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The Evolution of Awareness and Belief Ambiguity During the Process of High School Track Choice*

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Abstract

In this article, we provide novel survey evidence on mid schoolers' awareness and ambiguity perceptions and on how such perceptions evolve during the process of high school track choice. Children in our study display partial awareness about the set of available tracks. Additionally, children report substantial belief ambiguity about their likelihood of a regular high school path, especially for lower-ranked tracks. Students start 8th grade with greater information about their favorite alternatives and continue to concentrate their search on the latter during the months before pre-enrollment. Children from less advantaged families display lower initial perceived knowledge and acquire information at a slower pace, particularly about college-preparatory schools.

JEL Codes: D83, I24, J24.

Keywords: Subjective Beliefs, Learning under Ambiguity and Limited Awareness, School Choice.

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1 Introduction

Human capital is crucial for a wide range of private and social outcomes, including skill mismatch, long-term unemployment, and income inequality. Schooling and early career decisions of children and young adults mark key stages in the process of accumulation of human capital, at at time when students may still feel uncertain about future consequences of alternative paths and may have limited knowledge about specific aspects of the decision problem. In this article, we use new repeated survey measures of perceived awareness and belief ambiguity to study 1) how middle schoolers' awareness sets over available high school tracks, and 2) how their belief ambiguity about the likelihood of experiencing a regular academic path in high school evolves during the first semester of the 8th grade before students pre-enroll in high school.

Expectations of students and their families are important elements of schooling and early career decisions, as people evaluate their options by their perceived prospective outcomes. For example, a student's school choice may depend on the student's belief about their likelihood of successfully and timely completing the curriculum. It may additionally depend on the student's belief about their chances of continuing onto college and/or finding a job after graduation. A growing body of literature has measured and analyzed survey reports of youths' and (in fewer cases) parents' expectations for these outcomes and related ones, elicited on a probabilistic 0-100 scale. See the reviews of Hartog and Diaz-Serrano (2014) and Giustinelli and Manski (2015).

The majority of these studies has assessed the 'validity' of survey reports of probabilistic expectations and/or has used expectations measures to estimate microeconometric models of schooling or early career choices under uncertainty (e.g., Dominitz and Manski (1996), Fischhoff et al. (2000), Arcidiacono et al. (2012), Zafar (2013), Wiswall and Zafar (2015*a*), Giustinelli (2015)). A smaller set of papers has elicited repeated measures of expectations over schooling decisions and their consequences with the goal of studying how individuals form and update human capital-related expectations in real life rather than in the lab (e.g., Stinebrickner and Stinebrickner (2012, 2014), Wiswall and Zafar (2015*b*)). All of these studies have analyzed survey reports of probabilistic expectations within the Bayesian paradigms of decision making and learning under (subjective) risk.

In real life, however, individuals and their families assimilate information from government announcements and media reports (e.g., school directories); communication from friends, extended family, and experts (e.g., school teachers); personal experiences and observations of the experiences of others (e.g., older relatives and friends). The sampling process generating these forms of information is obscure and likely to vary across individuals and families. In addition, the chances associated to future outcomes of consequential human capital decisions might be perceived as partly unknown and to some extent unknowable by students and their families at the time of choice. If so, such perceptions of uncertainty and the subsequent choice behaviors appear more germane to economic theories of subjective *uncertainty* rather than 'pure risk.'

Since Ellsberg (1961), the theoretical literature on choice under uncertainty has recognized the need to relax the assumption that decision makers hold a single vector of beliefs. Recent successful frameworks postulate that agents have 'multiple priors' or, equivalently, that agents hold a set of probability distributions over states and, hence, over choice consequences (e.g., Camerer and Weber (1992) and Gilboa and Marinacci (2013)). In these frameworks, each probability distribution is seen as a 'model' and situations where agents hold multiple probability distributions over states are referred to as ones of *ambiguity* or *model uncertainty*. The empirical literature, on the other hand, has not yet taken these frameworks to the data. In particular, we are not aware of any empirical study in economics that has collected measures of subjective belief ambiguity outside the lab or analyzed the evolution of ambiguity perceptions in a real-word context.

Schooling and early career decisions are also shaped by students' and parents' awareness (or lack thereof) of the available choice alternatives. Building on the most recent theoretical literature, we use the term *limited awareness* to denote situations of incomplete knowledge about choice alternatives, choice consequences, or causal relationships (e.g., Karni and Vierø (2013*b*,*a*, 2015)). For example, some youths and their families might not be aware of the existence of specific schools or careers that may be good matches for them. Even when aware of their existence, they might not know of, or consider, relevant institutional attributes of the schools. Despite the obvious importance of (limited) awareness for human capital investment and decisions, empirical studies on this topic are scant at best. The body of empirical work on (un)awareness to date has developed outside economics. As a consequence, the existing literature does not use survey questions to address the relative explanatory power of economic theories in the data.[1]

In this article, we begin to fill the above gaps by generating and analyzing survey measures of the degree of awareness about existing high school tracks and the extent of belief ambiguity about the likelihood of graduating in time from high school, as perceived by a sample of Italian 8th graders at multiple times during the months preceding pre-enrollment in high school. We first assess the information content and relevance for choice of our new measures by documenting their correlation patterns with respondents' background characteristics and their predictive power on observed pre-enrollment decisions. Then, we quantify the extents of students' perceived awareness and belief ambiguity at the beginning of 8th grade and we document the evolution of those perceptions over the decision process.

[1]Outside economics, Schneider et al. (2000) and Neild (2005) respectively provide quantitative and qualitative survey evidence suggesting that families' knowledge and information gathering styles vary with their socioeconomic status and other characteristics. Dawes and Brown (2002) and Hoxby and Avery (2012) respectively analyze prospective students' awareness of college alternatives and their knowledge of the admission process to college.

Our unique dataset enables us to answer questions such as: At the start of 8th grade, what high school tracks are children aware of? Do children hold ambiguous or unambiguous beliefs about the likelihood of successfully and timely graduating from alternative high school tracks? Among children who start off with limited awareness, does perceived awareness increase or decrease over the course of the decision process? Similarly, does initial perceived ambiguity about future school performance decrease or increase over the decision process? Does evolution of children's perceived awareness and belief ambiguity vary by choice alternative or by respondents' demographic and socioeconomic characteristics? If so, how?

Only recently has the theoretical literature reached some consensus on how learning should be modeled in the presence of multiple priors and of limited awareness and, thus, on how ambiguous expectations or expectations with limited awareness evolve (e.g., Marinacci (2002), Epstein and Schneider (2003, 2007), Karni and Vierø (2013*b*,*a*, 2015)). We read our data in light of these recent paradigms. Students in our study display limited awareness and a significant amount of ambiguity, both of which are particularly concentrated among the low-ranked alternatives. Additionally, students' learning is incomplete and concentrated on their most preferred alternatives. We find important degrees of heterogeneity both in initial conditions and in learning patterns across demographic and socioeconomic characteristics. For example, conditional on a set of background and individual characteristics such as the GPA, children with a more educated mother display intensive learning and one that is quite differentiated across tracks. Conversely, children with a father working in a blue-collar occupation display a more focused learning pattern, concentrated on curricula of the technical and vocational (non-general) tracks. Foreign-born children start with smaller awareness levels and higher ambiguity levels relative to their Italian counterparts and follow a ('biased') learning pattern whereby their level of belief ambiguity about general curricula tend to increase over time.

The paper is organized as follows. In Section 2 we describe the study and the sample. In Section 3 we present our main survey measures; we study heterogeneity of responses across students and choice alternatives near pre-enrollment; and we establish the relevance of our measures for observed pre-enrollment choices. In Section 4 we define the main objects of our empirical analysis and the rules governing their evolution over time within a generalized Bayesian framework. In Section 5 we conduct the empirical analysis with a special emphasis on the evolution of students' awareness and belief ambiguity over the choice process and their main dimensions of heterogeneity. We conclude in Section 6.

2 The Study

Institutional Background Our study generates and analyzes new survey measures of students' and parents' perceived awareness about alternative schooling options and belief ambiguity about the likelihood of future consequences of alternative schooling decisions within the context of high school track choice in Italy, a country whose schooling system features curricular specialization or tracking. Curricular specialization makes track selection a consequential human capital decision for students and one subject to greater uncertainties the younger the students at the time of tracking.[2]

Enrollment of Italian students into high school tracks –general, technical, and vocational (with additional sub-categories)– occurs non-selectively ('open enrollment') by family choice during the final year of middle school (8th grade), aided by teachers' non-binding counseling. Italian tracking has both 'rigid' and 'flexible' features. On the one hand, different tracks or curricula are generally offered in separate schools and track-switching occurs infrequently and can be costly time-wise. On the other hand, graduation certificates from the majority of curricula (including vocational ones) enable students to continue onto college, albeit at the cost of training and, hence, skill mismatch.

Table 1 lists the main tracks and sub-tracks of the Italian secondary education in the school year of our study (2011-2012). Two of the general curricula, the Music & Choral *Liceo* and the Social Sciences *Liceo*, were newly introduced in the Italian secondary system at the time of the survey.

Table 1: HIGH SCHOOL TRACKS AND SUB-TRACKS OFFERED IN 2011-2012

Track	**Sub-Track (or Curriculum)**	
General	Humanities	
General	Languages	
General	Mathematics & Science	
General	Art	
General	Music & Choral	Newly
General	Learning and Social Sciences	introduced
Technical	Economic Sector	
Technical	Technology Sector	
Vocational	Services	
Vocational	Industry & Crafts	
Vocational	Professional Training	

Study Design Our study consists of 4 survey waves, which were fielded between October 2011 and April 2012 on a sample of 8th graders and their parents in the Italian city of Vicenza.[3] The study targeted the universe of 11 public middle schools of the Vicenza Municipality, 10 of which

[2]Curricular specialization and other forms of educational tracking are the norm among the OECD countries. Allocation of students into tracks and other institutional features of tracking, however, vary greatly across countries.

[3]Vicenza is a mid-size city of the Italian North-East region of Veneto. The surveys were designed by Giustinelli in collaboration with a research team at the nearby University of Verona.

endorsed the study and were used as a sampling frame for the students entering 8th grade in the fall of 2011 and their parents (approximately 900 families).[4]

The focus on 8th graders and the survey timeline were motivated by existing evidence suggesting that families concentrate their choice effort during the fall and winter of the final year of junior high school.[5] Thus, wave 1 was fielded during the first month of school; wave 2 was fielded right before the Christmas break; wave 3 was fielded during the week preceding the deadline for pre-enrollment into high school (February 20th 2012); and wave 4 was fielded during the spring of 2012, after pre-enrollment. This design implies that students' pre-enrollment choices were observed within the study.[6]

Each of the first 3 survey waves featured one questionnaire for the students and one for the students' parents, while in wave 4 only the students were interviewed. Each questionnaire was self-administered by respondents using paper and pencil and took approximately 60-75 minutes to complete. In each wave, respondents were given 10-15 days to individually and privately complete the questionnaire in their homes and return it to school in a sealed envelope. Trained interviewers introduced the study and described the first questionnaire to the students in class, with an emphasis on the subjective expectations questions (described below). The interviewers were personally in charge of distributing and collecting the questionnaires in participating schools and of answering any clarification questions respondents may have and contact them about.

To incentivize participation children who answered and returned all 4 questionnaires were entered a lottery awarding one scientific calculator in each participating school and class (47 participating classes in total). In addition, families whose parents took and returned all 3 questionnaires were entered a lottery awarding a 100 Euros voucher in each participating school and class to be spent toward purchase of 9th grade textbooks for the participating child. 649 students and 619 parents returned a fully or partially completed questionnaire in wave 1, corresponding to participation rates of approximately 72% and 68% respectively. These participation rates are highly satisfactory for mail surveys.[7]

Sample Characteristics and Selection Basic demographic and physical characteristics of children were measured through questions eliciting their gender, month and year of birth, country of birth, year in which they moved to Italy (if born abroad), location where they live in Vicenza, their

[4]At the end of 2010, the Municipality of Vicenza had approximately 116,000 inhabitants, 999 of which were 12 years-old. About 16% of residents of the Vicenza Municipality were foreign born at the time of the study.

[5]This was indicated by respondents to the qualitative in-depth interviews the research team fielded during the study's development.

[6]In principle families may change their choice during the summer preceding high school entry. In practice only a small fraction of families ($< 5\%$) modify their pre-enrollment decision.

[7]Unfortunately, in-class administration was not an option in this study, as school principals objected that the number and length of the surveys would take up too much of children's classroom time.

height and weight. In addition, the surveys collected extensive information on family composition and on demographic and socioeconomic characteristics of parents and siblings (wave 1) and grandparents (wave 2) (e.g., gender, age, country of birth, year in which each family member moved to Italy if applicable, main language spoken at home, educational attainment, fields of secondary and tertiary degrees if applicable, employment status, occupation, etc.).

Tables 19, 20-21, and 22 in Appendix B provide a snapshot of participating children and parents in waves 1 and 3. Specifically, Table 19 shows the sample distribution of respondents' self-reported identity. In each wave, parents could choose between taking the survey jointly (if both present) or having one parent respond. They were asked to record their choice on the survey. Tables 20-21 show the sample distributions of children's demographic and socioeconomic characteristics. Similarly, Table 22 shows the sample distribution of responding parents' background characteristics.

Sample sizes reported in the column headings (N) refer to respondents' participation in the corresponding waves. Children participated at a slightly higher rate than parents in all waves and participation decreased across waves due to attrition both among children and parents. The reported sample sizes do not include item non-response. Non-response rates specific to individual questions are shown under the corresponding sample distributions. Sample statistics shown in Tables 20-21 and 22 enable us to assess the selectivity of children's and parents' samples at the time of pre-enrollment (wave 3) relative to their baseline counterparts (wave 1), along basic observable characteristics. Wave 3 samples do look slightly selected in expected directions relative to wave 1 samples (e.g., wave 3 features more female, younger/'regular-in-school,' and higher SES children and less foreign-born children), but these differences are modest overall.

3 Main Survey Measures

This section introduces the three measures of awareness, subjective beliefs, and belief ambiguity that are the object of this study. For each measure, we describe the survey question used for elicitation and present the sample distribution of students' responses near pre-enrollment. To 'validate' our measures, we analyze how they correlate with background characteristics of the students, including their demographics, socioeconomic characteristics, and GPA, and we document the predictive power of our measures on observed pre-enrollment choices. Thus, this section focuses on the analysis of students' perceptions measured in wave 3, arguably the relevant time for choice. In Section 5, where we document and study the evolution of our measures over the decision process, we start our analysis from responses in wave 1.

3.1 (Un)Awareness of Choice Alternatives

In the initial section of each survey (waves 1-3, child and parent questionnaires), respondents were asked the following question with reference to each of the curricula listed in Table 1:

What high school curricula do you know or have you heard the name of? (Please mark one.)	
(Curriculum name)	◯ I know it ◯ I have heard the name only ◯ I have never heard of it

The mutually exclusive response categories seek to measure three main levels of awareness (or lack thereof). Specifically, 'I have never heard of [track K]' aims to measure complete unawareness by the respondent about existence of track K. 'I have heard [track K]'s name only' aims to identify respondents who are aware of K's existence but have no or very limited knowledge about its characteristics. Finally, 'I know [track K]' aims to identify respondents who are aware of K's existence and have fairly refined knowledge about its characteristics.[8]

Table 2 shows the sample distributions of perceived awareness levels about all high school curricula offered in Vicenza in 2011-2012, reported by responding children in wave 3, that is, at the time in which such perceptions may be relevant to observed pre-enrollment decisions.[9] Figures shown in Table 2 suggest that by the time of pre-enrollment, the fraction of children indicating being completely unaware ('Never heard of') is fairly small, 5.51% across all curricula. However, this number masks substantial heterogeneity in reported unawareness across curricula, ranging from 1-2% for traditional curricula of the general track to 4-7% for technical and newly-introduced general curricula and 10-13% for vocational curricula. Also the fractions of children selecting 'Know' and 'Heard of' responses vary across curricula, with nearly 85% of students reporting knowing the general Math & Science curriculum and less than 40% of them reporting knowing the vocational Industry & Crafts curriculum.

We next ask whether the documented heterogeneity across curricula characterizing students' awareness reports at the time of pre-enrollment is systematically related to particular characteristics of the respondents and their families. Specifically, we construct two sets of three outcome variables counting (I) the number of curricula the student reports being aware of ('Know' or 'Heard of') and (II) the number of curricula the student indicates knowing ('Know') out of (i) the total number of

[8] A similar question was asked with reference to the schools of Vicenza. Clearly, the act itself of asking the question may be thought of as an 'existence awareness' treatment, which in turn might prompt information seeking about schools' and tracks' characteristics. While no randomization was implemented to avoid low power, having 3 repeated measures of awareness across the 3 waves and the fact that the potential treatment induced by this question might be reasonably assumed to be homogeneous across curricula enables us to potentially assess the extent to which respondents' knowledge and behavior might be modified as a result.

[9] Throughout the paper wave 2 responses were used for respondents who took wave 2 but not wave 3.

Table 2: AWARENESS: SAMPLE DISTRIBUTION OF CHILD'S AWARENESS PERCEPTIONS ABOUT CHOICE ALTERNATIVES NEAR PRE-ENROLLMENT (WAVE 3), BY CURRICULUM

	Wave 3 Sample[a]		
	Children (N=452)		
Curriculum:	**'Know'**	**'Heard of'**	**'Never heard of'**
All	61.54	32.95	5.51
Gen, Humanities	77.43	21.02	1.55
Gen, Languages	78.54	20.35	1.11
Gen, Math & Science	84.73	13.50	1.77
Gen, Art	70.13	28.10	1.77
Gen, Music & Choral	47.79	45.13	7.08
Gen, Soc Sciences	62.39	33.63	3.98
Tech, Economic Sector	55.75	39.16	5.09
Tech, Technology Sector	60.84	34.51	4.65
Voc, Services	49.34	40.71	9.96
Voc, Industry & Crafts	39.82	47.35	12.83
Voc, Prof Training	50.22	38.94	10.84

[a]: Constructed from responses in wave 3. Wave 2 responses used for respondents who did not participate in wave 3.

curricula across the three tracks, (ii) the total number of curricula in the general track, and (iii) the total number of curricula in the technical and vocational tracks. In Table 3, we use these variables (one per column) as outcomes of Poisson regressions for count data on a range of covariates that may be related to the amount of information or knowledge about schooling alternatives held by 8th graders and their families.

The set of regressors listed in the first column of Table 3 includes dummies for child's gender (1=female), child's country of birth (1=foreign born), family structure (1=child lives with both parents), maternal education (college or higher and high school), whether the child has a stay-home mom, and whether the child has a blue-collar dad. Additionally, the set of predictors includes the child's number of older siblings (who therefore have already attended or are attending high school) and the child's GPA at the end of 7th grade.

Four of these regressors (female, college+ mom, stay-home mom, and GPA) are statistically associated to at least one of the count outcomes. Such associations are limited to the 'knowledge margin' (columns 2, 4, and 6) rather than to the 'awareness margin' (remaining columns). In particular, inspection of the estimates shown in column 2 reveals that female students and students with a stay-home mom report knowing a higher number of curricula across the three tracks, while students with a higher GPA at the end of 7th grade report knowing a lower number of curricula.

The latter correlation may appear counterintuitive. However, separate predictions of the number of curricula children report knowing within the general track (column 4) and within the technical and vocational tracks (column 6) indicate that the observed pattern only applies to the latter set of curricula, that is, the alternatives that are arguably less relevant for, and less likely to be chosen by, high GPA students on average. In Section 5 we analyze this hypothesis in more detail.

Finally, the track-specific analyses of columns 4 and 6 reveal also that the larger size of the knowledge set displayed by female students is concentrated within the alternatives of the general track. Moreover, conditioning the analysis on the general curricula reveals that children with a highly educated mother (college+) tend to report a larger knowledge set over these curricula than children with a lower educated mother.[10] Once again, this pattern suggests that children's reported knowledge of the available curricula is concentrated on those alternatives that are likely to be more relevant to them.

3.2 Probabilistic Beliefs and Measures of Model Uncertainty

The questions eliciting respondents' awareness levels about alternative curricula offered in Vicenza were followed by a sequence of questions eliciting their probabilistic expectations of choosing each curriculum and their expectations for a range of future outcomes or consequences of choosing alternative curricula. In particular, respondents were first asked to rank the curricula listed in Table 1 from their most preferred one to the least preferred one. Then, they were asked to assign a number between 0 and 100 to the chance that they would choose each of the listed curricula. Finally, respondents were asked their perception of the likelihood of a range of future events, some of which occur during high school (e.g., enjoyment, effort, graduation, etc.) and others are contingent on graduating from high school (e.g., enrolling in college or finding a job). The complete list of outcomes is shown in Table 23 of Appendix B.[11]

Each of the questions eliciting respondents' expectations for specific events following choice of alternative curricula was structured into three components or sub-questions. The first sub-question asked the respondent to assign an individual value between 0 and 100 percent to the likelihood of the event specified in the question ('point belief'). The second sub-question asked the respondent to indicate how sure they felt about their point belief. The third sub-question asked the respondents

[10]Consistent with existing empirical evidence, father's educational attainment does not have additional explanatory power over mother's education. Hence, we include the latter only.

[11]Choice of the vector of outcomes over which subjects' expectations were measured was informed by the literature on schooling and early career choice (reviewed by Hartog and Diaz-Serrano (2014)), by previous experience and findings from a related study by one of the authors (Giustinelli, 2015), and by respondents' answers to a set of qualitative in-depth interviews fielded during the development of the current study.

Table 3: Awareness: Predictors of Number of Alternatives the Child Reports Being Aware of Near Pre-Enrollment (Wave 3)

Poisson Regression of N of Alternatives the Child is Aware of near Pre-Enrollment:

	All Curricula		General		Technical & Vocational	
Predictors	**'Know' or 'Heard of'**	**'Know'**	**'Know' or 'Heard of'**	**'Know'**	**'Know' or 'Heard of'**	**'Know'**
female	0.00149 (0.03651)	0.10043** (0.04523)	0.00877 (0.04874)	0.13856** (0.05715)	−0.00789 (0.05513)	0.03039 (0.07419)
foreign born	−0.05516 (0.07000)	−0.14402 (0.08828)	−0.04337 (0.09295)	−0.15044 (0.11231)	−0.07044 (0.10642)	−0.13368 (0.14295)
lives with both parents	0.02708 (0.06253)	0.05694 (0.07799)	0.04163 (0.08368)	0.04329 (0.09655)	0.00829 (0.09412)	0.08119 (0.13235)
mom has college+ degree	−0.03861 (0.05797)	0.05390 (0.07220)	−0.00208 (0.07758)	0.16594* (0.09118)	−0.08521 (0.08727)	−0.14779 (0.12016)
mom has HS degree	−0.03145 (0.04911)	0.03008 (0.06106)	−0.00855 (0.06608)	0.07583 (0.07924)	−0.06000 (0.07342)	−0.03140 (0.09610)
has stay-home mom	0.00528 (0.04353)	0.09627* (0.05290)	0.00438 (0.05819)	0.05806 (0.06748)	0.00641 (0.06560)	0.15500* (0.08539)
has blue-collar dad	−0.01726 (0.04523)	−0.00166 (0.05546)	−0.02964 (0.06063)	−0.02918 (0.07139)	−0.00164 (0.06793)	0.04052 (0.08831)
n of older siblings	0.01599 (0.02522)	−0.02587 (0.03141)	0.00691 (0.03374)	−0.01786 (0.03944)	0.02760 (0.03798)	−0.03925 (0.05193)
7th-grade GPA	−0.00025 (0.02154)	−0.05114* (0.02659)	0.00084 (0.02874)	0.01780 (0.03340)	−0.00165 (0.03253)	−0.17180*** (0.04424)
constant	2.35354*** (0.17597)	2.19062*** (0.21699)	1.73670*** (0.23515)	1.14312** (0.27462)	1.57927*** (0.26533)	2.19532*** (0.35617)
F	2.06	13.57	0.91	15.64	2.43	32.07
Prob $> F$	0.9906	0.1385	0.9996	0.0747	0.9826	0.0002
R^2	0.0016	0.0083	0.0008	0.0124	0.0022	0.0249
Sample Size	304	304	304	304	304	304

***: significant at 1%, **: significant at 5%, *: significant at 10%.
Wave 2 responses used for respondents who did not participate in wave 3.

who reported being unsure to provide a range of chances, as shown below.[12]

Curriculum	**Number of Chances (between 0 and 100)**	**How sure do you feel about your answer? (Please mark one.)**
(Curriculum name)	- - -	◯ I am sure about my answer ◯ I am unsure about my answer **minimum** chances: ...... **maximum** chances: ...... ◯ I have no idea about the chances

We interpret 'I am sure about my answer,' as an expression of precise subjective beliefs or perceived lack of ambiguity; 'I am unsure about my answer,' as an expression of perceived partial ambiguity

[12] Manski (2004) argues in favor of allowing respondents to report their beliefs using ranges of chance. Manski and Molinari (2010) pilot the idea on the American Life Panel (ALP). Wallsten et al. (1983) review earlier measurement attempts in psychology using numerical ranges.

Table 4: POINT BELIEFS: SAMPLE DISTRIBUTION OF CHILD'S SUBJECTIVE BELIEF ABOUT THE PROBABILITY OF GRADUATING IN TIME FROM HIGH SCHOOL NEAR PRE-ENROLLMENT (WAVE 3), BY CURRICULUM

	Wave 3 Sample[a]						
	Children (N in 376-386)						
Curriculum:	**Mean**	**Std Dev**	**Q10**	**Q25**	**Median**	**Q75**	**Q90**
Gen, Humanities	46.64	30.38	0	20	50	70	90
Gen, Languages	52.56	31.77	3	25	50	80	98
Gen, Math & Science	53.72	32.46	2	30	50	80	99
Gen, Art & Music	53.10	32.65	0	20	50	80	100
Gen, Soc Sciences	53.22	31.89	5	30	50	80	100
Tech, Economic Sector	54.83	31.33	5	30	50	80	100
Tech, Technology Sector	56.27	30.43	5	40	50	80	100
Voc, Services	58.73	31.05	10	50	50	90	100
Voc, Industry & Crafts	55.46	31.77	5	40	50	80	100
Voc, Prof Training	60.49	33.45	8	50	50	95	100

[a]: Constructed from responses in wave 3. Wave 2 responses used for respondents who did not participate in wave 3.

quantified by the difference between the maximum and minimum percent-chance beliefs; and 'I have no idea about the chances,' as an expression of perceived total ambiguity.

In the current analysis, we focus on students' perceptions about the probability of passing all grades on the first try and graduating in the regular time from alternative high school curricula. Choice of this outcome yields an interpretation of our measure that is closest to a genuine subjective belief as defined in the theoretical section. Moreover, students' expectations reports about their future performance in high school have been shown to be significant predictors of observed high school track choices among Italian children (see Giustinelli (2015) and the analysis below).

Tables 4 through 6 present the main features of the distributions of responses to the above set of questions in the students' sample. Specifically, Table 4 displays the mean, standard deviation, and main quantiles of students' subjective probabilities of graduating in time from each curriculum offered in Vicenza. Table 5 shows for each curriculum the distribution of reported ambiguity levels about the point probability of graduating in time. Table 6 displays the mean, standard deviation, and main quantiles of subjective ranges among students who reported being unsure about the chances of graduating in time from the corresponding curricula.

The statistics shown in Tables 4-6 indicate that survey reports of probabilistic point beliefs and belief ambiguity vary substantially both across students and across curricula. In particular, students' reports of their subjective probability of successfully and timely graduating from high school

Table 5: AMBIGUITY I: SAMPLE DISTRIBUTION OF CHILD'S AMBIGUITY PERCEPTIONS ABOUT THE PROBABILITY OF GRADUATING IN TIME FROM HIGH SCHOOL NEAR PRE-ENROLLMENT (WAVE 3), BY CURRICULUM

	Wave 3 Sample[a]		
	Children (N in 376-386)		
Curriculum:	**'Sure'**	**'Unsure'**	**'No Idea'**
All	78.03	4.09	17.88
Gen, Humanities	82.55	4.69	12.76
Gen, Languages	83.16	4.92	11.92
Gen, Math & Science	83.12	5.97	10.91
Gen, Art & Music	84.46	3.63	11.92
Gen, Soc Sciences	81.15	3.14	15.71
Tech, Economic Sector	74.93	4.75	20.32
Tech, Technology Sector	75.98	4.18	19.84
Voc, Services	72.41	4.51	23.08
Voc, Industry & Crafts	70.29	3.18	26.53
Voc, Prof Training	71.81	1.86	26.33

[a]: Constructed from responses in wave 3. Wave 2 responses used for respondents who did not participate in wave 3.

Table 6: AMBIGUITY II: SAMPLE DISTRIBUTION OF CHILD'S SUBJECTIVE RANGES AROUND THE PROBABILITY OF GRADUATING IN TIME FROM HIGH SCHOOL NEAR PRE-ENROLLMENT (WAVE 3), CONDITIONAL ON 'UNSURE' REPORTS AND BY CURRICULUM

	Wave 3 Sample[a]							
	Children (N in 7-23)							
	Mean	**Std Dev**	**Q10**	**Q25**	**Median**	**Q75**	**Q90**	**N**
Gen, Humanities	42.78	33.88	10	20	30	60	100	18
Gen, Languages	51.32	36.51	10	20	40	100	100	19
Gen, Math & Science	45.22	36.99	10	10	35	100	100	23
Gen, Art & Music	40.86	35.29	10	17	25	60	100	14
Gen, Soc Sciences	48.42	40.02	5	17	35	100	100	12
Tech, Economic Sector	47.78	35.74	10	15	37	100	100	18
Tech, Technology Sector	36.50	33.74	7	12	25	45	100	16
Voc, Services	50.12	39.91	5	10	32	100	100	17
Voc, Industry & Crafts	48	40.64	7	10	34	100	100	12
Voc, Prof Training	38.57	30.78	10	10	30	50	100	7

[a]: Constructed from responses in wave 3. Wave 2 responses used for respondents who did not participate in wave 3.

span the whole 0-100 scale for most curricula. However, the curriculum-specific distributions of responses display a distinctive pattern, which reflects how challenging students believe each curriculum would be for them if they were to attend that curriculum. On average, students assign lower probabilities of graduating in the regular time to curricula of the general track (especially the Humanities curriculum) and somewhat higher probabilities to curricula of the vocational tracks (especially the Professional Training curriculum). This pattern suggests that students tend to view technical and vocational curricula as less academically challenging than curricula of the general track. At the same time, students display lower confidence (or greater ambiguity) on average about their subjective likelihood of graduating in time from any of the curricula of the technical or vocational tracks than from those of the general track.

Once again we are interested in whether the documented dimensions of heterogeneity in students' expectations reports and in their perceptions of belief ambiguity are systematically related to specific characteristics of respondents. To this end, Tables 7 and 8 present the estimates of two prediction exercises similar to the one performed in Table 3 for the awareness reports.

In Table 7, we estimate mean linear regressions of the subjective probability of graduating in time from alternative curricula (one per column) on a vector of covariates listed in the first column (same as in Table 3). Unsurprisingly –and in fact reassuringly– students' GPA in the end-of-year report of 7th grade is the most important predictor of students' subjective likelihood of graduating in time from high school, regardless of the curriculum. On the other hand, the gender and mother education dummies feature distinctive patterns, specific to particular curricula or tracks.

Consistent with previously documented under-confidence among girls about their ability or performance in scientific and technical disciplines, female gender negatively and significantly predicts children's subjective likelihood of a regular academic path in the general Math & Science curriculum, the technical curriculum of the Technology Sector, and the vocational Industry & Crafts curriculum. Having a college-educated mother positively and significantly predicts children's subjective probability of successfully and timely graduating from any curriculum of the general track; whereas having a mother with a high school diploma positively and significantly predicts children's subjective probability of graduating in time from technical or vocational curricula.

Next, we use students' ambiguity perceptions to construct two sets of three outcome variables counting the number of curricula for which each child reports having (I) a partially or fully ambiguous belief ('Unsure' or 'No idea'), or (II) a fully ambiguous belief ('No idea') about the probability of graduating in time, out of (i) the total number of curricula across the three tracks, (ii) the total number of curricula in the general track, and (iii) the total number of curricula in the technical and vocational tracks. In Table 8, we perform Poisson regressions of these variables (one per column) on the usual vector of covariates.

Table 7: Point Beliefs: Predictors of Child's Point Beliefs about the Probability of Graduating in Time from High School Near Pre-Enrollment (Wave 3)

Mean Linear Regression of Child's Point Belief of Graduating in Time from Curriculum:

Predictors	**Gen Hum**	**Gen Lang**	**Gen Math**	**Gen Art&Music**	**Gen Soc Sci**	**Tech Econ Sect**	**Tech Tech Sect**	**Voc Serv**	**Voc Ind**	**Voc Prof Train**
female	−3.19669	1.06568	−12.32106***	4.319253	4.72241	−0.00139	−7.90252**	−3.64416	−6.76359*	−2.94493
	(3.16978)	(3.41433)	(3.27637)	(3.76956)	(3.45525)	(3.54265)	(3.45876)	(3.78616)	(3.76533)	(4.07813)
foreign born	−2.86825	1.39694	3.80086	2.642378	1.45056	−6.94881	−11.1212*	2.39186	−0.94121	10.2415
	(6.07218)	(6.54065)	(6.27637)	(7.22115)	(6.61905)	(6.78647)	(6.62576)	(7.25295)	(7.21305)	(7.81227)
lives with both parents	−4.62434	0.29681	−11.67748**	−5.367129	−4.57072	−4.95079	−8.46522	−6.93368	−9.95766	−10.1297
	(5.39202)	(5.80801)	(5.57334)	(6.41229)	(5.87763)	(6.02630)	(5.88359)	(6.44052)	(6.40509)	(6.93719)
mom has college+ degree	13.2056**	14.8733***	11.91837**	17.27477***	18.8491***	11.6383**	8.99533	10.1826	6.58880	−1.84784
	(5.27587)	(5.68291)	(5.45329)	(6.2741)	(5.75102)	(5.89649)	(5.75685)	(6.30179)	(6.26713)	(6.78777)
mom has HS degree	6.47282	7.69546	4.169954	8.258499	7.35142	9.45384*	12.7380***	12.2767**	11.4673**	2.48369
	(4.51954)	(4.86823)	(4.67153)	(5.37473)	(4.92658)	(5.05119)	(4.93158)	(5.39839)	(5.36870)	(5.81470)
has stay-home mom	1.82237	6.16906	−.2890856	10.45195**	6.05124	6.14543	1.68818	4.01290	7.94084*	2.67237
	(3.80489)	(4.09844)	(3.93284)	(4.52485)	(4.14756)	(4.25247)	(4.15177)	(4.54477)	(4.51977)	(4.89525)
has blue-collar dad	0.13511	−1.51958	−4.543657	1.044239	−.248156	−8.45434*	−0.60805	−4.44544	−6.93639	−1.66939
	(3.93363)	(4.23711)	(4.06591)	(4.67795)	(4.28790)	(4.39636)	(4.29225)	(4.69855)	(4.67270)	(5.06088)
n of older siblings	1.02442	0.04197	2.372214	−1.358993	1.7645	−0.99312	0.29649	0.31079	−1.59732	2.07429
	(2.21363)	(2.38441)	(2.2880)	(2.63249)	(2.41299)	(2.47402)	(2.41543)	(2.64408)	(2.62953)	(2.84798)
7th-grade GPA/grade	16.0295***	15.1137***	18.0869***	11.47157***	12.8895***	10.1757***	10.9596***	6.98083***	9.06636***	7.49641***
	(1.87294)	(2.01744)	(1.93592)	(2.22733)	(2.04162)	(2.09326)	(2.04369)	(2.23714)	(2.22484)	(2.40966)
constant	−77.2605***	−73.7455***	−72.22443***	−42.90963**	−54.1290***	−23.7213	−23.5556	5.51424	−6.94051	13.2368
	(15.4791)	(16.6733)	(15.9996)	(18.4080)	(16.8732)	(17.3)	(16.8903)	(18.4891)	(18.3874)	(19.9149)
F	12.01	10	15.44	6.13	8.85	5.95	5.98	2.57	4.14	1.54
Prob $> F$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0075	0.0001	0.1351
R^2	0.3002	0.2632	0.3554	0.1795	0.2401	0.1753	0.1761	0.0842	0.1288	0.0521
Sample Size	262	262	262	262	262	262	262	262	262	262

***: significant at 1%, **: significant at 5%, *: significant at 10%. Wave 2 responses used for respondents who did not participate in wave 3.

The majority of regressors are statistically associated to at least one of the count outcomes. In columns 1-2, being female and being foreign born are positively and statistically significantly associated to the number of curricula for which participating students reported having a partially or totally ambiguous belief about the likelihood of experiencing a regular high school path. Whereas, living with both parents, having a college-educated mother or a mother with a high school diploma, and having a blue-collar father are negatively and statistically significantly associated to the number of curricula for which the respondents have totally (and in one case partially) ambiguous beliefs.

Estimates in columns 3-6 reveal that gender, immigration status, socioeconomic background, and GPA at the end of 7th grade display strong track-specific patterns. The higher level of ambiguity reported by female and foreign-born students and the lower level of ambiguity reported by students with a higher GPA are specific to curricula of the general track; whereas the lower level of ambiguity reported by children with a blue-collar father is specific to curricula of the technical and vocational tracks.

The regression coefficients of GPA and the blue-collar indicator point to significantly lower ambiguity at the time of pre-enrollment corresponding to the two subsets of curricula, general and technical/vocational respectively, that are likely to be more relevant to the sub-groups of respondents identified by those variables. Although not necessarily surprising, the pattern of the ambiguity perceptions of children with a blue-collar father might be viewed as troublesome from a policy perspective, especially if due to constraints underlying these children's socioeconomic conditions (e.g., liquidity constraints), rather than to their individual preferences (assuming the two can be separated). A similar argument can be made about the pattern observed for the ambiguity perceptions of foreign-born students (greater ambiguity for curricula of the general track), particularly as these patterns hold conditional on the remaining covariates (including GPA). We return to these issues in the second part of the paper, where we study the evolution of students' awareness and ambiguity reports.

3.3 Relevance for Choice

The main focus of this paper is to document and study the evolution of students' awareness and ambiguity perceptions over the process of high school track choice. A necessary condition for our analysis to be meaningful and relevant from both a theoretical and an empirical standpoint is that our measures of perceived awareness and belief ambiguity be relevant to high school choice. Therefore, before proceeding with the evolution analysis, we assess whether such perceptions measured at the time of pre-enrollment are statistically predictive of observed pre-enrollment decisions, unconditional and conditional on students' characteristics.

Table 8: AMBIGUITY: PREDICTORS OF NUMBER OF ALTERNATIVES THE CHILD REPORTS HAVING AMBIGUOUS BELIEF ABOUT THE LIKELIHOOD OF GRADUATING IN TIME FROM HIGH SCHOOL NEAR PRE-ENROLLMENT (WAVE 3)

Poisson Regression of N of Alt. for which the Child has Ambiguous Subjective Likelihood of Graduating in Time:

	All Curricula		General		Technical & Vocational	
Predictors	**'Unsure' or 'No Idea'**	**'No Idea'**	**'No Idea' or 'No Idea'**	**'Unsure'**	**'Unsure' or 'No Idea'**	**'No Idea'**
female	0.21185** (0.08497)	0.12449 (0.09354)	0.29581** (0.13782)	0.14386 (0.15928)	0.16081 (0.10799)	0.10995 (0.11574)
foreign born	0.31043** (0.14342)	0.30628* (0.16573)	0.47367** (0.20796)	0.42468* (0.25516)	0.15681 (0.19965)	0.20477 (0.21927)
lives with both parents	−0.54472*** (0.11818)	−0.68811*** (0.12845)	−0.55622*** (0.18626)	−0.80558*** (0.21174)	−0.52786*** (0.15304)	−0.61704*** (0.16186)
mom has college+ degree	−0.34171** (0.13720)	−0.60448*** (0.14570)	−0.42426* (0.22943)	−0.89188*** (0.26072)	−0.29317 (0.17136)	−0.46212*** (0.17749)
mom has HS degree	−0.17989 (0.11573)	−0.44635*** (0.12179)	−0.10992 (0.18769)	−0.47005** (0.20100)	−0.22165 (0.14708)	−0.41651*** (0.15353)
has stay-home mom	−0.12021 (0.10382)	−0.07554 (0.11516)	−0.31069* (0.17191)	−0.19968 (0.19812)	−0.00743 (0.13043)	−0.00941 (0.14138)
has blue-collar dad	−0.16276 (0.10480)	−0.33446*** (0.12016)	−0.18018 (0.16493)	−0.32649* (0.19459)	−0.14810 (0.13596)	−0.34413** (0.15371)
n of older siblings	0.03679 (0.05754)	−0.08857 (0.06687)	0.11033 (0.09026)	−0.07729 (0.11203)	−0.01703 (0.07495)	−0.09941 (0.08339)
7th-grade GPA	−0.03604 (0.04919)	−0.04788 (0.05449)	−0.21569*** (0.08052)	−0.34563*** (0.09596)	0.07438 (0.06259)	0.10336 (0.06711)
constant	1.70326*** (0.39697)	2.07660*** (0.43684)	2.04617*** (0.63577)	3.44953*** (0.73744)	0.39623 (0.51172)	0.36567 (0.54913)
F	38.33	48.51	40.15	47.74	17.42	28.31
Prob $> F$	0.0000	0.0000	0.0000	0.0000	0.0426	0.0008
R^2	0.0225	0.0312	0.0447	0.0632	0.0154	0.0264
Sample Size	262	262	262	262	262	262

***: significant at 1%, **: significant at 5%, *: significant at 10%.
Wave 2 responses used for respondents who did not participate in wave 3.

Table 9 reports the estimated coefficients of a sets of multinomial logit regressions of observed pre-enrollment choice on two alternative-specific indicators of awareness level (columns 1-2), on the alternative-specific graduation probabilities (column 3-4), and on one alternative-specific indicator of lack of belief ambiguity about the graduating likelihood (columns 5-6). Regressions in columns 1, 3, and 5 include only the latter measures and a vector of alternative-specific constants; whereas regressions in columns 2, 4, and 6 additionally condition on the usual set of covariates (coefficients not shown).[13] All regressions are run with the general Math & Science curriculum omitted, which is therefore used at the reference alternative.

[13]Because the conditioning background variables are not alternative-specific, the multinomial logit model implies a total number of associated coefficients equal to the number of regressors times the number of choice alternatives minus 1. In the interest of space and in order to focus the attention on the estimated coefficients of the key awareness, belief, and ambiguity measures, we do not report the estimated coefficients of the remaining covariates. These estimates are available from the authors upon request.

Table 9: AWARENESS, POINT BELIEF, AND AMBIGUITY MEASURES NEAR PRE-ENROLLMENT (WAVE 3) PREDICT OBSERVED PRE-ENROLLMENT CHOICES

Multinomial Logit Regression of Pre-enrollment Choices on Awareness, Point Belief, and Ambiguity Reports:[a]

	Awareness		**Point Belief**		**Ambiguity**	
Predictors	**Awareness Only**	**With Covariates[b]**	**Point Belief Only**	**With Covariates[b]**	**Ambiguity Only**	**With Covariates[b]**
Awarness: 'Know' dummy	3.47091*** (0.54614)	3.11250*** (0.59193)	–	–	–	–
Awarness: 'Heard of' dummy	0.12753 (0.29126)	0.01095 (0.34101)	–	–	–	–
Point Belief	–	–	0.07216*** (0.00779)	0.07542*** (0.00947)	–	–
Ambiguity: 'Sure' dummy	–	–	–	–	1.97775*** (0.41001)	2.09012*** (0.51577)
Wald χ^2	43.09	117.88	85.71	120.14	23.27	102.69
Prob $> \chi^2$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Sample Size	213	213	199	199	199	199

***: significant at 1%, **: significant at 5%, *: significant at 10%.

[a]: Wave 2 responses used for respondents who did not participate in wave 3.

[b]: Covariates: female; foreign born; lives with both parents; mom has college+ degree; mom has HS degree; has stay-home mom; has blue-collar dad; n of older siblings; 7th-grade GPA; constant.

The estimates shown in Table 9 confirm that our key measures are correlated with observed pre-enrollment choices and are strongly statistically predictive of those choices above and beyond main background characteristics of the students and their families. Consistent with intuition, knowing and being aware of a choice alternative positively predict choice of that alternative, although only the 'Know' margin is statistically significant. One's subjective likelihood of experiencing a regular academic path in a high school curriculum positively and statistically significantly predicts choice of that curriculum conditional on GPA and socioeconomic characteristics, once again confirming the relevance of the selected outcome for choice. Finally, reporting a non-ambiguous belief about the likelihood of successfully and timely graduating from a curriculum positively and statistically significantly predicts choice of that curriculum.

4 Belief Formation and Evolution: Interpretative Framework

This section provides a theory-based interpretative framework for the measures of awareness, probabilistic point beliefs, and belief ambiguity that we introduced and described in Section 3 and for how such measures evolve over time.

To lay out notation, let us consider an economic decision framework consisting of a set of *states* Ω (assumed to be finite), an associated set of *events* Σ (formed by all subsets of Ω), and a *model* m

consisting of a probability measure over Σ. There is a finite set of *consequences* Z and a finite set of acts A over Ω, which generates consequences for each realized state, $a : \Omega \to Z$. Thus, each act $a \in A$ induces a probability distribution p_a on the associated payoff-relevant events $\mathcal{Z}$ (the set of all subsets of Z) via m, as follows: $p^a(B) = m(\{\omega \in \Omega : a(\omega) \in B\})$, for all $B \in \mathcal{Z}$ (e.g., Kreps (1988), especially chapters 8-10).

Within this basic framework, track choice can be modelled as a subset of the set of acts, $\hat{A} = \{a_1, \ldots a_j, \ldots, a_N\} \subset A$.[14] In each of the N tracks, the student can either pass or fail. Let us define a partition of $\Omega = \Omega_1 \times \Omega_2$, where the typical state can be represented as a vector $\omega = (\omega_1, \omega_2)$. Ω_1 is the set of states directly related to the tracks and has 2^N elements.[15] Each $\omega_1 \in \Omega_1$ is a vector of length N of zeroes and ones, where a 1 in position j denotes 'pass in track a_j' and 0 in the same position denotes 'fail in track a_j'.

A probability distribution over Ω_1 can be described by a vector of marginal probability distributions $\Pi_0 = \{\pi_0^a\}_{a \in \hat{A}}$, where π_0^a denotes the probability of passing in track a. The zero subscript indicates that these probabilities are obtained using the unconditional probability according to model m. For each $j = 1, 2, \ldots, N$, let $C_j \subset \Omega$ be the 'pass-in-track-j set,' which is associated to act a_j. Then,

$$C_j = \{\omega \in \Omega : \omega_1^j = 1\}$$

and, hence,

$$\pi_0^{a_j} = m\left(C_j\right) = m(\{\omega \in \Omega : \omega_1^j = 1\}). \tag{1}$$

We now use the outlined framework to describe the 3 main theoretical paradigms we consider in the empirical analysis.

Subjective Expected Utility Child The standard subjective expected utility (SEU) model postulates that the child holds one probability model m over an immutable set of states Ω; it also assumes that the child knows about all feasible available tracks and is able to forecast all possible consequences of choosing each one of them.

Because the utility of the child is insensitive to states that do not concern the chosen track, we normalize the corresponding payoffs to zero. The expected payoff from choosing $a \in \hat{A}$ is thus

$$\beta^a \pi_0^a + \varepsilon^a,$$

where ε^a denotes the child's subjective taste for track a and β^a represents the additional payoff to the child associated to getting a degree from track a. β^a also accounts for the curvature of the

[14] We can of course allow for probabilistic choices.

[15] Of course, in presence of multiple agents, a set may be associated to each one of them. In this case a full description of the state of the world requires a set Ω_1 represented by the product of all individual sets. For notational simplicity, we focus on the decision problem of one child in isolation.

(Bernoulli) utility over consumption and wealth.

Although the realization of events which follow graduation from a specific track (i.e., elements in Ω_1) are revealed only after the child has chosen that track and taken all exams, other events in Σ (those sensitive to changes in elements in Ω_2) might occur before choice of a specific track. Such events constitute *signals* that may be correlated to events in Ω_1, including the arrival of new information about the grading standard applied by teachers of a specific school or the inclusion of a particular topic in the academic curriculum of the track in question.

With regard to the evolution of subjective beliefs over time, the standard model assumes that decision makers update their beliefs according to Bayes' Rule. This assumption is a necessary and sufficient condition for the described SEU preferences to be time consistent (e.g., Hammond (1988) and Machina (1989)). Similar to the vector Π_0 summarizing the (marginalized) subjective probabilities held by the child at date $t = 0$, $\Pi_t(\mathcal{I}_t)$ represents the vector of probabilities held by the child at times $t = 1, 2, 3$ (e.g., the three survey waves), which are obtained by marginalization using the conditional probabilities $m(\cdot|\mathcal{I}_t)$,

$$\pi_t^{a_j}(\mathcal{I}_t) = \frac{m\left(C_j \cap \mathcal{I}_t\right)}{m\left(\mathcal{I}_t\right)} = m(\{\omega \in \Omega : \omega_1^j = 1\}\ |\mathcal{I}_t), \tag{2}$$

where $\mathcal{I}_t$ indicates the child's 'information set' at date t, with $\mathcal{I}_0 = \Omega$.

The empirical literature on subjective expectations has collected and analyzed survey reports of probabilistic point beliefs having this standard framework in mind. Accordingly, the percent-chance measure of the student's subjective likelihood of a successful and timely graduation that we presented in Section 3 is a natural measure for π_t^a, as long as one maintains that the student has a *single probability model* in mind and is aware of all choice alternatives available to them. The possibility that some students may hold partially ambiguous beliefs about the likelihood of succeeding in alternative high school tracks and/or that they may not be aware of all of the tracks available in their location motivates the collection of the additional two measures of awareness and belief ambiguity introduced in Section 3. The latter measure has a natural interpretation within existing theoretical frameworks of choice and learning under ambiguity, to which we turn next.

Model Uncertainty and Ambiguous Child Let us now suppose that the child believes that m might not be the only possible probability model and that there may be a set M_0 of possible models (e.g., Gilboa and Marinacci (2013)) instead. A non-singleton M_0 may emerge whenever the child does not have enough information to compute a unique vector of probabilities. In this case, the probability of passing in track j is no longer a single number, but a set of probabilities.

Each of the models m in M_0 can be used to obtain a vector of probabilities $\Pi_0^m = \{\pi_0^{a,m}\}_{a \in \hat{A}}$. For each track $a \in \hat{A}$, let us define the upper and lower bounds of such probabilities, as follows:

$$\overline{\pi}_0^a = \max_{m \in M_0} \pi_0^{a,m} \quad \text{and} \quad \underline{\pi}_0^a = \min_{m \in M_0} \pi_0^{a,m}. \tag{3}$$

The difference $R_0^a = \overline{\pi}_0^a - \underline{\pi}_0^a$ gives a natural measure of model uncertainty (or ambiguity) perceived by the child regarding the likelihood of passing in track a. This difference is zero if an only if the child does not perceive any relevant amount of ambiguity regarding the likelihood of passing in track a. Most theories of decision with model uncertainty consider the upper and lower bounds we just defined the crucial determinants of choice.

Ambiguity Aversion. Based on the assumptions of Gilboa and Schmeidler (1989)'s seminal paper, the choice-relevant utility of graduating from track $a \in \hat{A}$ to an ambiguity-averse child (with $\beta^a \geq 0$) is

$$\beta^a \underline{\pi}_0^a + \varepsilon^a,$$

where ε^a and β^a have the same interpretation as above. Because obtaining a degree is a 'good' outcome (relative to failing to do so), in this model only the minimum probability of graduating matters for the choice of an ambiguous-averse child.

Ghirardato et al. (2004) propose an extension of Gilboa and Schmeidler (1989), called the α-maxmin model. This model implies the following payoff for the same choice:

$$\beta^a [\alpha \overline{\pi}_0^a + (1-\alpha)\underline{\pi}_0^a] + \varepsilon^a = \beta^a \underline{\pi}_0^a + \beta^a \alpha R_0^a + \varepsilon^a,$$

where $\beta^a \alpha$ captures the sensitivity of the decision maker to the 'degree on ambiguity' measured by the range R_0^a.

An important recent generalization of the ambiguity aversion model is given by the *variational preferences* framework developed in Maccheroni et al. (2006*a*,*b*). This class of models nests the original formulation of Gilboa and Schmeidler (1989) as well as the 'robust control' approach of Hansen and Sargent (2001) as special cases.[16]

Learning. In presence of ambiguity, the child may update his subjective probabilities for each model. He may also update the set of possible models. Recall that the 'marginalized' version of each model $m \in M_0$ at $t = 0$ on the curriculum-related entries is represented by the vector Π_0^m. For each $t \in \{1, 2, 3\}$, $\mathcal{I}_t \subset \Omega$, and $m \in M_0$, we obtain new probability measures over the set of

[16] Variational preferences imply the following payoff over chosen tracks: $\min_{m \in \Delta(\Omega)} \beta^a \pi_0^{a,m} + c_0(m) + \varepsilon^a$, where $\Delta(\Omega)$ represents the set of all possible probability distributions over Ω. The Gilboa and Schmeidler (1989)'s formulation is obtained for $c_0(m) = \infty$ if $m \notin M_0$ and $c_0(m) = 0$ if $m \in M_0$, implying $\min_{m \in \Delta(\Omega)} \beta^a \pi_0^{a,m} + c_0(m) = \min_{m \in M_0} \beta^a \pi_0^{a,m}$. In Hansen and Sargent (2001), the cost function takes the form $c(m) \equiv \theta d(m, m^*)$, where m^* is a reference probability and $d(m, m^*)$ represents the (possibly 'twisted') *relative entropy* of m with respect to m^*.

states by conditioning on the new information. Epstein and Schneider (2003) show that in order to preserve time consistency the new probability measures must be derived by updating the initial probabilities *model-by-model*.[17] If for each $m \in M_0$, the vector $\Pi_t^m(\mathcal{I}_t)$ represents the beliefs of a time-consistent child about the likelihood of passing conditional on information $\mathcal{I}_t$, this vector is obtained via marginalization on the sets $C^j, j = 1, 2, \ldots, N$, as in (1), according to the measure $m(\cdot|\mathcal{I}_t)$ as defined in (2). Note that learning (without forgetting) implies that we only consider information sets such that for $t' > t > 0$, $\mathcal{I}_{t'} \subset \mathcal{I}_t$. This in turn implies an absolute continuity ranking: if for $A \subset \Omega$, $m(A|\mathcal{I}_t) = 0$ and $t' > t$, then $m(A|\mathcal{I}_{t'}) = 0$. This is true for each $m \in M_0$.

The information contained in variations over the entries of Ω_2 also inform the child about the set of possible models. For simplicity, let us assume that Ω_2 can itself be described as the cartesian product of two sets, $\Omega_2 = \Omega_U \times \Omega_M$, where Ω_U governs the signals on unambiguous uncertainty and Ω_M contains information regarding the models. The information can thus be described as the product $\mathcal{I}_t = \mathcal{I}_t^U \times \mathcal{I}_t^M$. Let us further assume that model uncertainty is *only due* to the learnable characteristics in Ω_M.[18] In particular, if the single entry $\omega_M \in \Omega_M$ were known, the child would hold a single probability distribution over $\Omega_1 \times \mathcal{I}_t^U \subset \Omega_1 \times \Omega_U$. We may assume that updating over the set of models occurs very simply.[19] Whenever the arrival of new information implies a positive chance that a particular model is true, the child keeps that model in his set. Conversely, the child drops all models that –according to his information– cannot be true. The set of possible models is therefore a monotone function of the information regarding such models, $\mathcal{I}_t^M$.

Let $\mathcal{I}_t = \mathcal{I}_t^U \times \mathcal{I}_t^M$. Once we have the set of models and the conditional probabilities for each model m, the bounds $\overline{\pi}_t^a(\mathcal{I}_t)$ and $\underline{\pi}_t^a(\mathcal{I}_t)$ and the associated range $R_t^a(\mathcal{I}_t) = \overline{\pi}_t^a(\mathcal{I}_t) - \underline{\pi}_t^a(\mathcal{I}_t)$ are obtained from a generalization of (3), where for each $m \in M_0$ we use the elements of the vector $\Pi_t^m(\mathcal{I}_t^U)$ and where the min and the max are taken over the set $M_t(\mathcal{I}_t^M)$.

The discrete measure of perceived belief ambiguity described in Section 3 may be thought of as a screening tool for detection of student-track combinations featuring belief ambiguity ('Unsure' or 'No idea') about the likelihood of successful and timely graduation from the corresponding track. Students displaying belief ambiguity about the likelihood of graduating in time from specific curricula were then asked to report their subjective ranges (min and max probabilities) about the likelihood of graduating from those curricula, corresponding to the range widths R_t^a we just defined.

[17] Epstein and Schneider (2003) show that - within the Gilboa-Smeidler multi-prior setup - time consistency also implies 'rectangularity' of the subjective beliefs and that, in turn, rectangularity together with model-by-model Bayesian updating implies time consistency of preferences. A detailed relationship between this dynamic model and that of Hansen and Sargent (2001) in contained in Epstein and Schneider (2003), Sections 4.4. and 5; and Maccheroni et al. (2006*b*).

[18] Here we rely on the interpretation of ambiguity as lack of information. Ambiguity may also reflect the lack of confidence the child has on each model.

[19] This learning process could be obtained whenever the likelihood function describing the arrival of information over the parameters Ω_M is non-informative about the distribution over models but potentially informative over the support. Cerreia-Vioglio et al. (2013) contains an extensive analysis of learning over the set of models.

On the other hand, survey reports of point beliefs remain the relevant belief measures for those students and choice alternatives characterized by complete lack of ambiguity ('Sure') about the graduation probability.[20]

Recall that learning implies that the set $\mathcal{I}_t^M$ decreases during the learning process. Thus, the set of models $M_t(\mathcal{I}_t^M)$ can only shrink over time. While this feature might be intuitively related to a reduction in the size of the ranges, the cross-sectional size of ranges can change non-monotonically over time in our framework. Proposition A.1 in Appendix A provides sufficient conditions on the learning model under which the cross-sectional mean of the range widths should decrease over time according to what is predicted by the theory, even in presence of ex-ante heterogeneity. The proposition relies on three key assumptions. First, from the first observation (wave 1) onwards students can be divided into (observable or unobservable) types that do not change over time (fixed types). Second, conditional on the student's type and the set of models, relevant information over alternatives arrives independently across students. Third, for each given probability model and student's type, the information in $\mathcal{I}_t^U$ changes prior beliefs additively and uniformly across models.

Limited Awareness Let us now return to the case where the child has a single model m in mind. The tuple $W = (\Omega, m, A, Z)$ can be seen as the child's 'view of the world.' This view might be incomplete; nevertheless, the child firmly believes in W.[21] During the decision process, the child might discover a new high school track that was not in his awareness set A, a new consequence not contemplated in Z, or a new 'link' between tracks and consequences (i.e., a new state not in Ω). These discoveries change some of the elements in W, which gets replaced with a 'new view of the world, W', as a result.

Consider a situation in which the child learns about a new track whose existence he did previously ignore. This can be seen as an enlargement of the set of alternatives, which can be accommodated by extending Ω_1 to include a new slot with 0-or-1 entries, say in position $N+1$. Let us assume for simplicity that the set Ω_2 remains constant, the cardinality of Ω doubles as a result of this change. Of course, the probabilities over joint events –and hence the child's reading of the signals– must be adjusted accordingly.

Consider a track from the child's original awareness set A. In the new world W', the act of choosing track a_j is associated to the following set (note the new entry in the $N+1$ position): $C_j' =$

[20] Decision theory provides no guidance on how we should interpret the point probability reports π_t^a, given the students who subsequently indicate that they have ambiguous beliefs. A 'classical' approach is to assume that ambiguous students hold a uniform distribution over the marginalized probabilities, delivering the following interpretation: $\pi_t^a = \frac{1}{2}[\overline{\pi}_t^a(\mathcal{I}_t^i) + \underline{\pi}_t^a(\mathcal{I}_t^i)]$. In analogy to the α-maxmin model, we could generalize the previous formula and postulate that the preference weight α is also related to the response on the point probability question as follows: $\pi_t^a = \alpha\overline{\pi}_t^a(\mathcal{I}_t^i) + (1-\alpha)\underline{\pi}_t^a(\mathcal{I}_t^i)$.

[21] Unawareness has been defined epistemically as the union of the following logical statements regarding an event: 'I do not known it' and 'I do not know that I do not known it,' and so on at infinitum (Modica and Rustichini (1994, 1999)).

$\{\omega' \in \Omega' : \omega'_1 = (x_1, \dots x_{j-1}, 1, x_{j+1}, \dots, x_N, x_{N+1}), x_k \in \{0,1\} \forall k \neq j\}$ and has probability $\pi_0^{a_j} = m'\left(C'_j\right)$. Analogous expressions hold for the conditional measures.

Discovering a new relevant track induces a change in the set of states and increases the number of coordinates defining each state, i.e., $\Omega' \supset \Omega$. Additionally, it changes the probability model m to m'. When a new alternative is discovered, also the set of consequences Z changes to Z', with the addition of at least one new entry. The new probability measure p' over subsets of $\mathcal{Z}'$ induced by alternatives in A' is computed in the standard way (described at the beginning of the section), where m and Ω are replaced, respectively, by m' and Ω'.

Under the paradigm of *Reverse-Bayesianism* (Karni and Vierø, 2013*a*,*b*), the new probability measure m' can be obtained from the original m with a 'proportional' shift of the probability mass from states in Ω to the corresponding event in Ω', in such a way as to preserve the likelihood ratio of the events in Ω' and their projection in the original Ω. The main practical advantage of Reverse-Bayesianism is that it provides intuitive conditions under which we are entitled to study the evolution of beliefs and belief ambiguity without jointly keeping track of the curriculum and of the evolution of children's awareness. For example, in our framework this implies that pure changes in awareness *do not affect our measures of the marginal probabilities*, π_t^a, or their evolution for alternatives belonging to the awareness set in previous periods. For these alternatives, the likelihood ratio $\frac{\pi_t^a}{1-\pi_t^a}$ is simply reinterpreted within the 'new view of the world, W'. The articles by Karni and Vierø contain nice and concise reviews of the existing literature on the topic.

5 Evolution of Awareness and Belief Ambiguity: Evidence

In this section we analyze students' perceived levels of awareness and belief ambiguity at the start of 8th grade; we document how such perceptions evolve over time through the pre-enrolment's deadline; and we investigate the main patterns of heterogeneity in students' initial perceptions and their evolution over the decision process.

5.1 Perceived (Un)Awareness Over the Decision Process

Initial Awareness and Aggregate Evolution Columns 1-3 in the left panel of Table 10 present the sample distributions across curricula and by individual curriculum of students' awareness perceptions at the beginning of 8th grade (wave 1). On average, at the beginning of the school year less than half of the respondents (44.40%) indicates that they 'know' the curriculum they are being asked about. Although the majority of the remaining students expresses awareness accompanied by limited knowledge ('Heard of'), a sizeable fraction of over 15% of students reports being completely unaware ('Never heard of'). As previously noticed in Section 3 with regard to the distributions of perceived awareness in wave 3, the aggregate distribution masks substantial heterogeneity across curricula. In particular, the fraction of unaware students is generally small for the traditional curricula of the general tracks (3-4%) and sizably larger for technical curricula (13-15%) and vocational curricula (23%-36%). The Learning & Social Science and the Music & Choral curricula feature rates of perceived unawareness comparable to those reported for technical and vocational curricula (16-23%). The latter reflects the fact that these curricula had been recently introduced in the Italian secondary education system at the time of the survey.

How do students' awareness perceptions evolve between the beginning of the school year and pre-enrollment? To answer this question, columns 4-6 in the right panel of Table 10 present the sample distributions of students' awareness perceptions near pre-enrollment (wave 3). To avoid compositional effects and facilitate the comparison with the figures shown in the left panel for wave 1, all calculations were carried out using respondents who took both waves 1 and 3.

Perceived awareness increases markedly between waves for all curricula, as revealed by the substantially higher fractions of students selecting 'Know' and the corresponding lower fractions of 'Heard of' or 'Never heard of' responses observed in wave 3 relative to wave 1. A χ^2 test of equality of proportions rejects at a 99% confidence level the null hypothesis of equality of the distributions across the two waves.

Despite the marked increase in students' perceived awareness about all curricula, the distinctive pattern of heterogeneity in students' awareness reports observed in wave 1 persists through wave 3. Put differently, differences in reported awareness levels across curricula do not vanish by the time of choice. For instance, while only about 1% of students reports having never heard of (traditional) curricula of the general track at the time of pre-enrollment, over 9% of them still indicate having never heard of specific vocational curricula. Thus, children learn at different speeds about different high school alternatives. Compare, for example, the figures for the general Social Sciences curriculum with those for the technical Economic Sector curriculum. In wave 1, the two curricula display nearly the same fraction of 'Know's (37-38%). By wave 3, the fraction of children who report knowing the general Social Sciences curriculum is almost 8 percentage points higher than the fraction of children who indicate knowing the technical Economic Sector curriculum.

Awareness Transitions The evidence presented in Table 10 implies an unambiguous –although heterogeneous across curricula– increase of students' perceived awareness and knowledge levels over the 6-7 months preceding pre-enrollment. In fact, these aggregate patterns may mask heterogenous transitions in perceived awareness or knowledge across respondents, some of whom may experience increased awareness or knowledge and others may not. To investigate this possi-

Table 10: Awareness: Sample Distributions of Children's Awareness Perceptions about Choice Alternatives in Wave 1 vs. Wave 3, by Curriculum

	Wave 1 Sample[a]			Wave 3 Sample[b]			
	Children (N=405)			Children (N=397)			χ^2
Curriculum:	**'Know'**	**'Heard of'**	**'Never heard of'**	**'Know'**	**'Heard of'**	**'Never heard of'**	**Test[c]**
All	44.40	40.34	15.26	61.66	33.46	4.88	***
Gen, Humanities	64.20	32.84	2.96	78.84	20.15	1.01	***
Gen, Languages	70.12	25.93	3.95	80.60	18.39	1.01	***
Gen, Math & Science	75.06	20.99	3.95	85.89	13.10	1.01	***
Gen, Art	51.60	45.43	2.96	71.03	27.71	1.26	***
Gen, Music & Choral	29.14	47.90	22.96	47.36	46.35	6.30	***
Gen, Soc Sciences	37.28	46.42	16.30	62.97	34.01	3.02	***
Tech, Economic Sector	38.02	47.41	14.57	55.16	39.80	5.04	***
Tech, Technology Sector	45.93	40.99	13.09	60.96	34.51	4.53	***
Voc, Services	31.11	45.68	23.21	47.86	43.07	9.07	***
Voc, Industry & Crafts	18.27	45.93	35.80	38.79	48.87	12.34	***
Voc, Prof Training	27.65	44.20	28.15	48.87	42.07	9.07	***

[[a]]: Matched with wave 3 (or 2) sample.

[[b]]: Constructed from responses in wave 3. Wave 2 responses used for respondents who did not participate in wave 3. Matched with wave 1 sample.

[[c]]: Of equality of proportions. ***: 99% confidence level, **: 95% confidence level, *: 90% confidence level.

bility, in Table 11 we compute transition matrices of reported awareness states (Know, Heard of, Never heard) across pairs of waves (W1 to W2, W2 to W3, and W1 to W3). Conditional on a starting awareness level indicated by row, each of the first three entries of an individual matrix gives the fraction of students who moves from the starting awareness level to the level of awareness indicated in the corresponding column. The last column reports the total number of responses (individual-curriculum combinations) in each of the starting awareness levels.

The top row of matrices shown in Table 11 presents 'unconditional' transitions, which combine together awareness transitions for all curricula. Inspection of the top-right matrix displaying transitions between wave 1 and wave 3 reveals that learning is sizeable and monotone; children starting with a higher level of awareness or knowledge are more likely to end up at the highest level of perceived knowledge ('Know') by the time of pre-enrollment.

Table 11: AWARENESS: TRANSITION MATRICES OF CHILD'S AWARENESS PERCEPTIONS ACROSS WAVES (SAMPLE OF CHILDREN WHO RESPONDED TO EACH PAIR OF WAVES)

Unconditional

		W2 Know	W2 Heard of	W2 Never h.	N
W1	Know	0.83	0.16	0.01	1642
	Heard of	0.42	0.54	0.04	1501
	Never h.	0.26	0.52	0.22	553

		W3 Know	W3 Heard of	W3 Never h.	N
W2	Know	0.845	0.145	0.01	1342
	Heard of	0.38	0.57	0.05	947
	Never h.	0.18	0.57	0.25	153

		W3 Know	W3 Heard of	W3 Never h.	N
W1	Know	0.86	0.13	0.01	1333
	Heard of	0.47	0.48	0.05	1194
	Never h.	0.33	0.52	0.15	443

Conditional on Chosen Alternative

		Wave 2 Know	Wave 2 Heard of	Wave 2 Never h.	N
Wave 1	Know	0.98	0.02	0	170
	Heard of	0.62	0.38	0	26
	Never h.	0.53	0.29	0.18	17

		Wave 3 Know	Wave 3 Heard of	Wave 3 Never h.	N
Wave 2	Know	0.97	0.03	0	178
	Heard of	0.71	0.29	0	14
	Never h.	0.67	0.23	0	3

		Wave 3 Know	Wave 3 Heard of	Wave 3 Never h.	N
Wave 1	Know	0.98	0.02	0	188
	Heard of	0.84	0.16	0	31
	Never h.	0.63	0.27	0	19

Conditional on Ranked First in Start Wave

		Wave 2 Know	Wave 2 Heard of	Wave 2 Never h.	N
Wave 1	Know	0.97	0.03	0	267
	Heard of	0.715	0.225	0.06	49
	Never h.	0.50	0.375	0.125	16

		Wave 3 Know	Wave 3 Heard of	Wave 3 Never h.	N
Wave 2	Know	0.96	0.04	0	210
	Heard of	0.65	0.29	0.06	17
	Never h.	-	-	-	0

		Wave 3 Know	Wave 3 Heard of	Wave 3 Never h.	N
Wave 1	Know	0.95	0.05	0	216
	Heard of	0.68	0.26	0.06	35
	Never h.	0.38	0.62	0	13

Conditional on Alternatives Ranked Bottom or Unranked in Start Wave

		Wave 2 Know	Wave 2 Heard of	Wave 2 Never h.	N
Wave 1	Know	0.79	0.20	0.01	855
	Heard of	0.40	0.55	0.05	1041
	Never h.	0.24	0.52	0.24	380

		Wave 3 Know	Wave 3 Heard of	Wave 3 Never h.	N
Wave 2	Know	0.79	0.20	0.01	640
	Heard of	0.35	0.59	0.06	643
	Never h.	0.15	0.57	0.28	103

		Wave 3 Know	Wave 3 Heard of	Wave 3 Never h.	N
Wave 1	Know	0.83	0.16	0.01	685
	Heard of	0.45	0.50	0.05	863
	Never h.	0.29	0.53	0.18	301

In the interest of space, we do not report transitions specific to each of the main 10-11 curricula offered in Vicenza (available upon request). Nevertheless, some insight into individual learning heterogeneity across curricula can be obtained by grouping curricula according to whether they are of 'high vs. low relevance' to children. Accordingly, the matrices in the bottom section of Table 11 show the transition probabilities of awareness reports conditional on the (high-relevance) curriculum chosen at pre-enrolment (2nd panel); on the (high-relevance) curriculum the child ranked first in the survey of the start wave (3rd panel); and on the set of (low-relevance) curricula the child ranked bottom (4th or lower) or left unranked in the survey of the start wave (4th panel).

Children learn faster about the alternatives that are most relevant to them, as revealed by their pre-enrollment outcomes and by the choice preferences they report in the survey. There are stark differences between the entries of the matrices constructed conditional on the chosen or ranked-first alternative in the start wave and the corresponding entries of the unconditional matrices and of the matrices constructed conditional on the alternatives ranked bottom or unranked in the start wave. For example, the probabilities of ending up in the Know state are substantially larger for the high-relevance curricula than for the low-relevance curricula or across all curricula, regardless of the starting awareness state. Reassuringly, no children become completely unaware about the curriculum in which they are eventually observed to enroll and only a handful of children report becoming unaware of the curriculum they ranked first in the previous wave.

We tested the equality of the transition proportions across different conditionals. Table 24 in Appendix B reports the results of the test for all transition distributions (shown in individual rows of the transition matrices) across pairs of conditionals (i.e., across matrices in different panels of the table). For the majority of comparisons, the null hypothesis of equality of proportions is rejected at a 90% confidence level.

5.2 Perceived Belief Ambiguity Over the Decision Process

Initial Ambiguity and Aggregate Evolution Having documented the extent of students' self-reported awareness (or lack thereof) about alternative high school options and how such perceptions evolve between the start of 8th grade and the time of pre-enrollment, we now concentrate our analysis on the degree of ambiguity that students perceive with regard to a particular dimension of choice. Specifically, we document the extent of students' subjective ambiguity about the likelihood of successfully and timely graduating from alternative high school curricula.

As explained in the theoretical section, individuals may have multiple 'models' in mind, corresponding to a situation of ambiguity rather than to one of mere (physical or unambiguous) uncertainty. To determine whether students begin 8th grade with ambiguous or unambiguous subjective beliefs, and to assess whether students' ambiguity decreases or increases during the decision process, Table 12 shows for each curriculum the sample distributions of responses in waves 1 and 3 to the follow-up question that asks students to indicate whether they feel 'Sure,' 'Unsure,' or that they 'have No Idea' about the subjective graduation probability they gave in the preceding point belief question.

At start of 8th grade (wave 1), the proportion of students who report unambiguous beliefs ('Sure') is fairly high and ranges between 74% and 81% across curricula. Thus, such a proportion varies little across curricula. The complement proportion of students who report ambiguous beliefs ('Unsure' or 'No idea') follows a symmetric pattern. Within the latter group, however, subjective ambiguity levels ('Unsure' vs. 'No idea') vary sensibly across tracks. In particular, the fractions of students indicating feeling unsure about the chances of successful and timely graduation from curricula of the general track are almost double the fractions of students who say they have no idea about the chances. Such proportions are reversed for curricula of the technical and vocational tracks, that is, those curricula of which children are least aware to start with.

The near constancy of the aggregate fraction of students reporting being Sure masks differences in learning across tracks. Specifically, the fraction of students who report unambiguous belies increases moderately over time among curricula of the general track; whereas the fraction of Sure children decreases moderately over time among curricula of the technical and vocational tracks. As for the children reporting ambiguous beliefs, the fraction of Unsure decreases over time regardless of the curriculum; whereas the fraction of No Idea increases for all curricula.

The null hypothesis of equality of proportions between wave 1 and wave 3 is rejected for all curricula of the general and technical tracks, the vocational Industry & Crafts curriculum, and the vocational curriculum of Professional Training, whereas it cannot be rejected at standard levels of confidence for the vocational curriculum of Services.

Ambiguity Transitions We now analyze individual-level changes in subjective ambiguity by constructing transition matrices of ambiguity levels across pairs of waves, shown in Table 13. To streamline the presentation, we focus on the transitions between waves 1 and 3 (in the last column of matrices). Similar patterns are observed in the transitions between the remaining pairs of waves.

We start from the 'unconditional' matrices in the top panel, combining transitions for all of the alternatives. The higher probabilities appearing in the cells of the 'Sure' column relative to the remaining cells suggest an overall reduction in ambiguity between waves 1 and 3. Although most transitions go from higher to lower levels of ambiguity, a fraction of children moves in the opposite direction, from the Sure and Unsure states to the No idea state. A potential interpretation of this pattern is that in later waves children tend to select ambiguity categories with lower associated

Table 12: AMBIGUITY: SAMPLE DISTRIBUTIONS OF CHILDREN'S AMBIGUITY PERCEPTIONS ABOUT THE PROBABILITY OF PASSING IN WAVE 1 VS. WAVE 3, BY CURRICULUM

	Wave 1 Sample[a]			**Wave 3 Sample**[b]			
	Children (N=363-377)			**Children (N=334-343)**			χ^2
Curriculum:	**'Sure'**	**'Unsure'**	**'No Idea'**	**'Sure'**	**'Unsure'**	**'No Idea'**	**Test**[c]
All	77.91	9.16	12.93	77.35	4.10	18.55	***
Gen, Humanities	79.89	13.67	6.43	81.52	4.99	13.49	***
Gen, Languages	80.11	11.94	7.96	82.80	4.96	12.24	***
Gen, Math & Science	81.33	11.20	7.47	82.46	6.14	11.40	**
Gen, Art & Music	80.27	12.00	7.73	83.67	3.50	12.83	***
Gen, Soc Sciences	78.86	9.49	11.65	80.53	3.24	16.22	***
Tech, Economic Sector	77.24	8.94	13.82	74.11	4.76	21.13	***
Tech, Technology Sector	77.54	9.89	12.57	75.88	4.12	20	***
Voc, Services	76.69	4.61	18.70	71.56	4.49	23.95	
Voc, Industry & Crafts	76.65	4.67	18.68	69.16	2.99	27.84	**
Voc, Prof Training	74.10	4.41	21.49	71.26	1.80	26.95	*

[a]: Matched with wave 3 (or 2) sample.

[b]: Constructed from responses in wave 3. Wave 2 responses used for respondents who did not participate in wave 3. Matched with wave 1 sample.

[c]: Of equality of proportions. ***: 99% confidence level, **: 95% confidence level, *: 90% confidence level.

response burden (i.e., no follow-up).[22] However, conditional on starting Unsure, most transitions to a lower-burden answer end up into Sure rather than into No Idea. Similarly, the latter explanation might rationalize the observed limited movements from No idea to Unsure, but it does not convincingly rationalize the substantial fraction of transitions from No idea to Sure.

Children seem to learn faster in the alternatives most relevant to them, as revealed by a comparison of the transitions for the pre-enrollment and ranked-first alternatives (in the two middle panels) with the unconditional transitions (in the top panel) and the transitions for the alternatives ranked bottom or unranked (in the bottom panel). For example, let us compare the unconditional transitions between waves with the corresponding transitions for the pre-enrollment (chosen) alternative. The fractions of transitions to lower levels of ambiguity are larger for the chosen alternative than in the unconditional matrix. Similarly, when comparing transitions between waves for alternatives ranked first and ranked last, we observe that transitions from Sure or Unsure to No idea are more frequent for curricula ranked last than ones ranked first. Conversely, transitions from No idea or Unsure to Sure are less frequent for curricula ranked last than ones ranked first.[23]

[22]This hypothesis requires that children recall the relevant skip pattern for these questions from previous waves.

[23]The cells in the No Idea row for the enrollment and the ranked-first curricula have very small N. Hence, their entries should be interpreted with caution.

Table 13: AMBIGUITY: TRANSITION MATRICES OF CHILD'S AMBIGUITY PERCEPTIONS ACROSS WAVES (SAMPLE OF CHILDREN WHO RESPONDED TO EACH PAIR OF WAVES)

Unconditional

		Wave 2			
		Sure	Unsure	No Idea	N
Wave 1	Sure	0.86	0.02	0.12	2254
	Unsure	0.66	0.19	0.15	247
	No Idea	0.57	0.06	0.37	318

		Wave 3			
		Sure	Unsure	No Idea	N
Wave 2	Sure	0.87	0.02	0.11	1605
	Unsure	0.68	0.17	0.16	108
	No Idea	0.47	0.03	0.50	227

		Wave 3			
		Sure	Unsure	No Idea	N
Wave 1	Sure	0.86	0.02	0.12	1790
	Unsure	0.64	0.17	0.19	247
	No Idea	0.51	0.03	0.46	287

Conditional on Chosen Alternative

		Wave 2			
		Sure	Unsure	No Idea	N
Wave 1	Sure	0.92	0.02	0.06	163
	Unsure	0.77	0.19	0.04	26
	No Idea	0.72	0.14	0.14	7

		Wave 3			
		Sure	Unsure	No Idea	N
Wave 2	Sure	0.94	0.01	0.05	159
	Unsure	0.44	0.56	0	9
	No Idea	0.63	0.12	0.25	8

		Wave 3			
		Sure	Unsure	No Idea	N
Wave 1	Sure	0.94	0.01	0.05	177
	Unsure	0.68	0.22	0.10	31
	No Idea	0.33	0.11	0.56	9

Conditional on Ranked First in Start Wave

		Wave 2			
		Sure	Unsure	No Idea	N
Wave 1	Sure	0.92	0.02	0.06	248
	Unsure	0.68	0.22	0.10	40
	No Idea	1	0	0	5

		Wave 3			
		Sure	Unsure	No Idea	N
Wave 2	Sure	0.93	0.02	0.05	184
	Unsure	0.54	0.38	0.08	13
	No Idea	0.75	0.125	0.125	8

		Wave 3			
		Sure	Unsure	No Idea	N
Wave 1	Sure	0.92	0.02	0.06	198
	Unsure	0.62	0.23	0.15	34
	No Idea	0.20	0.30	0.50	10

Conditional on Alternatives Ranked Bottom or Unranked in Start Wave

		Wave 2			
		Sure	Unsure	No Idea	N
Wave 1	Sure	0.845	0.02	0.135	1519
	Unsure	0.67	0.17	0.16	141
	No Idea	0.55	0.05	0.40	263

		Wave 3			
		Sure	Unsure	No Idea	N
Wave 2	Sure	0.86	0.02	0.12	962
	Unsure	0.69	0.15	0.16	68
	No Idea	0.44	0.03	0.54	186

		Wave 3			
		Sure	Unsure	No Idea	N
Wave 1	Sure	0.84	0.02	0.14	1222
	Unsure	0.62	0.17	0.21	150
	No Idea	0.53	0.03	0.44	235

The results of χ^2 tests of equality of proportions similar to those performed in Table 24 for the awareness transitions are reported in Table 25 for the ambiguity transitions. The results are somewhat less conclusive than those obtained for the awareness transitions and should be taken with caution for transitions with small cell counts, especially those originating in Unaware. Caution notwithstanding, for several of the comparisons the null hypothesis of equality is rejected at a 90% level of confidence or higher.

Evolution of Subjective Ranges All students responding to the survey were asked their subjective probability of graduating in the regular time from alternative high school curricula. Figure 2 in Appendix B summarizes the evolution of point-belief responses across the three survey waves. Only students who indicated being unsure about their point-belief answer were asked to give a range of chances. In order to trace the evolution of range size over survey waves, we assign a range of size 0 (or lower bound=upper bound) to the curricula from which the respondent indicated being sure about his chances of graduating in time and a range of size 100 to the curricula from which the respondent indicated having no idea about the chances of graduating in time (or lower bound=0 and upper bound=100). Using the notation introduced in Section 4, $\overline{\pi}^a_S = \underline{\pi}^a_S$ (or $R^a_S = 0$) corresponds to a situation of no ambiguity and $\{\overline{\pi}^a_{NI} = 1, \underline{\pi}^a_{NI} = 0\}$ (or $R^a_{NI} = 1$) corresponds to maximal ambiguity.

Figure 1 summarizes the evolution of the mean range width over the three survey waves, using the range variables obtained by applying this assignment. The figure shows the time pattern of the mean range widths obtained by conditioning on specific sets of curricula. The horizontal axis of Figure 1 displays the conditionals, which include the cases in which the mean range width refers to the pre-enrollment choice, the alternatives ranked first in wave 1 and in wave 2, and the alternatives ranked last or unranked in wave 1 and in wave 2. These conditionals parallel those investigated in the analysis of the transition matrices. Figure 1 includes additional conditionals by track (general traditional, general newly introduced, technical, and vocational).

The decrease in the mean range width across waves for the alternative chosen at pre-enrollment confirms the reduction in subjective ambiguity observed in the transition matrices for this alternative. A similar pattern holds between waves 1 and 2 for the alternative ranked first in wave 1 and between waves 2 and 3 for the alternative ranked first in wave 2. While potentially puzzling at first, the increase in perceived ambiguity between wave 2 and wave 3 for the alternative ranked first in wave 1 is likely due to the substantial changes in the ranked-first alternative across waves, documented in Table 26 of Appendix B. As expected, the mean range size for alternatives ranked bottom or unranked in wave 1 follows the opposite pattern of that displayed by the mean range sizes for the chosen alternative and the alternative ranked first. Specifically, for alternatives ranked

Figure 1: Evolution of Subjective Ranges

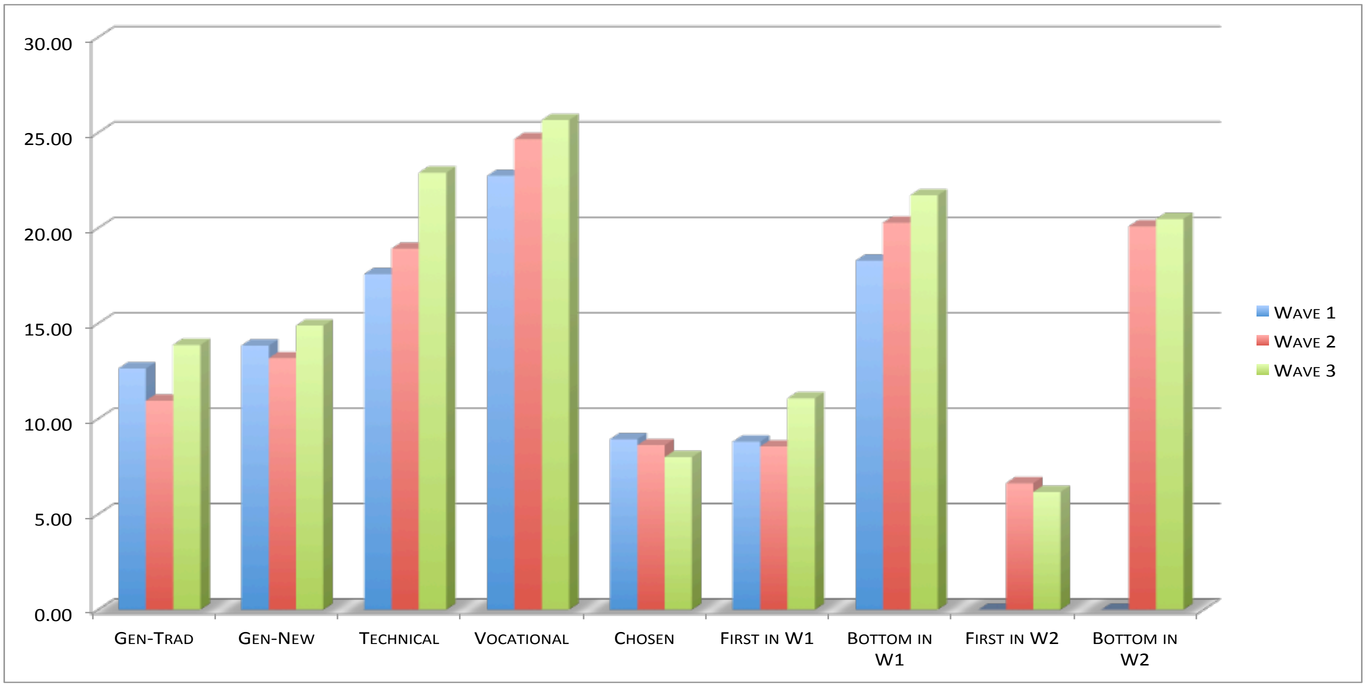

Figure 1: Mean range size in waves 1-3, conditional on the sets described in the horizontal axis.

bottom in wave 1 ambiguity tends to increase over time, consistent with the pattern observed in the transition matrices.

To complete the picture, in Figure 3 of Appendix B we plot the evolution of the mean range width by individual curricula. This figure confirms the main patterns emerging from the conditionals by track in Figure 1. Specifically, the mean range width features a non-monotone pattern in the level of ambiguity across waves for curricula of the general track, while the mean range width steadily increases across waves for curricula of the technical and vocational tracks.

5.3 Heterogeneity in Students' Initial Knowledge and Learning

The analyses of aggregate and individual-level transitions in the previous two subsections indicate that students' awareness and ambiguity perceptions at the beginning of 8th grade, and the evolution of such perceptions over the 6-7 months preceding the pre-enrollment decision, tend to vary systematically across curricula as a function of the curriculum's relevance for the student and of the track of the curriculum. In this subsection, we seek to better understand the documented patterns of heterogeneity in students' initial knowledge and learning over time by investigating whether and how the latter vary by observable characteristics of the students and their families.

Awareness Regressions To study heterogeneity in initial awareness levels across students' characteristics and tracks we begin our analysis by performing a series of Poisson regressions analogous to the ones presented in Table 3, with students' awareness reports at the time of pre-enrollment choice (wave 3) replaced with their reports at the beginning of the school year (wave 1). Specifically, in Table 14 we investigate the predictors of the total number of alternatives out of all available curricula that the student indicates being aware of (column 1) and knowing (column 2) at the start of 8th grade. In columns 3 (resp. 4) and 5 (resp. 6), we repeat the analysis by focusing on the initial number of alternatives the student is aware of (resp. knows) within the set of general curricula and within the set of technical and vocational curricula, respectively. The complete set of regressors is listed in the first column of Table 14 and coincides with vector of covariates used in related heterogeneity analyses of Section 3.

Inspection of Table 14 reveals that some of the main patterns of awareness heterogeneity among students observed at the time of pre-enrollment can be traced back to the beginning of the school year. For example, in wave 1 female students already display greater perceived knowledge (or a larger 'knowledge set') than their male counterparts, particularly within the set of general curricula. Similarly, having a higher GPA is already associated with lower perceived knowledge about vocational and technical curricula.

Some subtle differences between waves are also present. For instance, the negative and statistically significant differences in the levels of perceived awareness and knowledge about technical and vocational curricula observed between children with a highly educated mother (both college+ and high school diploma) and the remaining children in wave 1 are no longer significant by wave 3. On the other hand, the positive but statistically insignificant difference in the perceived level of knowledge about general curricula observed between children with a college-educated mother and the remaining children at the start of 8th grade becomes statistically significant by pre-enrollment.

While these comparisons between the predictors of students' awareness and knowledge levels in waves 1 and 3 begin to shed some light on the heterogeneity in students' learning between the two waves, in Table 15 we investigate the matter directly. Specifically, we construct the following outcome variables that capture the main sets of transitions in the level of awareness that students may have logically experienced between the beginning and the end of the relevant period. The first outcome variable (in column 1) is defined as the total number of curricula for which the student's perceived awareness level increases strictly between wave 1 and wave 3 from any starting level in wave 1 ('learning' for short). The second outcome variable (in column 2) is defined as the total number of curricula for which the student's perceived awareness in wave 1 decreases from the most refined level of knowledge (Know) to any lower level in wave 3 ('unlearning' for short). The outcome variables used in the remaining columns are defined similarly, but they refer to the subsets

Table 14: AWARENESS: PREDICTORS OF NUMBER OF ALTERNATIVES THE CHILD REPORTS BEING AWARE OF AT THE START OF 8TH GRADE (WAVE 1)

Poisson Regression of N of Alternatives the Child is Aware of at Start of 8th Grade:

	All Curricula		General		Techincal & Vocational	
Predictors	**'Know' or 'Heard of'**	**'Know'**	**'Know' or 'Heard of'**	**'Know'**	**'Know' or 'Heard of'**	**'Know'**
female	0.01325 (0.03863)	0.10535** (0.05276)	0.01028 (0.05034)	0.10957* (0.06444)	0.01751 (0.06024)	0.09223 (0.09208)
foreign born	−0.04558 (0.07545)	−0.11109 (0.10405)	−0.02633 (0.09759)	−0.16988 (0.13143)	−0.07310 (0.11896)	0.00478 (0.17090)
lives with both parents	0.06250 (0.06604)	0.05096 (0.08900)	0.02483 (0.08459)	−0.04477 (0.10353)	0.11915 (0.10573)	0.28876* (0.17502)
mom has college+ degree	0.09027 (0.06075)	−0.08589 (0.08249)	−0.02880 (0.07940)	0.12543 (0.10228)	−0.17860* (0.09455)	−0.53047*** (0.14698)
mom has HS degree	0.07015 (0.05133)	−0.06831 (0.06930)	−0.04352 (0.06779)	0.05395 (0.08929)	−0.10608 (0.07861)	−0.26050** (0.11129)
has stay-home mom	0.00543 (0.04594)	0.01612 (0.06214)	−0.01153 (0.06016)	−0.00199 (0.07725)	0.00293 (0.07118)	0.04252 (0.10477)
has blue-collar dad	0.01185 (0.04691)	0.06087 (0.06295)	0.00700 (0.06147)	−0.00034 (0.07929)	0.01830 (0.07260)	0.16331 (0.10446)
n of older siblings	0.00326 (0.02691)	−0.02079 (0.03671)	0.01338 (0.03486)	−0.01228 (0.04471)	−0.02750 (0.04236)	−0.03921 (0.06424)
7th-grade GPA	0.01855 (0.02271)	−0.00278 (0.03088)	0.02954 (0.02959)	0.05749 (0.03766)	0.00288 (0.03545)	−0.12735** (0.05445)
constant	2.10210*** (0.18567)	1.60779*** (0.25176)	1.48417*** (0.24180)	0.71945** (0.30852)	1.33444*** (0.28998)	1.40678*** (0.44228)
F	5.06	9.22	1.91	13.51	7.24	42.96
Prob $> F$	0.8288	0.4176	0.9928	0.1410	0.6126	0.0000
R^2	0.0037	0.0061	0.0017	0.0110	0.0063	0.0398
Sample Size	305	305	305	305	305	305

***: significant at 1%, **: significant at 5%, *: significant at 10%.
Wave 1 sample matched with wave 3 (or 2) sample.

of curricula of the general track (columns 3-4) and of the technical and vocational tracks (columns 5-6). The vector of covariates in column 1 is the same as before.

Inspection of columns 1-2 reveals that children with a more educated mother 'learn' significantly more (both college+ and high school diploma) and 'unlearn' significantly less (college+ only) than children with less educated mothers during the relevant period. Children with a stay-home mom unlearn significantly less as well. On the other hand, foreign-born children unlearn significantly more; whereas children with a blue-collar father learn significantly less. A negative association is also observed between the number of upward awareness transitions and GPA.

According to the estimates shown in columns 3 through 6, most of these patterns concern specific subsets of curricula. For example, the comparatively greater amount of unlearning displayed by foreign-born children is statistically significant only for general curricula. On the other hand, the comparatively greater learning and more contained unlearning of children with a more educated

Table 15: AWARENESS: PREDICTORS OF NUMBER OF TRANSITIONS IN CHILD'S AWARENESS PERCEPTIONS BETWEEN THE START OF 8TH GRADE AND PRE-ENROLLMENT (BETWEEN W1 AND W3)

Poisson Regression of N of Alternatives the Child's Initial Awareness Level:

Predictors	All Curricula		General		Technical & Vocational	
	Increases strictly	Decreases from 'Know'	Increases strictly	Decreases from 'Know'	Increases strictly	Decreases from 'Know'
female	-0.00767	0.05085	0.06030	0.04756	-0.06523	0.05531
	(0.06489)	(0.14265)	(0.09592)	(0.20325)	(0.08817)	(0.20038)
foreign born	0.04133	0.41749*	0.08877	0.57929*	-0.00060	0.23126
	(0.11912)	(0.22375)	(0.17027)	(0.29743)	(0.16680)	(0.34165)
lives with both parents	0.04417	-0.10158	0.22568	-0.31843	-0.09091	0.16836
	(0.11066)	(0.21398)	(0.17531)	(0.27806)	(0.14301)	(0.33843)
mom has college+ degree	0.29674***	-0.50877**	0.15781	-0.48817	0.40882***	-0.51970*
	(0.10946)	(0.22141)	(0.16136)	(0.32079)	(0.14961)	(0.30649)
mom has HS degree	0.24063**	-0.21418	0.22128	-0.10840	0.25823*	-0.30974
	(0.09483)	(0.18004)	(0.13624)	(0.26142)	(0.13215)	(0.24906)
has stay-home mom	0.07005	-0.80092***	0.05743	-1.01172***	0.07962	-0.62913**
	(0.07738)	(0.20438)	(0.11286)	(0.31667)	(0.10630)	(0.26899)
has blue-collar dad	-0.15015*	-0.03832	-0.15070	-0.06671	-0.15127	-0.01305
	(0.08267)	(0.16982)	(0.11975)	(0.24104)	(0.11429)	(0.23954)
n of older siblings	0.01556	0.02927	-0.01651	0.01216	0.04241	0.04278
	(0.04473)	(0.09880)	(0.06644)	(0.14138)	(0.06048)	(0.13839)
7th-grade GPA	-0.13525***	-0.12603	-0.18180***	-0.16548	-0.09516*	-0.08865
	(0.03844)	(0.08356)	(0.05685)	(0.11906)	(0.05224)	(0.11742)
constant	2.00651***	1.05102	1.45192***	0.79943	1.17509***	-0.13156
	(0.31189)	(0.66366)	(0.46339)	(0.93770)	(0.42292)	(0.94536)
F	24.33	29.89	16.54	24.54	15.65	9.50
Prob $> F$	0.0038	0.0005	0.0564	0.0035	0.0746	0.3927
R^2	0.0171	0.0376	0.0165	0.0488	0.0148	0.0191
Sample Size	301	301	301	301	301	301

***: significant at 1%, **: significant at 5%, *: significant at 10%.

mother are statistically significant only for technical and vocational curricula.

Ambiguity Regressions We now study heterogeneity in students' initial ambiguity and its evolution before pre-enrollment. Specifically, in Table 16 we perform Poisson regressions of the number of curricula for which in wave 1 the student has an ambiguous subjective expectation of successfully and timely graduating on the usual set of covariates. Separate regressions are performed for different levels of ambiguity (Unsure or No idea in the even-numbered columns and No idea only in the odd-numbered columns) and for different sets of curricula ('All' in columns 1-2, 'General' in columns 3-4, and 'Technical & Vocational' in columns 5-6). Once again, the regressions shown in Table 16 are analogous to the ones presented in Table 8, with students' ambiguity reports in wave 3 replaced with their reports in wave 1.

In Table 17, we directly investigate the predictors of the number of downward transitions in perceived ambiguity (or 'learning') between waves 1 and 3 from any starting level in wave 1 (columns

1, 3, and 5), and the predictors of the number of upward transitions from an unambiguous belief in wave 1 to an ambiguous one in wave 3 (or 'unlearning,' in columns 2, 4, and 6). Once again, the analysis is separately performed for all curricula (columns 2-3), the subset of general curricula (columns 4-5), and the subset of technical and vocational curricula (columns 6-7).

We begin with the main patterns of heterogeneity in students' ambiguity levels in wave 1 (Table 16) and their comparison with heterogeneity patterns in wave 3 (Table 8). Children who live with both parents, have a more educated mother, a stay-home mother, or a higher GPA start in wave 1 with statistically significant lower ambiguity than their counterparts for at least one ambiguity margin ('Unsure or No idea' or 'No idea') and one set of curricula (general, technical & vocational, or all). Conversely, female and foreign-born students start with higher ambiguity, although only the coefficients of the gender dummy are statistically significant.

With the exception of GPA, all of these patterns appear to be specific to particular subsets of curricula. The lower comparative ambiguity of children with a more educated mother and with a stay-home mother pertains to curricula of the general track. The lower comparative ambiguity of children who live with both parents and the higher ambiguity of female children pertain to curricula of the technical and vocational tracks.

The apparent differences in heterogeneity patterns between Table 16 (wave 1) and Table 8 (wave 3) suggest that specific groups of children identified by different values of the covariates follow different learning paths during the months before pre-enrollment. Inspection of Table 17 shows that students with a blue-collar dad, with a high GPA, and those living with both parents are the groups of individuals who learn comparatively more and/or unlearn less. The greater learning of children with a blue-collar dad is concentrated within the set of technical and vocational curricula; whereas their more limited unlearning applies across tracks. Similarly, the more limited unlearning of students with a higher GPA pertains the set of general curricula; whereas that of children living with both parents pertains the curricula of the technical and vocational tracks.

Foreign-born students tend to learn less across all curricula (statistically not significant) and to unlearn significantly more with respect to general curricula. Finally, female students display opposite learning patterns in different tracks. In particular, female students learn significantly more than male students over the set of technical and vocational curricula, but unlearn significantly more over the set of general curricula.

Summary Our analysis of heterogeneity identifies five main patterns of learning across different demographic and socioeconomic (SES) groups. We summarize these patterns, by organizing the discussion around whether each group of students starts off with more or less information than their counterparts; whether they learn more or less over time; and whether or not their learning

Table 16: AMBIGUITY: PREDICTORS OF NUMBER OF ALTERNATIVES WHOSE LIKELIHOOD OF GRADUATING IN TIME CHILD REPORTS HAVING AMBIGUOUS BELIEF ABOUT AT THE START OF 8TH GRADE (WAVE 1)

Poisson Regression of N of Alt. for which the Child has Ambiguous Subjective Prob. of Graduating at Start of 8th Grade:

	All Curricula		General		Technical & Vocational	
Predictors	**'Unsure' or 'No Idea'**	**'No Idea'**	**'Unsure' or 'No Idea'**	**'No Idea'**	**'Unsure' or 'No Idea'**	**'No Idea'**
female	0.18506** (0.08818)	0.44430*** (0.12455)	0.05224 (0.12989)	0.12474 (0.22436)	0.29803** (0.12060)	0.57513*** (0.15123)
foreign born	0.07068 (0.16301)	0.24856 (0.20097)	0.10444 (0.24261)	0.40870 (0.35747)	0.04160 (0.22016)	0.18672 (0.24361)
lives with both parents	−0.19295 (0.13455)	−0.38817** (0.17102)	−0.09290 (0.20710)	−0.50966 (0.31165)	−0.27277 (0.17720)	−0.34449* (0.20484)
mom has college+ degree	−0.25852* (0.13716)	−0.28417* (0.17110)	−0.36839* (0.20695)	−0.82743** (0.32979)	−0.17320 (0.18345)	−0.07143 (0.20586)
mom has HS degree	−0.07129 (0.11777)	−0.46149*** (0.14971)	−0.05708 (0.17500)	−0.67628*** (0.25746)	−0.08449 (0.15928)	−0.34171* (0.18592)
has stay-home mom	−0.33794*** (0.11186)	−0.06238 (0.14484)	−0.50902*** (0.17332)	−0.26699 (0.27936)	−0.20454 (0.14685)	0.02583 (0.16936)
has blue-collar dad	0.08209 (0.10444)	0.11110 (0.13953)	0.02829 (0.15652)	0.24872 (0.24424)	0.12658 (0.14030)	0.04755 (0.17091)
n of older siblings	0.09575 (0.05922)	0.05975 (0.08092)	0.08579 (0.08888)	−0.02467 (0.15525)	0.10413 (0.07939)	0.09369 (0.09461)
7th-grade GPA	0.15004*** (0.05089)	0.02769 (0.06851)	0.13463* (0.07600)	−0.20218 (0.13107)	0.16326** (0.06857)	0.11676 (0.08097)
constant	−0.29330 (0.42064)	0.21102 (0.55318)	−0.91311 (0.62843)	1.30610 (1.01443)	−1.05266* (0.56674)	−1.09685* (0.66479)
F	29.76	34.70	15.70	22.21	19.85	31.52
Prob $> F$	0.0005	0.0001	0.0733	0.0082	0.0189	0.0002
R^2	0.0187	0.0294	0.0178	0.0471	0.0187	0.0352
Sample Size	281	281	281	281	281	281

***: significant at 1%, **: significant at 5%, *: significant at 10%.
Wave 1 sample matched with wave 3 (or 2) sample.

pattern is tied to specific subsets of curricula.

(i) Gender. Female students start off with greater perceived knowledge about alternatives of the general track than male students, an advantage that persists throughout pre-enrollment. With regard to the specific dimension of future performance in high school, female students initially report higher ambiguity for non-general curricula, but this gap closes by pre-enrollment. Conversely, they tend to unlearn more about general alternatives than their male counterparts.[24]

(ii) GPA. Students with a higher GPA display lower levels of perceived knowledge about non-general curricula throughout the relevant period and tend to learn significantly less with respect to curricula of all tracks than children with lower GPA. Additionally, higher GPA students start off with higher levels of perceived ambiguity, but end up with significantly greater confidence (lower

[24]Borghans et al. (2009) find that woman tend to be more ambiguity tolerant than men, at least for small levels of ambiguity.

Table 17: AMBIGUITY: PREDICTORS OF NUMBER OF TRANSITIONS IN CHILD'S AMBIGUITY PERCEPTIONS BETWEEN THE START OF 8TH GRADE AND PRE-ENROLLMENT (BETWEEN W1 AND W3)

Poisson Regression of N of Alternatives the Child's Initial Ambiguity Level:

Predictors	All Curricula		General		Technical & Vocational	
	Decreases strictly	Increases from 'Sure'	Decreases strictly	Increases from 'Sure'	Decreases strictly	Increases from 'Sure'
female	0.24168* (0.12646)	0.23077* (0.12081)	0.08276 (0.17179)	0.38229* (0.20031)	0.42798** (0.18855)	0.15648 (0.15157)
foreign born	−0.06927 (0.24909)	0.42896** (0.20610)	0.06139 (0.33453)	0.70005** (0.29897)	−0.23005 (0.37416)	0.21792 (0.28699)
lives with both parents	−0.00343 (0.20488)	−0.79131*** (0.15693)	0.11876 (0.28734)	−0.90677*** (0.24208)	−0.15194 (0.29301)	−0.71540*** (0.20681)
mom has college+ degree	−0.01513 (0.21880)	−0.05203 (0.20397)	0.08193 (0.31445)	0.07162 (0.36207)	−0.09338 (0.30979)	−0.08054 (0.24714)
mom has HS degree	0.25825 (0.18611)	−0.00508 (0.17836)	0.39463 (0.27624)	0.30882 (0.31752)	0.14421 (0.25415)	−0.14997 (0.21651)
has stay-home mom	−0.27044* (0.15585)	0.03697 (0.14774)	−0.55709** (0.23542)	−0.42957 (0.27280)	−0.00448 (0.21193)	0.26681 (0.17805)
has blue-collar dad	0.23829* (0.14506)	−0.28806* (0.15400)	0.03366 (0.21062)	−0.26183 (0.24712)	0.44538** (0.20308)	−0.29186 (0.19743)
n of older siblings	0.08378 (0.08330)	0.04105 (0.08089)	−0.01340 (0.11953)	−0.01535 (0.13263)	0.18087 (0.11650)	0.06750 (0.10205)
7th-grade GPA	0.09723 (0.07292)	−0.09907 (0.06979)	0.09797 (0.10092)	−0.33146*** (0.11777)	0.10463 (0.10601)	0.02904 (0.08722)
constant	−0.96695 (0.61407)	1.54515*** (0.56103)	−1.58481* (0.85698)	2.21524** (0.92270)	−1.85014** (0.88828)	0.09009 (0.71182)
F	15.82	36.02	11.86	37.08	15.30	16.32
Prob $> F$	0.0707	0.0000	0.2211	0.0000	0.0829	0.0606
R^2	0.0165	0.0314	0.0201	0.0664	0.0257	0.0209
Sample Size	247	247	247	247	247	247

***: significant at 1%, **: significant at 5%, *: significant at 10%.

ambiguity) about curricula of the general track at the time of pre-enrollment. This differential is driven by the group of children who start off with larger sets of unambiguous beliefs, for within the latter group higher GPA students experience a smaller number of drops from no-ambiguity to ambiguity states.

(iii) Mother education (high SES). Students with a more educated mother display lower initial levels of awareness and knowledge about non-general curricula, but they learn over time and close the gap by pre-enrollment. These students learn comparatively more about general curricula as well, which results in them having a significantly higher level of perceived knowledge about general curricula at the time of pre-enrollment compared to children with a lower educated mother. Thus, while these children end up with comparatively greater knowledge about the curricula that they are also more likely to choose –whether according to their own preference or their parents'– there is no evidence suggesting the existence of mechanisms that prevent these children from learning also about the curricula that they tend to choose less frequently. Indeed, the learning pattern of this

group of children does not appear to be concentrated on any specific curriculum of track.

This is further corroborated by the time pattern of belief ambiguity in this group. These children display significantly less belief ambiguity over curricula of the general track at the beginning of the school year. While this differential persist throughout the relevant period, by the time of pre-enrollment it becomes statistically significant also with respect to non-general curricula.

(iv) Father occupation (low SES). The observed differences in perceived awareness and knowledge levels of these children relative to their counterparts are statistically insignificant in both waves. Nevertheless, these children learn significantly less over the period. Along the ambiguity metrics, these children experience a significant reduction in perceived belief ambiguity about the likelihood of a positive future performance in high school. However, this reduction is concentrated over curricula of the technical and vocational tracks. Thus, different from children of highly educated mothers, this group of students display a more focused learning pattern, concentrated over non-general curricula. It is worth noticing that the observed pattern holds conditional on a vector of individual characteristics which includes the student's GPA, pointing to the existence of some other mechanism which prompts these children to concentrate their attention and efforts on the acquisition of information about technical and vocational curricula.

(iv) Immigration status. Similar to the previous case, the lower levels of awareness and knowledge displayed by foreign-born students relative to their Italian counterparts at the beginning and at the end of the period are statistically insignificant. However, the transitions' analysis shows that these students tend to experience a significantly larger volume of knowledge drops within the set of general curricula. Furthermore, the higher but statistically insignificant level of belief ambiguity perceived by these children at the beginning of the school year becomes statistically significant by pre-enrollment for general curricula. This pattern is driven by the group of children who starts with a positive number of non-ambiguous beliefs at the beginning of the year and transitions into states of belief ambiguity for some of them. Thus, a major difference in the learning patterns of children of blue-collar fathers and foreign-born children is that the former tend to actively learn more about non-general curricula relative to their counterparts; whereas the latter tend to unlearn about general curricula.

6 Conclusion

In this article, we generate and analyze novel survey measures of perceived awareness about available high school tracks and belief ambiguity about the likelihood of a successful performance in alternative tracks, repeatedly collected in a sample of Italian 8th graders during the months preceding pre-enrollment in high school. After assessing the validity and relevance for choice of our new measures, we analyze students' perceived levels of awareness and belief ambiguity at the start of 8th grade; we document how such perceptions evolve over time; and we investigate the main patterns of heterogeneity in students' learning over the decision process.

Students in our study display limited awareness and substantial amounts of belief ambiguity, especially with regard to lower-ranked options. Students' learning is incomplete and concentrated on their most preferred alternatives. We find significant heterogeneity in students' initial knowledge and learning patterns across demographics and socioeconomic characteristics. For instance, conditional on a set of covariates which include GPA, children with a more educated mother learn intensively about curricula of all tracks. Conversely, children with a father working in a blue-collar occupation concentrate their learning on technical and vocational curricula. Foreign-born children start with lower awareness and higher ambiguity levels relative to their Italian counterparts and follow a ('biased') learning pattern, partly similar to that of children with a blue-collar father.

Documenting the evolution of individuals' awareness, subjective beliefs, and ambiguity perceptions in consequential real-life decisions under uncertainty can provide useful guidance to researchers on how to best specify choice models and perform relevant policy counterfactuals. Our analysis can also inform policy by pointing to particular combinations of family profiles and educational alternatives for which informational policies are more (or less) likely to effectively reduce skill misallocation and inequality.

Processes of information acquisition of the kind that we document in this paper may have important consequences for skill mismatch. On the one hand, the learning pattern of children who are observed to learn more slowly (if at all) about their low-ranked alternatives may be rationalized by the existence of a limit to the amount of attention that these children can apply to processing and storing new information. On the other hand, the learning pattern of children with a 'biased attention' toward top-ranked options can create important inefficiencies even when prompted by some kind of optimality criterion, as we find that learning is still incomplete at the time of choice.

Table 26 in Appendix B indicates that a sizeable fraction of students in our sample changes their preference ranking over alternatives during the 6-7 months before pre-enrollment. For example, the two matrices in the last column of the table show that 22-23% of the alternatives that were not ranked first in wave 1 become top-ranked by the time of pre-enrollment (wave 3). This evidence suggests that the learning process might be crucial for the choice of a sizable fraction of children, as these children do not yet know their most preferred alternative in wave 1. The 'selective learner' might become stuck with a coarse view of the world which associates a high degree of ambiguity to some initially unattractive options, thus reinforcing undesirability of the latter in later periods. For example, a talented musician might end up choosing a traditional curriculum of the general track, because he, his parents, or both have accurate information about general traditional curricula and rank them at the top when information is still diffuse. They subsequently acquire additional information about traditional curricula of the general track, neglecting or failing to collect relevant information about the newly-activated music & choral curriculum. An accurate measurement of such inefficiencies will be an import and quite challenging task that we leave for future research.

A second important venue for future research is the analysis of child-parent interactions. Table 18 presents preliminary evidence about existing within-family differences in awareness, optimism, and belief ambiguity between children and their parents. Figures in the top panel of Table 18 indicate that on average parents start with larger awareness sets than their children in wave 1; however, by wave 3 children reach higher levels of perceived awareness than their parents. Thus, children seem to learn faster than their parents, especially in non-general tracks. Figures in the middle panel suggest that in wave 1 parents' beliefs are more optimistic than those of their children, especially about the child's likelihood of a successful performance in general curricula. However, throughout the learning process parents become more pessimistic than their children with regard to the child's likelihood of a positive performance in general curricula, while reinforcing their (relative) optimism about the child's performance in non-general curricula. Finally, the statistics in the bottom panel indicate that on average parents start with a higher level of belief ambiguity than their children in wave 1. During the learning period, children's perceived ambiguity tends to polarize by either vanishing altogether ('Sure') or by reaching its upper limit ('No idea').

The reported sample averages of within-family differences in awareness, subjective expectations, and belief ambiguity between children and parents might look somewhat small at first sight; in fact, observed within-family differences in perceptions between children and parents display a significant amount of variability across families.

A detailed analysis to the parent-child relationship and of the decision process within the family may offer important new readings of the data. In particular, ambiguity perceptions and precision of respondents' beliefs across tracks are likely related to the family decision process and the nature of child-parent interactions within the family. This kind of analysis will be possible to us, as our dataset includes detailed information about the role of individual family members in the decision as well as the perceptions of each member about the choice preferences of the remaining members (see the questions listed in the bottom panel of Table 23).[25]

In our context, allowing for the possibility of conflicting objectives between altruistic parents and their children might also generate new non-paternalistic policy implications induced by the

[25]The literature studying child-parent interactions and parents' subjective beliefs about their children's production of human capital and the consequences of human capital investment is still in its infancy. Recent contributions include Cunha et al. (2013), Boneva and Rauh (2015), and Giustinelli (2015). See also the review by Giustinelli and Manski (2015).

possibility that parents manipulate adolescents' awareness sets, beliefs, and perceptions. While a few papers have addressed some of these aspects theoretically (e.g, von Thadden and Zhao (2012), Auster (2013), and Pavoni and Yazici (2015)), the empirical literature on these topics is nearly null.

Table 18: AWARENESS, POINT BELIEFS, AND AMBIGUITY: CHILD-PARENT COMPARISONS

	Wave 1 (matched with Wave 3)		**Wave 3[a] (matched with Wave 1)**	
	Mean Difference in Awareness Reports of Child-Parent Pairs			
	'Know'	**'Heard of'**	**'Know'**	**'Heard of'**
All curricula	-0.36283	0.30383	0.53915	-0.42469
General curricula	0.02654	0.22123	0.43975	-0.28012
Technical & vocational curricula	-0.38938	0.08259	0.09939	-0.14457
N	339	339	332	332
	Mean Difference in Point Belief Reports of Child-Parent Pairs			
Gen, Humanities	-4.82142		1.21705	
Gen, Languages	-2.91166		1.05405	
Gen, Math & Science	-1.67132		1.55343	
Gen, Art, Music & Choral	-2.53956		1.94961	
Gen, Soc Sciences	-2.86666		-0.84705	
Tech, Economic Sector	0.05323		0.07086	
Tech, Technology Sector	0.74906		-1.1891	
Voc, Services	-3.41472		-6.0806	
Voc, Industry & Crafts	-1.08267		-6.1951	
Voc, Prof Training	-0.72111		-5.7551	
N	251-286		241-262	
	Mean Difference in Ambiguity Reports of Child-Parent Pairs			
	'Unsure'	**'No Idea'**	**'Unsure'**	**'No Idea'**
All curricula	-0.32627	-0.25	-1.82978	1.67234
General curricula	-0.07627	-0.22881	-0.82127	0.53617
Technical & vocational curricula	-0.25	-0.02118	-1.00851	1.13617
N	236	236	235	235

[a]: Wave 2 responses used for respondents who did not participate in wave 3.

***Note:** The **top panel** shows the sample averages of the within-family differences between the number of alternatives the child reports knowing/having heard of and the number of alternatives the parent reports knowing/having heard of for all curricula and by track. A positive difference indicates that the child knows/has heard of more alternatives compared to the parent and viceversa. The minimum logical value for the child-parent difference in awareness is -11 across all curricula, -6 for curricula of the general track, and -5 for curricula for the non-general track. The maximum logical values are +11, +6, and +5, respectively. The **middle panel** shows the sample average of the within-family difference in point beliefs between children and parents disaggregated by alternative. In this case, a positive difference indicates that the child is more optimistic than the parent and viceversa. The minimum and maximum logical values for the child-parent difference in point beliefs are -100 and +100. The **bottom panel** presents the sample averages of the within-family differences between the number of alternatives the child has ambiguous beliefs about and the number of alternatives the parent has ambiguous beliefs about for all curricula and by track. The minimum (resp. maximum) logical value for the child-parent difference in ambiguity is -11 (resp. +11) across all curricula, -5 (resp. +5) for curricula of the general track, and -5 (resp. +5) for curricula of the non-general track.*

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A Ranges Behavior: Theory

Recall that we assumed that model uncertainty can only be due to the learnable characteristics in Ω_M. Let suppose, for simplicity, that we can associate one model to each element $\omega_M \in \Omega_M$, that is, $M_t = \mathcal{I}_t^M$. It is immediate to see that if the child were to learn over the set of models only (e.g., when the information set $\mathcal{I}^U$ is time constant), the range measure decreases with time for each a. From equation (3), we have $R_t^a(\mathcal{I}_t) = R_t^a(\mathcal{I}^U \times \mathcal{I}_t^M) = \max_{m \in M_t} \pi^{a,m}(\mathcal{I}^U) - \min_{m \in M_t} \pi^{a,m}(\mathcal{I}^U)$, where the max and the min are taken over the set $M_t = \mathcal{I}_t^M$, which 'shrinks' over time.

Let us now allow for time changing $\mathcal{I}^U$ and for ex-ante heterogeneity. Assume that the information $\{\mathcal{I}_t^U\}_t$, arrives *independently across students*, conditional on the student's type and the set of models; in particular, after the start of 8th grade there is no new aggregate information. Let us fix a model m. By the law of large numbers and by Bayes' rule, for each ex-ante homogeneous group $g \in G$, we have $\pi^{a,m,g}(\mathcal{I}_{t-1}^g) = \int \pi^{a,m,g}(\mathcal{I}_t^g) dh_t^g(\mathcal{I}_t^g)$, where $h_t^g(\cdot)$ is the distribution over new information in period t for group g, and $\pi^{a,m,g}(\mathcal{I}_{t-1}^g)$ and $\pi^{a,m,g}(\mathcal{I}_t^g)$ are the posteriors reported by the students in group g under model m, in period $t-1$ and t, respectively. Since this martingale condition holds for each group, if we let $f(\cdot)$ be the distribution of ex-ante heterogeneous types in the economy and assume that such a distribution is constant over time, we obtain $\hat{\pi}_{t-1}^{a,m} = \int_G \pi_{t-1}^{a,m,g} df(g) = \int_G \int \pi^{a,m,g}(\mathcal{I}_t^g) dh_t^g(\mathcal{I}_t^g) df(g) = \hat{\pi}_t^{a,m}$ $\forall a, t, m$. In other terms, given any model, Bayes' rule and cross-sectional independent signals jointly imply that the cross-sectional average of beliefs $\hat{\pi}_t^{a,m}$ is constant over time.

Since ranges are reported individually, for any two consecutive dates at which the set of models remains constant, $\mathcal{I}_t^M = \mathcal{I}_{t-1}^M = M$, and any collection of information sets, $\mathcal{I}_t^g \subset \mathcal{I}_{t-1}^g$, consistent with assumptions on the learning process, we have

$$
\begin{aligned}
\hat{R}_t^a &= \int_G \int R_t^{a,g}(\mathcal{I}_t^g \times I^M) dh_t^g(\mathcal{I}_t^g) df(g) = \int_G \int \left[\max_{m \in M} \pi^{a,m,g}(\mathcal{I}_t^g) - \min_{m \in M} \pi^{a,m,g}(\mathcal{I}_t^g) \right] df(g) \\
&\geq \int_G \left[\max_{m \in M} \int \pi^{a,m,g}(\mathcal{I}_t^g) dh_t^g(\mathcal{I}_t^g) - \min_{m \in M} \int \pi^{a,m,g}(\mathcal{I}_t^g) dh_t^g(\mathcal{I}_t^g) \right] df(g) \\
&= \int_G \left[\max_{m \in M} \pi^{a,m,g}(\mathcal{I}_{t-1}^{U,g}) - \min_{m \in M} \pi^{a,m,g}(\mathcal{I}_{t-1}^{U,g}) \right] df(g) = \int_G R_{t-1}^{a,g} df(g) = \hat{R}_{t-1}^a.
\end{aligned}
$$

The inequality in the second row follows immediately from the property of the maximum and the minimum, while the equality at the beginning of the last row uses the martingale property of beliefs.

This results implies that a decreasing range size is necessarily associated to higher knowledge; whereas an increasing range size has an ambiguous interpretation. To interpret an increase in the cross-sectional average of range size as a 'deterioration' of information, we need additional structure, which we provide next.

PROPOSITION A.1. *(i) If for some alternative $a \in \hat{A}$ and periods $t, t-1 \geq 0$, we observe a decreasing average range size, $\hat{R}_t^a > \hat{R}_{t-1}^a$, it must be that some or all children received some information over the set of models, that is, $\mathcal{I}_t^M \subset \mathcal{I}_{t-1}^M$ with strict inclusion.*
(ii) Moreover, for a fixed alternative a, if for all t, m, g, and $\mathcal{I}_t^U \subset \mathcal{I}_{t-1}^U$ consistent with the process of arrival of information, we have $\pi^{a,m,g}(\mathcal{I}_t^U) = \pi^{a,m,g}(\mathcal{I}_{t-1}^U) + \varepsilon^{a,g}(\mathcal{I}_t^U)$, then the cross-sectional average range measure $\hat{R}_t^a$ (weakly) decreases over time.

Proof. (i) This is a direct consequence of the fact that if I^M is constant, $\hat{R}_t^a$ can only increase.
(ii) For any two consecutive dates with fixed $\mathcal{I}_t^M = \mathcal{I}_{t-1}^M$ and each $g, \mathcal{I}_t^U \subset \mathcal{I}_{t-1}^U$, we have

$$\int R_t^{a,g}(\mathcal{I}_t^U \times \mathcal{I}_t^M) dh_t^g(\mathcal{I}_t^U) = R_{t-1}^{a,g}(\mathcal{I}_{t-1}^U \times \mathcal{I}_{t-1}^M).$$

The monotonicity is implied by the fact that for each $g, \mathcal{I}_t$, generated by the learning process we postulate the individual ranges $R_t^{a,g}(\mathcal{I}_t^U \times \mathcal{I}_t^M) = \max_{m \in \mathcal{I}_t^M} \pi^{a,m,g}(\mathcal{I}_t^U) - \min_{m \in \mathcal{I}_t^M} \pi^{a,m,g}(\mathcal{I}_t^U)$ decrease in $\mathcal{I}_t^M$ and $\mathcal{I}_t^M$ decreases with t. $\square$

B Additional Tables

Table 19: IDENTITY OF RESPONDENTS

	Wave 1 All		Wave 1 Matched with Wave 3[a]	
Respondent Identity	**Children Sample**	**Parents Sample**	**Children Sample**	**Parents Sample[b]**
Child (%)	649 (100%)	N.A.	410 (100%)	N.A.
Both parents (%)	N.A.	288 (47.84%)	N.A.	171 (48.44%)
Mother only (%)	N.A.	262 (43.52%)	N.A.	159 (45.04%)
Father only (%)	N.A.	47 (7.81%)	N.A.	23 (6.52%)
Other person (%)	N.A.	5 (0.83%)	N.A.	0 (0%)
N (%)	649 (100%)	602 (100%)	410 (100%)	353 (100%)

[a]: Based on subjects who responded to wave 3. Subjects who did not respond to wave 3 but responded to wave 2 were also included.

[b]: Parents sample in wave 3 is conditional on families where the same parent or parents responded across waves.

Table 20: CHILD'S BACKGROUND CHARACTERISTICS

	Wave 1 All Children Sample (N=649)	**Wave 1 Matched with Wave 3[a] Children Sample (N=410)**
Child's gender		
% male	46.53	43.17
% female	53.47	56.83
N (100%)	649	410
% item non-response/missing	0	0
Child's place of birth[b]		
% Italy	86.36	88.02
% other country	13.64	11.98
N (100%)	645	409
% item non-response/missing	0.62	0.24
Child's age[c]		
mean	13.0929	13.0732
std. dev.	0.4249	0.4072
min	12	12
median	13	13
max	15	15
N (100%)	646	410
% item non-response/missing	0.46	0
Child's age vs. school grade[d]		
% regular (born in 1998)	83.9	85.12
% ahead (born after 1998)	3.87	4.15
% behind (born before 1998)	12.23	10.73
N (100%)	646	410
% item non-response/missing	0.46	0
Child's GPA [e]		
mean	7.6541	7.7405
std. dev.	0.9663	0.9719
min	6	6
median	7.6	7.7
max	9.8	9.8
N (100%)	567	369
% item non-response/missing	12.63	10
Parent/s' child lives with[f]		
% both parents	87.84	88.48
% one parent	11.66	10.99
% none	0.51	0.52
N (100%)	592	382
% item non-response/missing	4.05	4.02
Number of older siblings[g]		
mean	0.6248	0.5594
std. dev.	0.7636	0.6966
min	0	0
median	0	0
max	3	3
N (100%)	581	379
% item non-response/missing	10.48	7.56

[a]: Based on subjects who responded to wave 3. Subjects who did not respond to wave 3 but responded to wave 2 were also included.
[b]: Constructed using multiple reports by child and parent/s.
[c]: Constructed from year of birth, using multiple reports child and parent/s.
[d]: Constructed from year of birth and current grade.
[e]: Constructed by averaging grades in 9 main subjects.
[f]: Constructed from co-residing question, using multiple reports by child and parent/s.
[g]: Constructed by censing up to 3 older siblings, using multiple reports by child and parent/s.

Table 21: CHILD'S BACKGROUND CHARACTERISTICS (CONTINUED)

	Wave 1 All Children Sample (N=649)	Wave 1 Matched with Wave 3[a] Children Sample (N=410)
Mother's country of birth[b]		
% Italy	87.79	82.7
% other country	19.21	17.3
N (100%)	609	393
% item non-response/missing	4.25	3.2
Father's place of birth[b]		
% Italy	81.16	83.03
% other country	18.84	16.97
N (100%)	584	383
% item non-response/missing	2.99	1.79
Language prevalently spoken at home[c]		
(conditional on one or multiple members being foreign-born)		
% Italian	47.2	56.79
% other language	52.8	43.21
N (100%)	125	81
% item non-response/missing	19.87	14.74
Mother's highest schooling degree[d]		
% elementary or less	2.37	1.85
% junior high school degree	20.14	18.78
% HS diploma (includes 3-years vocational degrees)	50.08	52.12
% college degree or higher (includes 3-years degrees)	27.41	27.25
N (100%)	591	378
% item non-response/missing	7.08	6.9
Father's highest schooling degree[d]		
% elementary or less	1.94	1.62
% junior high school degree	21.3	22.16
% HS diploma (includes 3-years vocational degrees)	50.35	50.81
% college degree or higher (includes 3-years degrees)	26.41	25.41
N (100%)	568	370
% item non-response/missing	5.65	5.13
Mother's working status[e]		
% works full-time	39.43	41.04
% works part-time	37.58	36.36
% does not work	22.90	22.60
N (100%)	596	385
% item non-response/missing	6.29	5.17
Father's working status[e]		
% works full-time	92.06	91.84
% works part-time	4.32	4.21
% does not work	3.63	3.95
N (100%)	579	380
% item non-response/missing	3.82	2.56
Mother's occupation[f]		
% stay-home mom	24.28	23.76
N (100%)	593	383
% item non-response/missing	3.93	2.46
Father's occupation[f]		
% blue collar	28.75	24.54
N (100%)	574	379
% item non-response/missing	4.65	2.82

[a]: Based on subjects who responded to wave 3. Subjects who did not respond to wave 3 but responded to wave 2 were also included.
[b]: Conditional on having one. Constructed from country of birth, using multiple reports by child and par/s.
[c]: Asked of child only.
[d]: Conditional on having one. Constructed from original question on educational attainment, using multiple reports by child and parent/s.
[e]: Conditional on having one. Constructed using multiple reports by child and parent/s.
[f]: Conditional on having one. Constructed from question on occupation, using multiple reports. Selected categories only.

Table 22: RESPONDING PARENTS' BACKGROUND CHARACTERISTICS

	Wave 1 All Children Sample (N=649)[b]	Wave 1 Matched with Wave 3[a] Children Sample (N=410)[b]
Responding mother's age		
mean	44.4614	44.6712
std. dev.	4.8075	4.4908
min	30	32
median	44	45
max	63	63
N (100%)	518	295
Responding father's age		
mean	47.6950	48.8984
std. dev.	5.9205	6.3393
min	28	31
median	47	48
max	73	73
N (100%)	318	128
Responding mother's place of birth		
% Italy	83.24	85.81
% other country	16.76	14.19
N (100%)	525	296
Responding father's place of birth		
% Italy	81.73	85.94
% other country	18.27	14.06
N (100%)	323	128
Responding mother's highest schooling degree		
% elementary or less	1.55	1.04
% junior high school degree	18.64	19.72
% HS diploma (includes 3-years vocational degrees)	50.68	50.52
% college degree or higher (includes 3-years degrees)	29.13	28.72
N (100%)	515	289
Responding father's highest schooling degree		
% elementary or less	1.89	0.79
% junior high school degree	21.45	24.41
% HS diploma (includes 3-years vocational degrees)	50.79	44.88
% college degree or higher (includes 3-years degrees)	25.87	29.92
N (100%)	317	127
Responding mother's working status		
% works full-time	39.65	40.96
% works part-time	37.72	36.18
% does not work	22.63	22.87
N (100%)	517	298
Responding father's working status		
% works full-time	92.26	90
% works part-time	4.02	5.38
% does not work	3.72	4.62
N (100%)	323	130

[a]: Based on subjects who responded to wave 3. Subjects who did not respond to wave 3 but responded to wave 2 were also included.
[b]: These statistics were constructed by matching responding parents' identity and parents' background characteristics.

Table 23: SELECTED EXPECTATIONS QUESTIONS ACROSS FORMATS, RESPONDENTS, AND WAVES

Type	Future Outcome	Format of Expectation Question	Respondent	Wave Asked
Stated choice/preference	Would choose for himself/herself/child today	Percent chance (point only)	Child; mother and father (or resp.parent)	W1 (only top 3 alternatives), W2, W3, W4 (all ranked alt.)
Event following choice	**Child graduates in time with passing (or higher) grades in all subjects**	Percent chance (point & range)	Child; responding parent(s)	**W1, W2, W3**
Event following choice	**Child's high school training permits a flexible college-work choice** (college &/or work, college only, work only)	Percent chance (point, with no idea option)	Child; responding parent(s)	**W1, W2, W3**
Event following choice	**Child's high school training permits a flexible college major choice** (hum. or soc. sciences; math & science or engineering; law or econ.)	Percent chance (point, with no idea option)	Child; responding parent(s)	**W1, W2, W3**
Event following choice	**Child's college-work choice** (college only, work only, college & work)	Percent chance (point, with no idea option)	Child; responding parent(s)	W3
Event following choice	Child's job after graduating from high school & no college	Expected job (job name, with no idea option)	Child; responding parent(s)	W3
Event following choice	Child's job after graduating from high school & college	Expected job (job name, with no idea option)	Child; responding parent(s)	W3
Family process	Parents would allow child's choice, without child explaining his/her choice	Percent chance (point, with no idea option)	Child; responding parent(s)	W1, W2, W3
Family process	Parents would allow child's choice, provided child explains his/her choice	Percent chance (point, with no idea option)	Child; responding parent(s)	W1, W2, W3
Family process	Curriculum mother would choose for the child today	Percent chance (point, with no idea option)	Child	W1, W2, W3 (top 3 ranked by mother)
Family process	Curriculum father would choose for the child today	Percent chance (point, with no idea option)	Child	W1, W2, W3 (top 3 ranked by father)
Family process	Curriculum child would choose for himself/herself today	Percent chance (point, with no idea option)	Child; responding parent(s)	W1, W2, W3 (top 3 ranked by child)
Family process	Curriculum other parent would choose for the child today	Percent chance (point, with no idea option)	Responding parent(s)	W1, W2, W3

Table 24: AWARENESS: χ^2 STATISTICS FOR TESTING EQUALITY OF ROWS IN THE AWARENESS TRANSITION MATRICES (SAMPLE OF CHILDREN WHO RESPONDED TO EACH PAIR OF WAVES)

	Wave 1 vs. Wave 2	**Wave 2 vs. Wave 3**	**Wave 1 vs. Wave 3**
Starting state	**Unconditional vs. Chosen**		
Know	***	***	***
Heard of	*	**	***
Never heard	**	*	**
Starting state	**Ranked First vs. Ranked Bottom (Ranked 4th or Below)**		
Know	***	***	***
Heard of	***	**	**
Never heard	*	NA	-

***: 99% confidence level, **: 95% confidence level, *: 90% confidence level.

Table 25: AMBIGUITY: χ^2 STATISTICS FOR TESTING EQUALITY OF ROWS IN THE AMBIGUITY TRANSITION MATRICES (SAMPLE OF CHILDREN WHO RESPONDED TO EACH PAIR OF WAVES)

	Wave 1 vs. Wave 2	**Wave 2 vs. Wave 3**	**Wave 1 vs. Wave 3**
Starting state	**Unconditional vs. Chosen**		
Sure	**	**	***
Unsure	-	***	-
No Idea	-	-	-
Starting state	**Ranked First vs. Ranked Bottom (Ranked 4th or Below)**		
Sure	***	***	***
Unsure	-	-	-
No Idea	*	**	***

***: 99% confidence level, **: 95% confidence level, *: 90% confidence level.

Figure 2: Evolution of Point Beliefs

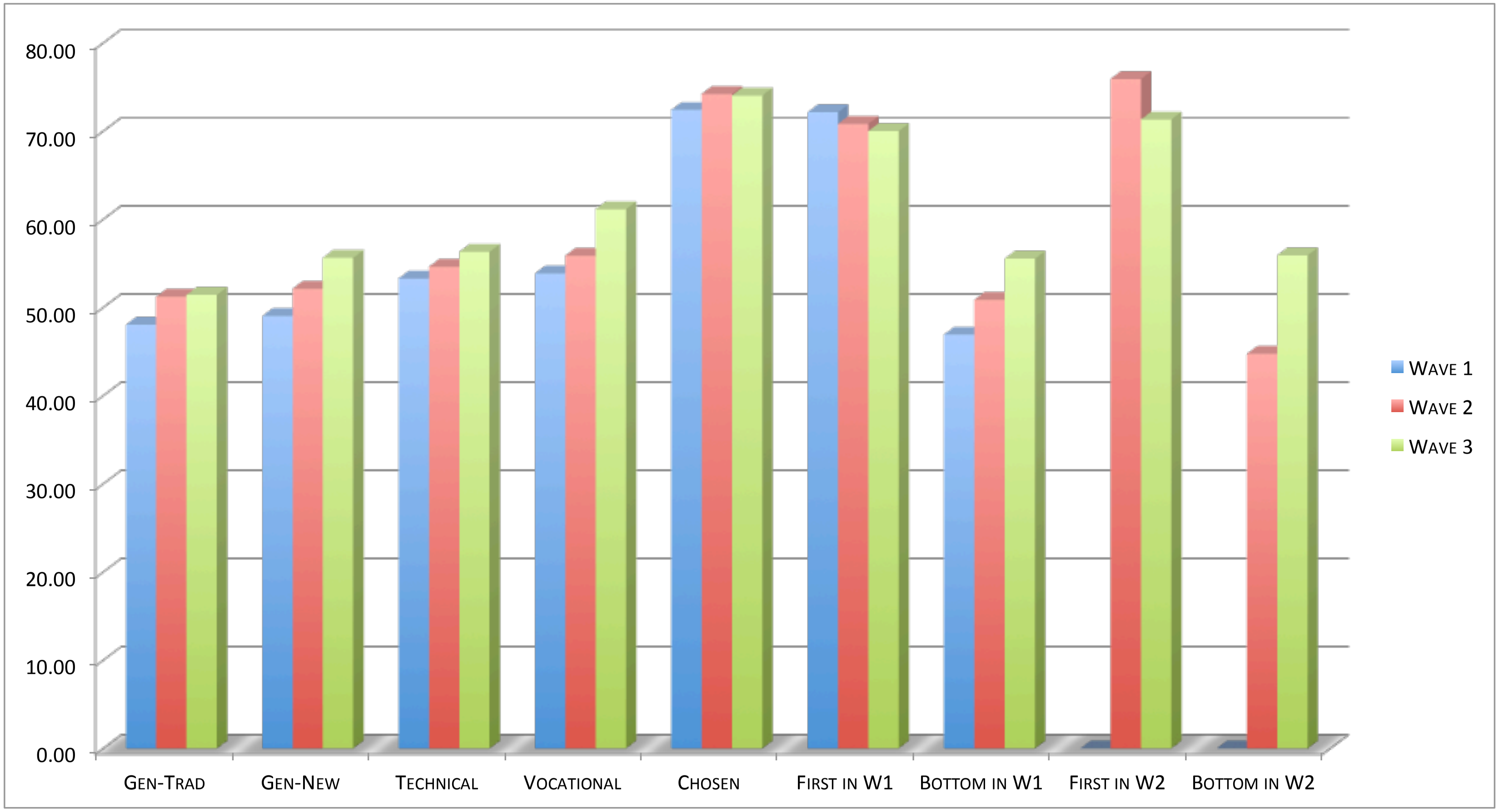

Figure 2: Mean point belief in waves 1-3, conditional on the sets described in the horizontal axis.

Figure 3: Evolution of Subjective Ranges by Curriculum

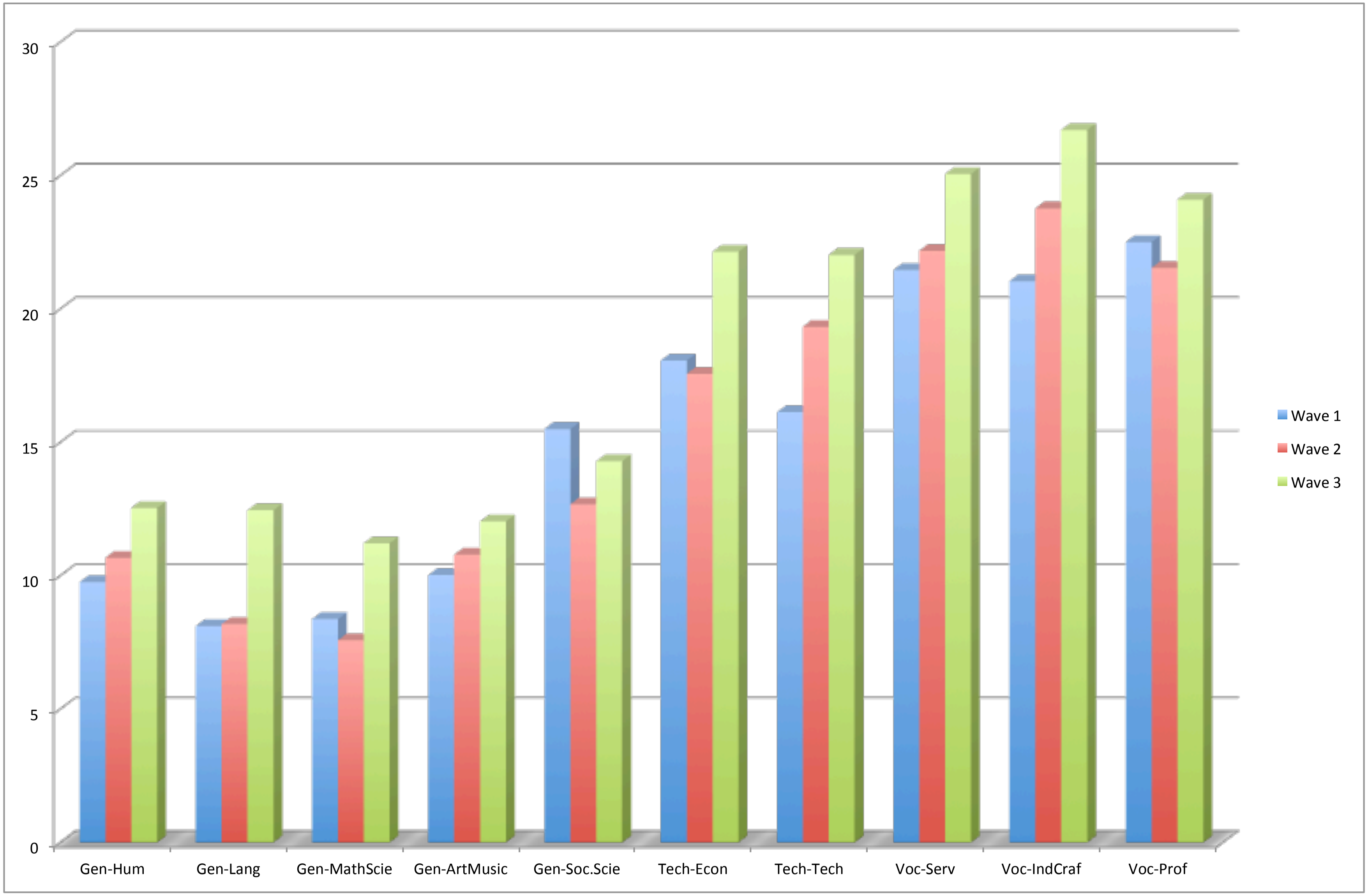

Figure 3: Mean range size in waves 1-3 by curriculum.

Table 26: Choice Preferences: Transition Matrices of Child's Preference Ranking Over Alternatives Across Waves

Waives Pairs Sample

		W2: Ranked 1st	W2: Not 1st	N
W1	Ranked 1st	0.71	0.29	301
W1	Not 1st	0.23	0.77	479

		W3: Ranked 1st	W3: Not 1st	N
W2	Ranked 1st	0.74	0.26	219
W2	Not 1st	0.18	0.82	291

		W3: Ranked 1st	W3: Not 1st	N
W1	Ranked 1st	0.63	0.37	254
W1	Not 1st	0.23	0.77	416

All Waves Sample

		W2: Ranked 1st	W2: Not 1st	N
W1	Ranked 1st	0.72	0.28	193
W1	Not 1st	0.22	0.78	307

		W3: Ranked 1st	W3: Not 1st	N
W2	Ranked 1st	0.76	0.24	209
W2	Not 1st	0.18	0.82	278

		W3: Ranked 1st	W3: Not 1st	N
W1	Ranked 1st	0.66	0.34	206
W1	Not 1st	0.22	0.78	330