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Taxing Carbon under Market Incompleteness

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Taxing Carbon under Market Incompleteness

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Abstract

This paper is the first attempt, to the best of our knowledge, to study the impact of a carbon tax by means of a heterogeneous agents DSGE model. The objectives of the paper are two: *i*) To assess how the results of a representative agent model compare to those coming from a model accounting for heterogeneity across agents when evaluating aggregate economic and environmental impacts of a carbon tax; *ii*) To assess the distributional implications of a carbon tax and how they can be mitigated through different recycling schemes. We find that heterogeneous agents models deliver different results from those based on the representative firm paradigm, the main tool used to guide policy making so far. In particular, we find evidence of a relatively sizable double dividend for several recycling schemes and carbon taxes as high as 50% of the energy price. In addition, we find the potential for redistributive channels related to carbon policies that can only be appreciated applying this type of modeling framework.

JEL codes: Q58, Q54, E2.

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1 Introduction

As the debate over the occurrence of climate change and its anthropogenic causes are finally settled (IPCC, 2013) the reality of a global convergence towards some form of regulation of carbon emissions seems unavoidable, although the timing and the form of this process are still open questions. Indeed, the slow pace of economic recovery in most developed countries is relegating climate change concerns to bottom positions of the political agenda. The current attempt of advocates and scholars is that of resuscitating the idea of carbon emissions mitigation by incorporating it in the discussion over public debt. A recent IMF (2012) report on *Fiscal Policy to Mitigate Climate Change* shows how the idea of using environmental taxation to contrast the current budget deficits is gaining momentum. This is not a new idea. It has long been argued that the imposition of environmental taxation may not necessarily imply welfare losses for the economy, even when environmental implications on welfare are not included in the analysis. Indeed, under specific circumstances, environmental taxation might also lead to a less distorted tax system, therefore partly or entirely compensating its costs. Sandmo (1975) suggested for the first time that revenues from Pigouvian taxation might be used to mitigate other distortionary taxes, thus reducing the cost of the environmental policy or even improving the non environmental welfare with respect to the no policy case.[1] Increasing government revenues might be a favorable argument to kick start the debate over climate change policy, but this does not provide definitive indication on which market based policy instrument should be adopted: Both a carbon tax and a cap and trade system, when permits are auctioned, imply revenues for the government. Similarly, how revenues should be allocated remains an open issue. Goulder et al. (2010) find that employing revenues from the auctioning of permits to finance cuts in marginal income tax rates might decrease the climate policy cost to about 33 percent than in the case where all allowances are freely allocated.

Politically, the debate on whether a price or a quantity based instrument should be adopted seemed definitely over with the adoption of an emission trading scheme by the European Commission and as the prescribed tool within the *Kyoto Protocol* framework. Nonetheless, the (rather lethargic) political discussion in the US leans more towards the idea of a carbon tax[2] and Australia has introduced a carbon tax in 2012.[3] If climate change talks are feeble, the debate over distribution and inequality is more than ever raging. It is therefore extremely important to study and carefully assess distributional implications of any climate policy being considered. For this reason we set out to study

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[1] See Bovenberg and Goulder (1996) for a detailed discussion on the counteracting effects of tax interaction and revenue recycling.

[2] See the SNL entry `http://www2.snl.com/Interactivex/article.aspx?CdId=A-25341985-12585`

[3] Although this is just a transitory phase towards the introduction of a cap-and-trade system to become effective in 2014-2015.

the macro-implications of climate policy by means of a modeling tool that allows to assess climate policy implications for heterogeneous actors.

There exists a large number of concerns related to the introduction of measures aiming at carbon mitigation. First and foremost, the potential impact climate policy might exert on businesses/sectors/countries that adopt climate regulation in the face of international competition. But also, the interaction with macro economic shocks and cycles and the potential distributional impacts across sectors, households and firms. Several studies have concentrated on the first concern, namely the impact on international competitiveness of firms located in countries that have adopted a unilateral climate policy, see for example the sectoral impact analysis in Alexeeva-Talebi et al. (2012) and in Aldy and Pizer (2011). A second, more recent strand of literature, has worked within the DSGE framework in order to assess the impact of pricing emissions in the presence of macroeconomic shocks. Specific attention has been devoted to the study of the optimal level of climate policy and to the performance of alternative economic instruments (carbon taxes versus quantity instruments with different allocation rules), by using Real Business Cycle models, both in a single sector (Fischer and Springborn, 2011, and Heutel, 2012) and in a multi-sector framework Dissou and Karnizova (2012). Heutel (2012) finds that optimal emissions are pro-cyclical and that the optimal emission policy should respond accordingly to economic fluctuations and cycles. Different policy tools are evaluated in Fischer and Springborn (2011), where the authors find that a cap system would achieve a given emission reduction with a slightly higher welfare cost than the tax, but it would ensure that the cut is achieved without lag, resulting in higher welfare if these additional reductions are valued; the cap system also features a lower level of labor variance than all other policies considered. These studies are based on a representative-agent framework, hence they are not designed to study distributional implications of climate policies. In addition they cannot explore the implications of taxes or tradeable permits when markets are incomplete.

A body of literature has looked into distributional implications of environmental policies. Parry (2004) looks into the regressivity of alternative allocations of permits and finds that a system of grandfathered emission permits can be highly regressive when compared to other instruments, as for example an environmental tax. He also finds that distributional effects are less of a concern for more stringent policies. Other papers look specifically into the distributional implications of a carbon tax Metcalf (2009), Rausch et al. (2011), and Fullerton and Monti (2013), among others. Rausch et al. (2011) in particular is nearest to our approach. Existing studies have typically used data on energy expenditure or household income groups in order to estimate the cost incidence of policies inducing energy cost increases, without looking to other general equilibrium effects. Similarly to our paper, Rausch et al. (2011) find that the income effect on distribution is progressive and completely offsets the regressive effects seen from just focusing on energy expenditure patterns.

In the present paper we employ a standard incomplete markets model with heterogeneous agents[4] - in the spirit of Huggett (1993) and Aiyagari (1994) - and we contrast the results obtained with a representative agent model with those derived through the inclusion of various idiosyncratic characteristics of agents. This allows us to address a set of crucial questions.

First, whether the introduction of heterogeneity across agents - under different forms - matters *per se* when analyzing the impact of a carbon tax on some aggregate indicator, as for example total emissions reductions, costs of a policy or ranking among alternative policy instruments. In Krusell and Smith (2007) it is indeed argued that in many of those instances when the interest is placed on aggregate variables, then a representative model can perform well enough. By means of our model we can test whether this is the case in the context of a carbon policy or, instead, some crucial features get lost when using the representative agent model typically employed in the literature. Second, along the lines of Fischer and Springborn (2011) and Heutel (2012), we contrast different instruments and schemes looking at their macro economic implications, but in our framework we are now able to study also the implications for the full distribution of agents. As our model can easily accommodate quantity based instruments and fully mimic emission trading among heterogeneous agents, in the final part of the paper we present this extension along with implications deriving from different assumptions on allocations. Finally, we dive deeper in the distributional dimension of these alternative climate policy instruments, investigating how revenues recycling schemes could alleviate unequal or distortionary impacts. Obviously, without a model where heterogeneity across agents is not spelled out this exercise would simply be impossible. By means of our model we are able to assess the level of regressivity of emission taxes and to test the double dividend hypothesis discussed in Bovenberg and Goulder (1996) and Bovenberg and van der Ploeg (1994). It is important to notice that in our analysis we abstract deliberately from any environmental benefit associated to climate policies, thus to the issue of optimally pricing carbon. Our assumption is that the environmental target will be set as a a result of a political negotiation process between science, various lobbies and actors affected differently from climate change. Our objective instead is to propose a tool to assess the distribution of the costs associated with such target.

We find that being able to portray heterogeneous endowments of capital and productivity does indeed make a difference in the aggregate welfare effects of pricing carbon, in many cases changing the sign of the aggregate non-environmental welfare implications of

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[4] In principle, heterogeneity among agents can be studied also in a complete markets framework: see for instance Chatterjee (1994). However, Carroll and Young (2009) show that the standard setting has some implausible theoretical implications, in particular in the presence of progressive taxation. Furthermore, the existing empirical evidence does not support the complete markets hypothesis (see Guvenen, 2011, Section 2, for a detailed discussion), and clearly identifies idiosincratic shocks to individual income (see Krueger et al., 2010, for a recent survey).

the policy. For instance, in our benchmark model, imposing a carbon tax in the $10\%-50\%$ range when no other climate policies are in place has a positive effect on social welfare - measured in consumption equivalent variation terms - that ranges from 0.24% to 0.49% of lifetime consumption.

In the next section of the paper we lay out the model structure, while in section three we discuss the calibration methodology. The fourth section describes results, while the last section concludes the paper by discussing future research developments.

2 The model

Time is discrete, indexed by $t \in \{0, 1, ..., \infty\}$. There exists a continuum of *ex-ante* identical and infinitely lived households, with total mass equal to one. Firms, directly owned by the households, produce a single homogeneous final good competitively, via a constant-returns-to-scale production function, using capital, labor and energy. Households own two factors of production, capital and labor, while energy is acquired on the international market. The final good can be used for consumption and investment, and is assumed to be the *numéraire*. Asset markets are incomplete: households are allowed to invest in physical capital only, and we assume that capital holdings cannot be negative. Hence, households cannot fully insure themselves against idiosyncratic shocks to their income. The next subsections will describe the model components more in detail. The recursive equilibrium is formally defined in the Appendix.

2.1 Households

2.1.1 Capital income

As in Angeletos (2007) and Covas (2006), each household owns a single private firm. Firms employ labor and purchase energy in competitive markets but use the capital stock accumulated by the respective owner. There is no market for physical capital, so households can invest capital only in the firm they own. Let us denote e_{it} the amount of energy used: we assume that emissions at the firm level are proportional to the use of e_{it} and units of emissions are chosen such that the quantity of emissions is equal to e_{it}. The individual firm's output is described by a constant elasticity function of energy and the Cobb Douglas composite of capital and labor:

$$q_{it} \equiv \kappa\phi_{it} \left[(1-\gamma_{it}) \left(k_{it}^{\alpha} n_{it}^{1-\alpha} \right)^{\frac{\omega-1}{\omega}} + \gamma_{it} e_{it}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{1}$$

where: k_{it} denotes the stock of capital in place at the beginning of period t, n_{it} the amount of labor hired, κ the (time-invariant) Total Factor Productivity, ϕ_{it} an idiosyncratic shock to plant-level productivity, $\gamma_{it} \in (0, 1)$ the (possibly household-specific) share of energy in gross output, $\alpha \in (0,1)$ the share of capital in the "Cobb-Douglas" composite of capital and labor, and $\omega > 0$ the elasticity of substitution between the "Cobb-Douglas" composite and energy. The idiosyncratic shock is realized at the beginning of period t, after capital is installed but before labor n_{it} and e_{it} are chosen. The log of productivity follows a stationary discrete Markov process, characterized by a transition matrix π_ϕ, which evolves independently across households; we assume that $\mathbb{E}(\phi) = 1$.[5]

The capital income of a generic household i, excluding the non-depreciated capital stock installed in the firm, is given by the firm's earnings net of factor costs:

$$\pi_{it} = q_{it} - (1+\tau_N)\, w_t n_{it} - (1+\tau_E)\, p_t e_{it} - z_t \left[e_{it} - \bar{\mathrm{e}}(k_{it})\right]$$

w_t is the wage rate, τ_N is the payroll tax, p_t the price of energy. When a price mechanism to curb carbon emissions is in place, τ_E represents the carbon tax. When a quantity instrument is in place, at the beginning of each period households obtain from the government an endowment of emissions permits denoted $\bar{\mathrm{e}}(k_{it})$ that possibly depends on the stock of installed capital, i.e. on the current size of the firm; emissions permits can be traded at a price z_t on a competitive secondary market.[6].

Since n_{it} and e_{it} affect only π_{it} in period t, and since they are chosen after k_{it} has been installed, $\bar{\mathrm{e}}_{it}$ has been obtained, and ϕ_{it} has been observed, the optimal n_{it} and e_{it} maximize π_{it} state by state. In other words, the firms solve the following maximization problem in each period:

$$\max_{\{n_{it},e_{it}\}} \; q_{it} - (1+\tau_N)\, w_t n_{it} - \left[(1+\tau_E)\, p_t + z_t\right] e_{it} + z_t \bar{\mathrm{e}}(k_{it}), \tag{2}$$

The individual factor demands and the firm's earnings are linear in k_{it}, because of constant returns to scale:

$$n_{it} = \mathfrak{n}_{it} k_{it}, \tag{3}$$

$$e_{it} = \mathfrak{e}_{it} k_{it}, \tag{4}$$

$$\pi_{it} = \mathfrak{r}_{it} k_{it} + z_t \bar{\mathrm{e}}(k_{it}), \tag{5}$$

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[5] This is an analytically (very) convenient way of introducing idiosyncratic capital income risk (entrepreneurial risk) in out setting. A second source of idiosincratic risk is highly desirable because the standard framework, where idiosincratic shocks affect only labor income, is unable to replicate neither the high level of inequality nor the fat tails that empirical distributions of wealth tend to display. Furthermore, it allows us to introduce idiosincratic firm-level volatility in the demand for energy. There is however a drawback: being the capital stock firm-specific in the short-run, adjustments in the use of capital services at the firm level are much slower than in more standard settings, where capital services are freely mobile across firms.

[6] If $\bar{\mathrm{e}}$ were sufficiently generous, no household would ever need to buy additional permits on the secondary market, and this would drive their equilibrium price to zero.

where:

$$\mathfrak{n}_{it} \equiv \Xi_{it}^{-\frac{1}{\alpha}} \left(\frac{\Lambda_{it}^{\omega-1}-\gamma_{it}}{1-\gamma_{it}}\right)^{\frac{1}{(1-\omega)\alpha}}, \tag{6}$$

$$\mathfrak{e}_{it} \equiv \Xi_{it}^{\omega}\mathfrak{n}_{it}^{1+(\omega-1)\alpha}, \tag{7}$$

$$\mathfrak{r}_{it} \equiv \alpha\left(1-\gamma_{it}\right)\kappa\phi_{it}\left[\left(1-\gamma_{it}\right)\mathfrak{n}_{it}^{(1-\alpha)\frac{\omega-1}{\omega}} + \gamma_{it}\mathfrak{e}_{it}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{8}$$

and:[7]

$$\Xi_{it} \equiv \frac{\left(1+\tau_N\right)w_t}{\left(1-\alpha\right)\left(1-\gamma_{it}\right)\kappa\phi_{it}\Lambda_{it}}, \tag{9}$$

$$\Lambda_{it} \equiv \frac{\left(1+\tau_E\right)p_t + z_t}{\kappa\phi_{it}\gamma_{it}}. \tag{10}$$

2.1.2 Labor income

At the beginning of each period, households receive a fixed time endowment, normalized to unity, whose productivity on the labor market is affected by an exogenous and idiosyncratic shock, denoted ε_{it}; this shock is modeled as a finite-state Markov process, characterized by a transition matrix π_ε, which evolves independently across households. We furthermore assume that $\mathbb{E}\left(\varepsilon\right) = 1$. After the realization of labor productivity, the household optimally allocates its time endowment between labor and leisure; for the sake of simplicity, we assume that the labor productivity shocks does not directly affect the utility function.

2.1.3 The optimization problem

Households' preferences over stochastic consumption and leisure streams are given by:

$$u_{it} \equiv \mathbb{E}_t\left[\sum_{j=t}^{\infty}\beta^{j-t}\left(\frac{c_{ij}^{1-\mu}-1}{1-\mu} - \xi\frac{l_{ij}^{1+\eta}}{1+\eta}\right)\right], \tag{11}$$

where c_{it} is the consumption level, $l_{it} \in [0,1]$ the share of time devoted to labor, $\beta \in (0,1)$ the intertemporal discount factor, $\mu > 0$ the reciprocal of the elasticity of intertemporal substitution, and $\eta > 0$ a parameter equal to the inverse of the Frisch elasticity of labor supply.

The stock of physical capital evolves over time according to the following accumulation equation:

$$k_{it+1} = \left(1-\delta\right)k_{it} + d_{it} - c_{it}, \tag{12}$$

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[7] By taking logs of (3) and using l'Hôpital's rule it can be shown that, in the "Cobb-Douglas" case, i.e. when $\omega = 1$, the expressions reduce to $\mathfrak{n}_{it} \equiv \Xi_{it}^{\frac{\gamma_{it}}{\alpha(1-\gamma_{it})}}\left[\frac{(1-\alpha)(1-\gamma_{it})\kappa\phi_{it}}{(1+\tau_N)w_t}\right]^{\frac{1}{\alpha(1-\gamma_{it})}}$, $\mathfrak{e}_{it} \equiv \Xi_{it}\mathfrak{n}_{it}$, and $\mathfrak{r}_{it} \equiv \alpha\left(1-\gamma_{it}\right)\kappa\phi_{it}\mathfrak{n}_{it}^{(1-\alpha)(1-\gamma_{it})}\mathfrak{e}_{it}^{\gamma_{it}}$.

where:

$$d_{it} \equiv \pi_{it} + w_t \varepsilon_{it} l_{it} - \mathcal{T}\left(\pi_{it} + w_t \varepsilon_{it} l_{it}\right) + G_t,$$

denotes household's disposable income, $\mathcal{T}(\cdot)$ the (possibly non-linear) tax function, $G_t \geq 0$ the per-capita government lump-sum transfers, and $\delta_K \in [0,1]$ a physical depreciation rate. As already mentioned, households also face a borrowing constraint: $k_{it+1} \geq 0$.

We can now put all the elements together; for given sequences of factor prices and transfers, the dynamic optimization problem of a generic household is as follows:

$$\max_{\{c_{ij}, l_{ij}, k_{ij+1}\}_{j=t}^{\infty}} \mathbb{E}_t \left\{ \sum_{j=t}^{\infty} \beta^{j-t} \left[\frac{c_{it}^{1-\mu} - 1}{1-\mu} - \xi \frac{l_{it}^{1+\eta}}{1+\eta} \right] \right\}, \tag{13}$$

$$\begin{aligned} \text{s.t.} \quad & k_{it+1} = (1-\delta) k_{it} + d_{it} - c_{it}, \\ & l_{it} \in [0,1], \\ & k_{it+1} \geq 0. \end{aligned} \tag{14}$$

The first order conditions can be combined to obtain the following inequalities:[8]

$$\xi l_{it}^{\eta} \leq c_{it}^{-\mu} \left(1 - \mathcal{T}_{y_{i,t}}\right) w_t \varepsilon_{it}, \tag{15}$$

$$c_{it}^{-\mu} \geq \beta \mathbb{E}_t \left\{ c_{i,t+1}^{-\mu} \left[1 - \delta + \left(1 - \mathcal{T}_{y_{i,t+1}}\right) \left(\mathfrak{r}_{i,t+1} + z_{t+1} \bar{\mathrm{e}}_{k_{i,t+1}} \right) \right] \right\}. \tag{16}$$

2.2 Aggregate variables

We concentrate our analysis on oil-using countries. As in Hassler and Krusell (2012), energy is imported from abroad, at a given international price $p_t = \bar{p}$, and its supply is perfectly elastic.[9] In other words, our economy can be characterized as a small open economy in the international market for energy; however, it should be remembered that households do not have access to international financial markets, and can only invest in physical capital. This implies that trade is balanced by assumption: energy imports are financed via final good exports. Note furthermore that the carbon tax acts effectively as a sales tax on energy imports. The absence of trade is certainly a limitation of our modelling framework: emissions embedded in imports are roughly 14% of total annual CO2 emissions for the group of OECD countries, as discussed in IPCC (2014). However, as we are not looking into potential leakage effects but are interested in distrubutional implication, we believe this is only a second order effect that can be abstracted form in this first version of the analysis.

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[8] In equation (15) we are anticipating an equilibrium outcome: given our utility function, it turns out that in equilibrium l_{it} remains always strictly positive, but not necessarily strictly below one. Hence, the marginal benefit in terms of utility of an additional hour of work can be greater (not lower) than the additional cost.

[9] Modeling energy as an imported good is a time-honored strategy in the literature: see Nordhaus (1977) among others.

At the beginning of each period, the government issues a total amount of emissions permits equal to $\bar{\mathrm{M}}_t$; an amount equal to $\bar{\mathrm{E}}_t \leq \bar{\mathrm{M}}_t$ is immediately distributed to the households for free, while the remaining permits are supplied to the secondary market by the government itself. Apart from this, the government plays a minimalist role, collecting tax revenues, selling permits, and paying everything back to the households via lump-sum sum transfers (capital letters denote aggregate variables):

$$G_t = T_t + \tau_N w_t N_t + \tau_E \bar{p} E_t + z_t \left(\bar{\mathrm{M}}_t - \bar{\mathrm{E}}_t \right). \tag{17}$$

3 Calibration

The parameters that characterize households' preferences are selected in the following way: the intertemporal discount factor and the reciprocal of the elasticity of intertemporal substitution are set to standard values in the literature, $\beta = 0.96$ and $\mu = 2$. Reichling and Whalen (2012) report that the Congressional Budget Office incorporates into its analysis an estimate of the Frisch elasticity of labor supply that ranges from 0.27 to 0.53: we set $\eta = 1.9$ in order to make the model reproduce a Frish elasticity equal to 0.53, and calibrate ξ so that the average hours worked are 40% of the time endowment. The depreciation rate is set to $\delta = 0.048$, while the share of capital in value added, α, is assumed to be 0.33. Finally, we normalize κ to one.

In our benchmark parametrization, we assume that the parameter governing the share of energy in gross output, γ_i, is a time-invariant household characteristic: more precisely, we divide the overall population into four technology types, characterized by their factor intensity and mass. Using sectoral data for the U.S. over the 1970-2005 period taken from the *EU-KLEMS* data-set, described in O'Mahony and Timmer (2009), we construct the long-run shares of energy in gross output for four aggregate sectors, namely *Agriculture*, *Services*, *High Energy-intensive Industrial Sectors*, and *Low Energy-intensive industrial Sectors*: the corresponding energy shares are 6%, 2%, 22%, and 4%, respectively. The shares of those sectors in total gross output are equal to 2%, 60%, 25%, and 13%, respectively. The values of γ_i and the mass of each type, denoted $\bar{\lambda}_i$, together with the price $\bar{p}$, are calibrated in order to reproduce in steady state the overall share of energy in gross output, equal to 7.3%, the reported sectoral energy shares, and the sectoral shares in gross output.

van der Werf (2008) reviews the literature on quantitative climate policy analysis and reports that the majority of models[10] that adopt the 2-level nesting structure implicit in our production function, described in (1), assume a unitary elasticity of substitution between capital and labor, and an elasticity of substitution between the capital-labor composite and energy in the $0.4 - 0.5$ range. Furthermore, he estimates the parameters

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[10] See Bosetti et al. (2006) for a recent example.

of different 2-level CES production functions using industry-level data from 12 OECD countries, and finds that the nesting structure where capital and labor are combined first fits the data best, and that the estimates of the elasticity of substitution between the capital-labor composite and energy are found in the range $0.17 - 0.65$, both at the country and industry level. More recently, Baccianti (2013) confirms that the elasticity of substitution between capital and labor is approximately one, but obtains a wider range of variation for estimates of the elasticity of substitution between the capital-labor composite and energy, equal to $0.00 - 0.82$, with a mean estimate of 0.25. Given this empirical evidence, and for comparability with the existing literature, we assume $\omega = 0.5$ in our benchmark parametrization, and perform some sensitivity analysis in the following.

The payroll tax, τ_N, amounts to 15% of wages, which is broadly in line with the average Social Security Payroll tax rate in the U.S. over the 2000-2011 period, as reported by the *OECD*. Following Conesa and Krueger (2006), we use a flexible functional form for the income tax function $\mathcal{T}$ that is theoretically motivated by the equal sacrifice principle, as discussed in Gouveia and Strauss (1994), and encompasses a wide range of progressive, proportional and regressive tax schedules:

$$\mathcal{T}(y) = a_0 \left[y - \left(y^{-a_1} + a_2 \right)^{-\frac{1}{a_1}} \right], \tag{18}$$

where $a_0 \geq 0$, $a_1 \geq 0$, and $a_2 \geq 0$.[11] Gouveia and Strauss (1994) estimate this tax function for the U.S., obtaining values of $a_0 = 0.258$ and $a_1 = 0.768$.[12] The parameter a_2 is calibrated so that total tax revenues, as described in (17), amount to 27% of GDP, a share in line with U.S. recent data.

For the sake of simplicity, the allocation rule for emission permits is assumed to take the general form:

$$\bar{\mathrm{e}}(k) = \mathrm{e}_0 + \mathrm{e}_1 k, \tag{19}$$

with $\mathrm{e}_0 \geq 0$ and $\mathrm{e}_1 \geq 0$, to be chosen on the basis of the allocation formula (see discussion in the following section).

The log of the individual labor productivity is assumed to follow an auto-regressive

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[11] Note that if $a_1 \to 0$, then $\mathcal{T}(y) \to a_0 y$, i.e. the tax schedule collapses to a pure proportional system. If $a_1 > 0$, the system becomes progressive, and the overall progressivity increases with a_1.

[12] These estimates are for tax year 1989, the last year reported in Gouveia and Strauss (1994).

Parameter	*Value*	Parameter	*Value*	Parameter	*Value*
β	0.96	γ_3	3.37%	a_0	0.258
γ	2	γ_3	40.94%	a_1	0.768
η	1.9	$\bar{\lambda}_1$	45%	a_2	2.086
ξ	9.852	$\bar{\lambda}_2$	23%	ρ_ε	0.98
δ	0.048	$\bar{\lambda}_3$	7%	σ_ε	0.11
α	0.33	$\bar{\lambda}_4$	25%	ρ_ϕ	0.40
γ_1	0.36%	$\bar{p}$	0.1852	σ_ϕ	0.45
γ_2	1.48%	τ_N	15%	ω	0.5

Table 1: Summary of the benchmark parametrization.

process of the form:[13]

$$\begin{aligned} \log \varepsilon_{t+1} &= \bar{\varepsilon} + \rho_\varepsilon \log \varepsilon_t + \epsilon_{\varepsilon,t+1}, \\ \epsilon_{\varepsilon,t} &\sim N\left(0, \sigma_\varepsilon^2\right). \end{aligned} \tag{20}$$

Borrowing the estimates provided in Karahan and Ozkan (2013), Table 3, we set $\rho_\varepsilon = 0.98$ and $\sigma_\varepsilon = 0.11$;[14] the aggregate labor endowment in steady state is normalized to one, and this implies, as already mentioned, that $\mathbb{E}(\varepsilon) = 1$: we set the parameter $\bar{\varepsilon}$ accordingly. This process is approximated with a 4-state discrete Markov chain computed using Rouwenhorst's method, as suggested in Kopecky and Suen (2010).

The log of plant-level productivity follows a similar process:

$$\begin{aligned} \log \phi_{t+1} &= \bar{\phi} + \rho_\phi \log \phi_t + \epsilon_{\phi,t+1}, \\ \epsilon_{\phi,t} &\sim N\left(0, \sigma_\phi^2\right). \end{aligned} \tag{21}$$

Abraham and White (2006), using a database that covers the entire U.S. manufacturing sector from 1976 until 1999, estimate plant-level TFP for a large number of plants using a specification similar to (21); borrowing their estimates, we set $\rho_\phi = 0.40$ and $\sigma_\phi = 0.45$. As already mentioned, we impose that $\mathbb{E}(\phi) = 1$ and set the parameter $\bar{\varepsilon}$ accordingly. As before, this process is approximated with a 4-state discrete Markov chain computed using Rouwenhorst's method.

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[13] Two somehow conflicting views on the nature of idiosyncratic income processes have emerged in the literature: as discussed in Guvenen (2009), one view holds that individuals are subject to large and very persistent shocks, while facing similar life-cycle income profiles. The alternative view holds that individuals are subject to income shocks with low persistence, while facing individual-specific income profiles. See also Carroll (1997) for a detailed discussion. Given that currently the jury still seems to be out, our choice of a very persistent labor income process is mainly driven by comparability with the existing literature and numerical convenience.

[14] The results of Karahan and Ozkan (2013) reported above have been obtained using data on annual earnings: hence, to correctly match this empirical evidence to our model we should take the endogeneity of labor supply into account, significantly complicating the calibration process. However, Karahan and Ozkan (2013) also report that using data on average hourly wages does not significantly change their results: we consider these findings reassuring, and introduce this shortcut for the sake of simplicity.

The parameter constellation is summarized in Table 1.

As far as the solution method is concerned, our approach is fairly standard. At the household level, we have to solve a stochastic dynamic optimization problem with occasionally binding constraints: this is done using fixed point iteration on the Euler equation.[15] At the aggregate level, we compute the ergodic distribution using the binning approach described in Young (2010).[16]

In order to assess the implications of climate policy under market incompleteness we perform a set of experiments by means of three modeling set-ups. The first is our benchmark setup, labeled from now on *Heterogeneous Agents Multi Technologies* (H.A.M.). The second is a single-sector, or single-technology, version of the benchmark setup, obtained by simply assuming a single γ, equal to 0.109, the weighted average of the γ_i used in our benchmark calibration, and labeled *Heterogeneous Agents Single Technology* (H.A.S.). The third is a *Representative Agent* (REP.A.) model, where no aggregate nor idiosyncratic shocks are considered: this model replicates perfectly the single-technology model described above, sharing the same parametrization but for the volatility of idiosyncratic shocks, which is set to zero.

4 Numerical Analysis and Results

4.1 Long-run analysis

Using each version of the model we perform a set of experiments in order to analyze implications of different policy measures. As we are agnostic on what the optimal level of climate taxation should be, we repeat each of these experiments for different carbon tax rates, ranging from 10% to 50% of the energy input price, in order to detect non-linear effects. Table 2 summarizes the different experiments and labels them appropriately.

The first experiment simply implies the imposition in the economy of a carbon tax. The increased revenues are then transferred to households through a lump-sum transfer. Experiments from two to four, instead, assume that the government budget is kept equal to the case without any carbon tax and revenues from the carbon tax are instead recycled in different ways. In particular, recycling scheme 1 (RS1) implies a reduction in payroll taxes, while scheme 2 (RS2) implies a proportional reduction in income taxes, i.e. a reduction in the parameter a_0.

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[15] Rendhal (2013) shows that time iteration on the Euler equation converges to the solution obtained with value function iteration also in the presence of occasionally binding constraints. Fixed point iteration is faster, but is not guaranteed to converge in general; in case of convergence, it converges by construction to the same solution obtained via time iteration.

[16] To solve for the policy functions, we discretize the state space using 800 uniformly-spaced nodes over the $[0, 100]$ interval, and employing linear interpolation to evaluate the functions at points that are not on the grid. The same grid is used to compute the stationary distribution. A further increase of the number of nodes does not substantially change the results.

Acronym	**Description**
TAX	Carbon Tax (10%-50%) Lump-sum transfer of revenues to household
TAX_RS1	Carbon Tax (10%-50%) Government Budget kept constant Recycling Scheme 1 (lower payroll tax rate)
TAX_RS2	Carbon Tax (10%-50%) Government Budget kept constant Recycling Scheme 2 (lower average tax rate on income)
CAP_EPC	Cap on emission consistent with emission reductions as in Tax Permits grandfathered on the basis of "equal per capita" principle
CAP_OUT	Cap on emission consistent with emission reductions as in Tax Permits allocated on the basis of output-based with updating

Table 2: Description of experiments.

The last two experiments described in Table 2 assume the imposition of an emission cap and trade system. In particular, we consider a cap on emissions that is consistent with the emission level achieved under the carbon tax scenarios, so that the two systems are completely equal in environmental terms, and a fully functioning emission permits market, without allowing for the banking of permits. In the first experiment emission permits are grandfathered to firms on the basis of an equal per capita principle ($e_0 = \bar{\mathrm{M}}$ and $e_1 = 0$ in equation 19). The second experiment assumes that permits allocation is proportional to capital ($e_0 = 0$ and $e_1 > 0$ in equation 19).[17] Note that, in both cases, $\bar{\mathrm{E}}_t = \bar{\mathrm{M}}_t$. Since (future) output is a control variable of the firm and the allocation of permits creates a subsidy to output, this creates an incentive to reduce emissions through conservation. In order to keep the overall emissions in line with the corresponding level of abatement under the other scenarios, we solve for the value of e_1 - the parameter controlling for the number of permits the government issues in proportion to capital - that generates an amount of emissions equal to the desired level.

4.1.1 Carbon Taxes

Let us start with the first research question we set out to address, namely comparing the aggregate environmental and economic effects of imposing a carbon tax as evaluated with the traditional, representative agent model and the model with heterogeneous agents. In Table 3 we report eight major variables of interest - namely GDP, the capital stock, hours worked, emissions, consumption, changes in social welfare (expressed in terms of consumption equivalent variations),[18] the tax revenues over GDP ratio, and the Gini

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[17] For the sake of analytical simplicity, we take the capital stock (a state variable) as a proxy for output.

[18] Following Aiyagari and McGrattan (1998), we assume a Benthamite welfare function, treating all households the same. The (aggregate) *Consumption Equivalent Variation* (CEV) is computed in the following way: denote as c_0, n_0, and λ_0 the policy functions and the ergodic distribution in the no-tax case, and as v_1 and λ_1 the value function and ergodic distribution in one of the alternative policy scenarios.

index for wealth (Gini_W) - for different levels of the carbon tax and for the three models. The first column in Table 3 reports the steady-state values of the variables under our benchmark parametrization, the no-tax case,[19] while the other columns report the results for the TAX experiments in terms of percentage, or absolute, deviations from the no-tax case.

We start off considering the simplest case where revenues from the carbon tax are recycled back through a lump-sum transfer to the household, we will then look into more complex recycling schemes. We also focus initially on the comparison between the H.A.S. model and the Rep.A. one (the second and third rows for each of the variables). As long as we consider aggregate emissions and GDP implications, the use of a representative agent model approximates well the result of the heterogeneous agents single technology model. Indeed, in both cases GDP and emissions decrease at the introduction of a price on carbon and react almost linearly to its increase. The modeling of heterogeneity however does affect the aggregate welfare in a very different way, as shown in Table 3. While the representative agent model shows a decrease in welfare as a result of the carbon tax, the model portraying heterogeneous agents shows non-negative implications for welfare, i.e. the so called "double dividend," at least for a range of carbon tax values below 20%. For carbon taxes implying increases in the price of energy larger than 20%, welfare decreases with respect to the no carbon policy case, albeit at a far lesser rate than in the representative agent setting. Before discussing the detailed mechanism at the basis of this difference, it is important to point out that, when assessing the welfare performance of each scenario, we are deliberately neglecting environmental implications of the policy but rather accounting for non-environmental welfare changes. Comparability of results across different climate policy instruments and alternative model specifications is ensured as we consider always a set of policy scenarios with identical environmental consequences. If we were to assess the optimal level of climate policy, then we would have to account for environmental welfare implications as well and this will always increase welfare associated to each climate policy scenario.

At the basis of aggregate welfare differences as computed with different model specifications is precisely the different ability of models to portray households/firms with het-

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Define, for a given scalar θ, the value function $v_0(\theta) = E_0\left\{\sum_{t=0}^{\infty} \beta^t u\left[(1+\theta) c_{0,t}, n_{0,t}\right]\right\}$. Then, solve for a unique (across households) θ such that $\int v_0(\theta)\, d\lambda_0 = \int v_1 d\lambda_1$. Note that these welfare measures do not take the transition between steady states into account, and therefore do not properly reflect the effects of switching from one policy to another. Transitions will be studied in the next Section.

[19] Note that the long-run properties of the representative agent model are not entirely replicating those of the other two models we are introducing. This derives from the fact that we are comparing a *deterministic* model (no aggregate nor idiosyncratic uncertainty) to alternative *stochastic* models (no aggregate uncertainty but potentially large idiosincratic one). The role of precautionary savings in the stochastic heterogeneous agents models can explain the differences among capital stocks and GDP levels across models. As we care about deviations from the benchmark, or no carbon policy scenario, across the three model versions, it is more important for us to maintain the same parametrization, while allowing for different long-run properties. This allows us to impute differences due to changes to the sole degree of heterogenoity.

		Carbon Tax **(as share of energy price)**					
		0%	10%	20%	30%	40%	50%
		Level	*% dev. from no-tax case*				
GDP	H.A.M.	**0.89**	0.42	0.57	0.59	0.55	0.47
	H.A.S.	**0.82**	-0.55	-1.09	-1.62	-2.15	-2.67
	Rep.A.	**0.60**	-0.63	-1.26	-1.88	-2.48	-3.11
Capital	H.A.M.	**2.06**	1.63	2.23	2.32	2.18	1.96
	H.A.S.	**1.95**	-1.66	-3.16	-4.55	-5.85	-7.07
	Rep.A.	**1.75**	-1.54	-2.97	-4.24	-5.32	-6.50
Hours	H.A.M.	**0.40**	-0.30	-0.40	-0.40	-0.39	-0.36
	H.A.S.	**0.41**	-0.34	-0.61	-0.85	-1.05	-1.22
	Rep.A.	**0.46**	-0.48	-0.93	-1.25	-1.42	-1.74
Emissions	H.A.M.	**0.38**	-34.69	-53.59	-63.69	-69.45	-72.94
	H.A.S.	**0.54**	-6.03	-11.22	-15.76	-19.78	-23.37
	Rep.A.	**0.53**	-6.22	-11.61	-16.27	-20.29	-24.03
Cons.	H.A.M.	**0.75**	0.17	0.19	0.14	0.07	-0.01
	H.A.S.	**0.69**	-0.40	-0.82	-1.24	-1.68	-2.11
	Rep.A.	**0.51**	-0.48	-0.98	-1.50	-2.01	-2.55
			Consumption Equivalent Variation (%)				
Welfare	H.A.M.		0.30	0.28	0.18	0.07	-0.03
	H.A.S.		0.04	0.01	-0.07	-0.21	-0.37
	Rep.A.		-0.23	-0.52	-0.86	-1.22	-1.63
			Δ in % from no-tax case				
$\frac{\textbf{Tax Rev.}}{\textbf{GDP}}$	H.A.M.	**27%**	0.32	0.46	0.53	0.60	0.67
	H.A.S.	**27%**	0.78	1.48	2.12	2.71	3.25
	Rep.A.	**24%**	1.06	2.00	2.87	3.67	4.39
Gini$_W$	H.A.M.	**77%**	1.27	2.55	3.56	4.36	4.98
	H.A.S.	**75%**	0.00	0.00	-0.01	-0.01	-0.01

Table 3: Summary of results for TAX experiment.

		Carbon Tax (as share of energy price)				
		10%	20%	30%	40%	50%
Households		*% dev. of welfare from no-tax case*				
Full sample		**0.08**	**0.02**	**-0.13**	**-0.37**	**-0.67**
Earning labor inc. only ($k = 0$)	*Low lab. inc.*	1.40	2.59	3.61	4.48	5.23
	High lab. inc.	-3.35	-6.55	-9.68	-12.89	-16.13
All others ($k > 0$)	*Low lab. inc.*	-0.21	-0.53	-0.93	-1.38	-1.88
	High lab. inc.	-0.69	-1.47	-2.32	-3.23	-4.18
Households		*Pop. share (%)*				
Earning labor inc. only ($k = 0$)	*Low lab. inc.*	17.6	17.5	17.4	17.3	17.1
	High lab. inc.	0.4	0.4	0.4	0.4	0.4
All others ($k > 0$)	*Low lab. inc.*	32.4	32.5	32.6	32.7	32.9
	High lab. inc.	49.6	49.6	49.6	49.6	49.6

Table 4: Welfare implications of TAX experiments for different groups of households in the *Heterogeneous Agents Single Technology* model.

erogeneous levels of capital and heterogeneous shocks to capital and labor productivity. Indeed, imposing a carbon tax on the macroeconomic system has (at least) three effects, that can be hardly disentangled in a representative agent general equilibrium model: *i*) the tax increases the price of one input of production, namely energy, and consequently decreases its demand; *ii*) it decreases wages, via general equilibrium effects; *iii*) it increases, *ceteris paribus*, households' disposable income via the lump-sum transfer of the carbon tax revenues. Depending on their level of capital and their productivity, households will be affected differently by these three effects. If we look into the implications for households endowed with small or zero capital, they can be divided into two groups: those with low labor income, because of low labor productivity, for which the benefits of the subsidy are larger than the wage implications and those with high labor income, for which the reverse holds true. Hence, the presence of a carbon tax implies, for firms endowed with low or zero capital, a redistribution favoring the poorer and less efficient households. As we move to households endowed with larger capital stocks both negative and positive effect, that might depend on the wages/subsidy trade off, are smoothed out and virtually go to zero, while the energy input cost increase effect prevails. The aggregate welfare implications, at least for low carbon taxes, are overall positive.

In order to clarify the point, we report in Table 4 the welfare implications[20] of our TAX experiment for different groups of households, namely the households that earn only labor income, because their capital holdings are zero, and the remaining ones, both further disaggregated into the ones that earn a labor income above the mean (high) and below it (low). We report also the corresponding populations shares. Imposing a carbon

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[20] Being the welfare levels negative, because of the form of the utility function, in order to compute percentage variations we adopt the following usual convention: $\Delta x\% = (x' - x) / |x| \cdot 100$.

tax always improves the welfare of low-income households that rely on labor income only, thanks to the redistributive nature of the lump-sum transfer of the corresponding tax revenues, while leaving essentially unaffected the high-income, labor-only households. The remaining households, that rely more or less on entrepreneurial income, see their welfare decrease, due to the increase in the price of energy. For low values of the carbon tax, essentially below $20\% - 30\%$, the welfare improvement obtained by the low-income, labor-only households, that represent around 17% of the population, is greater than the welfare loss incurred in by the remaining households, generating therefore an increase in the overall welfare level.

Note that the "double dividend" described in the previous paragraph is not the result of a reduction in other distortionary taxes made possible by the introduction of a revenue-neutral carbon tax, as emphasized in the literature (Bovenberg and Goulder (1996) and Bovenberg and van der Ploeg (1994) among others), but it is the outcome of a recycling scheme, i.e. the lump-sum redistribution of carbon-tax revenues, that serves as a partial substitute for missing insurance markets and enhances an equal distribution of economic welfare. This is somehow reminiscent of the results in Conesa and Krueger (2006), who study the optimal progressivity of the income tax code, and show that a moderate level of progressivity plays a very similar role.

If we look at the effect on the Gini indicator for wealth, shown in the bottom part of Table 3, we see that, overall, the distribution of wealth remains almost unchanged: though the policy slightly favors the poorest fraction of households, it does not impact capital accumulation and has very little implications on the dispersion of income. This is in stark contrast with the two recycling schemes affecting the propensity to invest and accumulate capital that we are going to discuss in the next section and which exert visible effects on the Gini indicator.

A large body of literature has focused on the distributional impacts of climate policy across households[21] and has emphasized a channel for a regressive impact of those regulations and policies that raise energy costs. Low level income groups would be disproportionally affected by the climate policy as their share of income devoted to energy expenditure is larger than that of higher income groups. In our analysis we have one homogenous final good, hence we are not able to mimic this effect, still accounting explicitly for heterogeneity helps us bringing forward an additional potential mechanism that has so far been neglected in the literature. Important extension of our work will be to include this second source of redistributive impact and compare the two, opposing, effects. Interestingly, some empirical evidence in Goulder et al. (2010) corroborates the idea that the regressivity of environmental taxation my be lower than predicted.

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[21] See Metcalf (2009), Rausch et al. (2011), and Fullerton and Monti (2013), among others.

		Carbon Tax (as share of energy price)				
		10%	20%	30%	40%	50%
Welfare		*Consumption Equivalent Variation (%)*				
TAX	H.A.M.	0.30	0.28	0.18	0.07	-0.03
	H.A.S.	0.04	0.01	-0.07	-0.21	-0.37
	Rep.A.	-0.23	-0.52	-0.86	-1.22	-1.63
RS1	H.A.M.	0.29	0.27	0.17	0.06	-0.04
	H.A.S.	0.03	-0.01	-0.10	-0.23	-0.39
	Rep.A.	0.03	-0.03	-0.17	-0.37	-0.61
RS2	H.A.M.	0.48	0.53	0.46	0.36	0.28
	H.A.S.	0.30	0.51	0.63	0.69	0.70
	Rep.A.	0.84	1.49	1.99	2.37	2.66
Gini$_W$		*Δ in % from no-tax case*				
TAX	H.A.M.	1.27	2.55	3.56	4.36	4.98
	H.A.S.	0.00	0.00	-0.01	-0.02	-0.03
RS1	H.A.M.	1.30	2.57	3.57	4.36	4.98
	H.A.S.	0.05	0.10	0.15	0.18	0.22
RS2	H.A.M.	1.37	2.67	3.67	4.45	5.08
	H.A.S.	0.24	0.45	0.62	0.78	0.92

Table 5: Summary of results for TAX-RS experiments.

4.1.2 Other recycling schemes

As the use of revenues from climate policies is crucial in defining their distributional implications, we resort to a more complex set of experiments where revenues from the carbon tax are recycled in different ways. Table A.1 summarizes the value of the calibrated parameters.

We start by looking at aggregate implications on welfare, which allows us to appreciate major differences across the two model setups. We then look into implications for redistribution at the aggregate level by reporting the Gini Index. Table 5 reports social welfare for an increasing carbon tax which is recycled, while keeping the government budget constant, either lowering the payroll tax rate (TAX_RS1) or the average tax rate on income (TAX_RS2), while the top row reports our benchmark: the lump-sum transfer scenario (TAX). When looking at aggregated welfare, the dominant recycling scheme is robust across the two model specifications (as we will see later this is actually also true when considering the Multi technology model as well) and it is RS2, namely the scheme implying a proportional reduction in income taxes. However, the two models disagree strikingly on the dimensions of these improvements: the Rep.A. model generates larger welfare increases than the H.A.S. one, more than twice as big. To understand this outcome, consider that in the Rep.A. model a decrease in a_0, the proportionality parameter in our tax function, affects directly the average and marginal tax rates faced by the single agent, stimulating this way further investment and capital accumulation, and therefore increasing the long-run output level. In the H.A.S. model, the decrease in a_0 will have a smaller effect due to the relatively large share of credit-constrained or low-income households. Furthermore, revenues from the carbon tax are smaller in the H.A.S. model in the first place, as evident from results presented in Table 3, and this allows for a smaller reduction in a_0, as described in Table A.1. It is this negative impact on the credit-constrained and low-income households that explains the worsening recorded by the Gini indicator for wealth under RS2, as reported in the bottom part of Table 5.

Scheme RS1 is strictly dominated by all other schemes, and the two models provide fairly similar results in terms of aggregate welfare implications. It is nonetheless interesting to look closer at the across-firms dynamics in the H.A.S. model, as the aggregate results fail to show important changes happening withing different household groups.

Tables A.2 and A.3 summarize the welfare implications of RS1 and RS2 for different groups of households, mirroring the disaggregation presented in Table 4. Consider Table A.2, i.e. the RS1 case. Credit-constrained households (the first two rows in Table A.2) loose from the carbon tax, and increasingly more so, as it implies a reduction in wages through general equilibrium effects, but now revenues are recycled through a reduction of the payroll tax, which has a negligible effect on this group when compared to lump-sum transfers. This also explains the worsening of the Gini wealth indicator reported in Table 5. As we move to households with increasing levels of capital, the reduced income tax positive effect tends to prevail, but for low carbon taxes only. The negative effect wins again for higher levels of the carbon tax, but for the subset of household with higher levels of labor income. It should be noted that RS1 implies a more modest deterioration of the Gini indicator than RS2; this trade-off between cost effectiveness and distribution had already been emphasized in Parry and Williams (2010), although he framework and type of analysis were completely different.

Consider now Table A.3, i.e. the RS2 case. Credit-constrained and low-income households (the first row in Table A.3) loose from the carbon tax, and increasingly more so, as it implies a reduction in wages through general equilibrium effects, but now revenues are recycled through a reduction of the average income tax rate which has a negligible effect on this group when compared to the lump-sum transfer case. Higher income credit-constrained households, instead, are almost entirely compensated for the the wage decrease implied by RS2, at least for moderate carbon tax rates. As we move to households with increasing levels of capital, the reduced income tax positive effect prevails.

		Carbon Tax (as share of energy price)				
		10%	20%	30%	40%	50%
		Consumption Equivalent Variation (%)				
H.A.M.	*Trans.*	**0.21**	**0.24**	**0.19**	**0.10**	**-0.02**
	S.S.	0.30	0.28	0.18	0.07	-0.03
H.A.S.	*Trans.*	**0.24**	**0.38**	**0.46**	**0.49**	**0.47**
	S.S.	0.04	0.01	-0.07	-0.21	-0.37

Table 6: Changes in social welfare: transition vs. steady state.

4.2 Transition dynamics

The previous Section focused on the long-run, comparing steady states across different policy scenarios. In order to better understand the mechanisms at play, we report in Figure 1 the adjustment dynamics during the transition to the new steady state for the H.A.S. model after a permanent and unexpected increase in τ_E from 0% to 10%.

On impact, imposing the carbon tax has a sizable negative effect on emissions, equal approximately to -5.8%, output, equal to -0.8%, and hours worked, equal to -0.5%, but a relatively small but negative effect on GDP, equal to no more than -0.25%. Consumption increases slightly, while revenues from income and payroll taxes drop substantially; revenues from the carbon tax, however, more than compensate for this drop, and therefore total tax revenues increase by almost 3%. The wage rate and the average (implied) rental rate, i.e. the marginal productivity of capital, drop by approximately 1%. Income inequality increases on impact, but wealth inequality is of course unaffected.

The subsequent dynamics of the model determines further reductions in GDP, output and emissions. After the initial drop, the average rental rate converges back to essentially the same initial value, while the wage rate further drops. The capital stock decreases significantly during the transition, dropping by -1.8% in the long run. Hours worked slightly recover the initial decrease, but remain significantly smaller in the long run. After the initial rise, the consumption level drops and converges to a permanently lower level in the long run. Total government revenues and payroll-tax revenues remain, respectively, larger and smaller than in the initial steady state. Wealth inequality initially increases over the transition, but then returns to the previous level in the long run, while income inequality remains permanently higher.

Table 6 reports the induced changes in social welfare, expressed again in terms of consumption equivalent variations, computed taking the transition fully into account,[22]

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[22] As before, denote as c_0, n_0, and λ_0 the policy functions and the ergodic distribution in the no-tax (i.e. pre-reform) steady state. Denote as v_1 the policy function at date 1, the date at which the policy reform is unexpectedly (and permanently) implemented, and note that in this case $\lambda_1 = \lambda_0$. The function v_1 represents the expected lifetime utility of an agent who has just been informed that there is a permanent policy change, and, under perfect foresight, takes the entire transition to the new steady state into account. Define, for a given scalar θ, the value function $v_0(\theta) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left[(1+\theta) c_{0,t}, n_{0,t} \right] \right\}$.

and compares them with changes computed focusing on steady states only. The differences are striking: once the transition towards the new steady state is factored in, the introduction of a positive carbon tax becomes clearly welfare-improving for al rates in the $10\% - 50\%$ range, and the increase in social welfare seems to reach a maximum around $\tau_E = 0.4$. Note that the equivalent variations in consumption range from a minimum of 0.24 percentage points to a maximum of 0.49 percentage points: these figures are far from negligible in relative terms, if we consider that they are of the same order of magnitude ($0.1 - 1.0$ percentage points) of the welfare gains computed by Krusell et al. (2009), who study the welfare effects of eliminating business cycles in a heterogeneous agents economy.

4.3 Extensions

4.3.1 Introducing Multi Technologies

Let us now consider the implications of adding complexity to our Heterogeneous agents model. In particular, we will assume the presence of four major technologies in the economy, replicating four clusters of sectors that differ in their use of energy. We will, in what follows, assume that households can respond to a change in the price of energy by leaving energy intensive capital to depreciate and providing work for more energy savvy technologies. This movement across technologies would in reality be, at least partially, constrained and costly. The required skills to migrate across technologies and the difficulty of adopting energy efficient technologies in some sectors are only two examples of such frictions. In the current analysis we disregard these costs and thus produce an extreme case of the potential of technical change in shaping the macroeconomic response to a carbon tax. If we move back to Table 3 and concentrate on results for the H.A.M. model, we immediately notice how emissions and GDP react with respect to the other two models. Emissions are curbed more aggressively in response to the presence of a price on carbon, while GDP stays almost unaltered. The presence of alternative technologies makes it possible for the economy to react more swiftly to climate policy than it is projected by the two other models. The mechanism at play is the following: As the price of carbon increases, production based on more energy intensive technologies is dramatically reduced as households endowed with such technologies reduce the scale of their operations, leave their capital to depreciate, and increase their labor supply in favor of households characterized by more energy efficient technologies. As the decrease in energy imports resulting from this change more than compensates the negative impact due to the carbon tax, GDP is almost unchanged. This mechanism has obvious repercussion on the reduction in emissions, which is way more pronounced that according to the other two models. Indeed, the effect on emissions reduction of a carbon tax that would increase by 50% the price of energy is almost in line with the 80% emission reduction that would be required

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To compute the consumption equivalent variation, we solve for a θ such that $\int v_0(\theta)\, d\lambda_0 = \int v_1 d\lambda_0$.

for the Copenhagen agreement target.

Now, what model is giving us a correct portray of the macroeconomic reaction to the imposition of a carbon tax? The multi-technology model more realistically depicts some features of reality, as we would clearly expect a change in production modes and technology types as a reaction to the introduction of a carbon tax, at least in the long run. What our multi-technology heterogeneous agents model is not realistically representing, though, are the costs of this technological transition, that would inevitably be larger than zero due to lock-in effects and other frictions.[23]

As the redistribution across technology is the main mechanism at play within the H.A.M. model, carbon revenues are much lower than in the case of the other two models. Hence, the implications of considering alternative recycling schemes are negligible, especially when compared to the other two models: see Table 5.

Figure 2 reports the adjustment dynamics for H.A.M. model, again after a permanent and unexpected increase in τ_E from 0% to 10%: some striking differences between the dynamics of the *Single* and *Multi Technology* models are evident. On impact, the main difference between the two models lies in the reaction of the average rental rate: in the H.A.S. model the rate was decreasing on impact, in the H.A.M. one, instead, the rate is increasing by 1%. Other differences are the smaller drop in GDP and output, the smaller increase in total government revenues, and the larger increase in income inequality. Also the transition dynamics turns out to be quite different between the tow models: in the *Single Technology* model emissions and output decrease further during the transition, but not dramatically so, while in the *Multi Technology* model this further reduction, at least in the case of emissions, is sizable. The reduction in the amount of energy used, and imported from abroad, is actually sufficiently larger than the reduction in output to make the GDP level increase in the long run. After the initial increase, the consumption level drops for a while, but then inverts the trend and converges to a permanently higher level in the long run, as does welfare. Also the capital stock converges to a higher level in the long run, while income taxes converge back to their initial value, as opposed to payroll taxes that end up being permanently lower. Finally, the income and wealth inequality steadily increases over the transition, more so for the latter. These striking differences are due to the previously described reallocation mechanism among technology types.

Table 6 reports the changes in social welfare computed taking the transition into account. For our H.A.M. model, the differences between these measures and the ones computed in steady state are less striking than for the H.A.S. model: for tax rates below 30%, the transitions seems to reduce slightly the overall welfare effect, which remains however strictly positive and relatively large, while for rates above 30% the opposite holds true.

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[23] See Acemoglu et al. (2012) for a discussion.

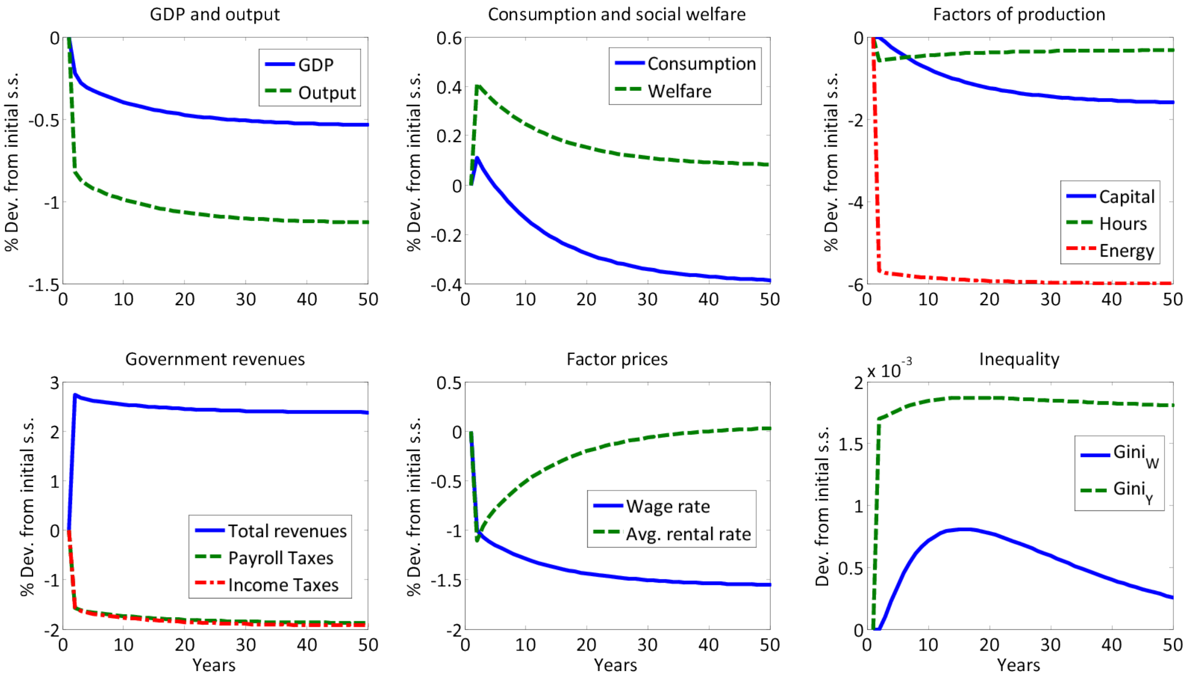

Figure 1: Adjustment dynamics in the *Heterogeneous Agents Single Technology* model after an increase in τ_E from 0% to 10%.

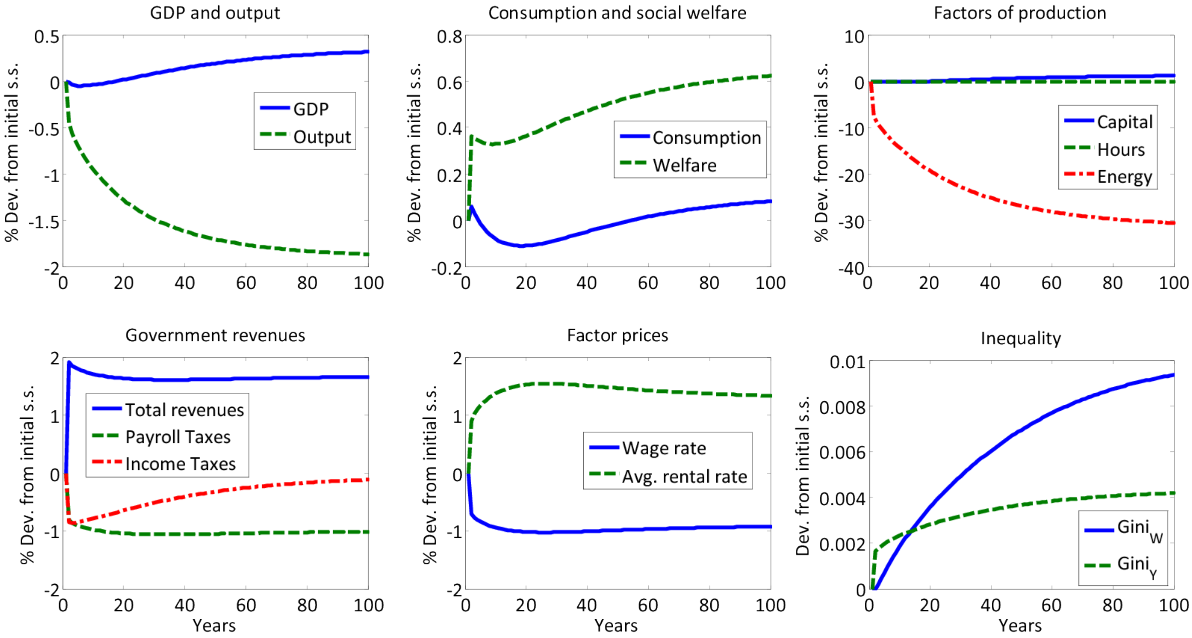

Figure 2: Adjustment dynamics in the *Heterogeneous Agents Multi Technology* model after an increase in τ_E from 0% to 10%.

	% dev. from no-policy case					
	GDP			**Cons.**		
	TAX	CAP_EPC	CAP_OUT	TAX	CAP_EPC	CAP_OUT
H.A.M.	0.57	0.57	1.76	0.19	0.18	1.02
H.A.S.	-1.09	-1.14	3.13	-0.82	-0.87	2.02
Rep.A.	-1.26	-1.35	3.20	-0.98	-1.05	1.62
	Hours			**Welfare (CEV)**		
H.A.M.	-0.40	-0.41	-0.54	0.28	0.28	0.64
H.A.S.	-0.61	-0.63	-1.39	0.01	-0.04	2.50
Rep.A.	-0.93	-0.91	-1.78	-0.52	-0.58	2.57
	Δ in % from no-policy case					
	$\frac{\textbf{Tax Rev.}}{\textbf{GDP}}$			$\textbf{Gini}_{\textbf{W}}$		
H.A.M.	0.46	-0.11	-0.07	2.55	2.61	2.39
H.A.S.	1.48	-0.24	-0.13	0.00	0.16	-0.87
Rep.A.	2.00	-0.29	-0.19			

Table 7: Impact of market-based instruments equivalent to a 20% Carbon tax, in terms of deviations from the no-policy case.

4.3.2 Environmentally Equivalent Caps

A long literature in environmental economics, beginning with the seminal paper by Weitzman (1974), has compared price and quantity instruments for regulating emissions. In Fischer and Springborn (2011), the authors set out to study the effectiveness of a tax versus two different allocation schemes in a cap system under macro-economics shocks and find that a cap system would achieve a given emission reduction with a slightly higher welfare cost than the tax, but it would ensure that the cut is achieved without lag, resulting in higher welfare if these additional reductions are valued; the cap system also features a lower level of labor variance than all other policies considered. Both the H.A.S. and H.A.M. models are particularly indicated to mimic emission trading as well as the emergence of a market price for emissions, given the initial endowment and the heterogeneity of firms. Table 7 reports the effects on GDP, the government budget/GDP ratio, hours worked, and welfare of the three policy instruments, in terms of percentage deviations from the benchmark case, when the goal is to mimic the reduction in emissions generated by a 20% carbon tax.

We consider climate policies characterized by increasing stringency, in line with the previous simulations, which are now imposed through either one of two emission trading schemes, each leading to the same environmental outcome of the TAX corresponding scenario (our benchmark simulation). Permits are allocated on a per capita base, CAP_EPC, or in proportion to output, CAP_OUT. As permit are not auctioned, government rev-

Carbon Tax **(as share of energy price)**		TAX	CAP_ECP	CAP_OUT
Households		*% dev. of welfare from no-tax case*		
Full sample		0.02	-0.07	4.35
Earning labor inc. only ($k = 0$)	*Low lab. inc.*	2.59	0.35	6.33
	High lab. inc.	-6.55	-4.74	30.85
All others ($k > 0$)	*Low lab. inc.*	-0.53	0.24	0.90
	High lab. inc.	-1.47	-0.81	6.96
Households		*Pop. share (%)*		
Earning labor inc. only ($k = 0$)	*Low lab. inc.*	17.5	17.9	16.9
	High lab. inc.	0.4	0.4	0.3
All others ($k > 0$)	*Low lab. inc.*	32.5	32.1	33.1
	High lab. inc.	49.6	49.6	49.7

Table 8: Welfare implications of market-based instruments equivalent to a 20% Carbon tax for different groups of households in the *Heterogeneous Agents Single Technology* model.

enues decrease rather than increasing (see the bottom part Table 7). Indeed, instead of an ex-post redistribution, allocation of permits imply an endowment of property rights to firms. In the case of the per capita allocation, CAP_EPC, the policy almost perfectly replicates the results of the TAX case.

The CAP_OUT simulation results clearly stand out as policy implies large benefits rather than costs, indifferently measured in terms of GDP or Welfare. This is robust to the different model formulations. CAP_OUT is also the preferred instrument if we care about wealth distribution. As before, depending on whether we consider the multi or the single-technology model we see a different effect on the Gini indicator depending on the assumptions about reactivity and mobility across technological categories. In the first case, the Gini indicator deteriorates, indicating that a large portion of households/firms stops producing using their own technology but rather allocates its labor to other households/firms endowed with more energy efficient technologies. As we have discussed, this mechanism increases inequity between energy efficient and energy inefficient firms. However, this is less so than with any of the other instruments/recycling schemes we have analyzed. In the case of the H.A.S. the Gini actually indicates an improvement in wealth distribution. This mainly depends on two factors. First, the allocation boosts capital accumulation across capital levels, hence it shifts the distribution of capital to the right. This, as shown in the bottom part of Table 8, decreases the share of capital constrained households. Second, general equilibrium effect positively affects wages, hence providing benefits to the remaining population whose income primarily depends on wage.

Table 9 reports again the changes in social welfare computed taking the transition into account; we limit ourselves to the CAP_EPC case, for the sake of brevity, and compare

			Carbon Tax (as share of energy price)				
			10%	20%	30%	40%	50%
			Consumption Equivalent Variation (%)				
H.A.M.	CAP_EPC	*Trans.*	**0.18**	**-0.14**	**-0.51**	**-0.83**	**-1.08**
		S.S.	0.29	0.28	0.18	0.07	-0.03
	TAX	*Trans.*	**0.21**	**0.24**	**0.19**	**0.10**	**-0.02**
		S.S.	0.30	0.28	0.18	0.07	-0.03
H.A.S.	CAP_EPC	*Trans.*	**0.24**	**0.40**	**0.47**	**0.50**	**0.47**
		S.S.	0.00	-0.04	-0.13	-0.27	-0.45
	TAX	*Trans.*	**0.24**	**0.38**	**0.46**	**0.49**	**0.47**
		S.S.	0.04	0.01	-0.07	-0.21	-0.37

Table 9: Changes in social welfare: transition vs. steady state under CAP_EPC.

the results with the ones obtained in the TAX case. Consider the H.A.S. model: we already noticed that the use of energy converges almost instantly to its new steady-state value, after the introduction of a carbon tax (see Figure 1). Hence, we intuitively expect the cap-and-trade scheme, which forces the aggregate use of energy to respect the given target from day one onwards, to generate similar results in terms of social welfare during the transition. Table 9 confirms this intuition: the welfare implications of TAX and CAP_EPC, for the H.A.S. model, are essentially identical. Consider now the H.A.M. model, instead: Figure 2 shows that it takes quite a long time for the use of energy to reach its new steady-state value. In this case, we expect that introducing a binding cap from day one should have relevant effects on the transition. As before, 9 confirms our intuition: in the TAX case, imposing a carbon tax has a significantly positive effect on welfare for rates below 50%, and taking the transition into account does not change the picture significantly. In the CAP_EPC case, the steady-state results are unsurprisingly in line with the TAX case, while taking the transition into account changes the outcomes dramatically: even if a cap equivalent to a 10% carbon tax still achieves an increase in welfare, more stringent policies cause a sizable decrease in social welfare, that reaches a 1.08% of lifetime consumption for a cap equivalent to a 50% carbon tax. This is in stark contrast with the TAX case, and suggest that, from a dynamic point of view, carbon taxes and cap-and trade schemes may have clearly different welfare implications, even if they obtain the same reduction in emission in the long run.

4.3.3 Sensitivity analysis

The elasticity of substitution between the “Cobb-Douglas” composite and energy in the production function (1) is a key parameter that may potentially heavily influence our results. As already anticipated, recent empirical evidence in Baccianti (2013) suggests that the range of variation for possible values of the elasticity of substitution between the capital-labor composite and energy could be as wide as $0.00 - 0.82$, with a point estimate of 0.25. In order to evaluate the role of this elasticity we perform some sensitivity analysis, and report in Table 10 the results of three sets of TAX experiments performed using the *Heterogeneous Agents Single Technology* model with three different values of the elasticity of substitution between capital-labor composite and energy; 0.2, 0.5, and 0.8.

Lowering the elasticity of substitution from 0.5 to 0.2 has no significant effect on the level of GDP in the no-policy case, but decreases the effects of carbon taxes by a 30% across the board, approximately. Increasing the elasticity to 0.8 has almost no effect on the level of GDP, but increases the effects of carbon taxes by a 33% across the board, again approximately. The effects on the capital stock are similar, but less quantitatively significant: the variations remain in the ±12% range. The same pattern emerges for consumption: in this case, the variations spread over the ±33% range. The effects on hours worked, instead, go in the opposite direction: when the elasticity of substitution drops to 0.2, the negative reaction of hours worked to carbon taxes becomes more pronounced, and this difference increases with the carbon tax from roughly 25% to 46%; when the elasticity rises to 0.8, instead, the reaction of hours worked becomes less pronounced, the difference ranging from −22% to −41%.

Varying the elasticity of substitution has important consequences on the sensitivity of energy use, and therefore emission, to climate policies: when the elasticity drops to 0.2, the effects of carbon taxes on emission are almost halved, with reductions that range from −48% to −44% as carbon taxes increase. When the elasticity rises to 0.8, instead, the effects are almost doubled, with increases that range from 49% to 40%.

The reaction of welfare is also significantly affected by changes in the elasticity of substitution: for an elasticity equal to 0.2, we observe an *increase* in welfare in all experiments, and the size of this increase is positively correlated with the carbon tax rate applied, while for an elasticity equal to 0.8 welfare consistently decreases in all experiments. This is in sharp contrast with the findings for an elasticity equal to 0.5, for which the welfare level increases for low levels of the carbon tax - below 20% of the price of energy - and decreases for higher levels.

The results clearly show that the lower the elasticity of substitution between the capital-labor composite and energy, the stronger the effect described in the previous Sections becomes. In particular, a low elasticity enhances the "double dividend" effect, at the cost, however, of implying, *ceteris paribus*, a smaller decrease in emissions.

5 Caveats and future research

In the present paper we study the implications of pricing carbon by means of a representative agent model and we contrast the results with those derived with a model portaying

		Carbon Tax (as share of energy price)					
	Elast. of subst.	**0%**	10%	20%	30%	40%	50%
		Level	*% dev. from no-tax case*				
GDP	$\omega = 0.2$	0.82	-0.37	-0.75	-1.14	-1.52	-1.92
	$\omega = 0.5$	0.82	-0.55	-1.09	-1.62	-2.15	-2.67
	$\omega = 0.8$	0.83	-0.72	-1.42	-2.09	-2.74	-3.36
Capital	$\omega = 0.2$	1.96	-1.43	-2.81	-4.13	-5.41	-6.65
	$\omega = 0.5$	1.95	-1.66	-3.16	-4.55	-5.85	-7.07
	$\omega = 0.8$	1.96	-1.86	-3.49	-4.94	-6.27	-7.48
Hours	$\omega = 0.2$	0.41	-0.40	-0.77	-1.12	-1.45	-1.75
	$\omega = 0.5$	0.41	-0.34	-0.61	-0.85	-1.05	-1.22
	$\omega = 0.8$	0.41	-0.25	-0.42	-0.54	-0.64	-0.71
Emissions	$\omega = 0.2$	0.54	-3.12	-5.95	-8.54	-10.93	-13.15
	$\omega = 0.5$	0.54	-6.03	-11.22	-15.76	-19.78	-23.37
	$\omega = 0.8$	0.54	-8.92	-16.34	-22.61	-28.01	-32.69
Cons.	$\omega = 0.2$	0.69	-0.27	-0.56	-0.85	-1.16	-1.48
	$\omega = 0.5$	0.69	-0.40	-0.82	-1.24	-1.68	-2.11
	$\omega = 0.8$	0.69	-0.54	-1.08	-1.62	-2.16	-2.68
			Consumption Equivalent Variation (%)				
Welfare	$\omega = 0.2$		0.39	0.70	0.93	1.09	1.20
	$\omega = 0.5$		0.04	0.01	-0.07	-0.21	-0.37
	$\omega = 0.8$		-0.23	-0.62	-1.13	-1.72	-2.37
			Δ in % from no-tax case				
$\frac{\textbf{Tax Rev.}}{\textbf{GDP}}$	$\omega = 0.2$	27%	0.81	1.58	2.32	3.02	3.69
	$\omega = 0.5$	27%	0.78	1.48	2.12	2.71	3.25
	$\omega = 0.8$	27%	0.76	1.41	1.96	2.44	2.87
Gini$_{\mathbf{W}}$	$\omega = 0.2$	75%	0.01	0.01	0.01	0.00	0.00
	$\omega = 0.5$	75%	0.00	0.00	-0.01	-0.01	-0.01
	$\omega = 0.8$	75%	-0.01	-0.02	-0.03	-0.04	-0.05

Table 10: Sensitivity analysis: the role of the elasticity of substitution between the "Cobb-Douglas" composite and energy in the *Heterogeneous Agents Single Technology* model.

incomplete markets with heterogeneous agents. Although some macro aggregate implications of this policy, as those on GDP and emissions, are fairly similar across the two models, projections concerning welfare implications differ substantially. This result hinges on the different ability of the two models to portray important channels through which a price on carbon might affect households and firms. Depending on their level of capital and their productivity, households will be affected differently by direct and indirect effects on input prices and wages. In addition, when we look into the implications of revenues redistribution (either through a lump-sum transfer or more complicated schemes), the difference across the two models becomes even starker. Difference are even larger if we consider an extension of the model where we allow for the presence of alternative production technologies. In addition to this, by allowing for heterogeneous agents not only we can mimic implications for different households but also we can compute inequality indicators that are a key dimension against which to assess alternative climate policies and implementation schemes.

There are some important caveats to the current version of the analysis that we are planning to address in future research. First, the modeling of multi- versus single-technology is currently very crude. We aim at integrating realistic frictions (by means of a CES structure governing the substitutability of different technologies) as well as the required expenditure in knowledge and physical capital that allow technological progress to less carbon intensive means of production. This will allow us to relax the rather crude assumption of equivalence between energy and emissions and it will possibly generate a solution that will lay in between the two current versions (single versus multi-technology) of the model. In addition, by introducing multiple final goods differening in their energy contentwe will be able to test and compare additional sources of distributional impact of the carbon policy.

References

Abraham, A. and K. White (2006, July). The Dynamics of Plant-Level Productivity in U.S. Manufacturing. Working Papers 06-20, Center for Economic Studies, U.S. Census Bureau.

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012, February). The Environment and Directed Technical Change. *American Economic Review 102*(1), 131–66.

Aiyagari, S. R. (1994, August). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics 109*(3), 659–84.

Aiyagari, S. R. and E. R. McGrattan (1998, October). The optimum quantity of debt. *Journal of Monetary Economics 42*(3), 447–69.

Aldy, J. E. and W. A. Pizer (2011, December). The Competitiveness Impacts of Climate Change Mitigation Policies. NBER Working Papers 17705, National Bureau of Economic Research, Inc.

Alexeeva-Talebi, V., C. Boehringer, A. Loeschel, and S. Voigt (2012, December). The value-added of sectoral disaggregation: Implications on competitive consequences of climate change policies. *Energy Economics 34*(S2), S127–S142.

Angeletos, G.-M. (2007, January). Uninsured Idiosyncratic Investment Risk and Aggregate Saving. *Review of Economic Dynamics 10*(1), 1–30.

Baccianti, C. (2013). Estimation of sectoral elasticities of substitution along the international technology frontier. ZEW Discussion Papers 13-092, ZEW.

Bosetti, V., C. Carraro, M. Galeotti, E. Massetti, and M. Tavoni (2006). A World Induced Technical Change Hybrid Model. *Energy Journal 27*, 13 – 37.

Bovenberg, A. L. and L. H. Goulder (1996, September). Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses. *American Economic Review 86*(4), 985–1000.

Bovenberg, A. L. and F. van der Ploeg (1994, November). Environmental policy, public finance and the labour market in a second-best world. *Journal of Public Economics 55*(3), 349–90.

Carroll, C. D. (1997, February). Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis. *The Quarterly Journal of Economics 112*(1), 1–55.

Carroll, D. R. and E. R. Young (2009, July). The Stationary Distribution of Wealth under Progressive Taxation. *Review of Economic Dynamics 12*(3), 469–478.

Chatterjee, S. (1994, May). Transitional dynamics and the distribution of wealth in a neoclassical growth model. *Journal of Public Economics 54*(1), 97–119.

Conesa, J. C. and D. Krueger (2006, October). On the optimal progressivity of the income tax code. *Journal of Monetary Economics 53*(7), 1425–50.

Covas, F. (2006, November). Uninsured idiosyncratic production risk with borrowing constraints. *Journal of Economic Dynamics and Control 30*(11), 2167–2190.

Dissou, Y. and L. Karnizova (2012). Emissions Cap or Emissions Tax? A Multi-sector Business Cycle Analysis. Working Papers 1210E, University of Ottawa, Department of Economics.

Fischer, C. and M. Springborn (2011, November). Emissions targets and the real business cycle: Intensity targets versus caps or taxes. *Journal of Environmental Economics and Management 62*(3), 352–66.

Fullerton, D. and H. Monti (2013, November). Can pollution tax rebates protect low-wage earners? *Journal of Environmental Economics and Management 66*(3), 539–53.

Goulder, L. H., M. A. Hafstead, and M. Dworsky (2010, November). Impacts of alternative emissions allowance allocation methods under a federal cap-and-trade program. *Journal of Environmental Economics and Management 60*(3), 161–81.

Gouveia, M. and R. P. Strauss (1994, June). Effective Federal Individual Income Tax Functions: An Exploratory Empirical Analysis. *National Tax Journal 47*(2), 317–39.

Guvenen, F. (2009, January). An Empirical Investigation of Labor Income Processes. *Review of Economic Dynamics 12*(1), 58–79.

Guvenen, F. (2011, Third Quarter). Macroeconomics with Heterogeneity: A Practical Guide. *Federal Reserve Bank of Richmond Economic Quarterly 97*(3), 255?326.

Hassler, J. and P. Krusell (2012, October). Economics And Climate Change: Integrated Assessment In A Multi-Region World. *Journal of the European Economic Association 10*(5), 974–1000.

Heutel, G. (2012, April). How Should Environmental Policy Respond to Business Cycles? Optimal Policy under Persistent Productivity Shocks. *Review of Economic Dynamics 15*(2), 244–64.

Huggett, M. (1993, September). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control 17*(5-6), 953–69.

IMF (2012, June). *Fiscal Policy to Mitigate Climate Change: A Guide for Policymakers.* IMF. 978-1-47553-659-1.

IPCC (2013). *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press.

IPCC (2014). *Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press.

Karahan, F. and S. Ozkan (2013). On the Persistence of Income Shocks over the Life Cycle: Evidence, Theory, and Implications. *Review of Economic Dynamics forthcoming.*

Kopecky, K. and R. Suen (2010, July). Finite State Markov-chain Approximations to Highly Persistent Processes. *Review of Economic Dynamics 13*(3), 701–14.

Krueger, D., F. Perri, L. Pistaferri, and G. L. Violante (2010, January). Cross-Sectional Facts for Macroeconomists. *Review of Economic Dynamics 13*(1), 1–14.

Krusell, P., T. Mukoyama, A. Sahin, and A. A. Smith, Jr. (2009, July). Revisiting the Welfare Effects of Eliminating Business Cycles. *Review of Economic Dynamics 12*(3), 393–402.

Krusell, P. and A. A. J. Smith (2007, June). Quantitative Macroeconomic Models with Heterogeneous Agents. In R. Blundell, W. Newey, and T. Persson (Eds.), *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress, Volume III*, Econometric Society Monographs. Cambridge University Press.

Metcalf, G. E. (2009). Designing a carbon tax to reduce US greenhouse gas emissions. *Review of Environmental Economics and Policy 3*(1), 63–83.

Nordhaus, W. D. (1977, February). Economic Growth and Climate: The Carbon Dioxide Problem. *American Economic Review 67*(1), 341–46.

O'Mahony, M. and M. P. Timmer (2009, June). Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database. *The Economic Journal 119*(538), F374–F403.

Parry, I. W. and R. C. Williams (2010, December). What are the Costs of Meeting Distributional Objectives for Climate Policy? *The B.E. Journal of Economic Analysis & Policy 10*(2), 1–35.

Parry, I. W. H. (2004, March). Are emissions permits regressive? *Journal of Environmental Economics and Management 47*(2), 364–87.

Rausch, S., G. E. Metcalf, J. M. Reilly, and S. Paltsev (2011). Distributional Impacts of a US Greenhouse Gas Policy. In *US Energy Tax Policy*, Chapter 3, pp. 52–112. Cambridge Univ. Press.

Reichling, F. and C. Whalen (2012, October). Review of Estimates of the Frisch Elasticity of Labor Supply. Working Paper 13, Congressional Budget Office.

Rendhal, P. (2013). Inequality Constraints and Euler Equation based Solution Methods. *The Economic Journal*, forthcoming.

Sandmo, A. (1975, March). Optimal Taxation in the Presence of Externalities. *The Swedish Journal of Economics 77*(1), 86–98.

van der Werf, E. (2008). Production functions for climate policy modeling: An empirical analysis. *Energy Economics 30*(6), 2964 – 79.

Weitzman, M. L. (1974). Prices vs. Quantities. *Review of Economic Studies 41*(4), 477–91.

Young, E. R. (2010, January). Solving the incomplete markets model with aggregate uncertainty using the Krusell-Smith algorithm and non-stochastic simulations. *Journal of Economic Dynamics and Control 34*(1), 36–41.

A Appendix

A.1 The recursive equilibrium

The idiosyncratic stochastic processes are independent across households, and can be jointly represented by a finite-state Markov process, denoted $\sigma \in \Sigma$, where $\Sigma \equiv E \times \Phi \times \Gamma$, and characterized by a transition matrix $\pi = \pi_\varepsilon \otimes \pi_\phi \otimes \pi_\gamma$ such that $\pi(j,i) \geq 0$ stands for the probability that $\sigma_{t+1} = \sigma_j$ if $\sigma_t = \sigma_i$, where, for the sake of notational convenience, $\sigma \equiv \{\varepsilon, \phi, \gamma\}$.

The vector of individual state variables $x_t \equiv \{k_t, \sigma_t\}$ lies in $X = [0, \infty) \times \Sigma$. The distribution of individual states across agents is described by an aggregate state, the probability measure λ_t. More precisely, λ_t is the unconditional probability distribution of the state vector x_t, defined over the Borel subset of X:

$$\lambda_t(\mathrm{k}, \mathrm{q}) = \lambda_t(\mathrm{x}) = \Pr(k_t = \mathrm{k}, \sigma_t = \mathrm{q}). \tag{22}$$

For the *Law of Large Numbers*, $\lambda_t(\mathrm{x})$ can be interpreted as the mass of agents whose individual state vector is equal to x. Being λ_t a probability measure, the total mass of agents is equal to one.

In a recursive equilibrium, the time-invariant individual policy functions will depend on the exogenous state, σ, on the beginning of period capital stock, k, and on the aggregate distribution λ. The aggregate wage rate w will depend on the distribution of individual wealth stocks. Hence, the exogenous Markov process for σ and the optimal policy function $c(x;\lambda)$ induce a law of motion for the distribution λ_t:

$$\lambda_{t+1}(\mathrm{x}) = \int_X \mathcal{I}(\mathrm{k}, k, \sigma)\, \pi(\mathrm{q}, \sigma)\, d\lambda_t, \tag{23}$$

where:

$$\mathcal{I}(\mathrm{k}, k, \sigma) = \begin{cases} 1 & \text{if } k'(x;\lambda_t) = \mathrm{k} \\ 0 & \quad \text{otherwise} \end{cases}. \tag{24}$$

Given the absence of aggregate uncertainty, in the long run the economy will reach a stationary equilibrium, i.e. steady state characterized by constant aggregate variables.

Definition 1. A stationary recursive equilibrium is a policy function $c(x;\lambda)$, a wage rate w, a price of emissions permits z, and a probability distribution λ such that:

1. The policy function solves the individual optimization problem (13).

2. The labor market clears:

$$\int_X \mathfrak{n} k\, d\lambda = N = \int_X \varepsilon l\, d\lambda.$$

3. The permits market clears:[24]

$$\int_X \mathfrak{e} k d\lambda = E \leq \bar{\mathrm{M}}.$$

4. The market for the final good clears:

$$C + K' - (1-\delta_K)\, K = Y = Q - \bar{p}E = \\ \Pi + (1+\tau_N)\, wN + \tau_E \bar{p}\bar{\mathrm{E}} + \mathrm{z}\left(\bar{\mathrm{M}} - \bar{\mathrm{E}}\right).$$

5. The distribution satisfies the induced law of motion:

$$\lambda\left(\mathrm{x}\right) = \int_X \mathcal{I}\left(\mathrm{k}, k, \sigma\right) \pi\left(\mathrm{q}, \sigma\right) d\lambda, \quad \forall \mathrm{x} \in X.$$

---

[24] Note that $z > 0$ if this equilibrium condition holds with equality.

		Carbon Tax (as share of energy price)				
		10%	20%	30%	40%	50%
RS1 (τ_N%)	H.A.M.	13.7	13.1	12.9	12.7	12.6
	H.A.S.	13.1	11.4	9.9	8.6	7.4
	Rep.A.	12.3	9.9	7.9	6.0	4.4
RS2 (a_0)	H.A.M.	0.248	0.244	0.242	0.241	0.240
	H.A.S.	0.243	0.230	0.218	0.207	0.196
	Rep.A.	0.234	0.213	0.194	0.176	0.159

Table A.1: Summary of calibrated fiscal parameters for recycling schemes 1-3.

		Carbon Tax (as share of energy price)				
		10%	20%	30%	40%	50%
Households		*% dev. of welfare from no-tax case*				
Full sample		**0.05**	**-0.02**	**-0.17**	**-0.40**	**-0.69**
Earning labor inc. only ($k = 0$)	*Low lab. inc.*	-0.56	-1.10	-1.63	-2.14	-2.63
	High lab. inc.	-2.34	-4.64	-6.96	-9.29	-11.62
All others ($k > 0$)	*Low lab. inc.*	0.48	0.78	0.95	1.02	1.01
	High lab. inc.	0.04	-0.08	-0.34	-0.70	-1.15
Households		*Pop. share (%)*				
Earning labor inc. only ($k = 0$)	*Low lab. inc.*	17.9	18.0	18.1	18.2	18.3
	High lab. inc.	0.4	0.4	0.4	0.4	0.4
All others ($k > 0$)	*Low lab. inc.*	32.1	32.0	31.9	31.8	31.7
	High lab. inc.	49.6	49.6	49.6	49.6	49.6

Table A.2: Welfare implications of TAX_RS1 experiments for different groups of households in the *Heterogeneous Agents Single Technology* model.

		Carbon Tax (as share of energy price)				
		10%	20%	30%	40%	50%
Households		*% dev. of welfare from no-tax case*				
Full sample		**0.54**	**0.90**	**1.11**	**1.22**	**1.23**
Earning labor inc. only ($k = 0$)	*Low lab. inc.*	-1.61	-3.03	-4.31	-5.50	-6.66
	High lab. inc.	0.12	0.16	0.01	-0.27	-0.73
All others ($k > 0$)	*Low lab. inc.*	1.55	2.76	3.72	4.49	5.14
	High lab. inc.	1.10	1.91	2.46	2.82	3.03
Households		*Pop. share (%)*				
Earning labor inc. only ($k = 0$)	*Low lab. inc.*	18.1	18.4	18.7	18.9	19.1
	High lab. inc.	0.3	0.3	0.3	0.3	0.3
All others ($k > 0$)	*Low lab. inc.*	31.9	31.6	31.3	31.1	30.9
	High lab. inc.	49.7	49.7	49.7	49.7	49.7

Table A.3: Welfare implications of TAX_RS2 experiments for different groups of households in the *Heterogeneous Agents Single Technology* model.