In the Appendix we show that our equilibrium is robust to different parameter specifications provided that $\frac{\tau}{v} \in (0, 0.1]$. Within this range the monotonicity of the β thresholds with respect to the relative tick size is guaranteed.

5 Tick Size and Market Quality

We start with the market (LM) -already presented in Section 4- characterized by a large tick size equal to τ, and we compare the resulting equilibrium trading strategies with those obtained when the tick size is set to $\frac{1}{3}\tau$ and the price grid is finer. Both price grids are shown in Table 2: on the LM the price grid is $\{A_2, A_1, B_1, B_2\}$, while on the small tick market (SM) it has five levels on both the ask and the bid side, a_l and b_l, with $l = 1, .., 5$.[15]

[15] The choice of the value $\frac{1}{3}\tau$ for the tick size of the SM is purely technical. This value is consistent with an additional price level between A_1 and B_1 on both sides of the market. At the same time it guarantees that there are four price levels on the SM grid (namely a_2, b_2 and a_5, b_5) that coincide with A_1, B_1 and A_2, B_2.

Given that the evolution of the state of the book is characterized by equation (3), the expected states of the book for the LM and the SM are given by:

$$E_{t-1}[S_t^{LM}] = S_{t-1}^{LM} + E[h_t^{LM}] \quad (8)$$

$$E_{t-1}[S_t^{SM}] = S_{t-1}^{SM} + E[h_t^{SM}] \quad (9)$$

where $E[h_t^{LM}]$ is the vector with elements $E[h_t^i] = \int_{\beta \in \{\beta : H_t(\beta) = \pm 1^i, 0\}} H_t(\beta)\frac{1}{2}d\beta$ with $i = A_k, B_k$, and $E[h_t^{SM}]$ the vector with elements $E[h_t^j] = \int_{\beta \in \{\beta : H_t^{SM}(\beta) = \pm 1^j, 0\}} H_t^{SM}(\beta)\frac{1}{2}d\beta$ with $j = a_l, b_l$. The main difference between the two markets is that in the SM both S_t^{SM} and h_t^{SM} consist of ten components rather than four. The trader's strategy space for the SM is much richer thanks to the finer price grid: $H_t^{SM} = \{\pm 1^j, 0\}$.

[**Insert Table 2 here**]

To compare the two markets, we build standard indicators of market quality based on traders' equilibrium strategies. For each period t, depth is measured as the number of shares available on the book at different price levels. More precisely, for the LM we define the following two depth indicators:

$$DPI_t^{LM} = E[Q_t^{A_{k'}} + Q_t^{B_{k'}}] \quad (10a)$$

$$DPT_t^{LM} = E[\sum Q_t^i] \quad (10b)$$

where DPI_t^{LM} is the average of the sum of the depth at the best quotes, and DPT_t^{LM} is the total depth measured by the sum of the average of the depth at all price levels. The average inside spread, SP_t, is computed as the expected difference between the best ask and the best bid prices:

$$SP_t^{LM} = E[A_{k'} - B_{k'}] \quad (11)$$

Finally, volume in period t is measured by the number of orders executed, while liquidity provision is measured by the number of limit orders submitted. Because at each period only one trader arrives, expected volume, VL_t, and liquidity provision, LP_t, are computed as the probability that a trader submits respectively a market order or a limit order at all price levels:

$$VL_t^{LM} = E[\sum_i \int_{\beta \in \{\beta : H_t = -1^i\}} 1 d\beta] \tag{12}$$

$$LP_t^{LM} = E[\sum_i \int_{\beta \in \{\beta : H_t = +1^i\}} 1 d\beta] \tag{13}$$

Indicators of market quality for the SM are computed in a similar way, but using $j = \{a_{1:5}, b_{1:5}\}$.

We then compute the difference between these indicators of market quality for the LM and the SM frameworks, and average them across periods:

$$\Delta y = \frac{1}{k} \sum_{t=t_1}^{t_k} [y_t^{SM} - y_t^{LM}] \tag{14}$$

where $y = \{DPI, DPT, SP, VL, LP\}$, $k = 2$ for $\{DPI, DPT, SP, LP\}$, and $k = 3$ for $\{VL\}$.[16]

To illustrate the effects of different tick sizes on both liquid and illiquid stocks, we use the initial state of the book as a proxy for liquidity and consider two cases: an empty book for illiquid stocks, and a book with one unit on the first (second) level of the LM (SM) price grid for liquid stocks.[17] The following Proposition summarizes the effects of a change in the tick size on both traders' strategies and market quality.

Proposition 1 *When moving from a large to a small tick market, changes in traders' order submission strategies and market quality depend on the initial state of the book.*

- *For liquid stocks liquidity provision increases and total volume decreases; spread, total depth and inside depth improve.*
- *For illiquid stocks the results are the opposite: liquidity provision decreases, total volume increases, and spread, total depth and inside depth deteriorate.*

[16] As in period t_3 the market closes and hence traders do not use limit orders ($LP_{t_3} = 0$), when computing liquidity provision, spread and depth, we only consider the initial two periods.

[17] To compare markets characterized by different tick sizes we need to start from the same initial state of the book. This implies that in the SM depth at the price levels not in common with the LM must be zero.

- *All the above effects are weaker for high priced stocks.*

[**Insert Figure 3**]

We begin by considering a liquid book that opens at t_1 with 1 share on both $A_1(a_2)$ and $B_1(b_2)$.[18] Results for this state of the book are shown in Figure 3, Panel A.

We use an example to explain how in our model a change in the tick size affects traders' choice between market and limit orders, and hence order flows and market quality. Consider a seller coming to the market at time t_1 with his own β_{t_1} who observes the LM characterized by the price grid presented in Table 2. Suppose that the LM book is liquid. As there is already one order standing at A_1, he excludes a limit order at this more aggressive price: he would still be queueing behind the order at A_1 but would be eventually executed at a worse price. So the trader's choice is between a market order at $B_1 = v - \frac{1}{2}\tau$ and a limit order at $A_2 = v + \frac{3}{2}\tau$.

Compared to the market order with immediate execution at B_1, the limit order at A_2 entails a 2-tick (2τ) price improvement, as well as an uncertain execution that depends on the execution probability $p_{t_1}(A_2|S_{t_1})$. We refer to Table 1 for the seller's profits from the two strategies. All else equal, the trader's evaluation depends on τ and v which influence the trade-off between price opportunity costs and non-execution costs. The price opportunity cost that the trader faces if he chooses a market order is equal to the price improvement he loses for not choosing a limit order, times the probability that the limit order is executed, $2\tau \times p_{t_1}(A_2|S_{t_1})$. The non-execution cost that the trader faces if instead he chooses a limit order is equal to the loss he realizes if his order is not executed, times the probability that the limit order is actually not executed, $[(1 - \beta_{t_1})v - \frac{1}{2}\tau] \times (1 - p_{t_1}(A_2|S_{t_1}))$.

When the tick size is smaller, i.e., $\frac{\tau}{3}$ rather than τ, two effects crucially influence traders' choice between market and limit orders. First, and all else equal, the price improvement that the trader would lose if he does not choose a limit order (price opportunity cost) is smaller, and therefore, the traders' incentive to choose limit orders falls. Second, since the price grid is finer and the trader has the option to post the limit order at the new more aggressive price levels, a_1 and b_1, he can

[18] We have also solved the model for a book that opens with two rather than one share at the inside spread. Results are consistent with the case of liquid stocks and presented in the Internet Appendix.

undercut the limit order standing at the top of the book and gain price priority. By posting the order at a more aggressive price, the limit order's execution probability increases with the result that non-execution costs decrease and price opportunity costs increase. The second effect outweighs the first one, and the result is that traders not only use more limit orders but they also become more aggressive in providing liquidity thus decreasing even further the submission of market orders.[19] The increased limit order submission rate increases market depth, both overall and at the inside spread, and the increased limit order aggressiveness results in a narrower spread. At the same time, the reduced liquidity demand preserves depth and keeps the spread narrow.

These effects become weaker as the stock price increases. In general, a higher v increases the non-execution costs that traders pay if limit orders remain unexecuted because the stock is now worth a higher amount. Therefore in equilibrium traders choose fewer limit orders, and the previously mentioned two effects that hinge on the use of limit orders are attenuated and so is the overall effect of the tick size reduction.

We now move to the case of illiquid stocks that we proxy by considering an empty book (Figure 3, Panel B). Going back to the previous example, when the stock is illiquid, the incoming seller has to choose between a market sell order at B_2 and a limit sell order at A_1 or at A_2. As before, a smaller tick size reduces price opportunity costs and increases non-execution costs thus reducing the incentive to choose a limit order. This time, however, the finer price grid that the smaller tick size entails does not generate any additional advantage for aggressive limit orders as there is no liquidity standing on the top of the book which can be undercut. Therefore when the book is empty traders switch from limit to market orders: both the reduced liquidity supply and the enhanced liquidity demand contribute to lower market quality in terms of smaller depth and wider spread.

As before, even in the scenario with an illiquid opening book, when the stock price increases, the effects that the smaller tick size has on traders' use of limit orders are weaker and all the resulting change in order flows and market quality become gradually smaller.

[19] This effects is even stronger when the book opens with 2 rather than 1 share at the top of the book. To economize space we present these results in the Internet Appendix, Table A1.

6 Dual Market Model and Sub-Penny Trading

In this Section we extend the previous framework by introducing competition between a large tick market that works like a PLB and a special type of venue, the SPV. We assume that the SPV has a smaller tick size similar to the SM described above and that only BDs are allowed to supply liquidity. Further, we assume that RTs have access to SORs and their orders therefore may also execute against the SPV limit orders. They will do so to the extent the standing orders in the SPV offer better executions. Sub-penny trading is carried out by BDs who can access SPVs to compete on price against the limit orders posted at the top of the PLB.

When traders post limit orders on the SPV to undercut existing liquidity on the PLB, liquidity provision and hence depth and spread on the PLB deteriorate. However, liquidity demand on the PLB also decreases. The reason is that, due to the existence of SORs, the better prices available in the SPV intercept market orders sent to the PLB. The reduced liquidity demand preserves depth and spread on the PLB. The net effect of the reduced supply and demand of liquidity on the quality of the PLB depends on the characteristics of the stock considered; specifically it depends on the initial state of the PLB, liquid or illiquid, as well as on the price of the stock. It also depends on the tick size that characterizes the PLB.

To discuss this setting, we need to further adapt our previous single market model to embed sub-penny trading. We assume that at each trading period one individual out of two groups of traders arrives at the market: with probability α the incoming trader is a BD and with the complementary probability, $1-\alpha$, he is a RT. While a BD can observe and use both the PLB and the SPV, a RT can only observe and provide liquidity to the PLB. The SPV is characterized by a smaller tick size so that the BD can undercut orders posted by other traders at the top of the PLB.[20] Furthermore, in order to approximate the nature of real SPVs, we assume that the SPV does not have a trading crowd sitting at a_5 and at b_5. Finally, while only BDs can post limit orders on the SPV, all traders can take advantage of the liquidity offered by both trading platforms. This assumption is consistent with a fast market in which a smart order routing technology allows all

[20] When the BDs' payoffs from trading across the two markets are the same, we assume that they submit orders to both markets with equal probability.

investors to simultaneously access multiple sources of liquidity. SORs allow traders to search the best quotes on the consolidated limit order book (PLB&SPV) but they do not necessarily reveal the state of the SPV.

To sum up, we consider two protocols: the benchmark (PLB), where only one trading platform is available to all traders, and the PLB&SPV framework, where a SPV competes with the PLB. The latter case further differentiates into a transparent and an opaque setting, where the lack of transparency refers to the visibility of the SPV by RTs. To introduce uncertainty on the state of the SPV, we assume that at t_1 the SPV opens either empty or with one unit on the first level of the book, with equal probability. The following Proposition presents the results.

Proposition 2 *When a SPV is added to a PLB, traders' order submission strategies and market quality change as follows.*

- *For liquid stocks, the market quality of the PLB, measured by depth and inside spread, improves, yet the PLB liquidity provision and volume worsen. The effects on market quality are weaker both for low priced stocks and when the SPV is opaque.*
- *For illiquid stocks, the market quality of the PLB deteriorates. The effect is stronger both for low priced stocks and when the SPV is opaque.*
- *The SPV is used more intensively by BDs when the stock is both liquid and low priced and when their proportion (α) increases.*

Figures 4 and 5 show how liquidity provision and market quality are affected by the competition of a SPV. In both Figures, Panel A provides results for the case of a PLB that opens with one share on both A_1 and B_1, and hence provides the intuition for liquid stocks; while Panel B focuses on the case of a PLB opening empty at t_1 and therefore is used to proxy illiquid stocks.

[**Insert Figures 4 and 5 here**]

For liquid stocks the main effect of sub-penny trading is to foster price competition. When the SPV platform is introduced, BDs submit limit orders to the SPV at a_1 (b_1) to undercut the existing depth at A_1 (B_1) (Figure 4, Panel A), thus intercepting incoming market orders away from the PLB. Therefore, the effect of BDs' undercutting is that both limit and market orders move from the PLB to the SPV thus reducing both liquidity provision and volume on the public venue (Figure 5, Panel A). The result is that depth increases and spread narrows. The reason for why market quality improves in the PLB despite the reduction of the liquidity provision is that when the book is liquid, traders use more market than limit orders and consequently the positive effect of the reduced liquidity demand - which helps preserve the low spread and high depth - prevails.

As discussed in Section 5, when the price of the stock is higher traders use more market than limit orders; hence, when the SPV is introduced the effect on liquidity demand is magnified and so is the effect on market quality. The result is that volume decreases even more on the PLB and market quality improves.[21]

An increase in the price of the stock magnifies the positive effect of the introduction of the SPV on the quality of the PLB; however, by reducing the incentive to supply liquidity, a higher asset price also reduces traders' incentive to post aggressive limit orders that undercut the existing liquidity on the top of the PLB (Figure 4, Panel A). This result is consistent with the empirical evidence reported in Figure 1. In real markets sub-penny trading in SPVs has increased substantially over the last decade, but not for high priced stocks. The increase is concentrated on low priced stocks.

Moreover, our results illustrate whether the effects of the competition from a SPV depend on the degree of transparency of the SPV. According to our model, when the stock is liquid, a decrease in the transparency of the SPV has only a moderate effect on traders' order submission strategies. This moderate effect has a subtle explanation. When the SPV turns opaque, RTs choose fewer market orders because uncertainty on the execution price increases price opportunity costs. They know that market orders can eventually be executed in the SPV but in this case they cannot observe the state of the SPV, and hence they cannot condition their order choice on the state of the SPV. Therefore volume in the SPV decreases and hence the execution probability of limit orders posted to the SPV also decreases with the result that the liquidity provision to the SPV declines.

[21] This positive effect is reinforced by the fact that the change in limit orders following the introduction of a SPV in a market for a high priced stock becomes almost irrelevant, and hence, compared to the $v = 1$ case, the reduction in liquidity supply becomes tiny.

However, even though total volume decreases as RTs switch from market to limit orders, the PLB volume increases. The reason is once more due to the uncertainty about the state of the SPV: because RTs cannot observe the liquidity available on the SPV, market orders bounce back to the PLB more frequently compared to the case with a transparent SPV. Moreover, because the stock is liquid and market orders play a dominant role, the feedback effect on the PLB volume - that increases - outweighs the small increase in the liquidity provision. So the overall positive effect on market quality of the competition from the SPV becomes weaker.[22]

We now turn to the framework with an empty opening book (Figure 5, Panel B). Even for illiquid stocks when the SPV is introduced both limit and market orders decrease on the PLB. However, contrary to the previous case, depth and spread worsen. In this scenario, the effect of the reduced liquidity supply outweighs the reduction in market orders resulting from their interception by the SPV. The reason is twofold. First, when RTs perceive the potential competition from BDs, they react by supplying less liquidity to the PLB. This is due to the fact that when the book is illiquid they are afraid of being undercut not only on the SPV but also on the PLB. Hence, the reduction of the liquidity supply in the PLB is stronger than for liquid stocks. Second, while traders generally use more market than limit orders for liquid stocks, when the stocks are illiquid traders prefer limit orders which therefore play a dominant role.

These effects become much weaker as the stock price increases. As explained above, when the stock price increases non-execution costs increase and traders switch from limit to market orders. This effect attenuates the negative reduction in the liquidity supply, and at the same time it makes the reduction of the liquidity demand more relevant: both effects have a positive impact on the quality of the PLB thus making the overall negative effect on market quality much smaller.

When instead the SPV is opaque, these effects become stronger because the uncertainty on the SPV depth and on the actual level of competition makes RTs even more reluctant to post limit orders to the PLB. Hence we can conclude that when the stock is illiquid, a reduction in the transparency of the SPV makes the negative effects on the quality of the PLB even more problematic, especially for low priced stocks.

[22] If the PLB volume is higher following the introduction of an opaque SPV, its reduction compared to the benchmark PLB is smaller and so is the effect on market quality.

By comparing traders' equilibrium strategies for liquid and illiquid stocks (Figure 4), we observe that BDs post orders to the SPV more intensively for liquid stocks in which competition for liquidity provision in the PLB is higher. This effect is stronger for low priced stocks for which non-execution costs are smaller and therefore providing liquidity is more convenient than taking liquidity.

Furthermore, when the proportion α of BDs increases from 10% to 20%, the sub-penny activity builds up in the SPV (Figure 6A) and hence the effects on the PLB spread and depth are magnified (Figure 6B).

[**Insert Figures 6A and 6B here**]

7 Tick Size and Inter-market Competition

Our model allows us to investigate the impact of a smaller tick size in a framework in which the PLB competes with the SPV. First, we show how the effects of the introduction of a SPV on the quality of the PLB change when the tick size that characterizes the PLB is $\tau/3$ rather that τ. Second, we show how a smaller tick size impacts the quality of a PLB that is already competing with a SPV before the reduction is implemented. The following Proposition summarizes the results.

Proposition 3 .

- *When the PLB has a smaller tick size, the effects of sub-penny trading are more positive for liquid stocks, and they are less negative for illiquid stocks.*
- *The effects of a smaller tick size on a PLB already competing with a SPV are that depth worsens, but spread improves.*

We first show how the introduction of sub-penny trading changes the quality of the PLB compared to the benchmark model without sub-penny trading when the tick size is smaller. To this end, we compare two regimes with a tick size equal to τ (Figure 5), and, all else equal, with a tick size equal to $\tau/3$ (Figure 7), respectively. As discussed in Section 5, when the tick size is smaller, price opportunity costs are smaller and therefore market orders become more convenient. For this reason traders switch from limit to market orders.[23] So, recalling the effects of the introduction of sub-penny trading discussed in Section 6, the result is that when the reduction in the demand for liquidity has a dominant role (liquid stocks), the positive effect on the liquidity of the PLB increases (Figures 5 and 7, Panel A).[24] When instead the driving factor of the market quality change is the reduced liquidity supply to the PLB (illiquid stocks), the negative effect on market quality is attenuated (Figures 5 and 7, Panel B).

Therefore we conclude that competition from a SPV is more beneficial when the PLB has a smaller tick size. The positive effects on liquid stocks are amplified and traders supply more liquidity to the SPV, whereas the negative effects on illiquid stocks are tempered and traders use the SPV less intensively.

[**Insert Figure 7 here**]

Let us now consider the impact of a smaller tick size in a market that is already competing with a SPV by studying the equilibrium outcome of the dual market model when the tick size is reduced. In this case we compare the quality of the PLB competing with a SPV before and after the reduction in the tick size and not -as in the previous case- the difference between the quality of the PLB in the dual market model and in the benchmark model under the two regimes with different tick size (diff-in-diff approach).

Due to the reduction in price opportunity costs a smaller tick size makes traders more reluctant to supply liquidity than demand liquidity. This decreases depth and increases volume on the public market (Figure 8). However, as explained above, with a smaller tick size the SPV competition is more beneficial irrespective of the state of the book and partly explains why the spread improves both for liquid and for illiquid stocks. In a market where the tick size is binding, a smaller tick size will automatically lead to a reduction of the spread. However, we believe that our model captures what goes on in today's fast trading platforms: the liquidity supplied by high frequency traders

[23] To economize space we do not report in the text the absolute values of LP^{PLB} and VL^{PLB} for the two cases of liquid and illiquid stocks, which are instead available in the Internet Appendix. By comparing Tables A2 and A3 with Tables A5 and A6 it is evident that LP^{PLB} decreases, while VL^{PLB} increases.

[24] By comparing Figure 5 and Figure 7, Panel A, one notes that the spread improvement is smaller rather than greater for liquid stocks. This effect, however, is due to the fact that in a market with a smaller tick size the PLB spread is technically smaller. Yet, the relative improvement due to the competition from the SPV is greater.

that characterizes the first few levels of the book tends to follow the movements of the price grid due to the variations in the tick size, thus making the spread narrower at the top of the book.

[**Insert Figure 8 here**]

8 Welfare

We now turn to the welfare analysis. Following Degryse et al. (2009) and Goettler et al. (2005), we measure expected welfare in each period, $E(W_t)$, as the sum of the agents' expected gains from both market (first term) and limit orders (second term):

$$E(W_t) = \sum_i \int_{\beta \in \{\beta : H_t = -1^i | S_{t-1}\}} |i - \beta v| \cdot f(\beta) d\beta + \sum_i \int_{\beta \in \{\beta : H_t = +1^i | S_{t-1}\}} |i - \beta v| \cdot p_t^*(i\, | S_t) \cdot f(\beta) d\beta \quad (15)$$

First, we use our measure of welfare to discuss the effects of a tick size reduction in the single market model. Our aim is to investigate how the change in the gains from trade that a smaller tick size entails realizes into a change in the welfare of market participants. We also aim to investigate whether the resulting changes in welfare are consistent with the effects on market quality discussed in Section 5.

To study the change in welfare after the reduction of the tick size we compare traders' expected welfare in the LM, $E(W_t^{LM})$, with that in the SM, $E(W_t^{SM})$, during each period of the trading game, and then average these differences across the three periods (Figure 9):

$$\Delta W_{SM-LM} = \frac{1}{3} \sum_{t=t_1}^{t_3} \frac{E(W_t^{SM} - W_t^{LM})}{E(W_t^{LM})} \quad (16)$$

We then replicate our welfare analysis for the dual market model and compare the expected welfare of market participants in the benchmark model, $E(W_t^{PLB})$ with that in the $PLB\&SPV$ framework, in which the PLB competes with the SPV, $E(W_t^{PLB\&SPV})$. As for the previous case, we then average the differences across the three periods (Figure 10):

$$\Delta W_{PLB\&SPV-PLB} = \frac{1}{3} \sum_{t=t_1}^{t_3} \frac{E(W_t^{PLB\&SPV} - W_t^{PLB})}{E(W_t^{PLB})} \quad (17)$$

The results obtained are summarized in the following Proposition.

Proposition 4 .

- *In the single market model a smaller tick size makes traders better off when the stock is liquid and worse off when it is illiquid. Changes in traders' welfare increase when the stock price decreases.*
- *Competition from a SPV makes all traders better off, especially when the SPV is transparent; the improvement decreases when the stock price is higher or, ceteris paribus, when the tick size is smaller.*

The effects on welfare of a change in the tick size depend on the state of the book (Figure 9). As we have shown in Section 5, when the market opens deep, a smaller tick size improves spread and makes the market deeper. The result of this improvement in market quality is that traders' welfare increases. When instead the book opens shallow, a smaller tick size worsens liquidity, and therefore it reduces traders' gains from trade, thus lowering total welfare. Finally, consistently with the results obtained on market quality, the effects of a tick size reduction are stronger for low priced stocks, whereas they tend to vanish when the price of the stock is high.

[**Insert Figure 9 here**]

To evaluate the effects on welfare of the opportunity to trade in a SPV characterized by a smaller tick size we have to consider BDs and RTs separately (Figure 10). The reason is that RTs cannot supply liquidity to the SPV, whereas BDs can.

As previously discussed, for liquid stocks the overall effect on market quality of the competition from a SPV is positive. Therefore, welfare increases for both BDs and RTs (Panel A). In essence, even though only BDs can post limit orders to the SPV, due to the existence of SORs, all traders can take advantage of the liquidity offered. Moreover, consistently with our previous discussion on market quality, the advantage of the SPV option is greatest when the SPV is visible to all market participants (transparent), and when the price of the stock is low.

In terms of the distribution of welfare, the model shows that, overall, RTs, who do not have complete access to the SPV, benefit somewhat less than BDs from the introduction of the SPV, especially when they cannot observe the SPV (opaque). Moreover, the welfare gains from the SPV option decrease substantially as the stock price increases, because, as expected, the gains from sub-penny trading are smaller. The welfare gains also decrease when the tick size is smaller because BDs have a lower incentive to provide liquidity on the SPV.

[**Insert Figure 10 here**]

Even for illiquid stocks the welfare effect of competition from the SPV is positive, both for BDs and for RTs (Panel B). As before the welfare gains are smaller when the SPV is opaque, especially for RTs, and when the stock is high priced. Interestingly, traders' welfare increases even though the quality of the PLB deteriorates following the introduction of the SPV. The reason is that all traders can take advantage of the liquidity available on the SPV which provides traders with a price improvement that is substantially greater than for liquid stocks.

Finally, by investigating the effects of a smaller tick size on a PLB that competes with a SPV we show that, because spread improves also welfare improves irrespective of the state of the book.

9 Empirical implications

The equilibrium strategies derived from our model generate several empirically testable predictions. First of all, we clarify how the trade-off between price opportunity costs and non-execution costs that governs traders' choice of limit and market orders crucially depends on the tick size, the price of the stock and the state of the limit order book.

Prediction 1 *Tick Size* - When a limit order book is liquid (illiquid), a smaller tick size increases (decreases) price opportunity costs and reduces (increases) non-execution costs so that traders switch from market to limit orders (limit to market orders). *Stock Price* - A higher stock price increases non-execution costs and hence traders' incentive to use market orders.

The model predictions for a variation in the tick size can be tested cross-sectionally by conducting event studies for stocks classified by different degrees of liquidity and different prices. Since we show that the trade-off depends not only on the tick size, but also on the price of the stock, when investigating the effects of a change in the tick size, it is important to control for variations in the price of the stock. A similar test can be run in a time-series framework by proxying for liquidity with the state of the book.

The next step is to test whether the relative changes in the demand and supply of liquidity actually generate the effects that the model predicts for market quality. Starting with the single market model, and consistently with our predictions for order flows, we show that a tick size reduction can improve or worsen the indicators of market quality, depending on the initial state of the book. These results lead to our second prediction.

Prediction 2 When the tick size is smaller, liquidity worsens if the book is shallow and it improves if it is deep. Both in deep and in shallow markets, the effect on order flows and liquidity of a smaller tick size gradually decreases as the price of the stock increases.

Most of the existing empirical evidence does not distinguish between liquid and illiquid stocks, and shows that when the tick size is reduced, the inside spread decreases but depth does not necessarily improve.[25] On the other hand, when stocks are classified according to their degree of liquidity, as in Bourghelle and Declerck (2004), results show that as liquidity decreases, the percentage spread increases and the quoted depth decreases. Similarly, our prediction is in line with the empirical findings of Goldstein and Kavajecz (2000), who sort NYSE stocks according to their levels of liquidity and price. They show that as the tick size is reduced, there is an overall improvement of the quoted spread and depth for liquid stocks, whereas market quality deteriorates for illiquid ones. Goldstein and Kavajecz also show that both effects become stronger as the stock price decreases, and that for low priced illiquid stocks the negative effect on the quoted spread and depth is the greatest.

[25] See Ahn et al. (1996), Bacidore (1997), Bessembinder (2003), Harris (1994), and Porter and Weaver (1997).

When we extend the analysis to a framework which includes inter-market competition between a primary market and a SPV, we obtain additional testable implications. The endogenous strategic interaction between RTs and more sophisticated BDs having access to dark platforms generates distinctive empirical patterns for liquidity supply and liquidity demand. Furthermore, competition from a SPV attracts both liquidity demand and liquidity supply away from the primary market and the effect on market quality depends both on the state of the book, and on the stocks' characteristics.

> **Prediction 3** Sub-penny trading, improves liquidity for liquid especially high-priced stocks, whereas it worsens liquidity for illiquid especially low-priced stocks. In this setting of competition between a PLB and a SPV, a more transparent SPV is overall beneficial for the quality of the PLB.

A sample of stocks stratified by market capitalization and price would be well suited to test our model's predictions on the effect of sub-penny trading on the quality of a PLB. However, as our model predicts, the quality of the market itself influences sub-penny trading. Therefore, the resulting endogeneity issue should be adequately tackled when investigating empirically the effects of sub-penny trading on spread and depth (Buti, Consonni, Rindi and Werner, 2013).

> **Prediction 4** The activity of the BDs on SPVs is more intensive for liquid and low priced stocks.

This prediction can be tested by examining how the fraction of sub-penny trading varies by liquidity and price in the cross-section of stocks. The key is to select a sample of stocks with enough independent variation in liquidity and in price. Consistently with our model's predictions, Buti, Consonni, Rindi and Werner (2013) show that queue-jumping on NYSE and NASDAQ is more intense for liquid and for low priced stocks.

Our last prediction deals with the effects of a smaller tick size on the quality of the PLB in a fast trading environment with inter-market competition.

> **Prediction 5** When the PLB competes with a SPV a reduction of the tick size decreases both spread and depth.

This prediction is consistent with the results from the recent BATS' Pan European Tick Size Pilot run in 2009.

10 Policy Discussion and Conclusions

Today's financial markets are characterized by extensive inter-market competition, both between fully transparent venues and between transparent and dark pools of liquidity. In such a world, the regulation of the minimum price increment is extremely important, as it defines how markets compete for liquidity. In this paper, we analyze the impact of the tick size for market quality and traders' welfare in a public limit order market. We then extend our analysis of a tick size variation to a dual market model in which a public limit order book competes for the provision of liquidity with a venue with a smaller price grid, which we label a Sub-Penny Venue (SPV). This venue can either be dark or fully transparent. With this extension we can also analyze how the entry of a competing venue that allows trading in sub-pennies (sub-penny trading) affects traders' strategies, and therefore results in changes in market quality and traders' welfare.

First of all, we discuss the effects of a smaller tick size within the context of a Public Limit order Book (PLB) and show that these effects depend on the liquidity and the price of the stock. The model's results show that market quality worsens with a smaller tick size for illiquid stocks, whereas for liquid stocks it improves. These effects are strong for low priced stocks, whereas they tend to vanish as the stock price increases. We also study the welfare ramifications of tick size changes. Following a reduction in the tick size, we find that traders' welfare increases for liquid stocks, while it decreases for illiquid stocks. Overall, the welfare effects are much stronger for low priced stocks. Our results suggest that the objective of tick size regulation should be the definition of a minimum tick size that is consistent with the stock's main attributes, specifically, the tick size should be related to its liquidity and price.

We then extend the model to include competition from a SPV. This dual market model accounts for the changing nature of liquidity provision around the world, which is now dominated by competition among regular and dark markets, and in which a growing number of dark venues allows Broker-Dealers (BDs) to post limit orders at fractions of the tick size. Therefore, this extension is well suited not only to investigate the effects of a smaller tick size in a more realistic setting, but also to discuss the issue of sub-penny trading, which is one of the main concerns addressed by the SEC in the April 2010 concept release on Equity Market Structure.

Our model shows that sub-penny trading undertaken by BDs on SPVs can have negative effects on the market quality for illiquid and especially for illiquid low priced stocks, while it is beneficial to the quality of the market for liquid and especially for liquid high priced stocks. However, our results show that sub-penny trading never hurts market participants – all traders' welfare increases when a SPV competes with the PLB. For liquid stocks competition from a SPV makes both BDs and Regular Traders (RTs) better off: even though only BDs can post limit orders to the SPV, in this fast market all traders can take advantage of the liquidity offered through Smart Order Routers (SORs). The access to SORs explains why the welfare increases also for illiquid stocks, despite the fact that market quality deteriorates following the introduction of a SPV.

An important caveat should be kept in mind. We assume that RTs, while they cannot post orders on SPVs, have access to a SOR and they can therefore tap the liquidity in the SPV. Clearly, if RTs do not have access to SORs, sub-penny trading in illiquid stocks (where the PLB spread widens) could be detrimental for RTs. We conjecture that if we extend the model to represent a slower market without SORs, RTs would be harmed by the SPV competition when trading illiquid stocks. We leave this extension for future work.

Finally, we find that within the more realistic dual market setting, a smaller tick size improves inside spread and total welfare, and worsens depth irrespective of the state of the book. We also show that with a smaller tick size when the SPV competition is detrimental (illiquid stocks), the negative effects are attenuated; when instead SPV competition is beneficial, the positive effects are amplified. Further, we find that a smaller tick size induces BDs to increase trading at sub-penny in liquid stocks and to reduce trading in illiquid stocks.

By drawing on our model we can comment on the conclusions of the SEC Report on Decimalization (2012), which was required by the JOBS Act (2012) and which states that "the Commission should not proceed with the specific rulemaking to increase the tick size." The JOBS Act advised the SEC to consider the possibility of increasing the tick size for low priced stocks and of eventually undertaking a pilot program, under which some small and mid-cap stocks would trade at increments greater than a penny. The objective of this proposal was to restore the economic incentive to support liquidity for these stocks, that over time had experienced a reduction of the inside spread. Our model suggests that a larger tick size would result in higher depth but a wider spread, and more importantly a larger tick size would be associated with lower welfare for all traders.

References

Ahn, H., C. Q. Cao, and H. Choe, 1996. Tick Size, Spread, and Volume. *Journal of Financial Intermediation* 5:2-22.

Angel J. J., 1997. Tick Size, Share Prices, and Stock Splits. *Journal of Finance* 52:655-681.

Anshuman, R. V., and A. Kalay, 1998. Market Making with Discrete Prices. *Review of Financial Studies* 11:81-109.

Bacidore, J., 1997. The Impact of Decimalization on Market Quality: an Empirical Investigation of the Toronto Stock Exchange. *Journal of Financial Intermediation* 6:92-120.

Barclay, M. J., T. Hendershott, and D. T. McCormick, 2003. Competition among Trading Venues: Information and Trading on Electronic Communications Networks. *Journal of Finance* 58:2637-2665.

BATS, 2009. Pan European Tick Size Pilot: An analysis of results.

Battalio, R., and C. W. Holden, 2001. A Simple Model of Payment for Order Flow, Internalization, and Total Trading Cost. *Journal of Financial Markets* 4:33-71.

Bessembinder, H., 2003. Trade, execution costs and Market quality after decimalization. *Journal of Financial and Quantitative Analysis* 38:747-777.

Bessembinder, H., and H. M. Kaufman, 1997. A Cross-exchange Comparison of Execution Costs and Information Flow for NYSE-listed Stocks. *Journal of Financial Economics* 46:293-319.

Biais, B., C. Bisiere, and C. Spatt, 2010. Imperfect Competition in Financial Markets: An Empirical Study of Island and Nasdaq. *Management Science* 56: 2237-2250.

Bourghelle, D., and F. Declerck, 2004. Why Markets Should not Necessarily Reduce the Tick Size. *Journal of Banking and Finance* 28:373-398.

Buti, S., B. Rindi, and I. M. Werner, 2013. Dark Pool Trading Strategies, Ohio State University Working Paper.

Buti, S., F. Consonni, B. Rindi, and I. M. Werner, 2013. Queue-Jumping and Mid-Crossing, mimeo.

Butler, G., 2010, Liquidity Aggregation: What Institutional Investors Need to Know, ITG Working Paper.

Chordia, T. and A. Subrahmanyam, 1995. Market Making, The Tick Size and Payment-for-Order-Flow: Theory and Evidence. Journal of Business, 68: 543-576.

Chowdhry, B., and V. Nanda, 1991. Multi-Market Trading and Market Liquidity. *Review of Financial Studies* 4:483-511.

Chung, K. H., C. Chuwonganant, and D.T. McCormick, 2004. Order Preferencing and Market quality on NASDAQ Before and After Decimalization. *Journal of Financial Economics* 71:581-612.

Cordella, T., and T. Foucault, 1999. Minimum Price Variations, Time Priority and Quote Dynamics. *Journal of Financial Intermediation* 8:141-173.

Degryse, H., M. Van Achter and G. Wuyts, 2009. Dynamic order submission strategies with competition between a dealer market and a crossing network. *Journal of Financial Economics* 91:319-338.

Delassus R., and S. Tyc, 2010. Subpenny trading in US equity markets. *BNP Paribas Working Paper.*

Foucault, T., O. Kadan, and E. Kandel, 2005. Limit Order Book as A Market for Liquidity. *Review of Financial Studies* 18:1171-1217.
Foucault, T. and A. J. Menkveld, 2008. Competition for Order Flow and Smart Order Routing Systems. *Journal of Finance* 63: 119-158.
Goettler, R. L., C. Parlour, and U. Rajan, 2005. Equilibrium in a Dynamic Limit Order Market. *Journal of Finance* 60:2149-2190.
Goldstein, M. A., and K.A. Kavajecz, 2000. Eighths, sixteenths, and the market depth: changes in tick size and liquidity provision on the NYSE. *Journal of Financial Economics* 56: 125-149.
Goldstein, M. A., A. V. Shiklko, B. F. Van Ness, and R. A. Van Ness, 2008. Competition in the market for NASDAQ securities. *Journal of Financial Markets* 11:113-143.
Griffiths, M. D., B. F. Smith, D. A. S. Turnbull, and R. W. White, 1998. The Role of Tick Size in Upstairs Trading and Downstairs Trading. *Journal of Financial Intermediation* 7:393-417.
Hansch, O., N. Y. Naik, and S. Viswanathan, 1999. Preferencing, Internalization, Best Execution and Dealer Profits. *Journal of Finance* 54:1799-1828.
Harris, L., 1994. Minimum Price Variations, Discrete Bid-ask Spreads and Quotation Sizes. *Review of Financial Studies* 7:149-178.
Hatheway, F., A. Kwan, and H. Zheng, 2013. An Empirical Analysis of Market Segmentation on U.S. Equities Markets. *Working Paper, SSRN FEN Library.*
He, C., E. Odders-White, and M. Ready, 2006. The Impact of Preferencing on Execution Quality. *Journal of Financial Markets* 9:246-273.
Hendershott, T., and C. Jones, 2005. Trade-Through Prohibitions and Market Quality. *Journal of Financial Markets* 8:1-23.
Hendershott, T., and H. Mendelson, 2000. Crossing Networks and Dealer Markets: Competition and Performance. *Journal of Finance* 55: 2071-2115.
Jarnecic, E., and M. R. Snape, 2010. An Empirical Analysis of High Frequency Trading on the London Stock Exchange. *Working Paper.*
Kadan, O., 2006. So, Who Gains from a Small Tick Size? *Journal of Financial Intermediation* 15:32-66.
Kandel, E. and L. Marx, 1999. Payments for Order Flow on Nasdaq. *Journal of Finance* 54, 35-65.
Kwan, A., R. Masulis, and T. McInish, 2013. Trading rules, competition for order flows and market fragmentation. *Working Paper, SSRN FEN Library.*
Kyle, A. S., 1985. Continuous Auctions and Insider Trading. *Econometrica* 53:1315-1335.
Parlour, C., 1998. Price Dynamics in Limit Order Markets. *Review of Financial Studies* 11:789-816.
Parlour, C., and D. Seppi, 2003. Liquidity-based Competition for Order Flow. *Review of Financial Studies* 16:301-343.
Ronen, T. and D. G. Weaver, 2001. Teenes' Anyone?. *Journal of Financial Markets* 4:231-260.
Rosenblatt Securities Inc., 2013. Trading Talk, Let there be light, February 26.
Seppi, D. J., 1997. Liquidity Provision with Limit Orders and a Strategic Specialist. *Review of Financial Studies* 10:103-150.
U.S. Securities and Exchange Commission (SEC), 2010. Concept Release on Equity Market Structure No. 34-61358.
U.S. Securities and Exchange Commission (SEC), 2012. Report to Congress on Decimalization.

Appendix

The following Lemma is necessary to prove Proposition 1:

Lemma 1 *For $\frac{\tau}{v} \in (0, 0.1]$, if at time $t \neq t_3$ at least one limit order strategy has positive execution probability, there will always exist a β value for which a limit order is optimally selected by the incoming trader.*

Traders' equilibrium β-thresholds, and hence strategies, crucially depend on the state of the book which affects the execution probability of limit orders. Consider the LM as an example. There are four possible scenarios: (a) there is room for limit orders on both sides of the market, i.e., $p_t^*(A_k|S_t) \neq 0$ & $p_t^*(B_k|S_t) \neq 0$ (b) the book opens full on the ask side, i.e., $p_t^*(A_k|S_t) = 0$ & $p_t^*(B_k|S_t) \neq 0$ (c) the book opens full on the bid side, i.e., $p_t^*(A_k|S_t) \neq 0$ & $p_t^*(B_k|S_t) = 0$ (d) the book is full on both sides, i.e., $p_t^*(A_k|S_t) = 0$ & $p_t^*(B_k|S_t) = 0$. The Lemma states that in scenarios (a), (b) and (c), i.e., unless the book is full on both sides, a limit order is always an equilibrium strategy on either both sides of the market, or the bid side, or the ask side, respectively. Because (b) and (c) are just a simplified version of (a), we discuss directly scenario (a) and assume without loss of generality that only one limit order strategy on each side of the market has positive execution probability. The possible available strategies are $H_t^{LM} = \{-1^{B_{k'}}, +1^{A_k}, +1^{B_k}, -1^{A_{k'}}, 0\}$ and the corresponding profits:

H_t^{LM}	$=$	$-1^{B_{k'}}$	:	$B_{k'} - \beta v$
H_t^{LM}	$=$	$+1^{A_k}$	:	$(A_k - \beta v) \cdot p_t^*(A_k\|S_t)$
H_t^{LM}	$=$	$+1^{B_k}$	:	$(\beta v - B_k) \cdot p_t^*(B_k\|S_t)$
H_t^{LM}	$=$	$-1^{A_{k'}}$	:	$\beta v - A_{k'}$
H_t^{LM}	$=$	0	:	0

The thresholds between these strategies are:

$$\begin{aligned}
\beta^{LM}_{-1^{B_{k'}},+1^{A_k},t|S_{t-1}} &= \frac{B_{k'}}{v} - \frac{p_t^*(A_k|S_t)}{1-p_t^*(A_k|S_t)} \cdot \frac{A_k - B_{k'}}{v} \\
\beta^{LM}_{+1^{A_k},+1^{B_k},t|S_{t-1}} &= \frac{p_t^*(A_k|S_t)A_k + p_t^*(B_k|S_t)B_k}{p_t^*(A_k|S_t)+p_t^*(B_k|S_t)} \cdot \frac{1}{v} \\
\beta^{LM}_{+1^{B_k},-1^{A_{k'}},t|S_{t-1}} &= \frac{A_{k'}}{v} + \frac{p_t^*(B_k|S_t)}{1-p_t^*(B_k|S_t)} \cdot \frac{A_{k'} - B_k}{v}
\end{aligned}$$

The β-thresholds are a function of the relative tick size, $\frac{\tau}{v}$, and degenerate to 1 when $\frac{\tau}{v} \to 0$. Consider for example $\beta^{LM}_{-1^{B_{k'}},+1^{A_k},t|S_{t-1}}$, for $S_{t-1} = S_{t_1}$, $B_{k'} = B_2$ and $A_k = A_1$. Examples of S_{t_1} are [0000], [1000], and [0001].[26]

$$\beta^{LM}_{-1^{B_2},+1^{A_1},t_2|S_{t_1}} = \frac{B_2}{v} - \frac{p_{t_2}^*(A_1|S_{t_2})}{1-p_{t_2}^*(A_1|S_{t_2})} \cdot \frac{A_1 - B_2}{v} = 1 - \frac{3}{2} \cdot \frac{\tau}{v} - \frac{\frac{1}{2} - \frac{\tau}{4v}}{1-(\frac{1}{2}-\frac{\tau}{4v})} \cdot \frac{2\tau}{v} = f(\frac{\tau}{v})$$

It is easy to show that $\beta^{LM}_{-1^{B_2},+1^{A_1},t_2|S_{t_1}} \to 1$ when $\frac{\tau}{v} \to 0$. To guarantee that a limit order is an equilibrium strategy provided that $p_t^*(A_k|S_t) \neq 0$ and $p_t^*(B_k|S_t) \neq 0$, we need to check that for $\frac{\tau}{v} \in (0, 0.1]$

[26] From here onwards, to simplify the notation, we omit commas. For example, [0, 0, 0, 0] is simplified into [0000].

the following inequalities hold:

$$\begin{aligned}\Delta d\beta_t(-1^{B_{k'}}, +1^{A_k}, +1^{B_k}) &= d\beta^{LM}_{-1^{B_{k'}},+1^{A_k},t|S_{t-1}}/d\tfrac{\tau}{v} - d\beta^{LM}_{+1^{A_k},+1^{B_k},t|S_{t-1}}/d\tfrac{\tau}{v} < 0 \\ \Delta d\beta_t(-1^{B_{k'}}, +1^{A_k}, +1^{B_k}) &= d\beta^{LM}_{+1^{B_k},-1^{A_{k'}},t|S_{t-1}}/d\tfrac{\tau}{v} - d\beta^{LM}_{+1^{A_k},+1^{B_k},t|S_{t-1}}/d\tfrac{\tau}{v} > 0\end{aligned}$$

for the ask and bid side, respectively. If these two requirements are satisfied, $\beta^{LM}_{-1^{B_{k'}},+1^{A_k},t|S_{t-1}} < \beta^{LM}_{+1^{A_k},+1^{B_k},t|S_{t-1}} < \beta^{LM}_{+1^{B_k},-1^{A_{k'}},t|S_{t-1}}$, so that $H_t = \{+1^{A_k}, +1^{B_k}\}$ are equilibrium strategies. We present graphically $\Delta d\beta_t$, and focus on the ask side, the bid side being symmetric. Figures A.1 and A.2 show the difference in the β-threshold derivatives at t_1 and t_2, respectively. For example, Figure A.1 shows that at t_1 for $\frac{\tau}{v} \in (0, 0.1]$, $\Delta d\beta_t(-1^{B_{k'}}, +1^{A_k}, +1^{B_k}) < 0$. Therefore in scenario (a) a limit sell order is always an equilibrium strategy at t_1. A similar result is obtained for t_2, by observing Figure A.2. The proof for the SM follows the same methodology and is omitted.

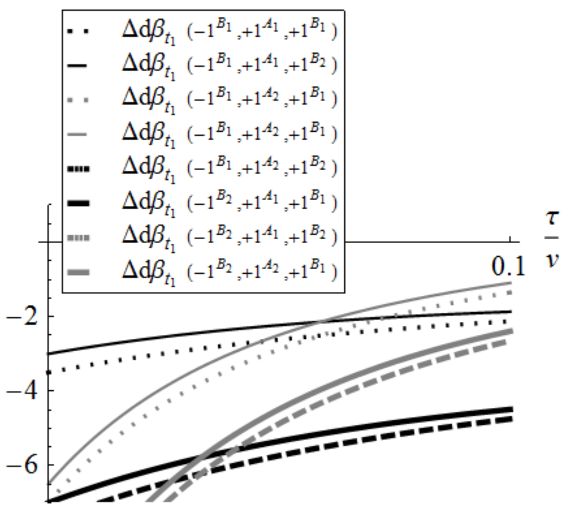


Figure A.1 - Difference in β-threshold derivatives at t_1.

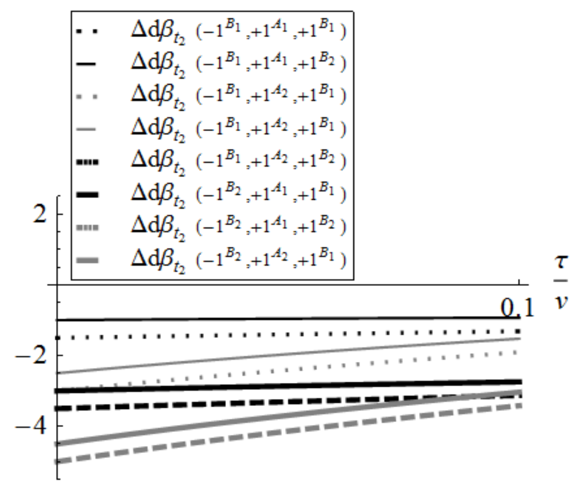


Figure A.2 - Difference in β-threshold derivatives at t_2.

A Proof of Proposition 1

We present the case for illiquid stocks. The cases for liquid stock and for very liquid stocks are presented in the Internet Appendix. The last case is a robustness check and is not discussed in the main text of the paper.

A.1 Large Market (LM)

At t_1 the book opens as $S_0 = [0000]$, and traders' strategy space is $\{-1^{B_2}, +1^{A_2}, +1^{A_1}, +1^{B_1}, +1^{B_2}, -1^{A_2}, 0\}$. Each strategy corresponds to an opening book at t_2 equal to [0000], [1000], [0100], [0010], [0001], and [0000], respectively. The model is solved by backward induction. We refer to pages 17-18 in the main text for equilibrium strategies at t_3, and focus directly on periods t_2 and t_1.

Period t_2. We present as an example the book that opens at t_2 with one share on A_2, i.e., $S_{t_1} = [1000]$. Traders' strategy space, $H_{t_2}^{LM}$, and the corresponding payoffs at t_2 are:

$$\begin{array}{|llcl|} H_{t_2}^{LM} = & -1^{B_2} & : & B_2 - \beta v \\ H_{t_2}^{LM} = & +1^{A_1} & : & (A_1 - \beta v) \cdot p_{t_2}^*(A_1|[1100]) \\ H_{t_2}^{LM} = & +1^{B_1} & : & (\beta v - B_1) \cdot p_{t_2}^*(B_1|[1010]) \\ H_{t_2}^{LM} = & +1^{B_2} & : & (\beta v - B_2) \cdot p_{t_2}^*(B_2|[1001]) \\ H_{t_2}^{LM} = & -1^{A_2} & : & \beta v - A_2 \\ H_{t_2}^{LM} = & 0 & : & 0 \end{array}$$

where the execution probabilities are given by (5) and (6). Because $p_{t_2}^*(A_1|S_{t_2}) \neq 0$ & $p_{t_2}^*(B_k|S_{t_2}) \neq 0$, following Lemma 1 equilibrium strategies include both limit and market orders.[27] Moreover, given that the book is empty on the bid side, traders must also choose the degree of aggressiveness of their limit buy orders. Traders will optimally choose a more aggressive limit order at B_1 rather than a limit order at B_2 if $\exists\beta$ s.t. $(\beta v - B_1) \cdot p_{t_2}^*(B_1|[1010]) > \max\{\beta v - A_2,\ (\beta v - B_2) \cdot p_{t_2}^*(B_2|[1001])\}$. This implies that the threshold between $H_{t_2} = -1^{A_2}$ and $H_{t_2} = +1^{B_1}$ must be larger than the threshold between $H_{t_2} = -1^{A_2}$ and $H_{t_2} = +1^{B_2}$. Specifically, as $\beta_{+1^{B_k},-1^{A_2},t_2|[1000]}^{LM} = \frac{A_2}{v} + \frac{p_{t_2}^*(B_k|S_{t_2})}{1-p_{t_2}^*(B_k|S_{t_2})} \cdot \frac{A_2 - B_k}{v}$, in order for $\beta_{+1^{B_1},-1^{A_2},t_2|[1000]}^{LM} > \beta_{+1^{B_2},-1^{A_2},t_2|[1000]}^{LM}$, the higher price (by one tick, τ) they pay if they choose a limit order at B_1 must be compensated by a higher execution probability. The relative tick size $\frac{\tau}{v}$ such that $\beta_{+1^{B_1},-1^{A_2},t_2|[1000]}^{LM} = \beta_{+1^{B_2},-1^{A_2},t_2|[1000]}^{LM}$ is $\frac{2}{3}(-5 + 2\sqrt{7}) \approx 0.19433$. Figure A.3 shows that for $\frac{\tau}{v} \in (0, 0.19433]$, $\beta_{+1^{B_1},-1^{A_2},t_2|[1000]}^{LM} < \beta_{+1^{B_2},-1^{A_2},t_2|[1000]}^{LM}$. Therefore, $H_{t_2} = +1^{B_1}$ is never optimal for $\frac{\tau}{v} \in (0, 0.1]$ if at t_2 the book opens empty on the bid side.[28] The equilibrium strategies at t_2 are:

$$H_{t_2}^{*LM}(\beta|[1000]) = \begin{cases} -1^{B_2} & \text{if } \beta \in [0, \beta_{-1^{B_2},+1^{A_1},t_2|[1000]}^{LM}) \\ +1^{A_1} & \text{if } \beta \in [\beta_{-1^{B_2},+1^{A_1},t_2|[1000]}^{LM}, \beta_{+1^{A_1},+1^{B_2},t_2|[1000]}^{LM}) \\ +1^{B_2} & \text{if } \beta \in [\beta_{+1^{A_1},+1^{B_2},t_2|[1000]}^{LM}, \beta_{+1^{B_2},-1^{A_2},t_2|[1000]}^{LM}) \\ -1^{A_2} & \text{if } \beta \in [\beta_{+1^{B_2},-1^{A_2},t_2|[1000]}^{LM}, 2] \end{cases}$$

where $\beta_{-1^{B_2},+1^{A_1},t_2|[1000]}^{LM} = \frac{B_2}{v} - \frac{p_{t_2}^*(A_1|[1100])}{1-p_{t_2}^*(A_1|[1100])} \cdot \frac{A_1 - B_2}{v}$, $\beta_{+1^{A_1},+1^{B_2},t_2|[1000]}^{LM} = \frac{p_{t_2}^*(A_1|[1100])A_1 + p_{t_2}^*(B_2|[1001])B_2}{p_{t_2}^*(A_1|[1100]) + p_{t_2}^*(B_2|[1001])} \cdot \frac{1}{v}$ and $\beta_{+1^{B_2},-1^{A_2},t_2|[1000]}^{LM} = \frac{A_2}{v} + \frac{p_{t_2}^*(B_2|[1001])}{1-p_{t_2}^*(B_2|[1001])} \cdot \frac{A_2 - B_2}{v}$. Considering that we are deriving the equilibrium strategies starting with a book that opens at t_2 with one share on A_2, we can now compute the execution probability of the strategy $H_{t_1} = +1^{A_2}$ that turned the book into $S_{t_1} = [1000]$:

$$\begin{aligned} p_{t_1}^*(A_2|[1000]) \quad = \quad & \frac{2 - \beta_{+1^{B_2},-1^{A_2},t_2|[1000]}^{LM}}{2} + \frac{\beta_{+1^{B_2},-1^{A_2},t_2|[1000]}^{LM} - \beta_{+1^{A_1},+1^{B_2},t_2|[1000]}^{LM}}{2} \cdot p_{t_2}^*(A_2|[1001]) \\ & + \frac{\beta_{-1^{B_2},+1^{A_1},t_2|[1000]}^{LM}}{2} \cdot p_{t_2}^*(A_2|[1000]) \end{aligned}$$

[27] Note that $p_{t_2}^*(A_2|S_{t_2}) = 0$ because the order already standing at A_2 has time priority: at t_3 the game ends and so there is only one trader left that could submit a market order to the ask side.

[28] Similarly, we can show that $H_{t_2}^{LM} = +1^{A_1}$ is never optimal if at t_2 the book opens empty on the ask side. An analogous result holds for the SM: if at t_2 the book opens empty, it is never optimal to post limit orders at a_l and b_l, for $l = 1, .., 4$.

The first term represents the probability that the limit sell order at A_2 gets executed at t_2 by a market order, whereas the other two terms are the probabilities of getting executed at t_3 after that at t_2 a limit order at B_2, or a market order to sell is submitted, respectively.

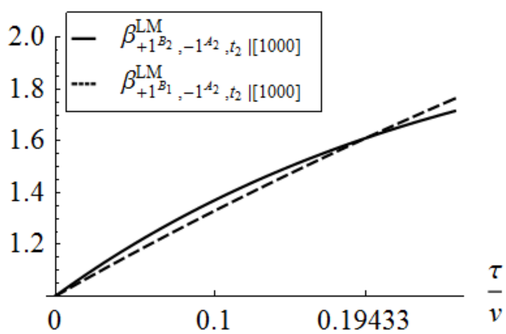


Figure A.3 - β-thresholds at t_2, $S_{t_1} = [1000]$.

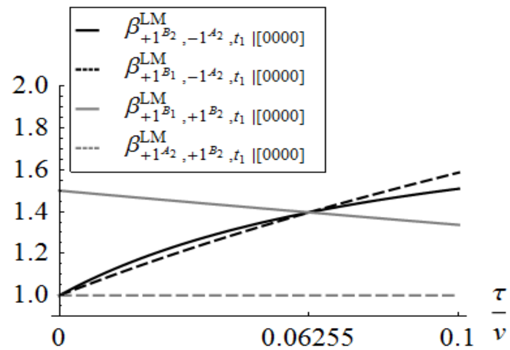


Figure A.4 - β-thresholds at t_1, $S_0 = [0000]$.

Period t_1. Analogously, we compute the execution probabilities of the other limit order strategies to compare all possible traders' payoffs at t_1, $H^{LM}_{t_1}=\{-1^{B_2}, +1^{A_2}, +1^{A_1}, +1^{B_1}, +1^{B_2}, -1^{A_2}, 0\}$. As for t_2, Lemma 1 ensures that both limit and market orders are equilibrium strategies because $p^*_{t_1}(A_k|S_{t_1}) \neq 0$ & $p^*_{t_1}(B_k|S_{t_1}) \neq 0$. However, traders need to determine the level of aggressiveness of their limit orders. Consider and compare again the strategies on the bid side. For $H_{t_1} = +1^{B_1}$ to be an equilibrium strategy, the threshold between $H_{t_1} = +1^{B_1}$ and $H_{t_1} = -1^{A_2}$ must be larger than the threshold between $H_{t_1} = +1^{B_2}$ and $H_{t_1} = -1^{A_2}$. We compute the value of the relative tick size such that the thresholds between a market buy order and a limit buy order at B_1 and B_2 respectively are equal, i.e., $\beta_{+1^{B_1},-1^{A_2},t_1|[0000]} = \beta_{+1^{B_2},-1^{A_2},t_1|[0000]}$, that is $\widehat{\frac{\tau}{v}}^{LM} \simeq 0.06255$. Figure A.4 shows that when $\frac{\tau}{v} \in (0, \widehat{\frac{\tau}{v}}^{LM})$, $\beta_{+1^{B_1},-1^{A_2},t_1|[0000]} < \beta_{+1^{B_2},-1^{A_2},t_1|[0000]}$ and $H_{t_1} = +1^{B_1}$ is not an equilibrium strategy. When instead $\frac{\tau}{v} \in [\widehat{\frac{\tau}{v}}^{LM}, 0.1]$, both $H_{t_1} = +1^{B_1}$ and $H_{t_1} = +1^{B_2}$ are equilibrium strategies: $\beta_{+1^{B_1},-1^{A_2},t_1|[0000]} > \beta_{+1^{B_2},-1^{A_2},t_1|[0000]}$, and $\beta_{+1^{B_1},+1^{B_2},t_1|[0000]} > \beta_{+1^{A_2},+1^{B_2},t_1|[0000]}$. In our numerical simulations, we assume that $\tau = 0.1$ so that $\frac{\tau}{v} \in [\widehat{\frac{\tau}{v}}^{LM}, 0.1]$ only for $v = 1$. So, at t_1 a limit order posted at B_1 conditional on $S_0 = [0000]$ is optimal for $v = 1$ while it is not optimal for $v = \{5, 10, 50\}$. We conclude that for $\frac{\tau}{v} \in (0, \widehat{\frac{\tau}{v}}^{LM})$ the equilibrium strategies are:

$$H^{*LM}_{t_1}(\beta, \tfrac{\tau}{v} \in (0, \widehat{\tfrac{\tau}{v}}^{LM})|[0000]) = \begin{cases} -1^{B_2} & \text{if } \beta \in [0, \beta^{LM}_{-1^{B_2},+1^{A_2},t_1|[0000]}) \\ +1^{A_2} & \text{if } \beta \in [\beta^{LM}_{-1^{B_2},+1^{A_2},t_1|[0000]}, \beta^{LM}_{+1^{A_2},+1^{B_2},t_1|[0000]}) \\ +1^{B_2} & \text{if } \beta \in [\beta^{LM}_{+1^{A_2},+1^{B_2},t_1|[0000]}, \beta^{LM}_{+1^{B_2},-1^{A_2},t_1|[0000]}) \\ -1^{A_2} & \text{if } \beta \in [\beta^{LM}_{+1^{B_2},-1^{A_2},t_1|[0000]}, 2] \end{cases}$$

where $\beta^{LM}_{-1^{B_2},+1^{A_2},t_1|[0000]} = \frac{B_2}{v} - \frac{p^*_{t_1}(A_2|[1000])}{1-p^*_{t_1}(A_2|[1000])} \cdot \frac{A_2-B_2}{v}$, $\beta^{LM}_{+1^{A_2},+1^{B_2},t_1|[0000]} = \frac{p^*_{t_1}(A_2|[1000])A_2+p^*_{t_1}(B_2|[0001])B_2}{p^*_{t_1}(A_2|[1000])+p^*_{t_1}(B_2|[0001])} \cdot \frac{1}{v}$, $\beta^{LM}_{+1^{B_2},-1^{A_2},t_1|[0000]} = \frac{A_2}{v} + \frac{p^*_{t_1}(B_2|[0001])}{1-p^*_{t_1}(B_2|[0001])} \cdot \frac{A_2-B_2}{v}$. When instead $\frac{\tau}{v} \in [\widehat{\frac{\tau}{v}}^{LM}, 0.1]$, submitting limit orders to the first level of the book is optimal, and the equilibrium strategies at t_1 are:

$$H_{t_1}^{*LM}(\beta, \tfrac{\tau}{v} \in [\widehat{\tfrac{\tau}{v}}^{LM}, 0.1]|[0000]) = \begin{cases} -1^{B_2} & \text{if } \beta \in [0, \beta^{LM}_{-1^{B_2},+1^{A_1},t_1|[0000]}) \\ +1^{A_1} & \text{if } \beta \in [\beta^{LM}_{-1^{B_2},+1^{A_1},t_1|[0000]}, \beta^{LM}_{+1^{A_1},+1^{A_2},t_1|[0000]}) \\ +1^{A_2} & \text{if } \beta \in [\beta^{LM}_{+1^{A_1},+1^{A_2},t_1|[0000]}, \beta^{LM}_{+1^{A_2},+1^{B_2},t_1|[0000]}) \\ +1^{B_2} & \text{if } \beta \in [\beta^{LM}_{+1^{A_2},+1^{B_2},t_1|[0000]}, \beta^{LM}_{+1^{B_2},+1^{B_1},t_1|[0000]}) \\ +1^{B_1} & \text{if } \beta \in [\beta^{LM}_{+1^{B_2},+1^{B_1},t_1|[0000]}, \beta^{LM}_{+1^{B_1},-1^{A_2},t_1|[0000]}) \\ -1^{A_2} & \text{if } \beta \in [\beta^{LM}_{+1^{B_1},-1^{A_2},t_1|[0000]}, 2] \end{cases}$$

where $\beta^{LM}_{+1^{A_1},+1^{A_2},t_1|[0000]} = \frac{p^*_{t_1}(A_1|[0100])A_1 - p^*_{t_1}(A_2|[1000])A_2}{p^*_{t_1}(A_1|[0100]) - p^*_{t_1}(A_2|[1000])} \cdot \frac{1}{v}$, $\beta^{LM}_{+1^{B_2},+1^{B_1},t_1|[0000]} = \frac{p^*_{t_1}(B_1|[0010])B_1 - p^*_{t_1}(B_2|[0001])B_2}{p^*_{t_1}(B_1|[0010]) - p^*_{t_1}(B_2|[0001])} \cdot \frac{1}{v}$.

A.2 Small Market (SM)

We solve the SM model in a similar way and directly present the equilibrium strategies at t_1 for the book that opens empty at t_1, that for brevity we indicate as $S_0 = [0]$. As for the LM, equilibrium strategies at t_1 differ depending on the relative tick size. By comparing the thresholds for the different limit order buy strategies, $H^{SM}_{t_1} = \{+1^{b_1}, +1^{b_2}, +1^{b_3}, +1^{b_4}, +1^{b_5}\}$, we obtain that when $\frac{\tau}{v} \in (0, \widehat{\frac{\tau}{v}}^{SM})$, where $\widehat{\frac{\tau}{v}}^{SM} \simeq 0.07162$ is the value of the relative tick size such that the thresholds between a market buy order and a limit buy order at b_1 and b_5 respectively are equal, i.e., $\beta_{+1^{b_1},-1^{a_5},t_1|[0]} = \beta_{+1^{b_5},-1^{a_5},t_1|[0]}$, traders submit limit orders only at b_5. Equilibrium strategies are: $H^{*SM}_{t_1}(\beta, \frac{\tau}{v} \in (0, \widehat{\frac{\tau}{v}}^{SM})|[0]) = \{-1^{b_5}, +1^{a_5}, +1^{b_5}, -1^{a_5}\}$. When instead $\frac{\tau}{v} \in [\widehat{\frac{\tau}{v}}^{SM}, 0.1]$, they increase the aggressiveness of their limit orders and equilibrium strategies at t_1 are: $H^{*SM}_{t_1}(\beta, \frac{\tau}{v} \in [\widehat{\frac{\tau}{v}}^{SM}, 0.1]|[0]) = \{-1^{b_5}, +1^{a_1}, +1^{a_5}, +1^{b_5}, +1^{b_1}, -1^{a_5}\}$.

A.3 LM vs. SM

Consider period t_1 as an example. The optimal strategies for a book that starts empty at t_1 are:

	Case 1	Case 2	Case 3
τ/v	$(0, 0.06255)$	$[0.06255, 0.07162)$	$[0.07162, 0.1]$
$H^{*LM}_{t_1}$	$-1^{B_2}, 1^{A_2}, 1^{B_2}, -1^{A_2}$	$-1^{B_2}, 1^{A_1}, 1^{A_2}, 1^{B_2}, 1^{B_1}, -1^{A_2}$	$-1^{B_2}, 1^{A_1}, 1^{A_2}, 1^{B_2}, 1^{B_1}, -1^{A_2}$
$H^{*SM}_{t_1}$	$-1^{b_5}, 1^{a_5}, 1^{b_5}, -1^{a_5}$	$-1^{b_5}, 1^{a_5}, 1^{b_5}, -1^{a_5}$	$-1^{b_5}, 1^{a_1}, 1^{a_5}, 1^{b_5}, 1^{b_1}, -1^{a_5}$

To compare the LM with the SM in terms of liquidity provision and executed volume, it is sufficient to compare the thresholds that make the trader indifferent between submitting a market order and the most attractive among the available limit orders. For the ask side, the equilibrium thresholds that we need to consider for the LM are $\beta^{LM}_{-1^{B_2},+1^{A_2},t_1|S_{t_1}}$ for Case 1, and $\beta^{LM}_{-1^{B_2},+1^{A_1},t_1|S_{t_1}}$ for Cases 2 and 3; while for the SM, $\beta^{SM}_{-1^{b_5},+1^{a_5},t_1|S_{t_1}}$ for Cases 1 and 2, and $\beta^{SM}_{-1^{b_5},+1^{a_1},t_1|S_{t_1}}$ for Case 3. As an example, we consider Case 3:

$$\beta^{LM}_{-1^{B_2},+1^{A_1},t_1|S_{t_1}} = \frac{B_2}{v} + \frac{p^*_{t_2}(A_1|S_{t_1})}{1-p^*_{t_2}(A_1|S_{t_1})} \cdot \frac{A_1 - B_2}{v} < \beta^{SM}_{-1^{b_5},+1^{a_1},t_1|S_{t_1}} = \frac{b_5}{v} + \frac{p^*_{t_2}(a_1|S_{t_1})}{1-p^*_{t_2}(a_1|S_{t_1})} \cdot \frac{a_1 - b_5}{v}$$

The opposite holds for the bid side. Consequently, volume is lower in the LM:

$$VL^{LM}_{t_1|[0000]} = \frac{2-\beta^{LM}_{+1^{B_1},-1^{A_2},t_1|[0000]}}{2} + \frac{\beta^{LM}_{-1^{B_2},+1^{A_1},t_1|[0000]}}{2} < VL^{SM}_{t_1|[0]} = \frac{2-\beta^{SM}_{+1^{b_1},-1^{a_5},t_1|[0]}}{2} + \frac{\beta^{SM}_{-1^{b_5},+1^{a_1},t_1|[0]}}{2}$$

When the book opens empty at t_1, $H_{t_1} = 0$ is never optimal and traders use only market and limit orders. Thus, because $VL^{LM}_{t_1|[0000]} < VL^{SM}_{t_1|[0]}$, we obtain that $LP^{LM}_{t_1|[0000]} > LP^{SM}_{t_1|[0]}$. Furthermore, when the book opens empty, total depth is equal to the liquidity provision so that $DPT^{LM}_{t_1|[0000]} > DPT^{SM}_{t_1|[0]}$. Inside depth also coincides with liquidity provision in this case, so that $DPI^{LM}_{t_1|[0000]} > DPI^{SM}_{t_1|[0]}$. Finally, we compute the spread:

$$\begin{aligned} SP^{LM}_{t_1|[0000]} &= E[A_{k'} - B_{k'}] \\ &= 3\tau - (\frac{\beta^{LM}_{+1^{A_1},+1^{A_2},t_1|[0000]} - \beta^{LM}_{-1^{B_2},+1^{A_1},t_1|[0000]}}{2} + \frac{\beta^{LM}_{+1^{B_1},-1^{A_2},t_1|[0000]} - \beta^{LM}_{+1^{B_2},+1^{B_1},t_1|[0000]}}{2}) \cdot \tau \\ &< 3\tau - (\frac{\beta^{SM}_{+1^{a_1},+1^{a_5},t_1|[0]} - \beta^{SM}_{-1^{b_5},+1^{a_1},t_1|[0]}}{2} + \frac{\beta^{SM}_{+1^{b_1},-1^{a_5},t_1|[0]} - \beta^{SM}_{+1^{b_5},+1^{b_1},t_1|[0]}}{2}) \cdot \frac{4\tau}{3} = SP^{SM}_{t_1|[0]} \end{aligned}$$

Because of the monotonicity of the β-thresholds, as visible for example in Figures A.1 and A.2, these results hold for $\frac{\tau}{v} \in (0, 0.1]$. Similarly, we obtain analogous results for periods t_2 and t_3.

B Proof of Proposition 2

We provide a proof for liquid stocks, i.e., a PLB that opens at t_1 as $S^{PLB}_0 = [0110]$. The illiquid case is solved in a similar way and omitted.

B.1 Transparent SPV (T)

When the SPV is transparent, RTs' equilibrium strategies depend on the initial state of the SPV. We present as an example the case with an empty SPV, $S^{SPV}_0 = [0]$, and refer to the Internet Appendix for the case in which the SPV has one share on the first level of the book, $S^{SPV}_0 = [1]$. To compute market quality indicators that are comparable with the opaque SPV case, we take the average of the values obtained for the two cases.

B.1.1 Equilibrium strategies

At t_1 the traders' strategy space, considering both RTs and BDs, is $\{-1^{\widehat{B}}, +1^i, +1^j, -1^{\widehat{A}}, 0\}$ with $i = \{A_{1:2}, B_{1:2}\}$, $j = \{a_{1:5}, b_{1:5}\}$, and $\widehat{A} = A_{k'} \wedge a_{l'}$ and $\widehat{B} = B_{k'} \vee b_{l'}$ the best prices across PLB and SPV. Therefore, at the beginning of t_2 there are 17 possible states of the books: one share added to the i-th level of the PLB and no shares added to the SPV (4 cases), one share added to the j-th level of the SPV and no shares added to the PLB (10 cases), one share taken from the PLB (2 cases) or no trading (1 case).

Period t_2. We compute the optimal equilibrium strategies of both RTs and BDs for each possible opening state of the book at t_2. Note that at t_3 the equilibrium strategies of RTs and BDs are the same: both can observe the best available price and, because traders submit only market orders, the BDs can't take advantage of their ability to post liquidity on the SPV. As a result, the orders' execution probabilities at t_2 do not depend on the type of trader arriving at t_3, for example: $p^{*RT}_{t_2}(A_k|S^{PLB}_{t_2}, S^{SPV}_{t_2}) = p^{*BD}_{t_2}(A_k|S^{PLB}_{t_2}, S^{SPV}_{t_2}) = p^{*}_{t_2}(A_k|S^{PLB}_{t_2}, S^{SPV}_{t_2})$. Suppose $H^{*}_{t_1} = +1^{A_2}$ so that at t_2 the book opens $S^{PLB}_{t_1} = [1110]$ and $S^{SPV}_{t_1} = [0]$. If a RT arrives at t_2, his payoffs are:

$H^{RT}_{t_2} = -1^{B_1}$	:	$B_1 - \beta v$
$H^{RT}_{t_2} = -1^{A_1}$	:	$\beta v - A_1$
$H^{RT}_{t_2} = 0$	:	0

and his equilibrium strategies are:

$$H^{*RT}_{t_2}(\beta|[1110],[0]) = \begin{cases} -1^{B_1} & \text{if } \beta \in [0, \beta^{RT}_{-1^{B_1},0,t_2|[1110],[0]}) \\ 0 & \text{if } \beta \in [\beta^{RT}_{-1^{B_1},0,t_2|[1110],[0]}, \beta^{RT}_{0,-1^{A_1},t_2|[1110],[0]}) \\ -1^{A_1} & \text{if } \beta \in [\beta^{RT}_{0,-1^{A_1},t_2|[1110],[0]}, 2] \end{cases}$$

By using the optimal β-thresholds associated with these strategies, we compute the execution probability of $H_{t_1} = +1^{A_2}$ conditional on a RT arriving at t_2:

$$p^{*RT}_{t_1}(A_2|[1110],[0]) = \frac{2-\beta^{RT}_{0,-1^{A_1},t_2|[1110],[0]}}{2} \cdot p^{*}_{t_2}(A_2|[1010],[0])$$

If instead a BD arrives at t_2, his payoffs are:

$H^{BD}_{t_2} = -1^{B_1}$	:	$B_1 - \beta v$
$H^{BD}_{t_2} = +1^{a_l}$	:	$(a_l - \beta v) \cdot p^{*}_{t_2}(a_l|[1110],[Q^{a_l} = 1])$
$H^{BD}_{t_2} = +1^{b_l}$	:	$(\beta v - b_l) \cdot p^{*}_{t_2}(b_l|[1110],[Q^{b_l} = 1])$
$H^{BD}_{t_2} = -1^{A_1}$	:	$\beta v - A_1$
$H^{BD}_{t_2} = 0$	:	0

where for example $[Q^{a_l} = 1]$ indicates a SPV with one unit at a_l and empty at all other price levels. As for the single market model, Lemma 1 ensures that both limit and market orders are equilibrium strategies because $p^{*}_{t_2}(a_l|[1110],[Q^{a_l} = 1]) \neq 0$ & $p^{*}_{t_2}(b_l|[1110],[Q^{b_l} = 1]) \neq 0$ for $l = 1,2$. Traders need to determine the level of aggressiveness of their limit orders, we consider again the bid side as an example. For $H_{t_2} = +1^{b_1}$ to be an equilibrium strategy, $\beta_{+1^{b_1},-1^{A_1},t_1|[1110],[0]} > \beta_{+1^{b_2},-1^{A_1},t_1|[1110],[0]}$. Figure A.5 shows that this is always the case for $\frac{\tau}{v} \in (0,1]$. Because $\beta_{+1^{b_2},+1^{b_1},t_1|[0000],[0]} > \beta_{+1^{a_2},+1^{b_2},t_1|[0000],[0]}$, also $H_{t_2} = +1^{b_2}$ is an optimal strategy. The equilibrium strategies for the BD are:

$$H^{*BD}_{t_2}(\beta|[1110],[0]) = \begin{cases} -1^{B_1} & \text{if } \beta \in [0, \beta^{BD}_{-1^{B_1},+1^{a_1},t_2|[1110],[0]}) \\ +1^{a_1} & \text{if } \beta \in [\beta^{BD}_{-1^{B_1},+1^{a_1},t_2|[1110],[0]}, \beta^{BD}_{+1^{a_1},+1^{a_2},t_2|[1110],[0]}) \\ +1^{a_2} & \text{if } \beta \in [\beta^{BD}_{+1^{a_1},+1^{a_2},t_2|[1110],[0]}, \beta^{BD}_{+1^{a_2},+1^{b_2},t_2|[1110],[0]}) \\ +1^{b_2} & \text{if } \beta \in [\beta^{BD}_{+1^{a_2},+1^{b_2},t_2|[1110],[0]}, \beta^{BD}_{+1^{b_2},+1^{b_1},t_2|[1110],[0]}) \\ +1^{b_1} & \text{if } \beta \in [\beta^{BD}_{+1^{b_2},+1^{b_1},t_2|[1110],[0]}, \beta^{BD}_{+1^{b_1},-1^{A_1},t_2|[1110],[0]}) \\ -1^{A_1} & \text{if } \beta \in [\beta^{BD}_{+1^{b_1},-1^{A_1},t_2|[1110],[0]}, 2] \end{cases}$$

It follows that, conditional on a BD arriving at t_2, the execution probability of the limit order posted at A_2 is:

$$p^{*BD}_{t_1}(A_2|[1110],[0]) = \frac{2-\beta^{BD}_{+1^{b_1},-1^{A_1},t_2|[1110],[0]}}{2} \cdot p^{*}_{t_2}(A_2|[1010],[0])$$

We compute the total execution probability of the limit order posted at A_2 at t_1 as the weighted average of the two conditional probabilities:

$$p^*_{t_1}(A_2|[1110],[0]) = \alpha p^{BD*}_{t_1}(A_2|[1110],[0]) + (1-\alpha)p^{RT*}_{t_1}(A_2|[1110],[0])$$

Similarly, we compute the equilibrium strategies for all the other possible states of the book at t_2 and obtain the execution probabilities of the different order types available at t_1.

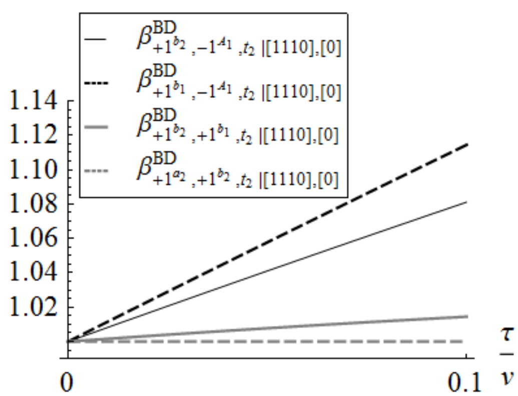


Figure A.5 - BD's β-thresholds at t_2, $S_{t_1} = [1110]$ and $SPV_{t_1} = [0]$.

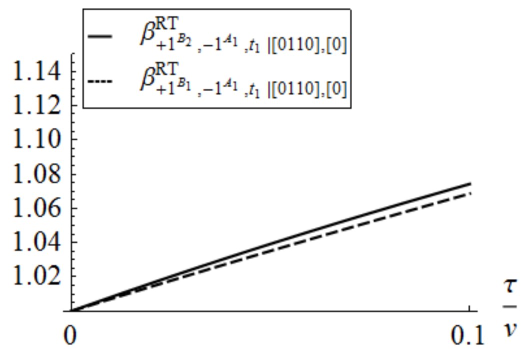


Figure A.6 - RT's β-thresholds at t_1, $S_0 = [0110]$ and $SPV_0 = [0]$.

Period t_1**.** If a RT arrives at t_1, his strategy space is $H^{RT}_{t_1} = \{-1^{\widehat{B}}, +1^{A_2}, +1^{A_1}, +1^{B_1}, +1^{B_2}, -1^{\widehat{A}}, 0\}$. Lemma 1 ensures that both limit and market orders are equilibrium strategies because $p^*_{t_1}(A_k|S_{t_1}) \neq 0$ & $p^*_{t_1}(B_k|S_{t_1}) \neq 0$. To determine the level of aggressiveness of traders' limit orders, we compare $\beta_{+1^{B_1},-1^{A_1},t_1|[0110],[0]}$ and $\beta_{+1^{B_2},-1^{A_1},t_1|[0110],[0]}$. Figure A.8 shows that for $\frac{\tau}{v} \in (0, 0.1]$, $\beta_{+1^{B_1},-1^{A_1},t_1|[0110],[0]} < \beta_{+1^{B_2},-1^{A_1},t_1|[0110],[0]}$. We conclude that submitting $H^{LM}_{t_1} = +1^{B_1}$ is not optimal. A similar derivation applies to the ask side. The equilibrium strategies for a RT are:

$$H^{*RT}_{t_1}(\beta|[0110],[0]) = \begin{cases} -1^{B_1} & \text{if } \beta \in [0, \beta^{RT}_{-1^{B_1},+1^{A_2},t_1|[0110],[0]}) \\ +1^{A_2} & \text{if } \beta \in [\beta^{RT}_{-1^{B_1},+1^{A_2},t_1|[0110],[0]}, \beta^{RT}_{+1^{A_2},+1^{B_2},t_1|[0110],[0]}) \\ +1^{B_2} & \text{if } \beta \in [\beta^{RT}_{+1^{A_2},+1^{B_2},t_1|[0110],[0]}, \beta^{RT}_{+1^{B_2},-1^{A_1},t_1|[0110],[0]}) \\ -1^{A_1} & \text{if } \beta \in [\beta^{RT}_{+1^{B_2},-1^{A_1},t_1|[0110],[0]}, 2] \end{cases}$$

If instead a BD arrives at t_1, he also selects a trading venue. Thus his strategy space is $\{-1^{\widehat{B}}, +1^i, +1^j, -1^{\widehat{A}}, 0\}$ with $i = \{A_{1:2}, B_{1:2}\}$ and $j = \{a_{1:5}, b_{1:5}\}$. Lemma 1 ensures again that both limit and market orders are equilibrium strategies. To determine the level of aggressiveness of BDs' limit orders, we compare in Figure A.7 the beta thresholds between a market order to buy at A_1 and a limit order to buy at all possible price levels on both the PLB and the SPV. We observe that for $\frac{\tau}{v} \in (0, 0.1]$, $\beta_{+1^{b_1},-1^{A_1},t_1|[0110],[0]} > \beta_{+1^{B_k},-1^{A_1},t_1|[0110],[0]}$ for $k = 1, 2$ and $\beta_{+1^{b_1},-1^{A_1},t_1|[0110],[0]} > \beta_{+1^{b_l},-1^{A_1},t_1|[0110],[0]}$ for $l = 2, .., 5$. We conclude that submitting $H^{BD}_{t_1} = +1^{b_1}$ is always optimal. By comparing the β-thresholds among all the limit order strategies, we obtain that $H^{BD}_{t_1} = \{+1^{b_2}, +1^{B_2}\}$ are also optimal. We show in Figure A.8 the thresholds for the equilibrium strategies only, and observe that the equilibrium is robust for $\frac{\tau}{v} \in (0, 0.1]$. A similar derivation applies to the ask side.

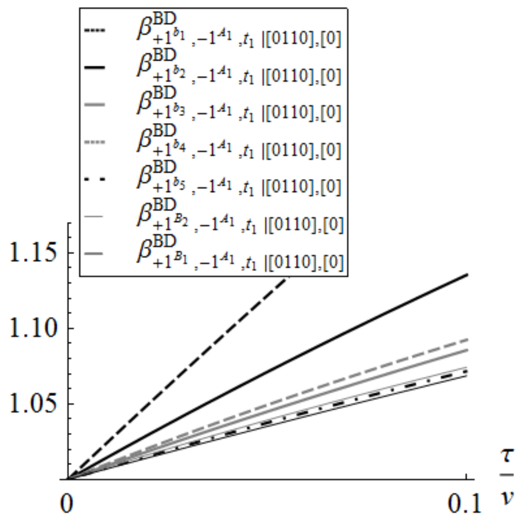


Figure A.7 - BD's β-thresholds at t_1 between $H_{t_1} = -1^{A_1}$ and $H_{t_1} = \{+1^{b_j}, +1^{B_i}\}$ for $j = 1, .., 5$ and $i = 1, 2$, $S_0 = [0110]$ and $SPV_0 = [0]$.

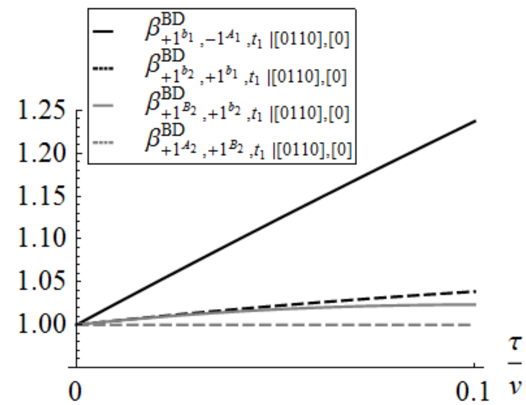


Figure A.8 - BD's equilibrium β-thresholds at t_1, $S_0 = [0110]$ and $SPV_0 = [0]$.

To summarize, the equilibrium strategies for a BD are:

$$H^{*BD}_{t_1}(\beta|[0110],[0]) = \begin{cases} -1^{B_1} & \text{if } \beta \in [0, \beta^{BD}_{-1^{B_1},+1^{a_1},t_1|[0110],[0]}) \\ +1^{a_1} & \text{if } \beta \in [\beta^{BD}_{-1^{B_1},+1^{a_1},t_1|[0110],[0]}, \beta^{BD}_{+1^{a_1},+1^{a_2},t_1|[0110],[0]}) \\ +1^{a_2} & \text{if } \beta \in [\beta^{BD}_{+1^{a_1},+1^{a_2},t_1|[0110],[0]}, \beta^{BD}_{+1^{a_2},+1^{A_2},t_1|[0110],[0]}) \\ +1^{A_2} & \text{if } \beta \in [\beta^{BD}_{+1^{a_2},+1^{A_2},t_1|[0110],[0]}, \beta^{BD}_{+1^{A_2},+1^{B_2},t_1|[0110],[0]}) \\ +1^{B_2} & \text{if } \beta \in [\beta^{BD}_{+1^{A_2},+1^{B_2},t_1|[0110],[0]}, \beta^{BD}_{+1^{B_2},+1^{b_2},t_1|[0110],[0]}) \\ +1^{b_2} & \text{if } \beta \in [\beta^{BD}_{+1^{B_2},+1^{b_2},t_1|[0110],[0]}, \beta^{BD}_{+1^{b_2},+1^{b_1},t_1|[0110],[0]}) \\ +1^{b_1} & \text{if } \beta \in [\beta^{BD}_{+1^{b_2},+1^{b_1},t_1|[0110],[0]}, \beta^{BD}_{+1^{b_1},-1^{A_1},t_1|[0110],[0]}) \\ -1^{A_1} & \text{if } \beta \in [\beta^{BD}_{+1^{b_1},-1^{A_1},t_1|[0110],[0]}, 2] \end{cases}$$

Market Quality Indicators. We focus again on period t_1, indicators for t_2 and t_3 are derived in a similar way. Expected volume on the PLB is:

$$\begin{aligned} VL^{PLB}_{t_1|[0110],[0]} &= \alpha[\Pr(H^{*BD}_{t_1} = -1^{B_1}) + \Pr(H^{*BD}_{t_1} = -1^{A_1})] + (1-\alpha)[\Pr(H^{*RT}_{t_1} = -1^{B_1}) + \Pr(H^{*RT}_{t_1} = -1^{A_1})] \\ &= \alpha(\frac{\beta^{BD}_{-1^{B_1},+1^{a_1},t_1|[0110],[0]}}{2} + \frac{2-\beta^{BD}_{+1^{b_1},-1^{A_1},t_1|[0110],[0]}}{2}) + (1-\alpha)(\frac{\beta^{RT}_{-1^{B_1},+1^{A_2},t_1|[0110],[0]}}{2} + \frac{2-\beta^{RT}_{+1^{B_2},-1^{A_1},t_1|[0110],[0]}}{2}) \end{aligned}$$

Because there is no volume executed in the SPV, we obtain that:

$$LP^{PLB}_{t_1|[0110],[0]} = 1 - VL^{PLB}_{t_1|[0110],[0]} - LP^{SPV}_{t_1|[0110],[0]}$$

We also compute the other market quality indicators for the PLB:

$$\begin{aligned} DPI^{PLB}_{t_1|[0110],[0]} &= 2 - VL^{PLB}_{t_1|[0110],[0]} \\ DPT^{PLB}_{t_1|[0110],[0]} &= 2 + LP^{PLB}_{t_1|[0110],[0]} - VL^{PLB}_{t_1|[0110],[0]} \\ SP^{PLB}_{t_1|[0110],[0]} &= \tau \cdot (LP^{PLB}_{t_1|[0110],[0]} + LP^{SPV}_{t_1|[0110],[0]}) + 2\tau \cdot VL^{PLB}_{t_1|[0110],[0]} \end{aligned}$$

B.1.2 PLB vs. PLB&SPV

We refer to Section A.2.3 in the Internet Appendix for the market indicators of the PLB benchmark framework, which coincide with the ones computed for the single market model, large tick case (LM). The market quality indicators for the PLB&SPV transparent framework are equal to the average of those obtained for $S^{SPV}_0 = [0]$ and $S^{SPV}_0 = [1]$. For example:

$$VL^{PLB,T}_{t_1} = \frac{VL^{PLB}_{t_1|[0110],[0]}}{2} + \frac{VL^{PLB}_{t_1|[0110],[1]}}{2}$$

We first analyze separately the two cases, and focus on t_1. $\mathbf{S}^{SPV}_0 = [\mathbf{0}]-$ Without BDs ($\alpha = 0$), the PLB&SPV model converges to the single market model. When α is positive, instead, limit orders submitted at t_1 to the PLB have a lower execution probability in the PLB&SPV model, because they are more frequently undercut by BDs. Therefore, RTs submit limit orders with a slightly lower probability than in the single market model while BDs submit limit orders more frequently but to the SPV, in order to undercut the best quote. We obtain that:[29]

$$\begin{aligned} \beta^{BD}_{-1^{B_1},+1^{a_1},t_1|[0110],[0]} &< \beta^{LM}_{-1^{B_1},+1^{A_2},t_1|[0110]} < \beta^{RT}_{-1^{B_1},+1^{A_2},t_1|[0110],[0]} \\ \beta^{BD}_{+1^{b_1},-1^{A_1},t_1|[0110],[0]} &> \beta^{LM}_{+1^{B_2},-1^{A_1},t_1|[0110]} > \beta^{RT}_{+1^{B_2},-1^{A_1},t_1|[0110],[0]} \end{aligned}$$

Because of the lower submission of limit orders to the PLB by both RTs and BDs, $LP^{PLB}_{t_1|[0110],[0]} < LP^{LM}_{t_1|[0110]}$. Moreover, even if RTs submit more market orders, this effect is dominated by the decreased submission of market orders by BDs so that $VL^{PLB}_{t_1|[0110],[0]} < VL^{LM}_{t_1|[0110]}$, and $VL^{LM}_{t_1|[0110]} - VL^{PLB}_{t_1|[0110],[0]} >$

[29] To avoid confusion between the PLB single market framework and the PLB in the PLB&SPV framework, we will still refer to the first one as "LM".

$LP^{LM}_{t_1|[0110]} - LP^{PLB}_{t_1|[0110],[0]}$. It is straightforward to show that:

$$\begin{aligned} DPI^{PLB}_{t_1|[0110],[0]} &> DPI^{LM}_{t_1|[0110]} \\ DPT^{PLB}_{t_1|[0110],[0]} &> DPT^{LM}_{t_1|[0110]} \\ SP^{PLB}_{t_1|[0110],[0]} &< SP^{LM}_{t_1|[0110]} \end{aligned}$$

$\mathbf{S}^{SPV}_0 = [\mathbf{1}]-$ All incoming market orders at t_1 are executed in the SPV, so that $VL^{PLB}_{t_1|[0110],[1]} = 0$. Furthermore, limit orders are only posted to the SPV as on the PLB they have a zero execution probability. It follows that $LP^{PLB}_{t_1|[0110],[1]} = 0$ and $SP^{PLB}_{t_1|[0110],[1]} = \tau$. Averaging over the two cases, $S^{SPV}_0 = [0]$ and $S^{SPV}_0 = [1]$, we obtain $VL^{PLB,T}_{t_1} < VL^{LM}_{t_1}$, $LP^{PLB,T}_{t_1} < LP^{LM}_{t_1}$, $DPT^{PLB,T}_{t_1} > DPT^{LM}_{t_1}$, $DPI^{PLB,T}_{t_1} > DPI^{LM}_{t_1}$, $SP^{PLB,T}_{t_1} < SP^{LM}_{t_1}$.

B.2 Opaque SPV (O)

In this proof we only highlight the differences with the transparent PLB&SPV framework, therefore we focus only on RTs. Consider again the PLB that opens [1110] at t_2. If a RT arrives, he will infer the state of the SPV from the observed PLB. The RT knows that $H_{t_1} = +1^{A_2}$ is never an equilibrium strategy for a BD if the state of the SPV is $S^{SPV}_0 = [1]$. So he will update the probabilities associated with $S^{SPV} = [0]$ and $S^{SPV} = [1]$ from $\frac{1}{2}$ to:

$$\Pr(S^{SPV}_{t_1} = [0]|S^{PLB}_{t_1} = [1110]) = \frac{\frac{1}{2}[\alpha \Pr(H^{*BD}_{t_1} = +1^{A_2}) + (1-\alpha)\Pr(H^{*RT}_{t_1} = +1^{A_2})]}{\frac{1}{2}\alpha \Pr(H^{*BD}_{t_1} = +1^{A_2}) + (1-\alpha)\Pr(H^{*RT}_{t_1} = +1^{A_2})} > \frac{1}{2}$$

$$\Pr(S^{SPV}_{t_1} = [1]|S^{PLB}_{t_1} = [1110]) = \frac{\frac{1}{2}(1-\alpha)\Pr(H^{*RT}_{t_1} = +1^{A_2})}{\frac{1}{2}\alpha \Pr(H^{*BD}_{t_1} = +1^{A_2}) + (1-\alpha)\Pr(H^{*RT}_{t_1} = +1^{A_2})} < \frac{1}{2}$$

To select his optimal trading strategy, he will then compute the expected payoffs using the Bayesian updated probabilities. For example:

$$H_{t_2} = -1^{\widehat{B}} : B_1 \Pr\{S^{SPV}_{t_1} = [0]|[1110]\} + b_1 \Pr\{S^{SPV}_{t_1} = [1]|[1110]\} - \beta v$$

To analyze the changes in the PLB after the introduction of a SPV, we compare the market quality indicators for an opaque SPV with those computed both for the benchmark PLB case and the case with a transparent SPV. We obtain:

$$\begin{aligned} DPI^{PLB,T}_{t_1} &> DPI^{PLB,O}_{t_1} > DPI^{LM}_{t_1|[0110]} \\ DPT^{PLB,T}_{t_1} &> DPT^{PLB,O}_{t_1} > DPT^{LM}_{t_1|[0110]} \\ SP^{PLB,T}_{t_1} &< SP^{PLB,O}_{t_1} < SP^{LM}_{t_1|[0110]} \end{aligned}$$

C Proof of Proposition 3

The results in Figures 7 and 8 are obtained by solving the dual market model for $\tau = \frac{0.1}{3}$ following the same methodology shown in the proof of Proposition 2, and by comparing the results of the numerical simulations with the ones derived before for a tick size $\tau = 0.1$. We consider two asset values: $v = \{1, 10\}$.

D Proof of Proposition 4

For the single market model, welfare values are computed using Eq. (15), and average change in welfare is given by Eq. (16). Results in Figure 9 are computed by running numerical simulations for $\tau = 0.1$ and $v = \{1, 5, 10, 50\}$. For the dual market model, we compute the welfare of BDs and RTs separately:

$$\begin{aligned}
E(W_t^{BD}) &= \sum_{i,j} \int_{\beta \in \{\beta : H_t^{*BD} = -1^{i,j} | S_{t-1}\}} (|i - \beta v| + |j - \beta v|) f(\beta) d\beta + \\
&\quad \sum_{i,j} \int_{\beta \in \{\beta : H_t^{*BD} = +1^{i,j} | S_{t-1}\}} (|i - \beta v| \cdot p_t^*(i|S_t) + |j - \beta v| \cdot p_t^*(j|S_t)) f(\beta) d\beta \\
E(W_t^{RT}) &= \sum_{i,j} \int_{\beta \in \{\beta : H_t^{*RT} = -1^{i,j} | S_{t-1}\}} (|i - \beta v| + |j - \beta v|) f(\beta) d\beta + \\
&\quad \sum_{i} \int_{\beta \in \{\beta : H_t^{*RT} = +1^{i} | S_{t-1}\}} |i - \beta v| \cdot p_t^*(i|S_t) f(\beta) d\beta
\end{aligned}$$

We then compute average welfare (AV) as:

$$E(W_t^{AV}) = \alpha E(W_t^{BD}) + (1 - \alpha) E(W_t^{RT})$$

Welfare changes over the three periods for BDs, RTs and on average are given by Eq. (17). Results in Figure 10 are obtained from numerical simulations for $\tau = \{\frac{0.1}{3}, 0.1\}$ and $v = \{1, 10\}$.

Table 1: Order Submission Strategy Space. This Table reports in column 3 the payoffs, $U(\cdot)$, of the order strategies H_t listed in column 1. In the case of market orders, $A_{k'}$ and $B_{k'}$ always refer to the best ask and bid prices.

Strategy	H_t	$U(\cdot)$
Market Sell Order	$-1^{B_{k'}}$	$B_{k'} - \beta v$
Limit Sell Order	1^{A_k}	$p_t(A_k \vert S_t) \cdot (A_k - \beta v)$
No Trade	0	0
Limit Buy Order	1^{B_k}	$p_t(B_k \vert S_t) \cdot (\beta v - B_k)$
Market Buy Order	$-1^{A_{k'}}$	$\beta v - A_{k'}$

Table 2: **Price Grid.** This Table shows the price grid for both the large tick market (LM) and the small tick market (SM). v is the asset value and τ is the tick size.

LM	**Price**	**SM**
A_2	$v + \frac{9}{6}\tau$	a_5
	$v + \frac{7}{6}\tau$	a_4
	$v + \frac{5}{6}\tau$	a_3
A_1	$v + \frac{3}{6}\tau$	a_2
	$v + \frac{1}{6}\tau$	a_1
	$v - \frac{1}{6}\tau$	b_1
B_1	$v - \frac{3}{6}\tau$	b_2
	$v - \frac{5}{6}\tau$	b_3
	$v - \frac{7}{6}\tau$	b_4
B_2	$v - \frac{9}{6}\tau$	b_5

Figure 1: NASDAQ Stocks - Queue-Jumping. This Figure shows the evolution of sub-penny trading over the past 10 years for different priced NASDAQ stocks. %Queue Jumping indicates the percentage of volume executed with a price increment smaller that 1 penny over total volume. Queue-Jumping does not include trades at the midpoint of the National Best Bid Offer. We use weekly statistics from Delassus and Tyc (2010), computed from Thomson Reuters tick-by-tick historical.

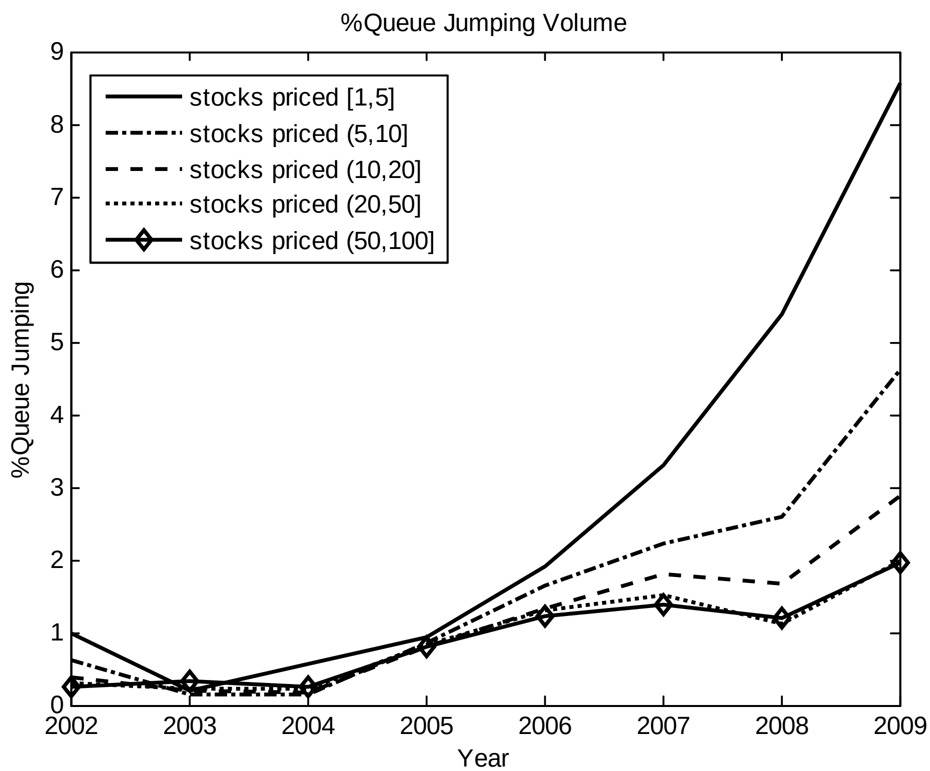


Figure 2: Single Market Model. Example from the extensive form of the game. The book opens at t_1 empty: $S_0 = [0, 0, 0, 0]$. H_t refers to the strategy of the trader who arrives at the market in period t. All available H_t are defined in Table 1.

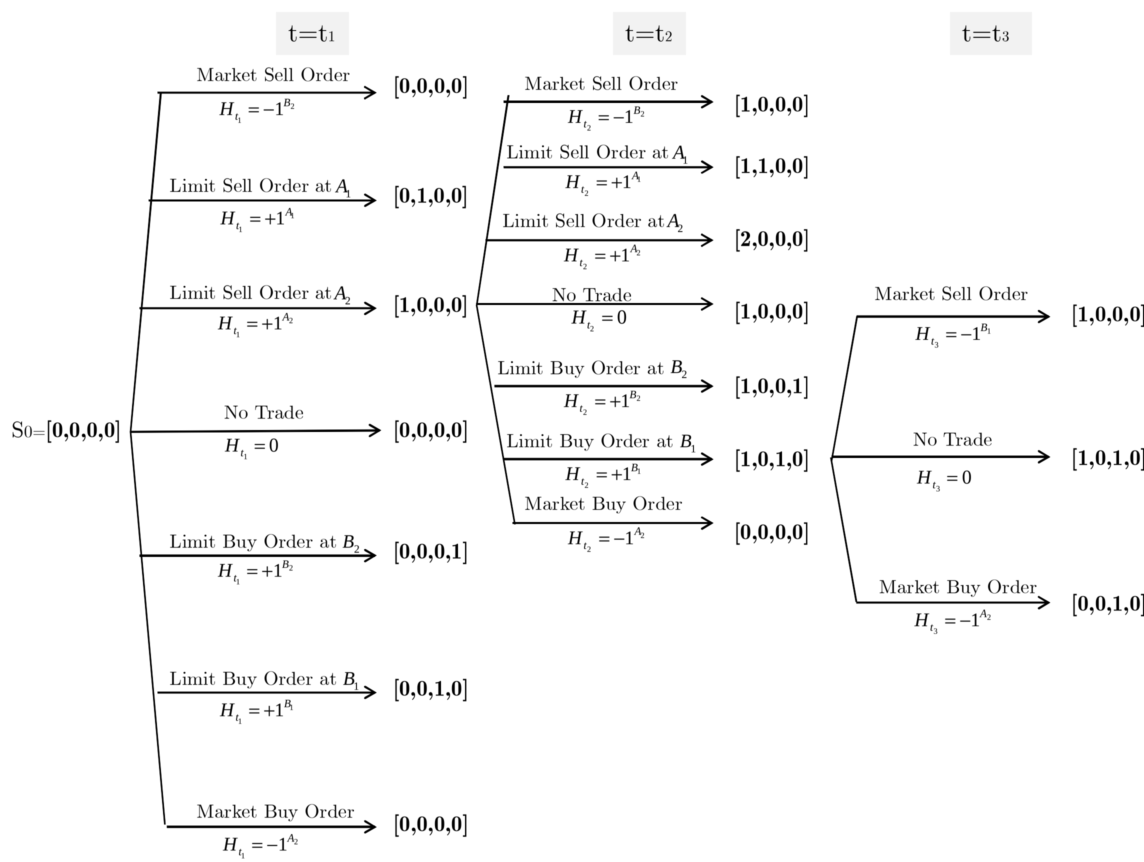
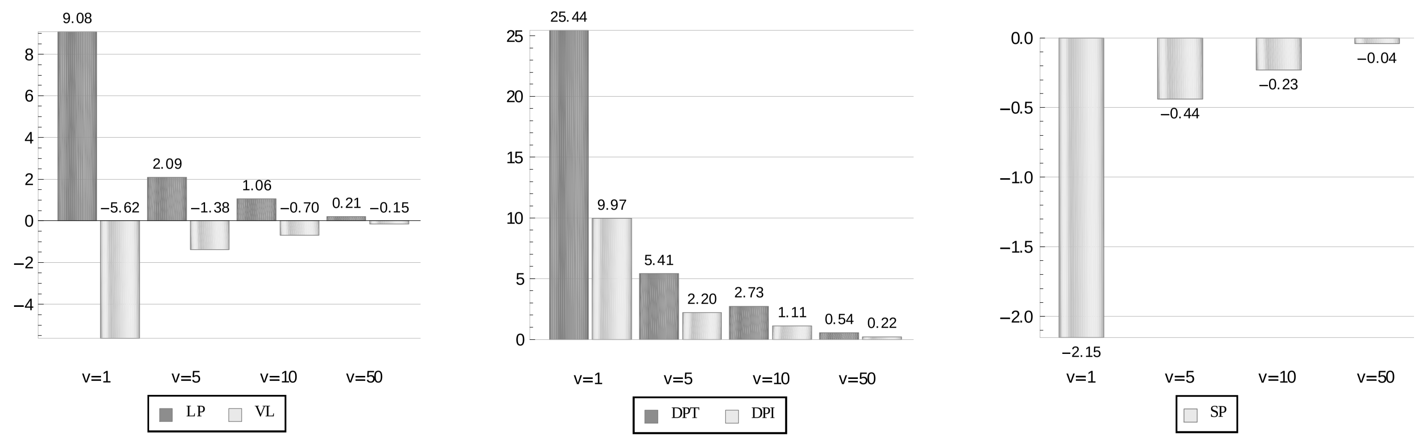


Figure 3 - Single Market Model

This Figure reports statistics for order flows and market quality that result from the comparison between the large tick market (LM) with $\tau = 0.1$, and the small tick market (SM) with $\tau = \frac{0.1}{3}$, under the two regimes of "Liquid stock" (Panel A) and "Illiquid stock" (Panel B), and for different asset values, v={1,5,10,50}. For order flows it reports statistics on liquidity provision (LP) and trading volume (VL); for market quality it reports statistics on total depth (DPT), depth at the best bid-offer (DPI) and spread (SP). The statistics are computed as the average difference between the value of the statistic in the SM, and the value of the same statistic in the LM: $\Delta y = \frac{1}{k}\sum_{t=t_1}^{t_k}[y_t^{SM} - y_t^{LM}] \times 100$, where y={DPI,DPT,SP,VL,LP}, k=2 for {DPI,DPT,SP,LP}, and k=3 for VL. Under the regime of "Liquid stock" at t_1 the book of the LM opens with one share on the first level (A_1,B_1), and the book of the SM opens with one share on the second level (a_2,b_2). Under the regime of "Illiquid stock" both the book of the LM and that of the SM open empty.

Panel A : Liquid stock



Panel B : Illiquid stock

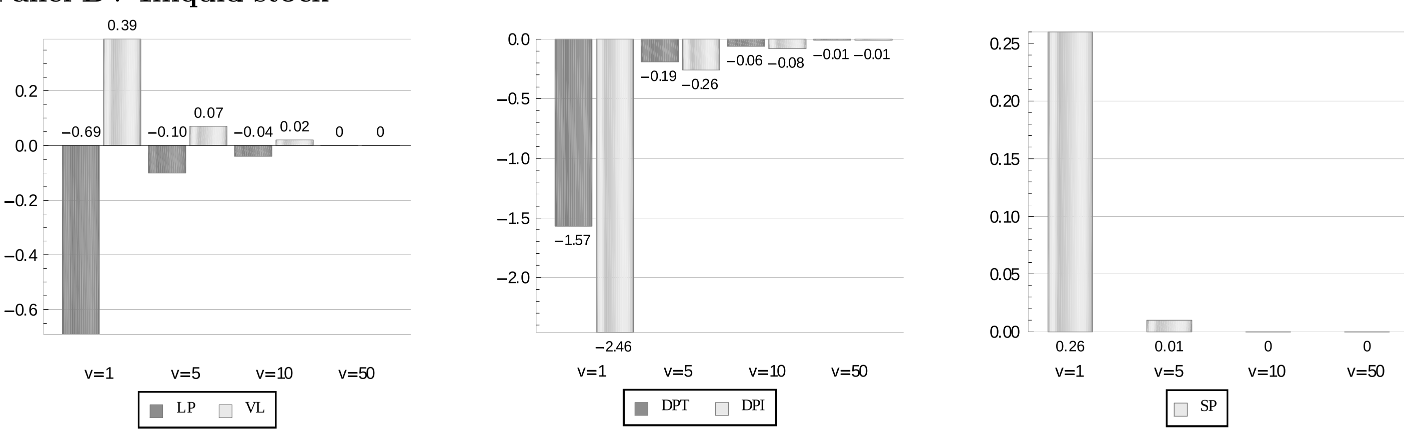


Figure 4 - Dual Market Model - Trading Strategies - $\tau = 0.1$, $\alpha = 10\%$

This Figure reports - for $t = t_1$ - the equilibrium submission probabilities of limit orders posted on the ask side at different levels of the book, under the two states of the book "Liquid Stock" (Panel A) and "Illiquid Stock" (Panel B), and for 2 different asset values, v={1,10}. The submission probabilities of limit orders for the Public Limit Order Book (PLB) refer to the equilibrium probabilities for the benchmark single market model with tick size τ, and price grid $\{A_1, A_2\}$. The submission probabilities of limit orders for the Dual Market Model are instead indicated as SPV when limit orders are posted to the Sub-Penny Venue with tick size $\frac{\tau}{3}$ and price grid $\{a_1, a_2, a_3, a_4, a_5\}$. They are indicated as (PLB&SPV) when they are posted to the PLB that competes with the (SPV) and has the same price grid as the PLB of the benchmark model, $\{A_1, A_2\}$. The circle indicates the price at which the order is submitted in equilibrium. Under the state of the book "Liquid stock", both the book of the PLB and the book of the PLB&SPV open at t_1 with one share on the first level (A_1, B_1); while under the regime of "Illiquid stock" both the book of the PLB and the book of the PLB&SPV open empty. In both cases the SPV opens with equal probability either empty or with one share on the first level (a_1, b_1). Results are reported for 2 regimes of transparency of the SPV, i.e., transparent (T) or opaque (O). The broker-dealer's arrival rate is $\alpha = 10\%$ and the tick size is $\tau = 0.1$.

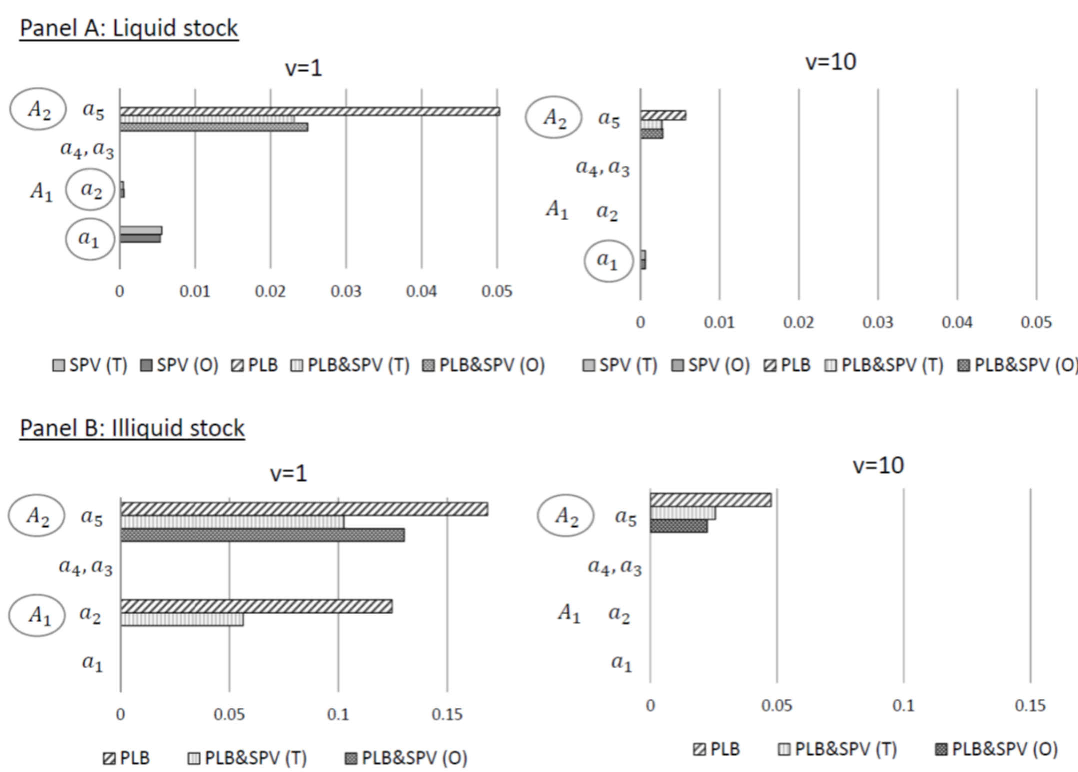


Figure 5 - Panel A - Dual Market Model - Liquid stock - $\tau = 0.1$, $\alpha = 10\%$

This Figure reports statistics for order flows and market quality that result from the comparison between the benchmark Public Limit Order Book (PLB) with tick size $\tau = 0.1$, and the Dual Market Model in which a PLB with the same tick size as the benchmark model (PLB&SPV) competes with a Sub-Penny Venue (SPV) characterized by a smaller tick size, $\tau = \frac{0.1}{3}$. We refer to "Liquid stock" as the regime under which both the benchmark PLB and the PLB that competes with the SPV open at t_1 with one share on the first level (A_1,B_1), and at the same time the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). We present results for the broker-dealers' arrival rate $\alpha = 10\%$ and for v={1,10}. For order flows this panel reports statistics on liquidity provision and trading volume both for the PLB (LP^{PLB} and VL^{PLB}) and for the SPV (LP^{SPV} and VL^{SPV}). For market quality we report statistics on total depth (DPT^{PLB}), depth at the best bid-offer (DPI^{PLB}) and spread (SP^{PLB}). The statistics are computed as the average difference between the value of the statistic in the PLB that competes with the SPV, and the value of the same statistic in the benchmark PLB: $\Delta y = \frac{1}{k}\sum_{t=t_1}^{t_k}[y_t^{PLB\&SPV} - y_t^{PLB}]\times 100$, where y={DPI^{PLB},DPT^{PLB},SP^{PLB},VL^{PLB},LP^{PLB},VL^{SPV},LP^{SPV}}, k=2 for {DPI^{PLB},DPT^{PLB},SP^{PLB},LP^{PLB},LP^{SPV}}, and k=3 for {VL^{PLB},VL^{SPV}}.

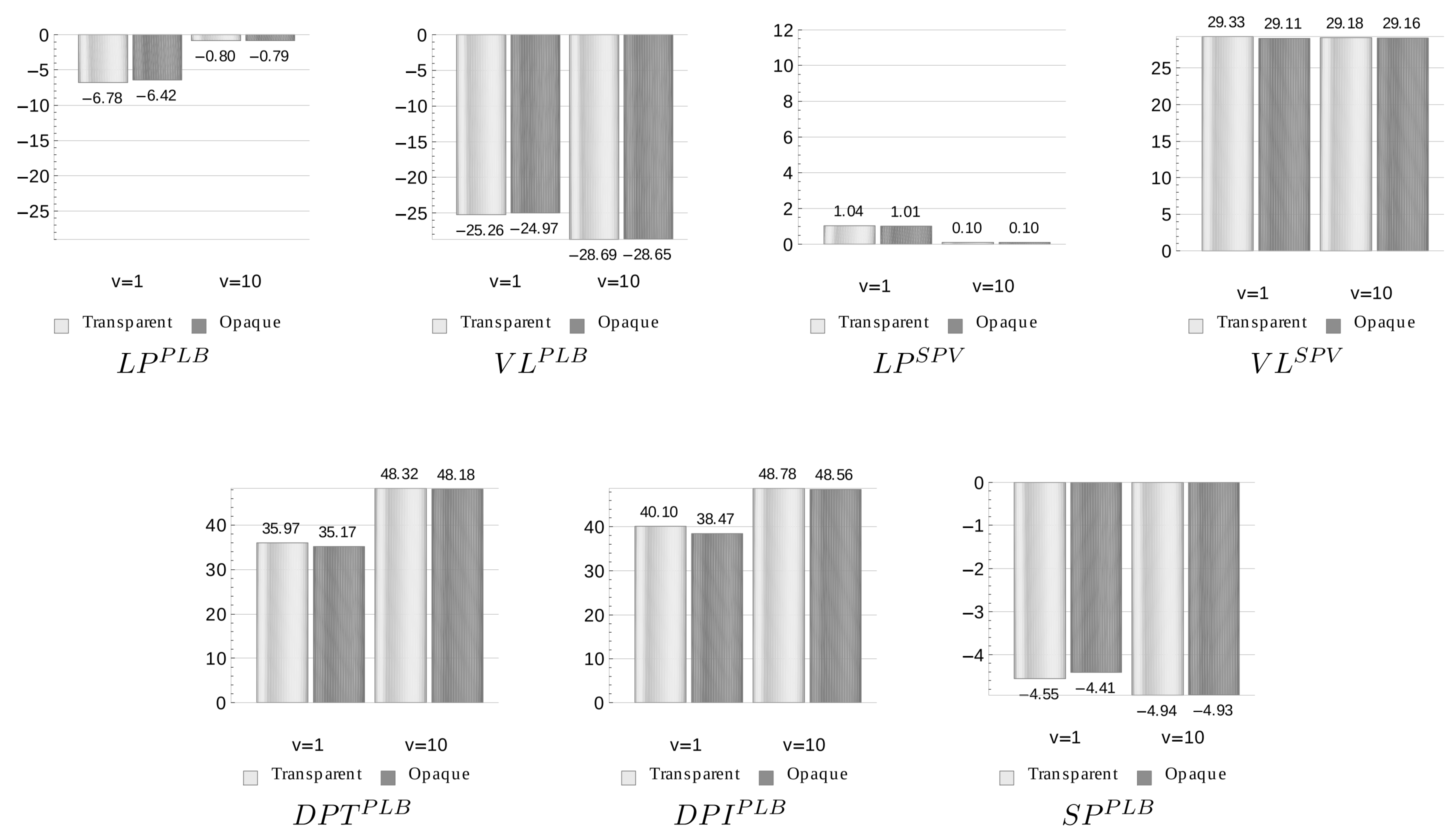


Figure 5 - Panel B - Dual Market Model - Illiquid stock - $\tau = 0.1$, $\alpha = 10\%$

This Figure reports statistics for order flows and market quality that result from the comparison between the benchmark Public Limit Order Book (PLB) with tick size $\tau = 0.1$, and the Dual Market Model in which a PLB with the same tick size as the benchmark model (PLB&SPV) competes with a Sub-Penny Venue (SPV) characterized by a smaller tick size, $\tau = \frac{0.1}{3}$. We refer to "Illiquid stock" as the regime under which both the book of the benchmark PLB and the book of the PLB that competes with the SPV open empty at t_1; and the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). We present results for the arrival rate of the broker-dealers $\alpha = 10\%$, and for v={1,10}. For order flows this panel reports statistics on liquidity provision and trading volume both for the PLB (LP^{PLB} and VL^{PLB}) and the SPV (LP^{SPV} and VL^{SPV}). For market quality it reports statistics on total depth (DPT^{PLB}), depth at the best bid-offer (DPI^{PLB}) and spread (SP^{PLB}). The statistics are computed as the average difference between the value of the statistic in the PLB that competes with the SPV, and the value of the same statistic in the benchmark PLB: $\Delta y = \frac{1}{k} \sum_{t=t_1}^{t_k} [y_t^{PLB\&SPV} - y_t^{PLB}] \times 100$, where $y = \{DPI^{PLB}, DPT^{PLB}, SP^{PLB}, VL^{PLB}, LP^{PLB}, VL^{SPV}, LP^{SPV}\}$, $k=2$ for $\{DPI^{PLB}, DPT^{PLB}, SP^{PLB}, LP^{PLB}, LP^{SPV}\}$, and $k=3$ for $\{VL^{PLB}, VL^{SPV}\}$.

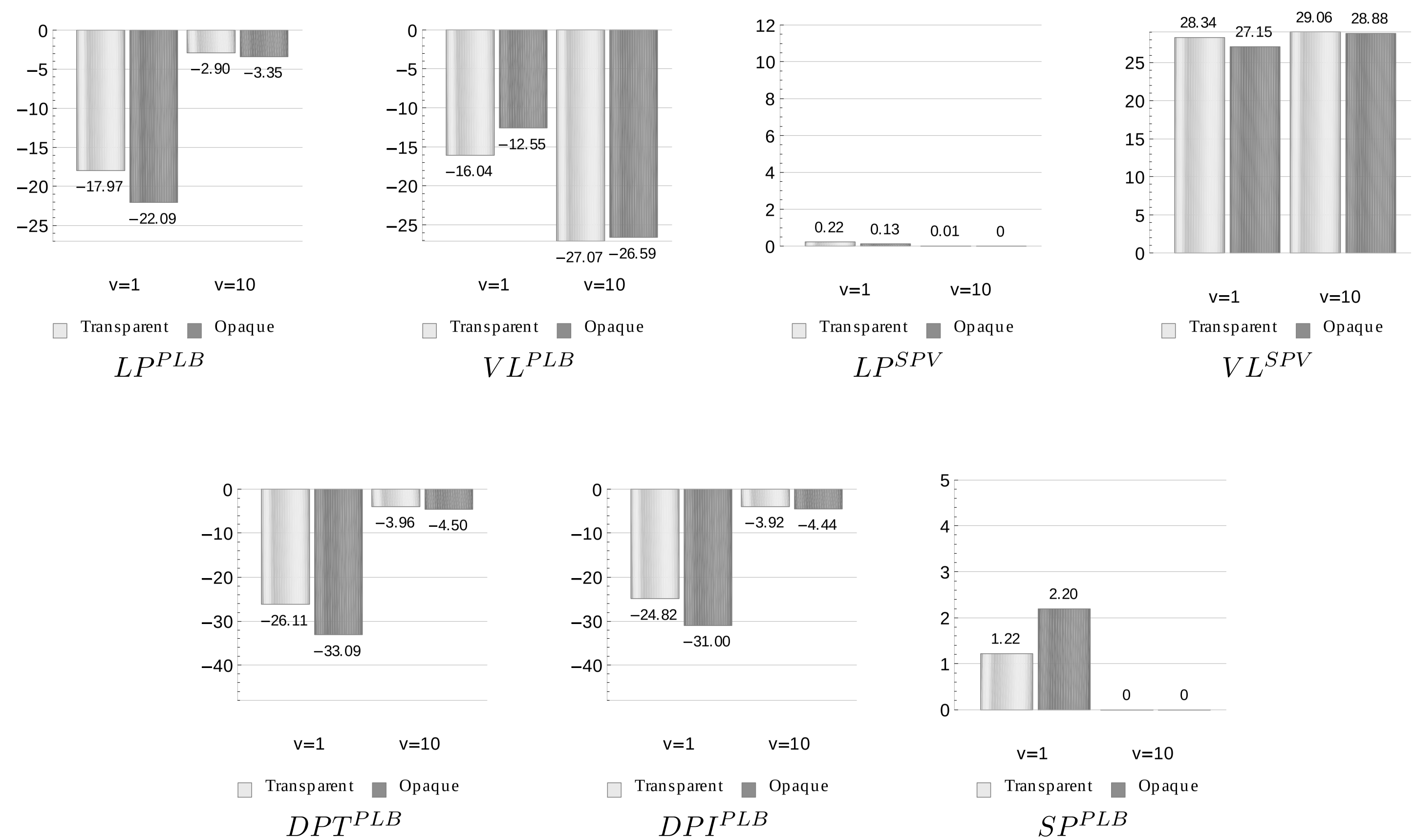


Figure 6A - Dual Market Model - Trading Strategies – Liquid stock - $\tau = 0.1$, $\alpha = 20\%$

This Figure reports - for $t = t_1$ - the equilibrium submission probabilities of limit orders posted on the ask side at different levels of the book, under the regime "Liquid stock" and for 2 different asset values, $v = \{1, 10\}$. Limit orders submission probabilities for the Public Limit Order Book (PLB) refer to the equilibrium probabilities for the benchmark single market model with tick size τ and price grid {A_1,A_2}. Equilibrium limit order submission probabilities for the Dual Market Model are indicated as (SPV) when limit orders are posted to the Sub-Penny Venue with tick size $\frac{\tau}{3}$ and price grid {a_1,a_2,a_3,a_4,a_5}, and as (PLB&SPV) when they are posted to the PLB that competes with the SPV and that has the same price grid as the PLB of the benchmark model, {A_1,A_2}. The circle indicates the price at which the order is submitted in equilibrium. Under the regime "Liquid stock" both the book of the PLB and the book of the PLB&SPV open at t_1 with one share on the first level (A_1,B_1), while the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). Results are reported also for 2 regimes of transparency of the SPV, i.e., transparent (T) or opaque (O). The arrival rate of teh broker-dealers is $\alpha = 20\%$, and the tick size is $\tau = 0.1$.

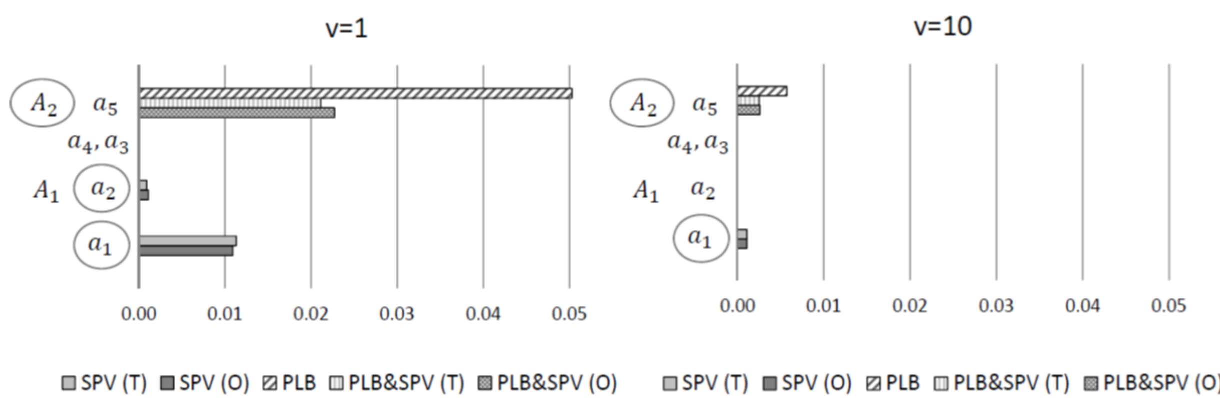


Figure 6B - Dual Market Model - Liquid stock - $\tau = 0.1$, $\alpha = 20\%$

This Figure reports statistics for order flows and market quality that result from the comparison between the benchmark Public Limit Order Book (PLB) with tick size $\tau = 0.1$, and the Dual Market Model in which a PLB with the same tick size as the benchmark model (PLB&SPV) competes with a Sub-Penny Venue (SPV) characterized by a smaller tick size, $\tau = \frac{0.1}{3}$. We refer to the regime of "Liquid stock" under which both the book of the benchmark PLB and the book of the PLB that competes with the SPV open at t_1 with one share on the first level (A_1,B_1), and the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). We present results for the arrival rate of the broker-dealers equal to $\alpha = 20\%$, and for v={1,10}. For order flows this panel reports statistics on liquidity provision and trading volume both for the PLB (LP^{PLB} and VL^{PLB}) and the SPV (LP^{SPV} and VL^{SPV}). For market quality it reports statistics on total depth (DPT^{PLB}), depth at the best bid-offer (DPI^{PLB}) and spread (SP^{PLB}). The statistics are computed as the average difference between the value of the statistic in the PLB that competes with the SPV, and the value of the same statistic in the benchmark PLB: $\Delta y = \frac{1}{k}\sum_{t=t_1}^{t_k}[y_t^{PLB\&SPV} - y_t^{PLB}]\times 100$, where y={DPI^{PLB},DPT^{PLB},SP^{PLB},VL^{PLB},LP^{PLB},VL^{SPV},LP^{SPV}}, k=*2* for {DPI^{PLB},DPT^{PLB},SP^{PLB},LP^{PLB},LP^{SPV}}, and k=*3* for {VL^{PLB},VL^{SPV}}.

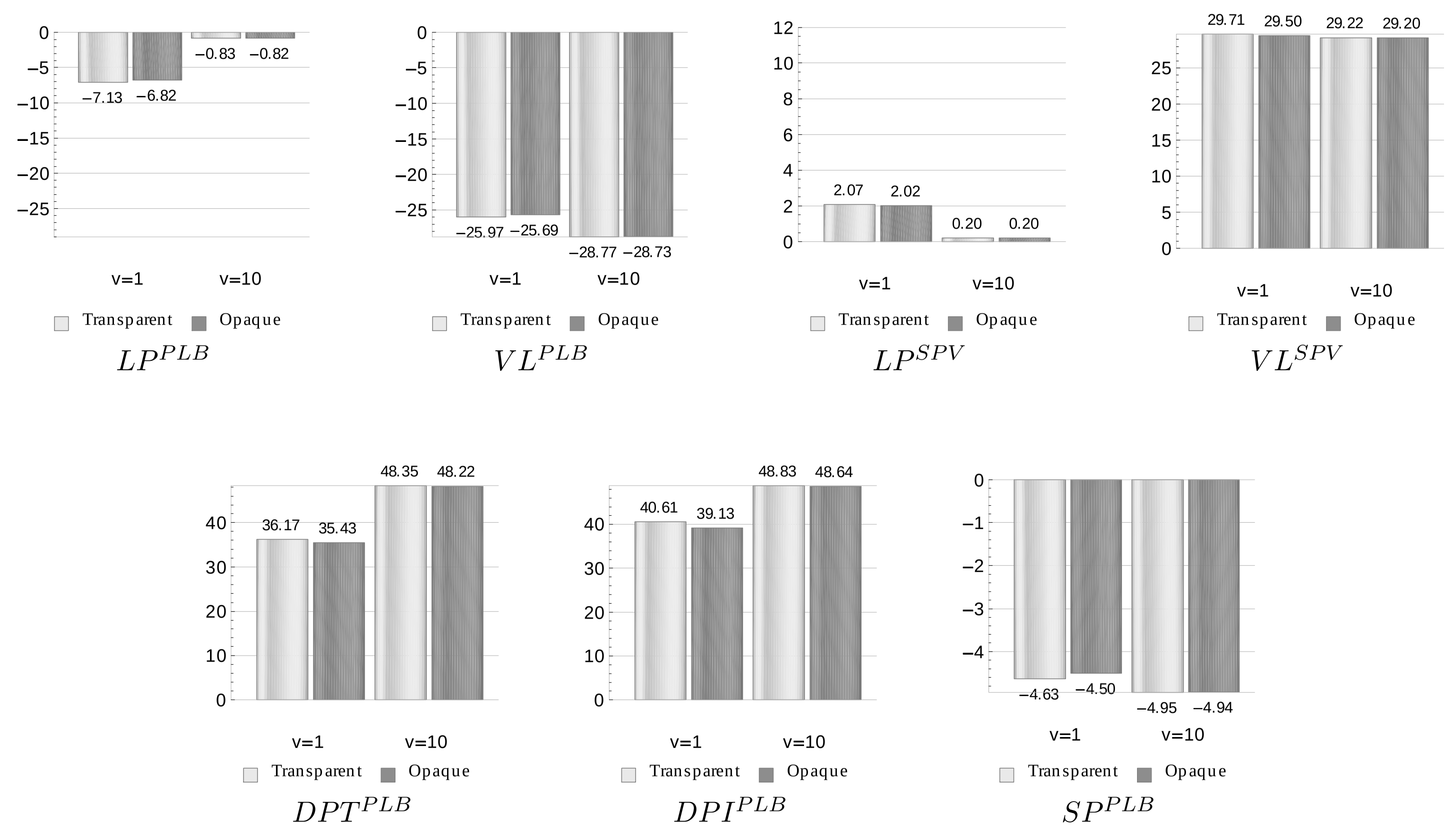


Figure 7 - Panel A - Dual Market Model - Liquid stock - $\tau = \frac{0.1}{3}$, $\alpha = 10\%$

This Figure reports statistics for order flows and market quality that result from the comparison between the benchmark Public Limit Order Book (PLB) with tick size $\tau = \frac{0.1}{3}$, and the Dual Market Model in which a PLB with the same tick size as the benchmark model (PLB&SPV) competes with a Sub-Penny Venue (SPV) characterized by a smaller tick size, $\tau = \frac{0.1}{9}$. We refer to the regime of "Liquid stock" under which both the book of the benchmark PLB and the book of the PLB that competes with the SPV open at t_1 with one share on the first level (A_1,B_1), and at the same time the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). We present results for the arrival rate of the broker-dealers equal to $\alpha = 10\%$, and for v={1,10}. For order flows this panel reports statistics on liquidity provision and trading volume both for the PLB (LP^{PLB} and VL^{PLB}) and the SPV (LP^{SPV} and VL^{SPV}). For market quality it reports statistics on total depth (DPT^{PLB}), depth at the best bid-offer (DPI^{PLB}) and spread (SP^{PLB}). The statistics are computed as the average difference between the value of the statistic in the PLB that competes with the SPV, and the value of the same statistic in the benchmark PLB: $\Delta y = \frac{1}{k} \sum_{t=t_1}^{t_k} [y_t^{PLB\&SPV} - y_t^{PLB}] \times 100$, where y={DPI^{PLB},DPT^{PLB},SP^{PLB},VL^{PLB},LP^{PLB},VL^{SPV},LP^{SPV}}, k=2 for {DPI^{PLB},DPT^{PLB},SP^{PLB},LP^{PLB},LP^{SPV}}, and k=3 for {VL^{PLB},VL^{SPV}}.

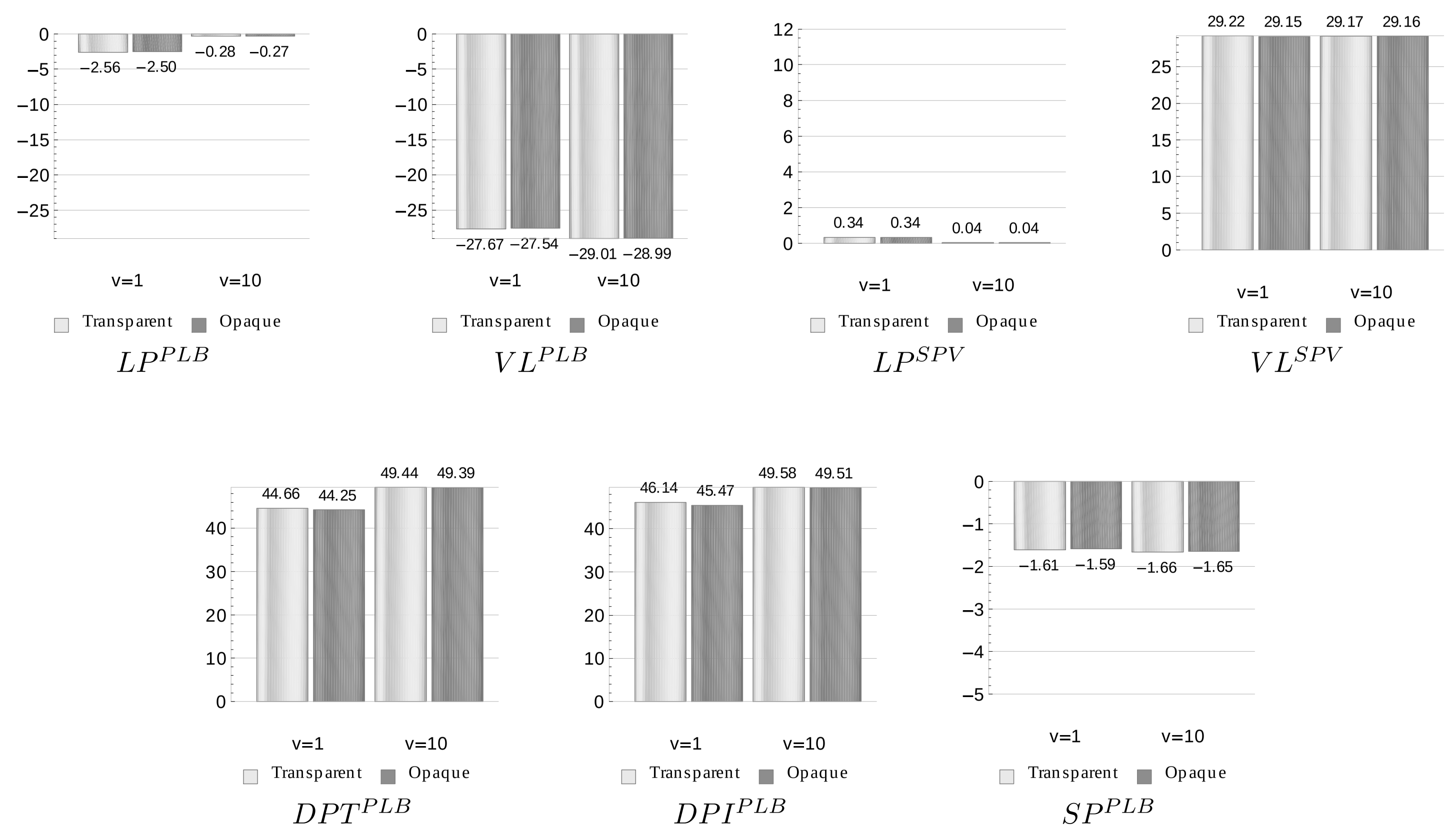
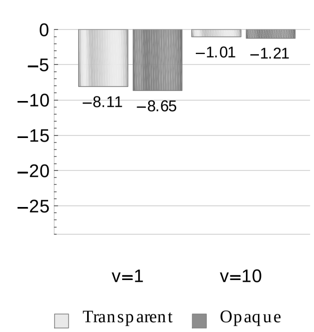
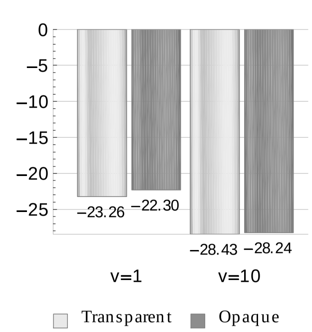


Figure 7 - Panel B - Dual Market Model - Illiquid stock - $\tau = \frac{0.1}{3}$, $\alpha = 10\%$

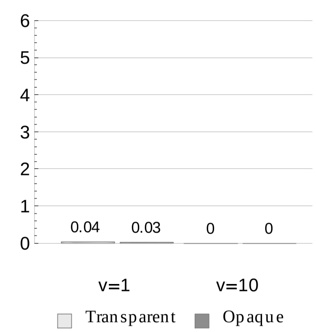
This Figure reports statistics for order flows and market quality that result from the comparison between the benchmark Public Limit Order Book (PLB) with tick size $\tau = \frac{0.1}{3}$, and the Dual Market Model in which a PLB with the same tick size as the benchmark model (PLB&SPV) competes with a Sub-Penny Venue (SPV) characterized by a smaller tick size, $\tau = \frac{0.1}{9}$. We refer to "Illiquid stock" as the regime under which both the book of the benchmark PLB and the book of the PLB that competes with the SPV open empty at t_1, and at the same time the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). We present results for the arrival rate of the broker-dealer equal to $\alpha = 10\%$ and for v={1,10}. For order flows this panel reports statistics on liquidity provision and trading volume both for the PLB (LP^{PLB} and VL^{PLB}) and the SPV (LP^{SPV} and VL^{SPV}). For market quality it reports statistics on total depth (DPT^{PLB}), depth at the best bid-offer (DPI^{PLB}) and spread (SP^{PLB}). The statistics are computed as the average difference between the value of the statistic in the PLB that competes with the SPV, and the value of the same statistic in the benchmark PLB: $\Delta y = \frac{1}{k}\sum_{t=t_1}^{t_k}[y_t^{PLB\&SPV} - y_t^{PLB}]\times 100$, where y={DPI^{PLB},DPT^{PLB},SP^{PLB},VL^{PLB},LP^{PLB},VL^{SPV},LP^{SPV}}, k=2 for {DPI^{PLB},DPT^{PLB},SP^{PLB},LP^{PLB},LP^{SPV}}, and k=3 for {VL^{PLB},VL^{SPV}}.



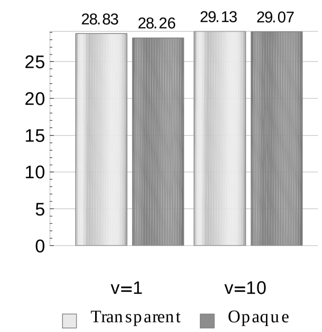
LP^{PLB}



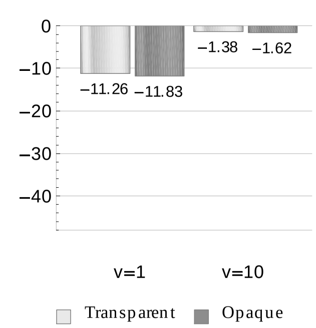
VL^{PLB}



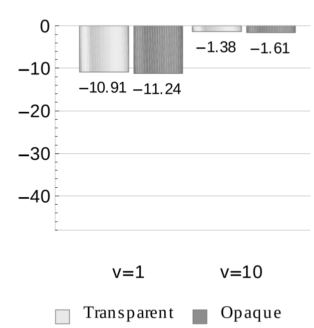
LP^{SPV}



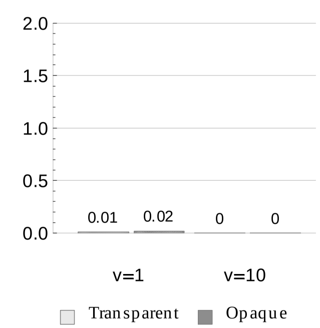
VL^{SPV}



DPT^{PLB}



DPI^{PLB}



SP^{PLB}

Figure 8 - Panel A - Dual Market Model - Competition from SPV - Liquid stock

This Figure reports statistics for order flows and market quality that result from the comparison between the Public Limit Order Book that competes with a Sub-Penny Venue (PLB&SPV) when the tick size is equal to $\tau = 0.1$, and the same PLB&SPV this time when the tick size is equal to $\tau = \frac{0.1}{3}$. We refer to "Liquid stock" as the regime under which the PLB&SPV opens at t_1 with one share on the first level (A_1,B_1), and at the same time the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). We present results for the arrival rate of the broker-dealers equal to $\alpha = 10\%$, and for v={1,10}. For order flows this panel reports statistics on liquidity provision and trading volume for the PLB&SPV ($LP^{PLB\&SPV}$ and $VL^{PLB\&SPV}$) and the SPV (LP^{SPV} and VL^{SPV}). For market quality we report statistics on total depth ($DPT^{PLB\&SPV}$), depth at the best bid-offer ($DPI^{PLB\&SPV}$) and spread ($SP^{PLB\&SPV}$). The statistics are computed as the average difference between the value of the statistic when $\tau = 0.1$, and when $\tau = \frac{0.1}{3}$: $\Delta y = \frac{1}{k} \sum_{t=t_1}^{t_k} [y_t^{PLB\&SPV|\tau=0.1} - y_t^{PLB\&SPV|\tau=\frac{0.1}{3}}] \times 100$, where y={$DPI^{PLB\&SPV}$,$DPT^{PLB\&SPV}$,$SP^{PLB\&SPV}$,$VL^{PLB\&SPV}$,$LP^{PLB\&SPV}$,LP^{SPV},VL^{SPV}}, k=2 for {$DPI^{PLB\&SPV}$,$DPT^{PLB\&SPV}$,$SP^{PLB\&SPV}$,$LP^{PLB\&SPV}$,LP^{SPV}}, and k=3 for {$VL^{PLB\&SPV}$,VL^{SPV}}.

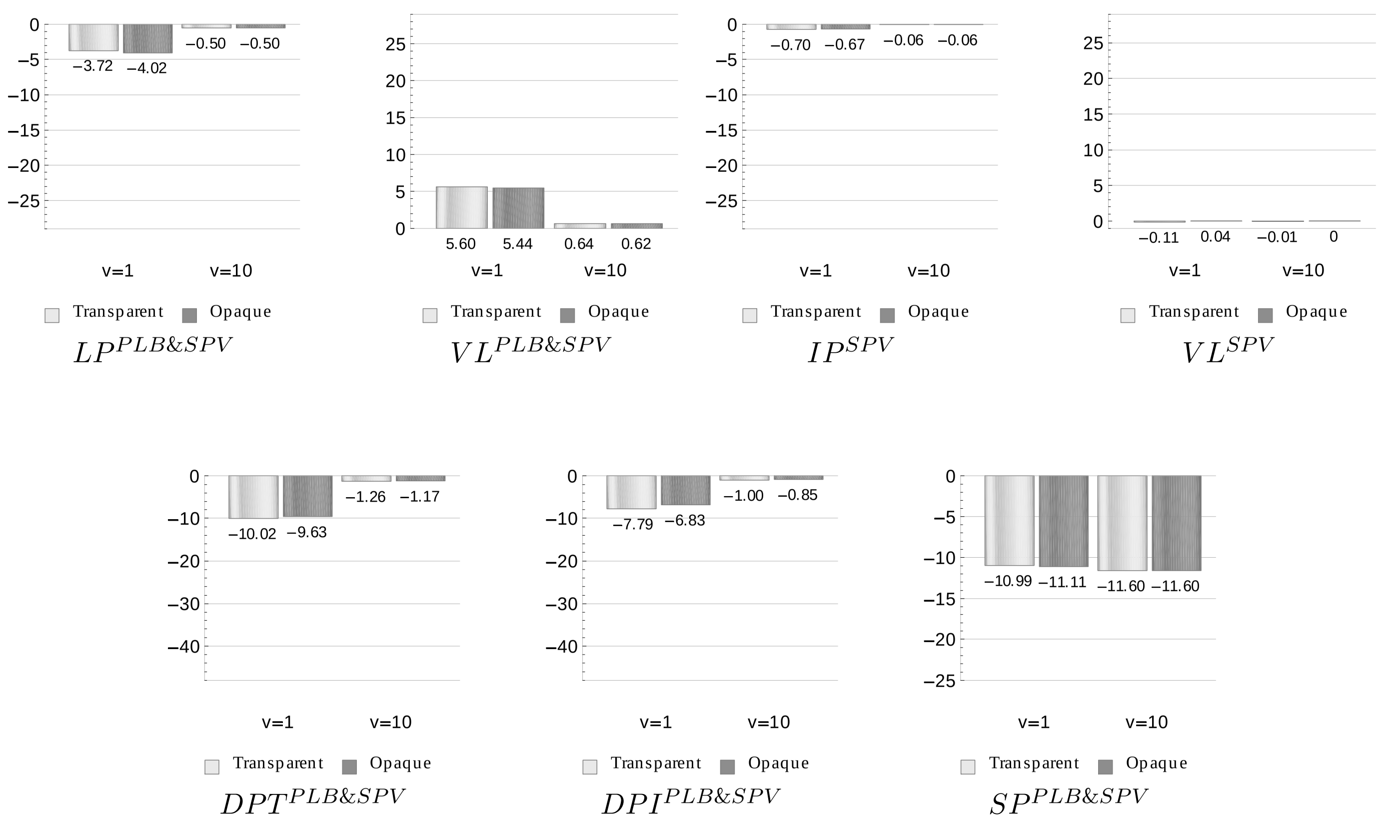


Figure 8 - Panel B - Dual Market Model - Competition from the SPV - Illiquid stock

This Figure reports statistics for order flows and market quality that result from the comparison between the Public Limit Order Book that competes with a Sub-Penny Venue (PLB&SPV) when the tick size is equal to $\tau = 0.1$, and the same PLB&SPV this time when the tick size is equal to $\tau = \frac{0.1}{3}$. We refer to "Illiquid stock" as the regime under which the PLB&SPV opens at t_1 empty, and at the same time the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). We present results for the arrival rate of the broker-dealers equal to $\alpha = 10\%$, and for v={1,10}. For order flows this panel reports statistics on liquidity provision and trading volume for the PLB&SPV ($LP^{PLB\&SPV}$ and $VL^{PLB\&SPV}$) and the SPV (LP^{SPV} and VL^{SPV}). For market quality we report statistics on total depth ($DPT^{PLB\&SPV}$), depth at the best bid-offer ($DPI^{PLB\&SPV}$) and spread ($SP^{PLB\&SPV}$). The statistics are computed as the average difference between the value of the statistic when $\tau = 0.1$, and when $\tau = \frac{0.1}{3}$: $\Delta y = \frac{1}{k}\sum_{t=t_1}^{t_k}[y_t^{PLB\&SPV|\tau=0.1} - y_t^{PLB\&SPV|\tau=\frac{0.1}{3}}]\times 100$, where $y=\{DPI^{PLB\&SPV}, DPT^{PLB\&SPV}, SP^{PLB\&SPV}, VL^{PLB\&SPV}, LP^{PLB\&SPV}, LP^{SPV}, VL^{SPV}\}$, $k=2$ for $\{DPI^{PLB\&SPV}, DPT^{PLB\&SPV}, SP^{PLB\&SPV}, LP^{PLB\&SPV}, LP^{SPV}\}$, and $k=3$ for $\{VL^{PLB\&SPV}, VL^{SPV}\}$.

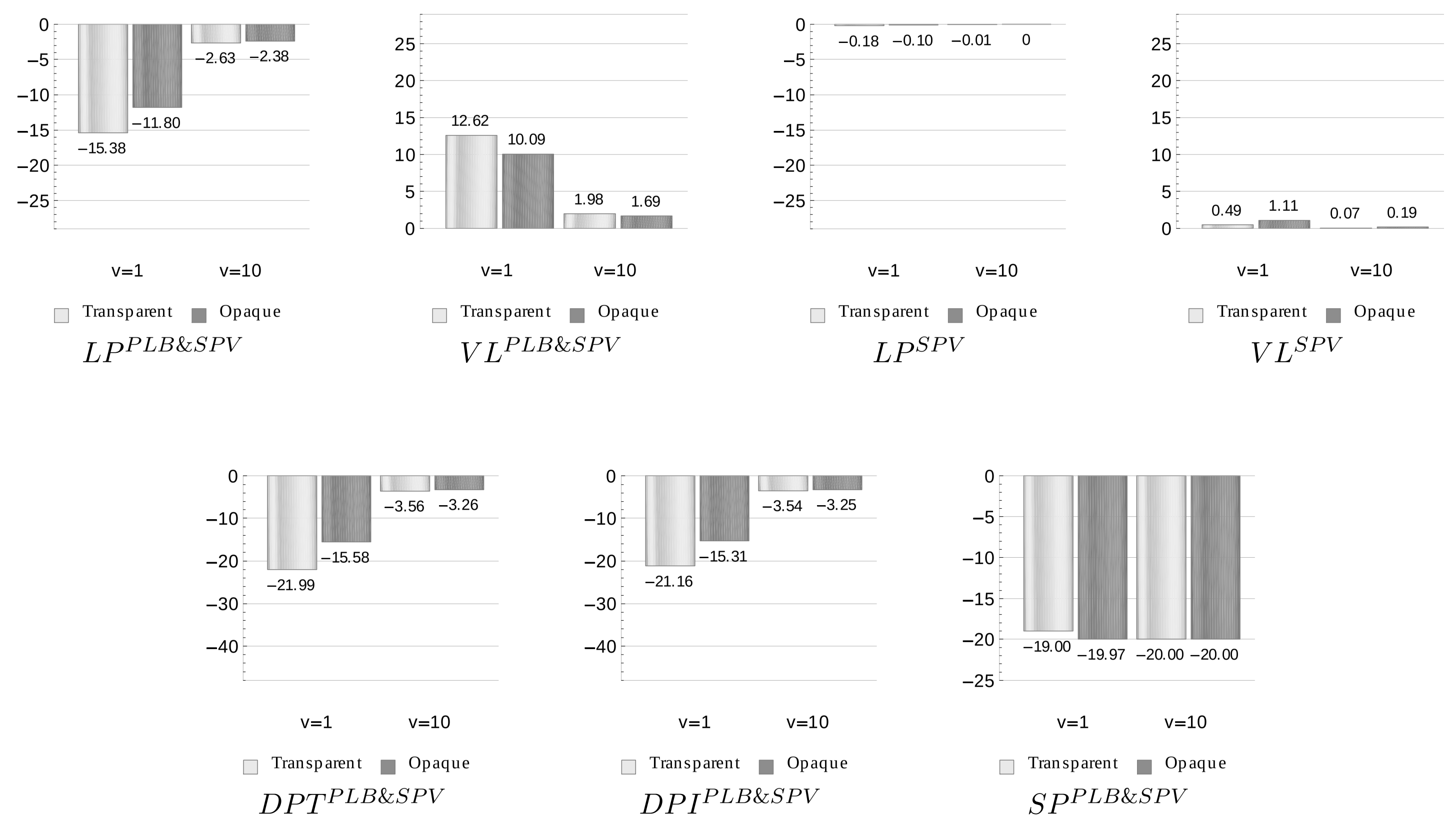


Figure 9 - Single Market Model - Welfare - $\tau = 0.1$

This Figure reports the average percentage difference in the gains from trade (W) over the three trading periods between the large tick market (LM), $\tau = 0.1$, and the small tick market (SM), $\tau = \frac{0.1}{3}$, computed as: $\Delta W_{SM-LM} = \frac{1}{3}\sum_{t=t_1}^{t_3}[E(W_t^{SM} - W_t^{LM})/E(W_t^{LM})] \times 100$. We consider two regimes "Liquid stock" and "Illiquid stock", and four different asset values, $v = \{1, 5, 10, 50\}$. Under the regime "Liquid stock" the book of the LM opens with one share on the first level (A_1, B_1), and the book of the SM opens with one share on the second level (a_2, b_2). Under the regime "Illiquid stock" both the book of the LM and that of the SM open empty at t_1.

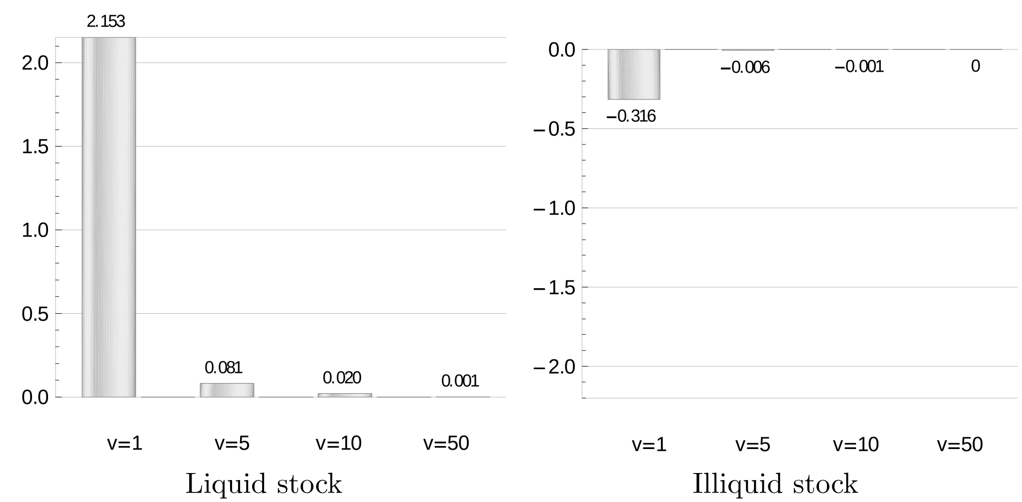
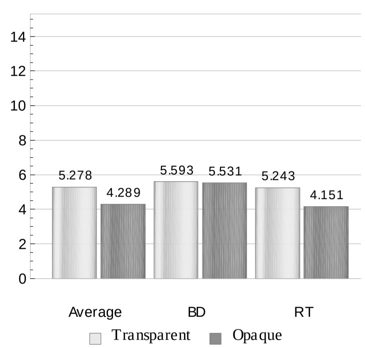


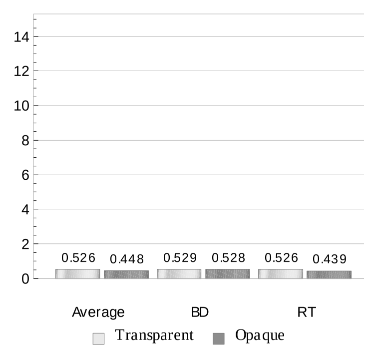
Figure 10 - Welfare - Dual Market Model - $\tau = 0.1$, $\alpha = 10\%$

This Figure reports the average percentage difference in the gains from trade (W) over the three trading periods between the model in which the PLB competes with the Sub-Penny Venue (PLB&SPV) and the single market model (PLB), when the arrival rate of the broker-dealers is $\alpha = 10\%$, and the tick size is $\tau = 0.1$. The difference is computed as: $\Delta W_{PLB\&SPV-PLB} = \frac{1}{3}\sum_{t=t_1}^{t_3}[E(W_t^{PLB\&SPV} - W_t^{PLB})/E(W_t^{PLB})] \times 100$. We consider two regimes, "Liquid stock" (Panel A) and "Illiquid stock" (Panel B), and two different asset values, v={1,10}. Under the regime of "Liquid stock", both the book of the PLB and the book of the PLB&SPV open at t_1 with one share on the first level (A_1,B_1). Uder the regime of "Illiquid stock", both the book of the PLB and the book of the PLB&SPV open empty. In both cases the SPV opens with equal probability either empty or with one share on the first level (a_1,b_1). Welfare in the PLB&SPV framework is computed for the two categories of agents - broker-dealers (BDs) and regular traders (RTs) -, and under the two regimes of transparency and opaqueness of the SPV.

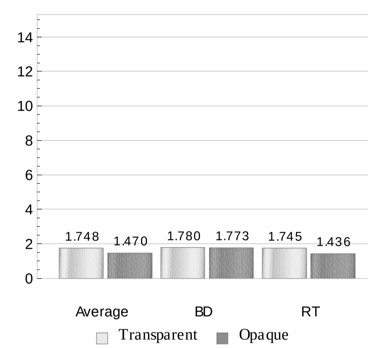
Panel A : Liquid stock



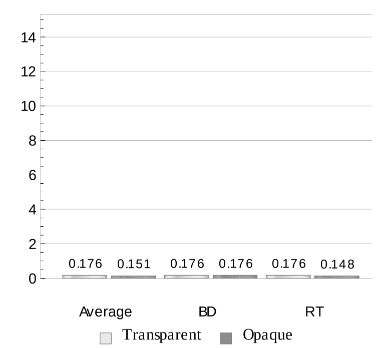
$v = 1,\ \tau = 0.1$



$v = 10,\ \tau = 0.1$

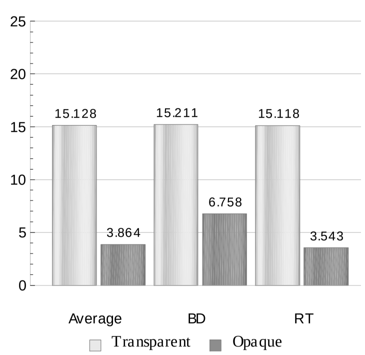


$v = 1,\ \tau = \frac{0.1}{3}$

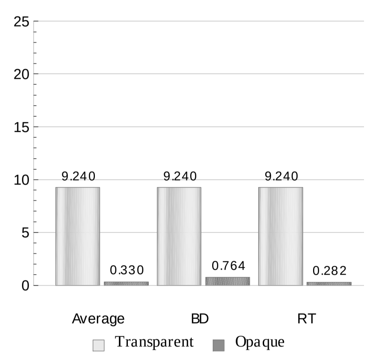


$v = 10,\ \tau = \frac{0.1}{3}$

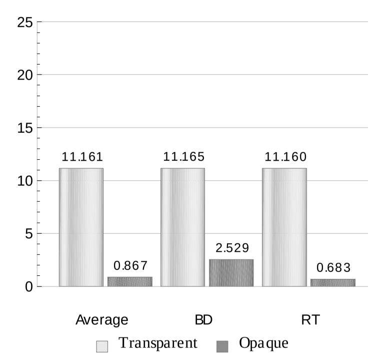
Panel B : Illiquid stock



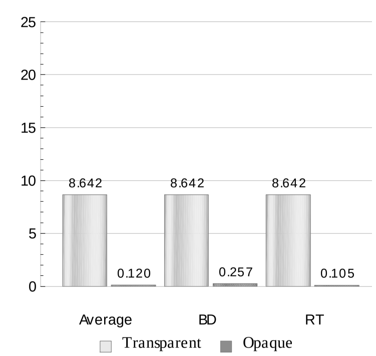
$v = 1,\ \tau = 0.1$



$v = 10,\ \tau = 0.1$



$v = 1,\ \tau = \frac{0.1}{3}$



$v = 10,\ \tau = \frac{0.1}{3}$