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Innovation, Pricing and Targeting in Networks*

Fabrizio Panebianco† Thierry Verdier‡ Yves Zenou§

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Abstract

Consider a network of firms where a firm T is given the opportunity to innovate a product (first-generation innovation). If successful, this firm can temporarily sell this innovation to her direct neighbors because this will give her access to a larger market. However, if her direct neighbors innovate themselves on top of firm T's innovation (second-generation innovations), then firm T loses the right to sell her initial innovation to the remaining firms in the market. We analyze this game where each firm (T and her direct neighbors) has to decide at which price they want to sell their innovation. We show that the optimal price policy of each firm depends on the level of property rights protection, the position of firm T in the network, her degree and the size of the market. We then analyze the welfare implications of our model where the planner that maximizes total welfare has to decide which firm to target. We show that it depends on the level of property rights protection and on the network structure in a non-trivial way.

JEL classification: D85, L1, Z13.

Keywords: Networks, diffusion centrality, targets, innovation.

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1 Introduction

The way the diffusion of a product or an idea operates is complex and depends on the innovation itself, adopters, communication channels, time, and the social system where the innovation takes place. In their meta reviews, Rogers (2003) and Greenhalgh et al. (2004) have identified several characteristics that are common among most studies. Potential adopters evaluate an innovation on its relative advantage (the perceived efficiencies gained by the innovation relative to current tools or procedures), its compatibility with the pre-existing system, its complexity or difficulty to learn, its trialability or testability, its potential for reinvention, and its observed effects. Rogers (2003) and Easley and Kleinberg (2010) highlight the fact that the diffusion of an innovation is strongly associated with the network where the innovation takes place since the communication channels allow the transfer of information from one unit to the other. The aim of this paper is to investigate further this issue by putting forward the role of the network in the diffusion of an innovation and how pricing strategies and targeting can affect this type of diffusion.

To be more precise, we consider a network of firms where a firm T has a new innovation and wants to sell it first to her direct neighbors. This new innovation is referred to as the *first-generation innovation*. In fact, firm T needs to sell first the innovation to her direct neighbors in order to have access to their market. This is because these direct neighbors are the firms with whom firm T has a direct contact with (word-of-mouth communication). These direct neighbors are the only firms in the network that can innovate on top of firm T (i.e. *second-generation innovation*). These two innovations (first and second-generation innovations) are *complementary* since the second-generation product requires the input of the first-generation innovation. In this framework, we consider two different environments.

In the first one, firm T writes a binding contract (that can be verified by a court) with each of her neighbors that gives them the right to innovate during a fixed period of time. In exchange, each of these neighbors has to pay a price (for example, a licencing fee) stipulated in the contract in order to be able to innovate. If we think, for example, of a software, then firm T will give the codes of the software to her neighbors during this period. If, at the end of this period of time, the neighbors have innovated, then firm T cannot sell her product to the firms belonging to the rest of the market (i.e. firm T have no rights on subsequent innovations) and each of the direct neighbors will sell their (second-generation) innovation to the firms belonging to their market. On the contrary, if, at the end of this period of time, the neighbors have *not* been able to innovate, then firm T will be the only one that will sell her product (first-generation innovation) to all the remaining firms in the market. However, because she has signed a binding contract with her direct neighbors that stipulates the price of the product, she will be obliged to sell her product at the same price to the remaining firms.

In the second environment, firm T does not write a contract but has an informal agreement with each of her neighbors that gives them the right to innovate during a fixed period of time. As above, each of these neighbors has to pay a price in order to be able to innovate. If, at the end of this period of time, the neighbors have innovated, then firm T cannot sell her product to the remaining firms in the market. If, at the end of this period of time, the neighbors have *not* innovated, then firm T will sell her product to the remaining firms at a price that can be different to the one charged to the direct neighbors. This is possible because there is no binding contract with a given price.

In this framework, it is important to understand the role of the network. Indeed, firm T can only sold her product to her direct neighbors in the network because these are the only firms she has directly access to and because these are the only firms that can improve her innovation since they are sufficiently technologically close to firm T. This means that the network can be viewed as representing the flow of information between connected firms but also as the technological proximity between firms. Thus, despite the threat that her neighbors can innovate and can therefore "steal" her product, firm T is willing to sell her innovation to her neighbors because it is the only way for her to have access to a large market. Of course, the price she will charge to her direct firms will depend on the environment she faces and will reflect the trade off between obtaining rents from her neighbors and inducing the possible remaining firms to buy a product.

More generally, what we have in mind here is the idea that innovations are the result of a *cummulative process*, which is typically true in complex industries such as biotechnologies, information technologies, electronics and software. Consider, for example, the 30GB version of Apple's fifth-generation iPod, the *Video iPod*, which went on sale in October 2005. For the 30GB Video iPod, the highest-value components are the *hard drive* and the *screen display*, which were both supplied by Japanese companies and not by Apple itself (Linden et al., 2009). These are intermediary products, not developed by Apple, which can be improved by innovative firms. In that case, each firm gets informed of an innovation in the process technology through a *word-of-mouth* process, so that firms can communicate among them and share information. Another example is the *software* industry. The software, computers, and semiconductors industries have been the most innovative industries of the last forty years. Even though these industries had weak patent protection, they have experienced a rapid imitation of their products. As argued by Bessen and Maskin (2009), for industries like software or computers, theory suggests that imitation may promote innovation and that strong patents (long-lived patents of broad scope) might actually inhibit it. Society and even the innovating firms themselves could well be served if intellectual property protection were more limited in such industries. As stated above, in these industries, innovation is both *sequential* and *complementary* (Bessen and Maskin, 2009). By sequential, we mean that each successive invention builds on the preceding one, in the way that the Lotus 1-2-3 spreadsheet built on VisiCalc, and Microsoft's Excel built on Lotus. And by "complementary," we mean that each potential innovator takes a different research line and thereby enhances the overall probability that a particular goal is reached within a given time. Our model fits well with this story since we model the degree of property right protections as a cost of innovation and the possibility that firms can imitate and innovate on top of the original innovation, i.e. innovation is both sequential and complementary. In this interpretation, our product could be a software that is developed by a firm T and sold to her "direct" neighbors, who can, in turn, improve this software and sell it to their own neighbors. Notice that, if the neighbors innovate, then the initial software sold by firm T will become "obselete" and the remaining firms in the market will not buy it; they will only buy the new software. If, of course, the neighboring firms did not succeed in innovating, then the remaining firms in the market will buy the initial software from firm T.

We first analyze the optimal price policy of firm T and of her neighboring firms when firm T must commit to a single price (because she has signed a binding contract that is easily verifiable) and when she does not need to commit to a single price (because she only has an informal agreement with her direct neighbors). In the first case, we show that firm T can only charge two prices. A high one that extracts all rents from the neighboring firms but, for which, the remaining firms will not buy T's product if her neighboring firms do not innovate. A low price that guarantees that all firms (both her neighboring firms and the remaining firms) will buy T's product. We also show under which condition firm T prefers a high price or a low price. Indeed, it should be clear that firm T faces the following trade off. If she charges a high price to her neighboring firms, then it is very likely that the remaining firms will not buy her product if her neighboring firms do not innovate. If she charges a low price, then she will not make that much profit from her neighboring firms and there is a risk that the remaining firms will not buy her product if her neighboring firms innovate. We show that the optimal price policy depends on the level of property rights protection, the position of firm T in the network, which indicates how large is her potential market and her degree, i.e. the number of direct neighbors/competitors of firm T. In the second case when firm T does not need to commit to a single price, we show that firm T will have a unique two-stage price policy, where, in the first stage, the price extracts all the rents from her neighboring firms while, in the second stage, the price extracts all the rents from the remaining firms, i.e. the firms belonging to the potential market of the neighboring firms.

In the second part of the paper, we analyze the welfare implications of our model. Our question is as follows: If a planner that maximizes total welfare wants to target a firm T, which one will she choose? Here, target means that the planner chooses the firm that would innovate first by providing, for example, some financial incentives. We show that which firm should be targeted depends on the institutional context (what kind of price commitment or contract prevails), the level of property rights protection, the position of firm T in the network and her degree. In particular, if the level of property rights protection is low so that it is easy to innovate and if firms must commit to a unique price, then the planner should always target the most peripheral firms in the network. On the contrary, if the level of property rights protection is high and if firms are obliged by contract to commit to a unique price, then the planner should always target the firms with the highest number of direct competitors. When firms do not need to commit to a unique price, we show that, independently of the level of property rights protection, the planner should always target the firm with the lowest degree. These results are due to the fact that the diffusion of the innovation can be beneficial or not for the society at large depending on the position in the network of the targeted firm and on the difficulty to innovate.

The rest of the paper unfolds as follows. In the next section, we relate our paper to the literature. In Section 3, we describe how the diffusion of the innovation in the network works. In Section 4, we consider the optimal price policy of the neighboring firms of firm T while, in Section 5, we focus on the pricing policy of firm T. Section 6 is devoted to the welfare analysis of the model where the planner has to decide which firm should be targeted. Finally, Section 7 concludes. All proofs can be found in the Appendix 1. In Appendix 2, we extend the model where firms only know partially the network.

2 Related literature

Our paper is related to several literatures.

Innovation There is a very important literature on innovation (for an overview, see Hall and Rosenberg, 2010). Our paper is mainly related to the literature on *sequential innovation*, which has been studied by Scotchmer (1991, 1996), Green and Scotchmer (1996), and Chang (1995) for the case of a single follow-on innovation and by Hunt (2004), O'Donoghue (1998), O'Donoghue et al. (1998), Bessen and Maskin (2009) and Cozzi and Galli (2014), who study a single invention with an infinite sequence of quality improvements. The focus of this literature is mainly on the trade off between innovation and patents and, in particular, how patents can enhance or reduce innovation. In particular, an important issue in this literature is the fact that the first-generation innovator may be granted some right on subsequent innovations and thus an *holdup* problem may arise. Indeed, if an early patent holder has a claim against subsequent innovators, the latter may be reluctant to invest in innovations (R&D) because they anticipae the expected cost of such claims (see Belleflamme and Peitz, 2015, Chap. 19, for an overview on these issues).

Our paper proposes a different framework as we model the sequential innovation in a network and study the effect of price policy, innovation diffusion and targeting in a network. In particular, we do not have the holdup problem mentioned above because the property-right protection level can vary from very low to very high and the structure of the network indicates which firm has access to which market. As a result, the first-generation innovator (firm T) is ready to disclose her innovation to her neighbors (technologically close firms) because she hopes that this will give her access to the market of these firms. Firm T will be able to sell her first-generation innovation to the whole market only if her neighbors do not innovate. In our model, what is interesting is that, in the context of sequential innovation, firms create their own competitors and are ready to lose some rights on their innovation if this gives them access to a bigger market.

Pricing in networks[1] There is an important literature on pricing in networks (for a recent survey, see Bloch, 2016). Ballester et al. (2006) provide a tractable approach to study network games with strategic complementarities amongst players using linear-quadratic utility functions. They show that the Nash equilibrium in effort is proportional to the "Katz-Bonacich centrality" of each player. Two recent contributions by Bloch and Quérou (2013) and Candogan et al. (2012) incorporate the pricing decisions into the framework of Ballester et al. (2006) where externalities come from consumption.[2] They independently show that if the monopoly firm can price discriminate among players and the network effects are symmetric (undirected), the resulting optimal prices do not take into account the network structure. Fainmesser and Galeotti (2016) use a somewhat different framework and consider the possibility that agents (firms and consumers) only know partially the network and have information only about the consumers' in-degrees or/and out-degrees (i.e., how influential they are or how significantly they influence by their neighbors). If the level of influence and level of susceptibility are perfectly positive correlated, Fainmesser and Galeotti (2016) show that the optimal pricing is also independent of the network. In a complete-information setting, Chen et al. (2015) extend this framework by considering two firms that compete with each other and consumers that can consume different goods, which can be substitutes or complements. They show that the structure of the network matters and that firms tend to charge lower price to more connected consumers.

[1]There is a growing literature on the economics of networks. For overviews, see Jackson (2008), Easley and Kleinberg (2010), Jackson and Yariv (2011) and Jackson et al. (2016).

[2]See also Shi (2003), Deroian and Gannon (2006), Banerji and Dutta (2009), Jullien (2011), Billand et al. (2014), Carroni and Righi (2015), Currarini and Feri (2015), Ushchev and Zenou (2015), Leduc et al. (2016).

Our model is very different from the ones developed in this literature. We have neither a pure monopoly or a duopoly since firms have a local monopoly on their own products but still compete indirectly with each other. In fact, we highlight a different trade off in pricing policies. Firms can charge a high or low price depending on their position in the network (in most of the papers above, firms are not located in the network, only consumers are) and on the probability of innovation of their direct neighbors.

Diffusion and targeting in networks There is a growing literature on targeting in networks, both from non-economists (Mayzlin, 2016) and economists (Bloch, 2016; Zenou, 2016). The literature has been divided in two parts: the targeting of agents to diffuse information or opinions in a social network (Bloch, 2016) and the targeting of agents where the objective is not to seed the network to start a diffusion process but to remove a node in order to reduce or increase activity (Zenou, 2016). Our paper is clearly related to the first strand of this literature where firms take the network of social interactions as given and consider how to optimally leverage social effects to introduce new products or maximize profits. There are few papers with analytical results on targeting. Exceptions include Galeotti and Goyal (2009) and Campbell (2013). To be more precise, Galeotti and Goyal (2009) assume that the firm only knows the degree distribution of consumers in the social network, and the degree of each consumer. Their main question is: should consumers with a low degree (who are not influenced by many agents) receive higher advertising than consumers with a high degree (who are influenced by many other agents)? Among other results, they show that consumers with higher in-degree (i.e. who influence more other agents) should receive more advertising expenditures. Campbell (2013) considers a somehow similar network to study monopoly pricing and targeting in the presence of word of mouth communication. A monopoly sets a price, and consumers decide whether to purchase or not, according to a random valuation. Campbell (2013) shows that the emergence of a giant component depends on the price charged by the monopolist, and that there exists a critical price such that a giant component emerges when price is below the critical price. In this framework, he shows that the firm should target the consumer with the highest degree only if her price is below to this critical price and should instead target the consumer with the smallest number of connections if it is above the critical price.[3]

Our framework is quite different in the sense that firms are themselves located in the network and their position in the network, captured by both their degree and their diffusion

[3]See also Lim et al. (2016) who examine how a profit-maximizing firm tries to diffuse a product in a network by picking a seed and a price. They show that the structure of the network affects the optimal policy. For example, on a line, the optimal price policy is positive while, when a complete graph becomes large, the optimal price goes to zero. For overviews on diffusion in networks, see Jackson and Yariv (2011) and Lamberson (2016).

centrality (a concept introduced by Banerjee et al., 2013, that we define below), are crucial in determining the price policy of the firms. As a result, the way we define the network is different. Each firm gets informed of an innovation in the process technology through a word-of- mouth process, so that firms can communicate among them and share information. More importantly, the aim of targeting a firm in our framework is to maximize total welfare in the network and not necessary diffusion. We also obtain clear-cut analytical results on the targeting of firms. Finally, it is important to observe that the main focus of our paper is not on diffusion itself but rather on optimal pricing for the firms and targeting for the planner.

3 The network of innovation diffusion

3.1 Building the network

Consider a finite set of firms $N = \{1, \ldots, n\}$, each firm being a monopolist on her own market. Think, for example, of firms operating in different sectors, producing different products. Consider firms using a common technology (e.g. software, managerial technologies, organizational structure, consumer tracking policies, etc.) that we call *process technology.* We can think of firms working in health services, logistics, banking and so on, but all are using a common software to manage their data. This software is the process technology. In other words, the product sold by firm is an intermediary product (such as a software) that can be used to produce a final good. In our model, these are the types of intermediary products that we are thinking of and that can be improved and innovated upon.

Because innovation will be a profit-enhancing activity, each firm wants to innovate so that she can improve this technology and sell it to a higher price. Because this process technology is compatible among the different firms (e.g. software), it can be transmitted from one firm to the other, as it is or when it is improved after an innovation. Each firm gets informed of an innovation in the process technology through a *word-of-mouth* process, so that firms can communicate among them and share information. The *network* thus keeps tract of this *word-of-mouth* process so that information flows between linked firms. It also keep tract of how technologically close firms are.

Network definitions The channels by which firms communicate and, potentially, spread the process technology is modeled as a fixed undirected network $\boldsymbol{g}$. The corresponding adjacency matrix $\mathbf{G} = [g_{ij}]$ is such that $g_{ij} = 1$ if firms i and j are linked together and $g_{ij} = 0$ otherwise, with the convention that $g_{ii} = 0$. For analytical simplicity, we assume that the network $\boldsymbol{g}$ have no cycles and thus is a tree.[4] Define the set N_i as the neighborhood of agent i in network $\boldsymbol{g}$, i.e. $N_i = \{all\ j \mid g_{ij} = 1\}$. The *degree* d_i of firm i (i.e. the number of links of i) is: $d_i = \sum_j g_{ij}$. The links represent the possible channels of *information flows* among firms. The *outdegree* of firm i, denoted by d_i^-, is defined as $d_i^- = \sum_j g_{ji}$. The kth power $\mathbf{G}^k = \mathbf{G} \times \overset{(k\ times)}{\cdots} \mathbf{G}$ of the adjacency matrix $\mathbf{G}$ keeps track of indirect connections in $\mathbf{G}$. More precisely, the coefficient $g_{ij}^{[k]}$ in the (i, j) cell of $\mathbf{G}^k$ gives the number of walks[5] of length k in $\mathbf{G}$ between i and j.

Assuming that each firm i is a monopolist on her own market means that the profits of all n firms derived from the production of the good are independent of the process technology innovation used by other firms in the network. We denote by firm T (targeted firm) the firm that has the opportunity to develop this product first. In other words, firm T is an innovator that creates a new product, for which there are no close substitutes, and acts as a monopolist. If we think of the product as a software, or a hard drive or a screen display, as for example in the case of the *Video iPod*, then firm T is the firm that has been given the opportunity to develop this product first. Denoting this firm T as a targeted firm will be useful when dealing with optimal-target policies.

The timing is as follows. At time $t = 0$, all firms have access to the same technology and all obtain a profit π_0, normalized to 0. At time $t = 1$, one firm (firm T) comes up with an innovation about the process technology which, if successfully developed, can reduce her production costs, thus leading to higher profits. As stated above, we refer to this firm as firm T or the T-firm as if targeted by nature or by a policy maker, to potentially improve the industry technology. As for every technological innovation process, innovation is costly, and we model this in the next sections. At time $t = 2$, starting from firm T, the innovated technology can be diffused and, potentially, further ameliorated by the direct neighbors of firm T, following the links of the network. The innovation can flow along the paths of the links in the originally undirected network $\mathbf{g}$. Then the innovation process stops and all firms in the network may benefit or not of using this new product, depending on the price at which it is sold.

To be more precise, given a $T-$firm, a *T-induced directed network* $\boldsymbol{z}_T$ is derived from the undirected network $\boldsymbol{g}$ imposing specific directions on the links of $\boldsymbol{g}$ where we only consider *outdegrees*. Denote by N_T the set of neighbors of firm T. Then, the *T-induced directed network* $\boldsymbol{z}_T$, defined by the matrix $\mathbf{Z}_T = [z_{ij}]$, is built in the following way:

- Start with a generic firm T and build *directed* links in the direction of all its neighbors

[4] In fact, our model can be extended to networks with cycles. The analysis becomes much more cumbersome but the main results and intuitions remain the same.

[5] A *walk* in a network $\mathbf{g}$ refers to a sequence of nodes, $i_1, i_2, i_3, \ldots, i_{K-1}, i_K$ such that $i_k i_{k+1} \in \mathbf{g}$ for each k from 1 to $K - 1$. The *length* of the walk is the number of links in it, or $K - 1$.

i in $\boldsymbol{g}$. Formally, for all $i \in N_T$, set $z_{Ti} = 1$ and $z_{iT} = 0$;

- Given any firm i reached by a directed link, create directed links towards its remaining neighbor firms in $\boldsymbol{g}$. Formally, for all $i \in N_T$, and for all $j \in N_i \setminus T$, set $z_{ij} = 1$ and $z_{ji} = 0$;
- Continue this way, step by step, until all terminal nodes are reached.

One can see that, by construction, the matrix $\mathbf{Z}_T$ is *asymmetric* and only captures *outdegrees*. Figure 1 provides two examples of how, selecting a firm T in a specific undirected network, leads to two *induced directed networks*.

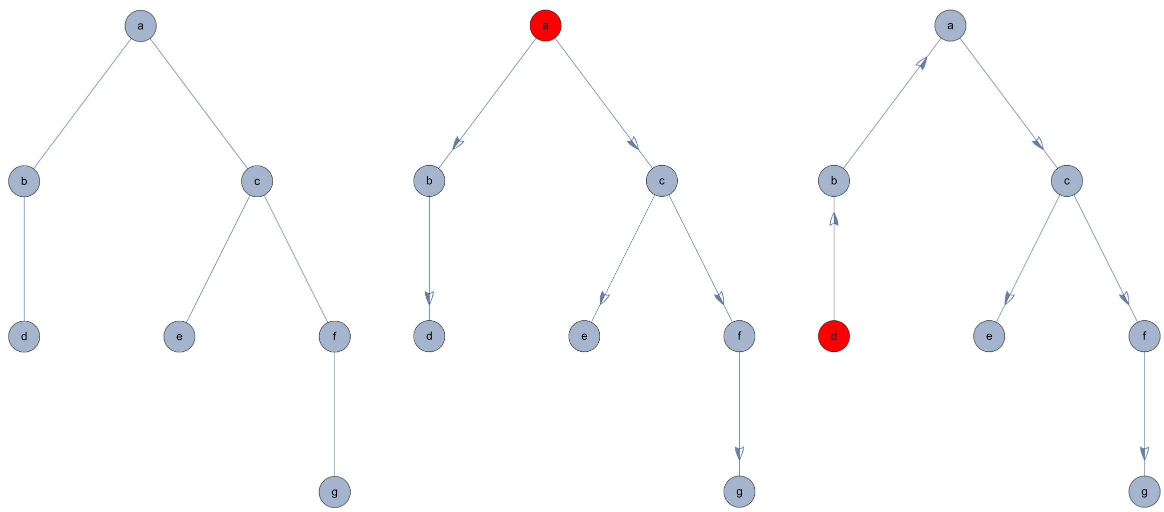

Figure 1: Deriving the $T-$induced directed networks

Indeed, start with the left (undirected) network described in Figure 1. Its (symmetric) adjacency matrix $\mathbf{G}$ is equal to (the first row corresponds to node a, the second row to b, etc.):

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Let us first consider node a as firm T. The middle network in Figure 1 displays the a-induced directed network. For that, we build directed links starting from the node $T = a$, that is the links ab and ac (i.e. $z_{ab} = 1$ and $z_{ac} = 1$). Then, we create directed links from b and c, which are links bd, ce and cf. Since f is the only node, which is *not* terminal, we continue this process by building the link fg. We stop here because we have reached all the peripheral nodes. In that case, $\boldsymbol{z}_a$, the *a-induced directed network* is: $\boldsymbol{z}_a = \{a, b, c, d, e, f, g\}$, i.e. it is the whole network, and the cardinality of $\boldsymbol{z}_a$ is $|\boldsymbol{z}_a| = n = 7$.

Let us now consider node d as the firm T in the left (undirected) network of Figure 1. The process is the same as before and we obtain the d-induced directed network, $\boldsymbol{z}_d$, which is displayed in the right network in Figure 1. The adjacency matrices for the a-induced and the d-induced networks are given by

$$\mathbf{Z}_a = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{Z}_d = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3.2 Network relevant characteristics

Before presenting the details of the diffusion process, we introduce two *measures* summarizing some characteristics of the topology of the network that will be useful for our analysis. In particular, these two measures relate to centrality measures already well-known in the existing literature on *diffusion in network*.

Given a firm T and the corresponding *T-induced directed network* $\boldsymbol{z}_T$, the number of firms that can be reached in $\boldsymbol{z}_T$ starting from a generic firm i is exactly the number of nodes in the *i-induced directed subnetwork* of $\boldsymbol{z}_T$ minus 1 (firm i itself). Consider, for example, the middle network in Figure 1 where firm $T = a$ is targeted. Then, the number of firms that can be reached from, for example, firm c is $4 - 1 = 3$, where 4 is the number of firms belonging to the *c-induced directed subnetwork* of $\boldsymbol{z}_a$ (namely, firms c, e, f, g). The cardinality of any i-induced directed subnetwork can be easily derived from the notion of *diffusion centrality*, introduced by Banerjee et al. (2013). Define $\mathbf{Z}_t^k$ as the k-th power matrix of $\mathbf{Z}_T$ and $\mathbf{1}$ as a vector of 1.

Definition 1 (Banerjee et al. (2013))**.** *Given a network* $\mathbf{Z}_T$ *and a scalar* $q \in (0,1]$*, the diffusion centrality vector is given by:*

$$\mathbf{c}(\mathbf{Z}_T, q, K) = \left[\sum_{k=1}^{K} (q\mathbf{Z}_T)^k\right] \mathbf{1}$$

Diffusion centrality captures how much a given piece of information is diffused to the network starting from a given node i. If the information is transmitted with probability q, then each entry i in the vector of diffusion centrality counts the expected number of firms of the network $\mathbf{Z}_T$ a given firm i can reach during a number K of iterations. In our framework of innovation diffusion, we have a directed network without cycles, so that the number of iterations after which diffusion stops cannot be larger than $n-1$. Moreover, we consider the case of $q = 1$, where some piece of innovation is always transmitted among firms, when available. Thus, in what follows, we call *diffusion centrality* the vector

$$\mathbf{c}(\mathbf{Z}_T, 1, n-1) = \left[\sum_{k=1}^{n-1} (\mathbf{Z}_T)^k\right] \mathbf{1} \tag{1}$$

and we refer to $c_{T,i}$ as the diffusion centrality of firm i when firm T has been targeted, i.e. the i-th entry of $\mathbf{c}(\mathbf{Z}_T, 1, n-1)$. Denote by $\boldsymbol{z}_{T,i}$, the $i-$induced directed subnetwork of $\boldsymbol{z}_T$, and $|\boldsymbol{z}_{T,i}|$ its cardinality. Namely, $\boldsymbol{z}_{T,i}$ is the part of the directed network $\boldsymbol{z}_T$ that starts from i. Then, it is easily checked that

$$c_{T,i} = |\boldsymbol{z}_{T,i}| - 1$$

Consider, for example, the middle network in Figure 1 where firm a is the $T-$firm. First, the $a-$induced directed subnetwork of $\boldsymbol{z}_a$, denoted by $\boldsymbol{z}_{a,a}$, is basically $\boldsymbol{z}_a$ (the $T-$induced directed subnetwork of $\boldsymbol{z}_T$ is $\boldsymbol{z}_T$ itself), i.e. $\boldsymbol{z}_{a,a} = \boldsymbol{z}_a = \{a, b, c, d, e, f, g\}$, with $|\boldsymbol{z}_{a,a}| = 7$. Then, $c_{a,a} = |\boldsymbol{z}_{a,a}| - 1 = 7 - 1 = 6$. Indeed, for any target firm T, $c_{T,T} = n - 1$ since a target firm can reach, directly or indirectly, the whole network. Second, the $b-$induced directed subnetwork of $\boldsymbol{z}_a$, denoted by $\boldsymbol{z}_{a,b}$, is $\boldsymbol{z}_{a,b} = \{b, d\}$, with $|\boldsymbol{z}_{a,b}| = 2$. As a result, $c_{a,b} = |\boldsymbol{z}_{a,b}| - 1 = 1$. Third, $\boldsymbol{z}_{a,c} = \{c, e, f, g\}$ with $|\boldsymbol{z}_{a,c}| = 4$. Thus, $c_{a,c} = |\boldsymbol{z}_{a,c}| - 1 = 3$. Finally, $\boldsymbol{z}_{a,d} = \{d\}$ and thus $c_{a,d} = |\boldsymbol{z}_{a,d}| - 1 = 0$. It also easily verified that $c_{a,e} = c_{a,g} = 0$ and $c_{a,f} = 1$.[6]

When firm d is the $T-$firm (see the right network in Figure 1), it is easily verified that $c_{d,a} = 4$, $c_{d,b} = 5$, $c_{d,c} = 3$ and $c_{d,d} = 6$. To summarize, to calculate the diffusion centrality of a given node i, one first needs to know the targeted firm T, then derive $\boldsymbol{z}_T$, the $T-$induced directed network, and, finally, calculate the diffusion centrality of node i defined

[6]Since $n - 1 = 6$, it is easily verified that (the first row of the matrix and vector corresponds to node a,

as $c_{T,i} = |\boldsymbol{z}_{T,i}| - 1$, where $\boldsymbol{z}_{T,i}$ is the $i-$induced directed subnetwork of $\boldsymbol{z}_T$.

It turns out that diffusion centrality will play a crucial role in shaping incentives to innovate in our model. This is because $c_{T,i}$ represents the number of firms firm i can pass on or sell (directly or indirectly) its innovation to. In other words, $c_{T,i}$ is the size of firm i's *potential market*. It is now convenient to define also the *Market Dispersion Index* M_T, as a function of the diffusion centralities of T's neighbors. We have:

Definition 2 (Market Dispersion Index - MDI). *The Market Dispersion Index of firm T is defined as:*

$$M_T = \sum_{i \in N_T} (c_{T,i})^2 \tag{2}$$

The Market Dispersion Index (MDI) measures how the diffusion centralities of T's neighbors are shaped. Indeed, given a firm T and its direct neighbors (whose size is d_T), M_T shows how the remaining $n - 1 - d_T$ firms are distributed in the different subgraphs originated by each T's neighbor. From its definition, we can see that M_T increases when there are few large subgraphs and a lot of other small subgraphs while it decreases when the firms are evenly distributed among the different subgraphs. From a graphical point of view, it gives an idea of how much T is in a symmetric position in the network. To better understand how this index relates to the network topology, Figure 2 gives the values of the MDI for each firm of the (undirected) left network in Figure 1.

The MDI of each node is computed assuming that the firm is considered to be the T firm, then computing the index c_i of all her neighbors in z_T to obtain the value of M_T. Take, for example, the left network of Figure 1. Let us first calculate M_a. For that, we need to assume that firm a is the targeted firm, i.e. $T = a$ (see the left network in Figure 1 to see the label of each node). Then, we need to calculate the diffusion centralities of only firm a's neighbors, that is firms b and c. For that, we need to know $\boldsymbol{z}_a$, the $a-$induced directed network, which is given by the graph displayed in the middle of Figure 1. Consider, first, firm b. The $b-$induced directed subnetwork of $\boldsymbol{z}_a$, denoted by $\boldsymbol{z}_{a,b}$, is $\boldsymbol{z}_{a,b} = \{b, d\}$, with $|\boldsymbol{z}_{a,b}| = 2$. As a result, $c_{a,b} = |\boldsymbol{z}_{a,b}| - 1 = 1$. Consider, now, node c. We have: $\boldsymbol{z}_{a,c} = \{c, e, f, g\}$ with

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the second row to node b, etc.):

$$\mathbf{c}(\mathbf{Z}_a, 1, 6) = \left[\sum_{k=1}^{6} \mathbf{Z}_a^k\right] \mathbf{1} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

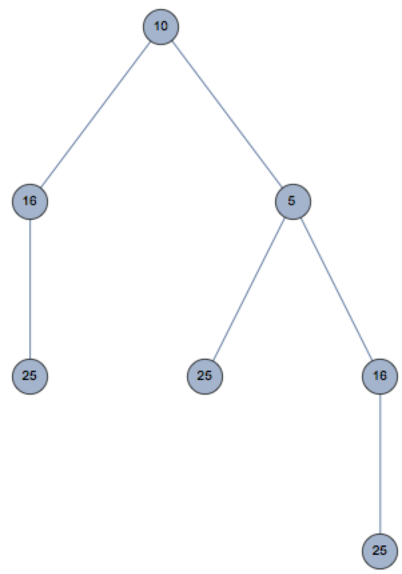

Figure 2: Computing the Market Dispersion Index (MDI) for the left network in Figure 1

$|\boldsymbol{z}_{a,c}| = 4$. Thus, $c_c = 3$. As a result, $M_a = 1^2 + 3^2 = 10$, which is the number given for firm a in Figure 2.

Let us now calculate the MDI of node c. For that, assume that c is the targeted firm, i.e. $T = c$. Firm c has three direct neighbors: a, e and f. To calculate their diffusion centralities, we need to derive the $c-$induced directed network and determine $\boldsymbol{z}_{c,a}$, $\boldsymbol{z}_{c,e}$ and $\boldsymbol{z}_{c,f}$. It is easily verified that $\boldsymbol{z}_{c,a} = \{a, b, d\}$, $\boldsymbol{z}_{c,e} = \{e\}$ and $\boldsymbol{z}_{c,f} = \{f, g\}$. As a result, $c_{c,a} = 2$, $c_{c,e} = 0$ and $c_{c,f} = 1$. Thus, the MDI of node c is given by: $M_c = 2^2 + 0^2 + 1^2 = 5$. Finally, if we want to calculate the MDI of the *peripheral* firm d so that d is the $T-$firm, we see that d's only neighbor is firm b, for which $c_{d,b} = 5$, so that $M_d = 5^2 = 25$. More generally, peripheral firms (such as firms d, e and g in the left network of Figure 1) always have the largest MDI value. In particular, if firm T is a peripheral node, it has only one neighbor, which can reach the remaining $n - 2$ firms in the network. Then, it should be clear that $(n - 2)^2$ is the maximal value of the MDI, i.e. $\max_T M_T = (n-2)^2, \forall \boldsymbol{z}_T$. As a result, the MDI is lower, the more even is the distribution of nodes in the subgraphs originated from T's neighbors. Thus, moving to the center of the network means having more balanced subnetworks, and, therefore, a lower MDI. This is simply due to the fact that, what matters, is also the number of neighbors the $T-$firm has, and moving towards the center does not mean having necessarily a larger number of neighbors. Still, we have that terminal nodes always have the highest MDI given by $(n - 2)^2$, while the more we move to symmetric positions, the lower is the MDI.

As for the diffusion centrality, the MDI has an immediate economic meaning. The higher the MDI, the lower is the number of neighbors a firm has so that it has access to a large market. This will turn out to be crucial in our analysis since, as we will see below, each neighbor is the mean by which each $T-$firm can have its technology innovation spread. The incentives for these neighbors to innovate on top of T's innovation will then crucially depend on whether these neighbors have access to a large or to a small market. In other words, the MDI will signal how much each neighbor is a potential valuable link to spread the initial innovation and, at the same time, a potential competitor.

As we will see below, the position in the network of each firm, summarized by her diffusion centrality and her MDI, determines both her incentives to innovate and her pricing policies.

3.3 Diffusion process

We now describe the technology diffusion process in the network. To ease the exposition, we first pretend that innovation happens at an exogenous rate and that it is sold at an exogenous price. This exercise is useful to understand the mechanism behind the diffusion process. Once the main mechanism is described, in the next sections, we will analyze the general problem when firms can choose both their innovation effort and price policy. Note that the case of exogenous efforts and prices is a useful benchmark since it relates to standard literature of diffusion in networks (e.g. diffusion of opinions) where the diffusion process is mainly mechanical and exogenous (Jackson and Yariv, 2011).

Consider the diffusion mechanism described in Figure 3 where the payoffs correspond to that of the $T-$firm (first payoff), to any firm $i \in N_T$ (second payoff) and to any firm $j \in \boldsymbol{z}_{T,i}$ (third payoff). As it will become clear below, the payoffs given in Figure 3 are gross payoffs, i.e. payoffs obtained from using the technology without substracting the price paid by firms $i \in N_T$ and by firms $j \in \boldsymbol{z}_{T,i}$ when acquiring the new technology. Let us start with firm T, i.e. the targeted firm. With the original technology and before any innovation, the technology is available to all firms in the network and each firm earns the same profit of $\pi_0 = 0$. Then, if T's innovation is not successful, which occurs with probability $1 - e_T$, then nothing changes in the network and each firm earns a profit $\pi_0 = 0$.

With probability e_T, which is equal to firm's T innovation effort, T successfully provides a technology innovation (i.e. first-generation innovation). Thanks to the improved technology, this gives each firm in the network using this technology a profit of $\pi_1 > 0$. Firm T can sell her innovation at some price $p_{T,i}$ to her neighboring firms $i \in N_T$.[7] As stated above, each of the neighboring firms that uses this new technology will earn a profit π_1. Consider

[7] If firm T can sell her innovation to all other firms, then the network $\mathbf{g}$ is by definition a complete network.

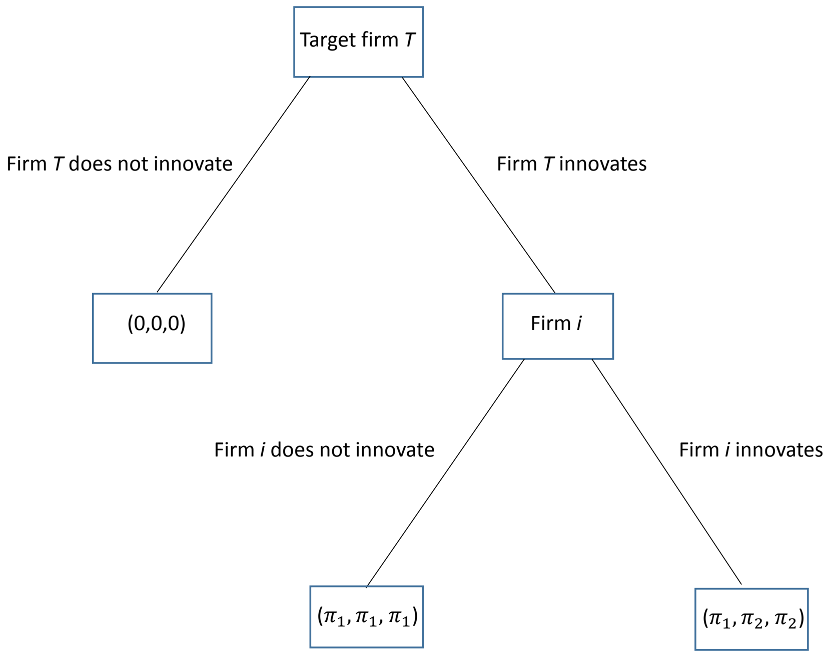

Figure 3: Structure of the game and payoffs

now any firm $i \in N_T$ that bought the innovation from firm T. As in the previous step, firm i can innovate further the new technology (second-generation innovation) with probability e_i, which is equal to her innovation effort. If not successful, which occurs with probability $1 - e_i$, then all firms in the network (i.e. both firms $i \in N_T$ and firms $j \in \boldsymbol{z}_{T,i}$) will obtain a profit of π_1 and buy the technology at price $p_{T,i}$ from firm T. This just means that, if firm i does not innovate, then she just transmits the available technology to her neighbors. If, on the contrary, firm i is successful in innovating so that the initial product/technology is improved, firm i will obtain a profit of $\pi_2 > \pi_1$ and will be able to sell her innovation at some price to her neighbors $j \in \boldsymbol{z}_{T,i}$ in the $i-$induced directed subnetwork. In that case, firm T is losing the market of the $i-$induced directed subnetwork (i.e. all firms $j \in \boldsymbol{z}_{T,i}$) to her neighbors, firms $i \in N_T$, who will sell their improved technology/product to the firms from their induced directed subnetwork. We assume that, after two steps of innovation, no further innovation is possible. This assumption enables us to have a tractable model that analyzes the main forces of diffusion of innovation while keeping the main mechanism at work: a targeted firm T that innovates can sell to her neighbors her technology but runs the risk that these neighbors innovate themselves, which can eventually have a higher profit than firm T. In other words, this allows us to model the trade off between innovation and competition so that innovating firms can sell their innovation to their possible competitors. More generally, the two-step innovation process enables us to capture the countervailing effects of neighbors and neighbors' neighbors' innovation incentives, once we endogeneize prices and innovation efforts. As we explain below, firm T needs neighbors to access the market since the larger is the market, the higher are the incentives to innovate for firm T. At the same time, T's neighbors are also T's competitors, and thus neighbors with a large market also represent a threat to T. These two opposite incentives create interesting outcomes. As we will see below, these two forces are exactly captured by the diffusion centrality and the MDI defined above and will shape the incentives to innovate and the pricing policies. Having more that two steps of innovation will *not* change this main intuition but will make the analysis much more complicated.

Let us clarify the profit that each firm obtains in the different cases, as described in Figure 3. For that, consider the $T-$firm, any firm $i \in N_T$ and any firm $j \in \boldsymbol{z}_{T,i}$ and consider the profits profile (π_T, π_i, π_j). Assume $\pi_2 > \pi_1 > \pi_0 = 0$. Then

- If firm T does not innovate, the profit profile is $(\pi_0 = 0, \pi_0 = 0, \pi_0 = 0)$;
- If firm T innovates and firm i does not innovate, the profit profile is (π_1, π_1, π_1);
- If firm T innovates and firm i also innovates, then the profit profile is (π_1, π_2, π_2).

Notice that, if firm T innovates and also firm $i \in N_T$ innovates, then firm i does not sell her own technology to firm T, but just to the remaining firms of the i-induced (directed) subnetwork (of $\boldsymbol{z}_T$), i.e. $\boldsymbol{z}_{T,i}$. This can be justified by the presence of some costs that make it unprofitable to switch technology if it had just been adopted in the recent past.

4 Innovation efforts and pricing: Firm T's neighbors

Let us now model how firms choose innovation effort and their pricing policy by focussing first on firms $i \in N_T$, i.e. firm T's neighbors. Indeed, consider a generic firm $i \in N_T$, where

T is the targeted firm. We know from Figure 3 that firm $i \in N_T$ has the opportunity to innovate only if the $T-$firm has already innovated. In that case, we have the following result.

Lemma 1. *Consider a firm $i \in N_T$, where T is the targeted firm. Then, if, first, firm T has innovated and, then, firm $i \in N_T$ has innovated, firm i will sell her new technology to all firms in the i-induced subnetwork $\boldsymbol{z}_{T,i}$ at a price $p_i^* = \pi_2$.*

Indeed, if firm i innovates (with probability e_i), then she improves the process technology and obtains a profit of $\pi_2 > \pi_1$ from production. Since firm i is the owner of the new technology, she can sell it to all firms in the i-induced subnetwork $\boldsymbol{z}_{T,i}$. Since these firms cannot innovate on top of that (by assumption), the value they assign to this innovation is exactly π_2. This is because they just give to this innovation the value they get from production (a profit of π_2) without any possibility of improving and reselling it. Thus firm i sets a price equal to π_2, which is the value for which all the firms in the i-induced subnetwork are indifferent between buying and not buying the technology.[8] As in a Bertrand game, firm i extracts all the rents from this new technology so that all firm belonging to the i-induced subnetwork have a zero utility.

Let us now determine the (expected) utility of a firm $i \in N_T$, with innovation effort e_i, assuming that firm T has innovated and sold her technology to firm i at a price $p_{T,i}$ (determined below). It is given by:

$$U_i(e_i) = e_i \pi_2 (1 + c_i) + (1 - e_i)\pi_1 - \frac{\beta}{2} e_i^2 - p_{T,i} \quad (3)$$

where $\beta > 0$. If firm i innovates (with probability e_i), then she improves the process technology and obtains a profit of $\pi_2 > \pi_1$ from production. At the same time, firm i is the owner of the new technology and thus can sell it to all firms in the i-induced subnetwork $\boldsymbol{z}_{T,i}$. As stated in Lemma 1, firm i sets a price equal to π_2 and all the firms in the i-induced subnetwork buy it. Observe that the number of firms in this subnetwork is exactly equal to i's diffusion centrality index c_i, defined in (1). If firm i does not succeed in innovating (with probability $1 - e_i$), then she just obtains the profit π_1 derived from using T's technology. The quadratic term $\frac{\beta}{2} e_i^2$ captures the cost of exerting innovation effort. Observe that the parameter β represents how difficult for a firm i it is to provide a successful innovation that can be sold to other firms. As a result, β can represent (i) the *technological complexity* of the industrial sector or (ii) the level of *intellectual property protection*. We have the following straightforward result.

[8] We assume that when firms are indifferent between buying and not buying the technology, they always buy the technology.

Proposition 1. *Take any firm $i \in N_T$, then*

$$e_i^* = \frac{\pi_2 - \pi_1 + \pi_2 c_i}{\beta} \tag{4}$$

so that the effort e_i^* is increasing in c_i, decreasing in β and increasing in the technological profit $(\pi_2 - \pi_1)$. Furthermore, the equilibrium (expected) utility of each firm $i \in N_T$ is equal to:

$$U_i^* \equiv U_i(e_i^*) = \pi_1 + \frac{(\pi_2 - \pi_1 + \pi_2 c_i)^2}{2\beta} - p_{T,i} \tag{5}$$

Proposition 1 characterizes the equilibrium behavior of any firm that is a neighbor of firm T and that does not need to worry about possible competitors on her own innovation. The results of this proposition are very intuitive but interesting since they relate to the position of firm i in the network (captured by c_i) to effort and utility. Indeed, the higher is c_i, the diffusion centrality of firm i, the higher is the effort e_i^*. Since c_i is a measure of the market size, this results states that a firm innovates more the larger is the market for her own innovation. Moreover, the innovation effort is decreasing in the property rights protection (or technological complexity), which is captured by β. Finally, innovation effort is increasing in the technological progress provided by the innovation since if $(\pi_2 - \pi_1)$ is higher then the gain from production are higher.

Notice that, for firm i, we can identify two types of incentives for a given effort: incentives related to profit and innovation difficulties, independently of the network structure (captured by β and $\pi_2 - \pi_1$), and incentives related to the possible extraction of rents from the network (c_i), which is determined by the position of the firm in the network. As a result, firms can decide to innovate more or less depending on which branch in the network they are located in. Consider, for example, the middle network in Figure 1 where $T = a$, and compare the efforts of firms b and c, which are both neighbors of firm $T = a$. It should be clear that $e_b^* < e_c^*$ since $c_{a,b} = 1 < 3 = c_{a,c}$.

5 Innovation efforts and pricing: Targeted firm T

The problem faced by the $T-$firm is more complex. As for each $i \in N_T$, firm T has to decide its optimal effort e_T and a vector of prices to sell her innovation if successful. For the firm T, each branch of the network generated by each neighbor firm i (each i-induced subnetwork for each $i \in N_T$) represents a different market.[9] Let us focus on each of these markets. As

[9]Since the network has no cycles, the different branches of the network do not communicate with each other and, thus, the same piece of innovation can only arrive to a firm through a unique link.

stated in the Introduction, we consider two different environments: either firm T and each firm $i \in N_T$ sign a binding verifiable contract that stipulates the price of the innovation and thus firm T must commit to this price $p^*_{T,i}$ even if she eventually sell her innovation to the remaining firms $j \neq i$ belonging to the i-induced subnetwork $\boldsymbol{z}_{T,i}$, or firm T and each firm $i \in N_T$ have an informal agreement so that firm T can change her initial price $p^*_{T,i}$ to attract the firms $j \neq i \in \boldsymbol{z}_{T,i}$. The latter means that the firm T can charge different prices to firms $i \in N_T$ and to firms $j \neq i \in \boldsymbol{z}_{T,i}$.

5.1 Committed prices (fixed prices)

Because of a binding contract, firm T cannot modify her price so that once she commits on her price she cannot change it. This implies that, if firm T has sold her technology to firms $i \in N_T$ and the latter did not innovate, she cannot change here price in order to attract the remaining firms in the network, i.e. all firms $j \neq i \in \boldsymbol{z}_{T,i}$. In that case, she will face the following trade-off. On the one hand, she wants to charge a high price $p^*_{T,i}$ to her direct neighbors, i.e. all firms $i \in N_T$, to maximize her utility. However, if her direct neighbors do not innovate, there is a risk that the remaining firms (all firms $j \neq i \in \boldsymbol{z}_{T,i}$) will not buy her product because it is too expensive. As a result, the price set by firm T will depend on the position of firm T and firms $i \in N_T$ in the network since these positions affect market diffusion through the Market Dispersion Index, M_T, for firm T and through $c_{T,i}$, the diffusion centrality of firm i. For example, if a firm $i \in N_T$ has a very high (low) $c_{T,i}$, then firm T has a large incentive to set a low (high) price because, if firm i does not innovate, firm T will be able to sell her innovation to a very (small) large market.

In this framework, two cases may arise: either firm T will set a *low price* of $p_{T,i} = \pi_1$ since, in that case, all firms in the network (i.e. all firms $i \in N_T$ and, if firms i do not innovate, all firms $j \neq i$ in their i-induced subnetworks) will be able to buy T's technology or a *high price* of $p^*_{T,i} = \pi_1 + \frac{1}{2\beta}(\pi_2 - \pi_1 + \pi_2 c_i)^2 > \pi_1$, which is determined such that each firm $i \in N_T$ has a zero expected utility $U^*_i = 0$, since, in that case, only firms $i \in N_T$ can buy the new technology (i.e. even if firms i do not innovate, firms $j \neq i \in \boldsymbol{z}_{T,i}$ will not be able to buy T's product because their utility will be negative). We have:

Lemma 2. *Suppose that firm T has innovated and consider the case when firm T has to choose a unique price p^i_T to sell her technology to her direct neighbors $i \in N_T$ and to all members of the i-induced subnetwork $\boldsymbol{z}_{T,i}$. Then, firm T can only set two prices:*

(i) Either

$$p^*_{T,i} = \pi_1$$

(*ii*) *or*

$$p^*_{T,i} = \pi_1 + \frac{1}{2\beta}(\pi_2 - \pi_1 + \pi_2 c_i)^2 \equiv \overline{U}_i \tag{6}$$

Indeed, in case (i), firm T sets a price of $p^*_{T,i} = \pi_1$ so that all firms in the network accept to buy her technology. Given that T's technology provides a profit of π_1 to everyone, firms that cannot innovate on top of that and that cannot extract further rents, are not willing to pay more that π_1. In particular, if this price is chosen and firm i is not successful in innovating, T's technology can still be transmitted further since all other agents of the i-induced subnetwork would buy it.

In case (ii), firm T sets a price of $p^*_{T,i} = \pi_1 + \frac{1}{2\beta}(\pi_2 - \pi_1 + \pi_2 c_i)^2 \equiv \overline{U}_i$, which is higher than π_1, and is calculated so that each firm $i \in N_T$ has a zero expected utility $U^*_i = 0$ (see (5)), and guarantees that firm i will always accept to buy the technology from firm T. In that case, firm T extracts all the expected rents from firms i. Observe that this implies that firm T charge different prices to firms $i \in N_T$ since they have different $\overline{U}_i$ depending on their position in the network, captured by their diffusion centrality c_i. In fact, in that case, the price set by firm T is higher, the higher is c_i, i.e. *firms with more central positions are charged a higher price.* Observe also that, if firms $i \in N_T$ do not innovate, technology diffusion stops since the price is too high for all the other firms.

Let us now determine under which condition firm T will set a price of $p^*_{T,i} = \pi_1$ or a price of $p^*_{T,i} = \overline{U}_i$. For that, we need to study each case separately.

5.1.1 Case (i) $p^*_{T,i} = \pi_1$

When firm T set a price of $p^*_{T,i} = \pi_1$, her expected utility can be written as follows:

$$U^{\pi_1}_T = e_T \left\{ \pi_1 + \sum_{i \in N_T} [e^*_i \pi_1 + (1 - e^*_i)\pi_1(1 + c_i)] \right\} - \frac{\beta}{2} e^2_T \tag{7}$$

First, notice that firm T obtains something positive only if she successfully innovates since, if innovation is not successful, she obtains a profit of $\pi_0 = 0$. When innovation is successful (which occurs with probability e_T), she gets π_1 as her own profit from production. Then, for each of her neighbors $i \in N_T$, firm T obtains $p_{T,i} = \pi_1$ if i innovates (which occurs with probability e^*_i), since, in that case, a new technology will be sold by i to the rest of the i-induced subnetwork, whereas, if i does not innovate, firm T gets π_1 from each firm $i \in N_T$ and π_1 from each firm $j \neq i$ in the i-induced subnetwork, which is determined by the diffusion centrality c_i of firm i.

Denote by d_T, the degree (i.e. number of direct links) of firm T and by M_T, the Market Dispersion Index (defined by (2)). Using (4), we have the following result:

Proposition 2. *Assume*

$$n > \frac{\pi_2 M_T - (\pi_2 - \pi_1)(1 + d_T)}{\beta - (\pi_2 - \pi_1)} \tag{8}$$

If firm T sets a price $p^*_{T,i} = \pi_1$, for all $i \in N_T$, then her optimal effort and equilibrium expected utility are given by:

$$e_T^{\pi_1} = \frac{\pi_1}{\beta}\left[n - \frac{(\pi_2 - \pi_1)(n - 1 - d_T)}{\beta} - \frac{\pi_2}{\beta} M_T\right] \tag{9}$$

and

$$U_T^{\pi_1} = \frac{\pi_1^2}{2\beta}\left[n - \frac{(\pi_2 - \pi_1)(n - 1 - d_T)}{\beta} - \frac{\pi_2}{\beta} M_T\right]^2 \tag{10}$$

which are both decreasing in the Market Dispersion Index M_T of firm T and increasing in d_T, the degree of firm T. Moreover, the effect of β on the equilibrium effort and utility is ambiguous.

Let us focus on the equilibrium effort $e_T^{\pi_1}$ determined in (9) since the comparative statics of the (expected) utility are the same as long as condition (8) holds (which guarantees that $e_T^{\pi_1} > 0$). The term π_1/β represents the different incentives to innovate since π_1 is both the profit and the price from the innovation while β captures, on the contrary, the difficulty to innovate. This ratio is increasing in π_1, the gains in profit from the innovation for firm T while it is decreasing in β, the difficulty to innovate due to property rights or to the technological complexity of the innovation. This is independent of the presence of other firms and of the position of the firm in the network.

The most interesting aspect of $e_T^{\pi_1}$ is captured by the term in parentheses, which keeps track of all the effects derived from the network and the possible diffusion of technology. First, n represents how large is the overall market for T's innovation, so that the larger is the potential market, the higher is effort $e_T^{\pi_1}$ because of larger potential gains. These gains are, however, affected by the topological structure of the network. The parameter β enters in the second and third term by lowering the optimal effort. This may seem counterintuitive since we have just seen that a high β increases own innovation costs. However, from T's perspective, a high β also increases the innovation costs of her competitors (i.e. all $i \in N_T$) so that a high β makes it easier, once T has innovated, to avoid further innovation. Overall, β has a negative effect on effort when considering own costs to innovate, and a positive effect when considering competitors' costs to innovate. As a result, the effect of β on $e_T^{\pi_1}$ is ambiguous. The second term $-(\pi_2 - \pi_1)(n - 1 - d_T)/\beta$ captures the fact that a high

$(\pi_2 - \pi_1)$ induces a large incentive for firms i to exert effort, since they can resell their own innovation at a higher price (π_2 instead of π_1) to the rest of the firm population ($n-1-d_T$ firms) gaining higher rents, thus discouraging T's effort.

Finally, the effort $e_T^{\pi_1}$ decreases with the Market Dispersion Index M_T of firm T. Indeed, if M_T is low, then each of T's competitors $i \in N_T$ has a small market, and thus has a low incentive to innovate since they would not gain much from it. On the contrary, if M_T is high, then there are few competitors having access to a large market (to the extreme, one neighbor firms accessing all the remaining $n-2$ firms), and this creates a great threat to T, thus reducing her effort $e_T^{\pi_1}$. We can thus characterize the equilibrium effort in terms of position of firms in the network. Even if there is no clear correlation between M_T and d_T, it is straightforward to notice that M_T is maximal when d_T is minimal, that is to say at the very periphery of the network.

Let us now go back to the role of β, which captures property rights protection (or industry technological complexity). Consider, first, the case when β is very low. Then, it is very likely that firms $i \in N_T$ will exert an effort e_i close to 1 so that they will always innovate. As a consequence, firm T will innovate much less since she would lose part of the rents. If we consider the equilibrium effort $e_T^{\pi_1}$ given by (9), we can see that a very low β would imply that $e_T^{\pi_1^*} = 0$, so that the overall process is blocked. In this respect, if firm T chooses $p_{T,i}^* = \pi_1$, having low property rights protection would mean that there will be no diffusion of the innovation. On the other side, if β is very high, it is more likely that e_i, firm i's effort, would be close to zero, which implies that a second innovation will not take place.

5.1.2 Case *(ii)* $p_{T,i}^* = \overline{U}_i$

Consider, now, the case when $p_{T,i}^* = \overline{U}_i$, so that firm T decides to impose a price that only her direct neighbors can afford. Then, T's expected utility can be written as follows:

$$U_T^{\overline{U}_i} = e_T \left(\pi_1 + \sum_{i \in N_T} \overline{U}_i \right) - \frac{\beta}{2} e_T^2 \tag{11}$$

When firm T innovates (probability e_T), she obtains π_1 as her own profit from production. Then, she obtains $p_{T,i}^* = \overline{U}_i$ from all firms $i \in N_T$ since all other firms cannot afford to buy her technology. Indeed, in that case, firm T extracts the highest possible rents from each i, but, since $\overline{U}_i > \pi_1$, no firm outside of i can buy T's innovation if i fails to innovate. Using (6), we have:

Proposition 3. *If a firm T sets a price of $p^*_{T,i} = \overline{U}_i$, then her equilibrium effort and expected utility are given by:*

$$e_T^{\overline{U}_i} = \frac{1}{\beta}\left[\pi_1 + \frac{\pi_2(\pi_2 - \pi_1)(n-1)}{\beta} + \left(\frac{2\beta\pi_1 + \pi_1^2 - \pi_2^2}{2\beta}\right) d_T + \frac{\pi_2^2 M_T}{2\beta}\right] \tag{12}$$

$$U_T^{\overline{U}_i} = \frac{1}{2\beta}\left[\pi_1 + \frac{\pi_2(\pi_2 - \pi_1)(n-1)}{\beta} + \left(\frac{2\beta\pi_1 + \pi_1^2 - \pi_2^2}{2\beta}\right) d_T + \frac{\pi_2^2 M_T}{2\beta}\right]^2 \tag{13}$$

which are both increasing in the Market Dispersion Index M_T of firm T and decreasing in the level of intellectual property protection β. Furthermore, $e_T^{\overline{U}_i}$ and $U_T^{\overline{U}_i}$ are increasing (resp. decreasing) in d_T, the degree of firm T, if and only if $2\beta > (\pi_2^2 - \pi_1^2)/\pi_1$ (resp. $2\beta < (\pi_2^2 - \pi_1^2)/\pi_1$).

Before discussing the optimal effort $e_T^{\overline{U}_i}$, it is worth noticing how incentives change in this case. The most important change with respect to the previous case is that firm T makes each firm i pays for the entire expected utility she obtains. When i is in a position of the network that induces a high effort and, thus, a high expected rent, then T can extract all of it. While, in the previous case, this was a threat for T, it is now an opportunity to extract higher rents from firms $i \in N_T$. In particular, if i has access to a large market and thus has a high e_i^*, then this is good news for T since it can extract the entire rent. Moreover, a high e_i^* reduces the probability that i does not innovate. In this case, given the high price $\overline{U}_i > \pi_1$, the innovation cannot be transmitted further. However, a high e_i^* reduces this possibility since it is likely that i innovates and thus transmits her own innovation at a price π_2, which is affordable to firms $j \in \mathbf{z}_{T,i}$. In this respect, having a high d_T reduces the number of firms that cannot buy the technology, and thus increases the incentives to innovate. At the very same time, a firm T with a high M_T means that firm T has neighbors with a large market, and thus with a high incentive to provide e_i^* and to innovate. As we just argued, this has the effect to increase the rent to extract.

If we look more closely at (12), $1/\beta$ captures the easiness to innovate for firm T. The first two terms in parentheses can be rewritten as $\pi_1 + d_T[\pi_1 + \frac{1}{2\beta}(\pi_2 - \pi_1)^2]$. The first term captures the incentives to innovate for T due to the gain of π_1. The second term captures the fact that, for each neighbor $i \in N_T$, firm T can extract some rents that depend on the gains that i can obtain from innovating on top of T. These gains are given by π_1 plus a measure of profitability to innovate given by $(\pi_2 - \pi_1)^2$. Notice that now a high β, by lowering the chances of i to innovate, decreases the rents that T can extract. This is common also to all the other terms of (12). The third term in (12) positively depends on $n - 1 - d_T$, which gives the number of firms that are non T's neighbors and T itself, i.e. all firms $j \neq i \in \boldsymbol{z}_{T,i}$. Clearly, the higher is the number of these firms, the larger is the market for T's neighbors, and the higher is their effort and their expected utility, and therefore, the higher is the rent that firm T can extract from them. This leads to more incentive to exert effort for firm T. The last term of (12) captures the idea that these effects are magnified, and thus the rents extracted are higher, if some neighbors have access to very large market, so that their probability of success increases.

5.1.3 Price choice with commitment

The analysis of the two previous cases shows how the pricing policies dramatically change the incentives for T to innovate. In particular, we have seen that if T chooses $p^*_{T,i} = \pi_1$, then she faces the competition of further technological innovation by direct neighbors. Thus, the more firms $i \in N_T$ have incentives to innovate, the less T wants to innovate. On the contrary, when firm T sets a price of $p^*_{T,i} = \overline{U}_i$, then she extracts all rents from her neighbors and thus the more firms $i \in N_T$ have incentives to innovate, the more T wants to innovate. In what follows, we show how the pricing choices are affected by firms' positions in the network and by the parameter β.

If firm T has to choose between $p^*_{T,i} = \pi_1$ or $p^*_{T,i} = \overline{U}_i$, what would be her choice? In terms of innovation diffusion, the two pricing policies are very different. In fact, if $p^*_{T,i} = \pi_1$, then each node of the network has access to T's technology, if T innovates. On the contrary, if $p^*_{T,i} = \overline{U}_i$ and T's neighbor i does not innovate, then the price is too high for all the other agents in the i-induced subnetwork, and the diffusion process is stopped, which means that nobody benefits from the new technology. The following proposition determines under which condition firms set $p^*_{T,i} = \pi_1$ or $p^*_{T,i} = \overline{U}_i$.

Proposition 4. *There exist a $\overline{\beta}$ and a $\overline{M}_T$ such that it is optimal for firm T to set a price $p^*_{T,i} = \overline{U}_i$ (resp. $p^*_{T,i} = \pi_1$) for all $i \in N_T$ if and only if $\beta < \overline{\beta}$ and/or $M_T > \overline{M}_T$ (resp. $\beta > \overline{\beta}$ and/or $M_T < \overline{M}_T$).*

The proof of this proposition stems from the fact that the difference in utilities $U_T^{\overline{U}_i} - U_T^{\pi_1}$ decreases in β but increases in $\overline{M}_T$. Let us first focus on the role of β, the degree of property right protection. The proposition states that if β is high enough, then firm T chooses $p^*_{T,i} = \pi_1$, otherwise it chooses $p^*_{T,i} = \overline{U}_i$. Indeed, if β is high, then it is very hard to innovate. As a result, once T innovates, her direct neighbors have a low probability of success due to high β. This implies that competitors are not much of a threat for T, which can impose a low price π_1 and hope that neighbors do not innovate so that the rest of the firms in the network adopt her technology by paying T a price of π_1. On the contrary, if β is low, then innovating is very easy and, thus, it is very likely that T's neighbors innovate, which means that firm T will not be able to see her technology adopted by other firms in the network. In that case, it is better for T to extract immediately the rents by choosing $p^*_{T,i} = \overline{U}_i$.

In a similar way, we can analyze the effect of M_T (Market Dispersion Index) on pricing policies. When M_T has a large value, firm T chooses a high price. Remember that M_T acts as a threat in the case of $p^*_{T,i} = \pi_1$, while it increases the utility of T in the case of $p^*_{T,i} = \overline{U}_i$. Now, if M_T is high, meaning that there is a large market diffusion, the choice of $p^*_{T,i} = \overline{U}_i$ makes firm T better off since it can extract all expected rents immediately. Moreover, since M_T is high, it is likely that there are some neighboring firms with access to a large market that innovate, and this increases further the expected rents firm T can get. On the other hand, if M_T is low, then each neighboring firms represents a low expected rent to be extracted when setting $p^*_{T,i} = \overline{U}_i$. Moreover, each of these neighboring firms has a low incentive to innovate since they have access to a small market. As a consequence, firm T finds it more profitable to set a low price $p^*_{T,i} = \pi_1$. Thus, with high probability, the neighboring firms will not innovate and firm T can obtain $p^*_{T,i} = \pi_1$ from all the other firms in the network. However, in terms of technology diffusion, the two price schemes are not equivalent since, if $p^*_{T,i} = \overline{U}_i$, and the neighboring firms do not innovate, then technology diffusion is blocked in that specific branch of the network. The fact that $p^*_{T,i} = \overline{U}_i$ is chosen by nodes with a higher MDI, enables us to provide the following interpretation of the results in terms of the position of the firm T in the network with respect to pricing choices.

Corollary 1. *A peripheral firm T tends to charge high prices thus potentially blocking technology diffusion while a firm T with a more central positions in the network tends to charge low prices thus fostering technological diffusion.*

This is an interesting result that shows how firm T's position in the network affects her price policy. We can now understand both the previous proposition and corollary by looking at the role of β. Recall the previous discussions about β. In particular, in order not to block the innovation process, it is best to have low β (low property rights protection) under $p^*_{T,i} = \overline{U}_i$, and middle-valued β under $p^*_{T,i} = \pi_1$. Then, the corollary states that peripheral firms set $p^*_{T,i} = \overline{U}_i$ and more central firms set $p^*_{T,i} = \pi_1$. As a consequence, in an industry characterized by a low β it seems better to target peripheral firms, while for higher levels of β, it seems better to target more central firms.

Example: Network in Figure 1 To understand these results, consider the left network described in Figure 1 with 7 firms. Consider, first, the case when the *central firm* c is targeted (which has the lowest M_T since $M_c = 5$) and then when the *peripheral firm d* is targeted (which has the highest M_T since $M_d = 25$). Assume that $\pi_1 = 1$ and $\pi_2 = 2$ so that the incentives are fixed and let us focus on the role of M_T and β.

Observation 1. *Consider the network in Figure 1 with $n = 7$ and assume that $\pi_1 = 1$ and $\pi_2 = 2$.*

(*i*) *When $0 < \beta < 10.167$, both targeted firms c and d charge high prices equal to $p^*_{c,i} = p^*_{d,i} = \overline{U}_i$.*

(*ii*) *When* $10.167 < \beta < 23.1$, *firm $T = c$ (**central firm**;* low value of $\overline{M}_T$*) finds it optimal to set a* low price *of $p^*_{c,i} = \pi_1$ while firm d (**peripheral firm**;* high value of $\overline{M}_T$*) sets a* high price *of $p^*_{d,i} = \overline{U}_i$.*

(*iii*) *When $\beta > 23.1$, then both targeted firms c and d charge a* low price *of $p^*_{c,i} = p^*_{d,i} = \pi_1$.*

This confirms the results above described in Proposition 2 and Corollary 1. When β is sufficiently small, then firms tend to set high prices because they are afraid that their direct neighbors will innovate (low β), which means that they will lose the remaining firms in the market while, when β is sufficiently high, both firms tend to set low prices since there is little chance that their direct neighbors will innovate. For intermediary values of β, then it is the Market Dispersion Index M_T that matters. When the M_T of the targeted firm T is *high* so that this firm tends to be *peripheral*, she sets a *high price* because she benefits from a very large market. On the contrary, when the targeted firm T has a low M_T so that she is more *central* in the network, she sets a *low price* to attract more firms to buy her product. Interestingly, when firm T sets the high price $p^*_{T,i} = \overline{U}_i$, she discriminates between different firms $i \in N_T$. Indeed, when firm $T = c$, there will be three different prices since firm c has three direct neighbors (see the left network of Figure 1). In fact, we have (see the proof of Observation 1 in Appendix 1):

$$p^*_{c,a} = \overline{U}_a = 1 + \frac{25}{2\beta}, \quad p^*_{c,e} = \overline{U}_e = 1 + \frac{1}{2\beta}, \quad p^*_{c,f} = \overline{U}_f = 1 + \frac{9}{2\beta}$$

where $p^*_{c,a} > p^*_{c,f} > p^*_{c,e}$. This just confirms what we noticed before: *firm T charges higher prices to more central firms*, i.e. firms with higher diffusion centrality ($c_a = 2 > c_f = 1 > c_e = 0$). In the case when firm $T = a$, there will be only one price, which is equal to: $p^*_{d,i} = \overline{U}_b = 1 + 121/(2\beta)$. Observe that since any price $p^*_{T,i} = \overline{U}_i > \pi_1$, no firm $j \neq i$ belonging to the $i-$induced network will buy T's product if firms i do not innovate.

5.2 Non-committed prices (flexible prices)

Assume now that there an informal agreement between firm T and her neighbors so that firm T can set distinct prices to direct and indirect neighbors. The following lemma gives the optimal price policy of firm T.

Lemma 3. *Suppose that firm T has innovated and consider the case when she can charge different prices to direct and indirect neighbors. The optimal price policy for firm T is to set a price of*

$$p_{T,i}^{*} = \overline{U}_i \equiv \pi_1 + \frac{1}{2\beta}(\pi_2 - \pi_1 + \pi_2 c_i)^2 \tag{14}$$

to all firms $i \in N_T$ and, then, if firms i do not innovate, to set a price equal to $p_{T,j}^{*} = \pi_1$ to all the other firms in the network, i.e. all firms $j \neq i$ belonging to the i-induced subnetwork $\boldsymbol{z}_{T,i}$.

It should be clear that this is the best policy for firm T since she can extract all the rents from her neighbors, firms $i \in N_T$, and then still sell her product to the remaining firms at a lower price if firms i do not innovate.

The (expected) utility function is now given by:

$$U_T^{NC} = e_T \left\{ \pi_1 + \sum_{i \in N_T} \left[\overline{U}_i + (1 - e_i^{*}) c_i \pi_1 \right] \right\} - \frac{\beta}{2} e_T^2 \tag{15}$$

As above, when firm T innovates (probability e_T), she obtains π_1 as her own profit from production. Then, she obtains $p_{T,i}^{*} = \overline{U}_i$ from all firms $i \in N_T$. What is new is that, even if a firm $i \in N_T$ does not innovate (probability $1 - e_i^{*}$), firm T can still sell her product to all firms $j \neq i \in \boldsymbol{z}_{T,i}$ at a lower price π_1. Indeed, if we compare the utility (15) with the one given in (11), we see that, for a given firm T, this new utility is higher since there is now the possibility to obtain some rents even in the case when firm i does not innovate since firm T can now lower her price from $\overline{U}_i$ to π_1 to be able to sell her product to the remaining firms in the network (i.e. all firms $j \neq i \in \boldsymbol{z}_{T,i}$).

Proposition 5. *Assume that*

$$2n \left[\beta \pi_1 + (\pi_2 - \pi_1)^2 \right] > (\pi_2 - \pi_1)^2 (2 + d_T) + (\pi_2 - 2\pi_1) \pi_2 M_T \tag{16}$$

If firm T does not need to commit to a single price, then the equilibrium effort and utility are given by:

$$e_T^{NC} = \frac{1}{\beta} \left[n\pi_1 + \frac{(\pi_2 - \pi_1)^2 (n-1)}{\beta} - \frac{(\pi_2 - \pi_1)^2 d_T}{2\beta} - \frac{(\pi_2 - 2\pi_1) \pi_2 M_T}{2\beta} \right] \tag{17}$$

and

$$U_T^{NC} = \frac{1}{2\beta} \left[n\pi_1 + \frac{(\pi_2 - \pi_1)^2 (n-1)}{\beta} - \frac{(\pi_2 - \pi_1)^2 d_T}{2\beta} - \frac{(\pi_2 - 2\pi_1) \pi_2 M_T}{2\beta} \right]^2 \tag{18}$$

which are both decreasing in d_T, the degree of firm T and increasing (resp. decreasing) in M_T, the Market Dispersion Index of firm T, if and only if $\pi_2/\pi_1 < 2$ (resp. $\pi_2/\pi_1 > 2$). The effect of β on the equilibrium effort and utility is ambiguous.

Consider equation (17), which gives the equilibrium value of e_T^{NC}, and compare it with (12), which gives the optimal effort $e_T^{\overline{U}_i}$ when the firm T commits to a single price. The first two terms are unchanged since they capture the benefits for T from selling her product to her neighbors $i \in N_T$. The third term is different because it captures the incentives obtained from the $n-1-d_T$ firms that belong to the $i-$induced subnetworks $\mathbf{z}_{T,i}$. The gains are now higher when firm T does not commit because she takes into account the fact that, with some probability, she will obtain some profit by selling her product to the firms belonging to the $i-$induced subnetworks. The last term captures the effect of M_T, which enters negatively in $e_T^{\overline{U}_i}$ and has an ambiguous effect on e_T^{NC} depending whether or not π_2/π_1 is greater or smaller than 2. Namely, if $\pi_2/\pi_1 < 2$, then M_T has a positive impact on e_T^{NC} while, if $\pi_2/\pi_1 > 2$, the effect is negative. This is due to the fact that, if π_2 is high relatively to π_1, then the incentives of firms $i \in N_T$ to produce effort are quite high. Then, these effects are magnified for i's with a large market. This, in turn, increases the rents that firm T can immediately extract from firms i, and thus increases the incentives for T to exert effort. This implies that effort will be higher for peripheral firms T. On the contrary, if π_2 is not very large compared to π_1, firm T would have higher expected rents if firms i have a low probability to innovate, thus having a higher chance to obtain π_1 for the rest of firms in the network. In that case, T's effort is higher for firms in more symmetric positions of the network.

Let us now consider the role of β. Consider, first, the case in which $\pi_2/\pi_1 > 2$ so that impact of M_T on e_T^{NC} is positive. This means that the overall effect of β on the terms in brackets is negative. In fact, if β is low, firms $i \in N_T$ will be more likely to innovate but then T would also gain from this since she would find it easy to innovate and, additionally, she would extract lots of rents. This would be truer for more peripheral firms because they have a high M_T and thus the effect would be magnified. Notice that, however, this is not always necessarily verified since peripheral firms have a low d_T, and this lowers the effect of β looking at the other terms. Then the overall effect strictly depends on the topology of the network. On the contrary, if β is very high, firms will not innovate, and both efforts would be very low. However, it would be best to target peripheral firms since choosing the firm with the highest M_T would minimize the negative effect on innovation. Consider, now, the case in which $\pi_2/\pi_1 > 2$. Then M_T has a negative effect on e_T^{NC} and, therefore, the effect of β can be ambiguous. This can be most likely to happen for high M_T firms, which are peripheral firms. Then, in order to reduce this countervailing effect, it would be best to target more central firms.

Corollary 2. *If a targeted firm T could freely choose between the three different pricing* *mechanisms, independently of her position in the network, she would always choose not to commit and thus the prices will be determined as in Lemma 3.*

This result is straightforward since, by not committing to a single price, firm T will be better off than committing to $p^*_{T,i} = \overline{U}_i$ because firm T can also benefit when the neighboring firms do not innovate or committing to $p^*_{T,i} = \pi_1$ since she can charge a higher price to neighboring firms and let π_1 being the price for the remaining firms.

The three mechanisms, however, are different also in terms of distribution of rents. In fact, in the non-committed case, and in the committed price with $p^*_{T,i} = \overline{U}_i$, the firm T obtains the entire welfare. When firm T commits to $p^*_{T,i} = \pi_1$, part of the surplus goes to T's neighbors since, if they innovate, they sell their innovation at π_2, getting a total revenue of $\pi_2 c_i$, while having paid only π_1. This what we investigate now.

6 Targeting policies

Consider a policy maker, whose aim is to maximize total welfare in the network, and has to decide with firm to target, i.e. which firm T to choose? We assume that after the planner has chosen firm T, then the latter decides upon her price policy. The timing is thus as follows. In the first stage, the planner decides which firm T to target. Then the timing of the game is as above. Here, targeting means that the planner gives some financial incentives to form T so that she is the first that can innovate. As in the previous sections, we will consider two cases: when firm T can commit to the same price or when she cannot.

6.1 Committed prices (fixed prices)

Assume that firm T must commit to a single price as in Section 5.1 because of a binding contract between firm T and her neighbors. This implies that firm T cannot change her price once firms $i \in N_T$ have not innovated. In Lemma 2, we have seen that firm T can only set two prices: either $p^*_{T,i} = \pi_1$ or $p^*_{T,i} = \pi_1 + \frac{1}{2\beta}(\pi_2 - \pi_1 + \pi_2 c_i)^2 \equiv \overline{U}_i$. We start with the latter since the results are easier to establish.

6.1.1 Firm T commits to a unique price $p^*_{T,i} = \overline{U}_i$

When firm T always set a price of $p^*_{T,i} = \overline{U}_i$, the expected utility of firms $i \in N_T$ is equal to zero (by definition of the pricing rule). Also, if firm i innovates, she charge a price of π_2 to $j-$firms (which leads to a zero utility for $j-$firms) and if i does not innovate, $p^*_{T,i} = \overline{U}_i$ is too high and thus firms j do not buy the product. As a result, the utility of $j-$firms is always equal to zero. Therefore, the total welfare $\mathcal{W}^{\overline{U}_i}$ is just equal to the utility of the targeted firm, i.e.

$$\mathcal{W}^{\overline{U}_i} = U_T^{\overline{U}_i} = \frac{1}{2\beta}\left[\pi_1 + \frac{\pi_2\left(\pi_2 - \pi_1\right)(n-1)}{\beta} + \left(\frac{2\beta\pi_1 + \pi_1^2 - \pi_2^2}{2\beta}\right)d_T + \frac{\pi_2^2 M_T}{2\beta}\right]^2 \tag{19}$$

We can now use the result of Proposition 3 to obtain the following result.

Proposition 6. *Assume that firm T commits to a unique price $p_{T,i}^* = \overline{U}_i$ so that she charges this price to both firms $i \in N_T$ and firms $j \in \mathbf{z}_{T,i}$. Then, the planner should always target the more peripheral firms, i.e. the firms with highest M_T. Furthermore, if two firms T_1 and T_2 have the same Market Dispersion Index, i.e. $M_{T_1} = M_{T_2}$, then the planner should target the firm with the highest degree if $\left(\pi_2^2 - \pi_1^2\right)/\pi_1 < 2\beta$ and the one with the lowest degree if $\left(\pi_2^2 - \pi_1^2\right)/\pi_1 > 2\beta$.*

We omit the proof of this proposition since it is a direct application of Proposition 3. Indeed, when firm T sets a high price $p_{T,i}^* = \overline{U}_i$ to all firms, maximizing total welfare is equivalent to maximizing $U_T^{\overline{U}_i}$, the utility of firm T. In that case, the planner wants to target the peripheral firms, which have the highest Market Dispersion Index M_T, because these are the firms with exert the highest effort and have the highest utility in the network.

6.1.2 Firm T commits to a unique price $p_{T,i}^* = \pi_1$

When firm T sets a fixed price of $p_{T,i}^* = \pi_1$, the expected utility of firms $j \in \mathbf{z}_{T,i}$ is equal to zero (by definition of the pricing rule) but the utility of firms $i \in N_T$ is strictly positive. Therefore, the total welfare $\mathcal{W}^{\pi_1}$ is equal to the utility of the targeted firm T and the sum of utilities of firms $i \in N_T$, i.e.

$$\mathcal{W}^{\pi_1} = U_T^{\pi_1} + \sum_{i \in N_T} U_i^*$$

We have the following result.

Proposition 7. *Assume condition (8) holds. Assume that firm T commits to a unique price $p_{T,i}^* = \pi_1$ to both firms $i \in N_T$ and firms $j \in \mathbf{z}_{T,i}$. Then, the planner should always target the firms with the highest degree d_T. Moreover, if $\pi_2/\pi_1 < 2$, the planner should target the most central firms, i.e. the firms with the lowest Market Dispersion Index M_T. If, on the contrary, $\pi_2/\pi_1 > 2$, then it is not clear which firm should be targeted.*

When firm T commits to a low price, then both firms $i \in N_T$ and firms $j \in \mathbf{z}_{T,i}$ will buy her product. The planner wants to choose a firm T that maximizes both the utility of firm

T and the sum of utilities of firms $i \in N_T$. We have seen (Proposition 2) that the utility of firm T increases with d_T, the degree of firm T. Indeed, when d_T increases, firm T is better off because she can extract more rents from her direct neighbors. Furthermore, $\sum_{i \in N_T} U_i^*$ increases mechanically with d_T since the higher d_T, the higher is the number of firms $i \in N_T$. As a result, the planner wants to target the firm that has the highest degree. What about the Market Dispersion Index M_T, which measures the size and the diffusion of the market faced by firm T? If $\frac{\pi_2}{\pi_1} < 2$, which means that firms $i \in N_T$ will not make that much profit if they innovate, the planner wants to target a firm T with a low M_T because, in that case, the competitors of firm T have a small market and will be less likely to innovate.

Corollary 3. *Assume that firm T commits to a unique price $p_{T,i}^* = \pi_1$ to both firms $i \in N_T$ and firms $j \in \mathbf{z}_{T,i}$. Then, if n is large enough, the planner should always target the most central firms, i.e. the firms with the lowest Market Dispersion Index M_T. If two firms have the same M_T, the planner should target the one that has the highest degree.*

This Corollary just confirms what we just said but by looking at another exogenous variable: n, the number of firms belonging to the network. The impact of d_T on total welfare is still positive whatever the value of n. However, the effect of M_T on total welfare depends on n. If n has a high value, then the market for both firm T and firms $i \in N_T$ will be large and thus the firm will want to target a central firm that has a low M_T.

6.2 Non-committed prices (flexible prices)

In that case, firm T will charge different prices to firms $i \in N_T$ and firms $j \in \mathbf{z}_{T,i}$. We have the following result.

Proposition 8. *Assume condition (16) holds and that firm T can charge different prices to firms $i \in N_T$ and firms $j \in \mathbf{z}_{T,i}$. Then, if $\pi_2/\pi_1 < 2$, the planner should target more peripheral firms, i.e. firms with higher M_T. On the contrary, if $\pi_2/\pi_1 > 2$, the planner should target more central firms, i.e. firms with lower M_T. Finally, if two firms have the same Market Dispersion Index M_T, then the planner should target the firm with the lowest degree d_T.*

When firm T can charge different prices to firms $i \in N_T$ and firms $j \in \mathbf{z}_{T,i}$, we have seen in Lemma 3 that the optimal price policy for firm T is to set a price of $p_{T,i}^*$ to all firms $i \in N_T$ and, then, if firms i do not innovate, to set a price equal to $p_{T,j}^* = \pi_1$ to all the other firms in the network, i.e. all firms $j \neq i$ belonging to the i-induced subnetwork $\boldsymbol{z}_{T,i}$. This implies that all firms but firm T will have an (expected) utility of zero and, as a result, the total welfare will be reduced to the (expected) utility of firm T, i.e. $\mathcal{W} = U_T(e) = \frac{\beta}{2}(e_T^{NC})^2$.

Therefore, maximizing total welfare is equivalent to maximize e_T^{NC}, which is defined by (17). In Proposition 5, we have seen that the equilibrium effort is decreasing in d_T and increasing (resp. decreasing) in M_T if and only if $\pi_2/\pi_1 < 2$ (resp. $\pi_2/\pi_1 < 2$). Proposition 8 is a direct consequence of this result.

6.3 Comparing policies with different price schemes

In Proposition 6 (firm T commits to a unique price $p^*_{T,i} = \overline{U}_i$), Proposition 7 and Corollary 3 (firm T commits to a unique price $p^*_{T,i} = \pi_1$) and Proposition 8 (firm T charge distinct prices to different firms), we have seen that the planner, which maximizes total welfare, would target different firms. This is an interesting result since it implies that depending on the market structure (especially β) and on the incentives of the firms (price strategies), different firms will be targeted. Using Proposition 4, we have the following result:

Proposition 9. *We have:*

- *(i) Assume that firms must commit to one price (binding contract).*
  - *(i1) If β is small enough, then the planner should always target the more peripheral firms, i.e. the firms with highest M_T. However, if two firms T_1 and T_2 have the same Market Dispersion Index, i.e. $M_{T_1} = M_{T_2}$, then the planner should target the firm with the highest degree if $\left(\pi_2^2 - \pi_1^2\right)/\pi_1 < 2\beta$ and the one with the lowest degree if $\left(\pi_2^2 - \pi_1^2\right)/\pi_1 > 2\beta$.*
  - *(i2) If β is large enough, then, the planner should always target the firms with the highest degree d_T. Moreover, if $\frac{\pi_2}{\pi_1} < 2$, then the planner should target the most central firms, i.e. the firms with the lowest Market Dispersion Index M_T.*
- *(ii) Assume that firms can charge distinct prices to different firms (informal agreement). Then, the planner should target the firm with the lowest degree d_T.*

To illustrate this result, let us go back to the left network in Figure 1. Which firm should the planner target? First, consider a market where firm T cannot charge distinct prices to different firms because of a binding contract. If we assume that property rights protections are very low (i.e. low β), then the planner should target the *peripheral firms* d, e, g since they all have a $M_T = 25$, for $T = d, e, g$ (see Figure 2), which is the highest in the network, and all have the same degree $d_T = 1$, for $T = d, e, g$. If, on the contrary, property-right protections are very high in this market, then the planner should target the *central firm* c because she has the highest degree, $d_c = 3$. Second, consider a market where firm T can charge different prices, then the planner should always target the *peripheral firms* d, e, g since they have the lowest degree. It is interesting to see that, depending on the market structure and the property-right protections, the targeted firms that maximize total welfare can be very different, from very central to very peripheral.

If we now consider the specific example of Observation 1 for the left network in Figure 1 with $n = 7$, $\pi_1 = 1$, $\pi_2 = 2$, and where we assume that firm T commits to only one price, then we have the following result:

Observation 2. *Assume that firms must commit to one price. Consider the network in Figure 1 with $n = 7$ and assume that $\pi_1 = 1$ and $\pi_2 = 2$.*

(i) When $0 < \beta < 10.167$, the planner should target the peripheral firm d.

(ii) When $10.167 < \beta < 23.1$, the planner should target the peripheral firm d.

(iii) When $\beta > 23.1$, the planner should target the central firm c.

First, observe that, given our results in Proposition 9, in the left network in Figure 1, the planner will either target the most peripheral firm (firm d)[10] or the most central firm (firm a). This is why in Observation 2, we only focus on these two firms. Second, we have seen in Observation 1 that, when $0 < \beta < 10.167$, both firms a and d, if targeted, will charge the high price. In that case, the firm with highest M_T, i.e. firm d, should be targeted. When $\beta > 23.1$, we have the opposite result since both firms a and d will charge a low price and, in that case, the firm with highest d_T, i.e. firm a, should be targeted. When $10.167 < \beta < 23.1$, firm d sets a high price of $p^*_{d,i} = \overline{U}_i$ while firm c sets a low price of $p^*_{c,i} = \pi_1$. As a result, it is not clear which firm should be targeted. In the proof of Observation 2, we show that total welfare is higher when firm c rather than firm d is targeted.

7 Conclusion

In this paper, we develop a model where a firm T has a new (intermediary) product in the market and wants to sell it first to her direct neighbors. These direct neighbors could be firms with whom she has a direct contact with (word-of-mouth communication if the network represents the flow of information between firms) or firms that are close technologically (in that case, the network will represent the technology space). To sell her new product to her neighbors, firm T will either sign a binding observable contract (committed price) or will

[10] or firm e or firm g since they are identical to firm d because they all have the same d_T and the same M_T.

just have an informal agreement with her direct neighbors (non-commited price). This will depend on the institutional context firm T operates in. Then, the neighboring firms can further improve the product of firm T (sequential innovation) and sell it to her own market, i.e. all the firms that she can communicate with or the firms that are technologically close. If the neighboring firms are not successful in innovating, then firm T can still sell her product to the firms belonging to the market of her neighboring firms.

We first analyze the optimal price policy of firm T and of her neighboring firms when firm T must commit to a single price and when she does not need to commit to a single price. We show that the optimal price policy depends on the level of property rights protection, the position of firm T in the network and on her degree, i.e. her number of direct neighbors/competitors. We then analyze the welfare implications of our model by determining which firm should be targeted if the planner wants to maximize total welfare. Which firm should be targeted depends on the institutional context (the type of contract that prevails), the level of property rights protection, the position of firm T in the network and on her degree. In particular, if the level of property rights protection is low so that it is easy to innovate and if firms must commit to a unique price, then the planner should always target the more peripheral firms. On the contrary, if the level of property rights protection is high and if firms must commit to a unique price, then the planner should always target the firms with the highest number of direct competitors. When firms do not need to commit to a unique price, we show that, independently of the level of property rights protection, the planner should always target the firm with the lowest degree.

In Appendix 2, we extend our model by assuming that the network is only partially known to the firms. To be more precise, firms only know their own degree (i.e. their direct competitors) and all firms share the same common belief about the degree of the other firms in the network. We show that most of our results prevail in this case. In particular, when the firm commits to a fixed price, we show that she charges a high price if the level of property rights protection is low enough and a high price if it is high enough. We also show how the network structure affects the pricing policy.

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Appendix 1: Proofs

Proof of Lemma 2: It should be clear that firm T can only charge two prices: $p^*_{T,i} = \pi_1$ or $p^*_{T,i} = \overline{U}_i$. Let us show that these are the only prices that can arise. Indeed, all prices belonging to $[0, \pi_1) \cup (\pi_1, \overline{U}_i) \cup (\overline{U}_i, \infty)$ can never be equilibrium prices for firm T. Indeed, any $p_{T,i} \in [0, \pi_1)$ is dominated by π_1 since firms that can afford $p_{T,i} \in [0, \pi_1)$ can also afford π_1. Any $p_{T,i} \in (\pi_1, \overline{U}_i)$ is dominated by either π_1 or $\overline{U}_i$, depending on which of the two is the optimal price. Finally, any $p_{T,i} \in (\overline{U}_i, \infty)$ is dominated by $\overline{U}_i$ since no firm would accept $p_{T,i} > \overline{U}_i$ and thus the firm T would not be able to sell her product to anybody in the network.

Proof of Proposition 2: The maximization of (7) leads to:[11]

$$\beta e_T^{\pi_1} = \pi_1 + \sum_{i \in N_T} \left[e_i^* \pi_1 + (1 - e_i^*) \pi_1 (1 + c_i) \right]$$

Consider (4). Then, notice that:

$$\begin{aligned} & \sum_{i \in N_T} \left[e_i^* \pi_1 + (1 - e_i^*) \pi_1 (1 + c_i) \right] \\ = & \sum_{i \in N_T} \left[\pi_1 (1 + c_i) - e_i^* \pi_1 c_i \right] \\ = & \sum_{i \in N_T} \left[\pi_1 (1 + c_i) - \left(\frac{\pi_2 - \pi_1 + \pi_2 c_i}{\beta} \right) \pi_1 c_i \right] \\ = & \pi_1 \left(\sum_{i \in N_T} 1 + \sum_{i \in N_T} c_i \right) - \frac{1}{\beta} \sum_{i \in N_T} (\pi_2 - \pi_1) \pi_1 c_i - \frac{1}{\beta} \sum_{i \in N_T} \pi_2 \pi_1 c_i^2 \\ = & \pi_1 (n - 1) - \frac{(\pi_2 - \pi_1) \pi_1 (n - 1 - d_T)}{\beta} - \frac{\pi_1 \pi_2}{\beta} M_T \end{aligned}$$

By plugging this value in the first-order condition above, we obtain:

$$\begin{aligned} \beta e_T^{\pi_1} &= \pi_1 + \pi_1 (n - 1) - \frac{(\pi_2 - \pi_1) \pi_1 (n - 1 - d_T)}{\beta} - \frac{\pi_1 \pi_2}{\beta} M_T \\ &= \pi_1 \left[n - \frac{(\pi_2 - \pi_1)(n - 1 - d_T)}{\beta} - \frac{\pi_2}{\beta} M_T \right] \end{aligned}$$

[11]It should be clear that $U_T^{\pi_1}(e_T)$ is stritcly concave in e_T so that there is a unique maximum.

This leads to (9). Now, we have:

$$
\begin{aligned}
U_T^{\pi_1} &= e_T^{\pi_1}\left\{\pi_1 + \sum_{i\in N_T}\left[e_i^*\pi_1 + (1-e_i^*)\pi_1(1+c_i)\right]\right\} - \frac{\beta}{2}\left(e_T^{\pi_1}\right)^2 \\
&= e_T^{\pi_1}\left\{\pi_1 + \sum_{i\in N_T}\left[\pi_1(1+c_i) - e_i^*\pi_1 c_i\right]\right\} - \frac{\beta}{2}\left(e_T^{\pi_1}\right)^2 \\
&= e_T^{\pi_1}\left\{\pi_1 + \pi_1(n-1) - \frac{(\pi_2-\pi_1)\,\pi_1\,(n-1-d_T)}{\beta} - \frac{\pi_1\pi_2}{\beta}M_T\right\} - \frac{\beta}{2}\left(e_T^{\pi_1}\right)^2 \\
&= e_T^{\pi_1}\left\{\pi_1 n - \frac{(\pi_2-\pi_1)\,\pi_1\,(n-1-d_T)}{\beta} - \frac{\pi_1\pi_2}{\beta}M_T\right\} - \frac{\beta}{2}\left(e_T^{\pi_1}\right)^2
\end{aligned}
$$

Now using the value of $e_T^{\pi_1}$ in (9), we have:

$$
\begin{aligned}
U_T^{\pi_1} &= e_T^{\pi_1}\left\{\pi_1 n - \frac{(\pi_2-\pi_1)\,\pi_1\,(n-1-d_T)}{\beta} - \frac{\pi_1\pi_2}{\beta}M_T\right\} - \frac{\beta}{2}\left(e_T^{\pi_1}\right)^2 \\
&= \frac{\pi_1^2}{\beta}\left[n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T\right]^2 - \frac{\pi_1^2}{2\beta}\left[n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T\right]^2 \\
&= \frac{\pi_1^2}{2\beta}\left[n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T\right]^2
\end{aligned}
$$

which is (10).

We need to check that $e_T^{\pi_1} > 0$. Using (9), this is equivalent to:

$$
n > \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} + \frac{\pi_2}{\beta}M_T
$$

which, after rearranging some terms leads to (8).

When (8) holds, apart from β, the comparative statics of $U_T^{\pi_1}$ is equivalent to that of $e_T^{\pi_1}$. We have:

$$
\frac{\partial e_T^{\pi_1}}{\partial d_T} > 0,\ \frac{\partial e_T^{\pi_1}}{\partial M_T} < 0
$$

which implies

$$
\frac{\partial U_T^{\pi_1}}{\partial d_T} > 0,\ \frac{\partial U_T^{\pi_1}}{\partial M_T} < 0
$$

Denote

$$
A \equiv n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T > 0
$$

Then,

$$\frac{1}{\pi_1^2}\frac{\partial U_T^{\pi_1}}{\partial \beta} = -\frac{1}{2\beta}A^2 + \frac{1}{\beta}A\left[\frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta^2} + \frac{\pi_2}{\beta^2}M_T\right]$$

Thus

$$\begin{aligned}
\frac{\partial U_T^{\pi_1}}{\partial \beta} &\gtreqless 0 \Leftrightarrow 2(\pi_2-\pi_1)(n-1-d_T) + 2\pi_2 M_T \gtreqless A\beta^2 \\
&\Leftrightarrow 2(\pi_2-\pi_1)(n-1-d_T) + 2\pi_2 M_T \gtreqless \left[n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T\right]\beta^2 \\
&\Leftrightarrow (2+\beta)\left[(\pi_2-\pi_1)(n-1-d_T) + \pi_2 M_T\right] \gtreqless \beta^2 n \\
&\Leftrightarrow n\left[(2+\beta)(\pi_2-\pi_1) - \beta^2\right] + (2+\beta)\pi_2 M_T \gtreqless (2+\beta)(\pi_2-\pi_1)(1+d_T)
\end{aligned}$$

Finally, it is easily verified that:

$$sign\frac{\partial^2 U_T^{\pi_1}}{\partial\beta\partial M_T} = sign\frac{\partial^2 U_T^{\pi_1}}{\partial\beta\partial d_T} = sign\left[A'_\beta\beta^2 - 2\beta A\right]$$

where

$$A \equiv n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T$$

$$A'_\beta = \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta^2} + \frac{\pi_2}{\beta^2}M_T$$

and

$$A'_\beta\beta^2 - 2\beta A = 3(\pi_2-\pi_1)(n-1-d_T) + 3\pi_2 M_T - 2\beta n$$

As a result,

$$\frac{\partial^2 U_T^{\pi_1}}{\partial\beta\partial M_T} \gtreqless 0 \Leftrightarrow n\left[3(\pi_2-\pi_1) - 2\beta\right] + 3\pi_2 M_T \gtreqless 3(\pi_2-\pi_1)(1+d_T)$$

$$\frac{\partial^2 U_T^{\pi_1}}{\partial\beta\partial d_T} \gtreqless 0 \Leftrightarrow n\left[3(\pi_2-\pi_1) - 2\beta\right] + 3\pi_2 M_T \gtreqless 3(\pi_2-\pi_1)(1+d_T)$$

Proof of Proposition 3: The maximization of (11) leads to:[12]

$$\beta e_T^{\overline{U}_i} = \pi_1 + \sum_{i\in N_T}\overline{U}_i$$

[12]It should be clear that $U_T^{\overline{U}_i}(e_T)$ is strictly concave in e_T so that there is a unique maximum.

Then, using (6), notice that:

$$\begin{aligned}
& \pi_1 + \sum_{i \in N_T} \overline{U}_i \\
= \; & \pi_1 + \sum_{i \in N_T} \left[\pi_1 + \frac{1}{2\beta}(\pi_2 - \pi_1 + \pi_2 c_i)^2 \right] \\
= \; & \pi_1 + \sum_{i \in N_T} \pi_1 + \frac{1}{2\beta} \sum_{i \in N_T} (\pi_2 - \pi_1 + \pi_2 c_i)^2 \\
= \; & \pi_1(1 + d_T) + \frac{1}{2\beta} \sum_{i \in N_T} \left[(\pi_2 - \pi_1)^2 + 2\pi_2 (\pi_2 - \pi_1) c_i + \pi_2^2 c_i^2 \right] \\
= \; & \pi_1(1 + d_T) + \frac{1}{2\beta} \sum_{i \in N_T} (\pi_2 - \pi_1)^2 + \frac{\pi_2 (\pi_2 - \pi_1)}{2} \sum_{i \in N_T} c_i + \frac{\pi_2^2}{2\beta} \sum_{i \in N_T} c_i^2 \\
= \; & \pi_1(1 + d_T) + \frac{(\pi_2 - \pi_1)^2 d_T}{2\beta} + \frac{\pi_2 (\pi_2 - \pi_1)(n - 1 - d_T)}{\beta} + \frac{\pi_2^2}{2\beta} M_T
\end{aligned}$$

By plugging this value in the first-order condition above, we obtain:

$$\beta e_T^{\overline{U}_i} = \pi_1(1 + d_T) + \frac{(\pi_2 - \pi_1)^2 d_T}{2\beta} + \frac{\pi_2 (\pi_2 - \pi_1)(n - 1 - d_T)}{\beta} + \frac{\pi_2^2 M_T}{2\beta}$$

This leads to (12). Now, we have:

$$\begin{aligned}
U_T^{\overline{U}_i} &= e_T^{\overline{U}_i} \left(\pi_1 + \sum_{i \in N_T} \overline{U}_i \right) - \frac{\beta}{2} \left(e_T^{\overline{U}_i} \right)^2 \\
&= \beta \left(e_T^{\overline{U}_i} \right)^2 - \frac{\beta}{2} \left(e_T^{\overline{U}_i} \right)^2 \\
&= \frac{\beta}{2} \left(e_T^{\overline{U}_i} \right)^2 \\
&= \frac{1}{2\beta} \left[\pi_1(1 + d_T) + \frac{(\pi_2 - \pi_1)^2 d_T}{2\beta} + \frac{\pi_2 (\pi_2 - \pi_1)(n - 1 - d_T)}{\beta} + \frac{\pi_2^2 M_T}{2\beta} \right]^2
\end{aligned}$$

which is (13). The effect of M_T, d_T and β on $e_T^{\overline{U}_i}$ and $U_T^{\overline{U}_i}$ are straightforward to obtain.

Proof of Proposition 4: Consider (10) and (13), which we rewrite for the sake of the exposition:

$$U_T^{\pi_1} = \frac{\pi_1^2}{2\beta} \left[n - \frac{(\pi_2 - \pi_1)(n - 1 - d_T)}{\beta} - \frac{\pi_2}{\beta} M_T \right]^2$$

$$U_T^{\overline{U}_i} = \frac{1}{2\beta}\left[\pi_1(1+d_T) + \frac{(\pi_2-\pi_1)^2 d_T}{2\beta} + \frac{\pi_2(\pi_2-\pi_1)(n-1-d_T)}{\beta} + \frac{\pi_2^2 M_T}{2\beta}\right]^2$$

It is immediate to see that $U_T^{\overline{U}_i} - U_T^{\pi_1}$ is increasing in M_T and decreasing in β. As a result, firm T will choose $p_{T,i}^* = \overline{U}_i$ if and only if $U_T^{\overline{U}_i} > U_T^{\pi_1}$ and the result follows. Similarly, firm T will choose $p_{T,i}^* = \pi_1$ if and only if $U_T^{\pi_1} > U_T^{\overline{U}_i}$ and the result follows.

Proof of Observation 1: When firm c is the $T-$firm, we have:[13]

$$U_c^{\pi_1} = \frac{1}{2\beta}\left(7 - \frac{13}{\beta}\right)^2 \quad \text{and} \quad U_c^{\overline{U}_i} = \frac{1}{2\beta}\left(4 + \frac{35}{2\beta}\right)^2$$

Therefore,

$$U_c^{\pi_1} \gtreqless U_c^{\overline{U}_i} \Leftrightarrow \Phi_1(\beta) \equiv 132\beta^2 - 1288\beta - 549 \gtreqless 0$$

It is easily verified that there is a unique positive solution to $\Phi_1(\beta)$ given by $\beta = 61/6 = 10.167$. As a result, since $\Phi_1(0) < 0$, for $0 < \beta < 10.167$, $\Phi_1(\beta) < 0$, i.e. $U_c^{\pi_1} < U_c^{\overline{U}_i}$ so that $p_{c,i}^* = \overline{U}_i$, for $i = a, e, f$,[14] whereas for $\beta > 10.167$, $\Phi_1(\beta) > 0$, i.e. $U_c^{\pi_1} > U_c^{\overline{U}_i}$ so that $p_{d,i}^* = \pi_1 = 1$.

Now, when firm d is the $T-$firm, we have:

$$U_d^{\pi_1} = \frac{1}{2\beta}\left(7 - \frac{55}{\beta}\right)^2 \quad \text{and} \quad U_d^{\overline{U}_i} = \frac{1}{2\beta}\left(2 + \frac{121}{2\beta}\right)^2$$

Therefore,

$$U_d^{\pi_1} \gtreqless U_d^{\overline{U}_i} \Leftrightarrow \Phi_2(\beta) \equiv 180\beta^2 - 4048\beta - 2541 \gtreqless 0$$

It is easily verified that there is a unique positive solution to $\Phi_2(\beta)$ given by $\beta = 23.1$. As a result, since $\Phi_2(0) < 0$, for $0 < \beta < 23.1$, $\Phi_2(\beta) < 0$, i.e. $U_d^{\pi_1} < U_d^{\overline{U}_i}$ so that $p_{d,i}^* = \overline{U}_b = 1 + 121/(2\beta)$, whereas for $\beta > 23.1$, $\Phi_2(\beta) > 0$, i.e. $U_d^{\pi_1} > U_d^{\overline{U}_i}$ so that

[13] Indeed, for any $T-$firm, we have:

$$U_T^{\pi_1} = \frac{1}{2\beta}\left[7 - \frac{(6-d_T)}{\beta} - \frac{2M_T}{\beta}\right]^2$$

and

$$U_T^{\overline{U}_i} = \frac{1}{2\beta}\left[1 + d_T + \frac{d_T}{2\beta} + \frac{2(6-d_T)}{\beta} + \frac{2M_T}{\beta}\right]^2$$

[14] We have:

$$\overline{U}_a = 1 + \frac{25}{2\beta}, \quad \overline{U}_e = 1 + \frac{1}{2\beta}, \quad \overline{U}_f = 1 + \frac{9}{2\beta}$$

$p^*_{d,i} = \pi_1 = 1$. The results follow directly.

Proof of Proposition 5: From the proof of Proposition 3, we have:

$$\sum_{i\in N_T} \overline{U}_i = \pi_1 d_T + \frac{(\pi_2-\pi_1)^2 d_T}{2\beta} + \frac{\pi_2(\pi_2-\pi_1)(n-1-d_T)}{\beta} + \frac{\pi_2^2}{2\beta}M_T \tag{20}$$

Then, using (20), (4) and (14), the maximization of (15) with respect to e_T leads to:

$$\begin{aligned}
\beta e_T^{NC} &= \pi_1 + \sum_{i\in N_T}\left[\overline{U}_i + (1-e_i^*)c_i\pi_1\right] \\
&= \pi_1 + \sum_{i\in N_T}\overline{U}_i + \pi_1\sum_{i\in N_T}(1-e_i^*)\,c_i \\
&= \pi_1 + \sum_{i\in N_T}\overline{U}_i + \pi_1\sum_{i\in N_T}\left[1-\frac{(\pi_2-\pi_1)+\pi_2 c_i}{\beta}\right]c_i \\
&= \pi_1 + \sum_{i\in N_T}\overline{U}_i + \pi_1\sum_{i\in N_T}c_i - \frac{\pi_1(\pi_2-\pi_1)}{\beta}\sum_{i\in N_T}c_i - \frac{\pi_1\pi_2}{\beta}\sum_{i\in N_T}c_i^2 \\
&= \pi_1 + \pi_1 d_T + \frac{(\pi_2-\pi_1)^2 d_T}{2\beta} + \frac{\pi_2(\pi_2-\pi_1)(n-1-d_T)}{\beta} + \frac{\pi_2^2}{2\beta}M_T \\
&\quad + \pi_1(n-1-d_T) - \frac{\pi_1(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_1\pi_2}{\beta}M_T \\
&= \pi_1(1+d_T) + \frac{(\pi_2-\pi_1)^2 d_T}{2\beta} + \frac{\left[(\pi_2-\pi_1)^2+\beta\pi_1\right](n-1-d_T)}{\beta} - \frac{(\pi_2-2\pi_1)\,\pi_2 M_T}{2\beta}
\end{aligned}$$

which leads to (17). The utility (15) is given by:

$$U_T^{NC} = e_T^{NC}\left\{\pi_1 + \sum_{i\in N_T}\left[\overline{U}_i + (1-e_i^*)c_i\pi_1\right]\right\} - \frac{\beta}{2}\left(e_T^{NC}\right)^2$$

We have seen above that:

$$\beta e_T^{NC} = \pi_1 + \sum_{i\in N_T}\left[\overline{U}_i + (1-e_i^*)c_i\pi_1\right]$$

As a result,

$$\begin{aligned} U_T^{NC} &= \frac{1}{\beta}\left(\pi_1 + \sum_{i\in N_T}\left[\overline{U}_i + (1-e_i^*)c_i\pi_1\right]\right)^2 - \frac{1}{2\beta}\left(\pi_1 + \sum_{i\in N_T}\left[\overline{U}_i + (1-e_i^*)c_i\pi_1\right]\right)^2 \\ &= \frac{1}{2\beta}\left(\pi_1 + \sum_{i\in N_T}\left[\overline{U}_i + (1-e_i^*)c_i\pi_1\right]\right)^2 \\ &= \frac{\beta}{2}(e_T^{NC})^2 \end{aligned}$$

We now need to verify that $e_T^{NC} > 0$. Denote

$$A \equiv n\pi_1 + \frac{(\pi_2-\pi_1)^2(n-1)}{\beta} - \frac{(\pi_2-\pi_1)^2 d_T}{2\beta} - \frac{(\pi_2-2\pi_1)\pi_2 M_T}{2\beta}$$

We need to show that $A > 0$, which is equivalent to:

$$2n\left[\beta\pi_1 + (\pi_2-\pi_1)^2\right] > (\pi_2-\pi_1)^2(2+d_T) + (\pi_2-2\pi_1)\pi_2 M_T$$

which is condition (16).

Apart from β, if (16) holds, the comparative statics of U_T^{NC} are the same as the ones of e_T^{NC}. We have:

$$\frac{\partial U_T^{NC}}{\partial M_T} \gtreqless 0 \Leftrightarrow \frac{\pi_2}{\pi_1} \lesseqgtr 2$$

$$\frac{\partial U_T^{NC}}{\partial d_T} < 0$$

Furthermore, it is easily verified that:

$$\begin{aligned} sign\frac{\partial U_T^{NC}}{\partial \beta} &= sign\left[\frac{1}{\beta}AA'_\beta - \frac{1}{2\beta^2}A^2\right] \\ &= sign\left(A'_\beta - \frac{1}{2\beta}A\right) \end{aligned}$$

where

$$A = n\pi_1 + \frac{(\pi_2-\pi_1)^2(n-1)}{\beta} - \frac{(\pi_2-\pi_1)^2 d_T}{2\beta} - \frac{(\pi_2-2\pi_1)\pi_2 M_T}{2\beta}$$

$$A'_\beta = -\frac{(\pi_2-\pi_1)^2(n-1)}{\beta^2} + \frac{(\pi_2-\pi_1)^2 d_T}{2\beta^2} + \frac{(\pi_2-2\pi_1)\pi_2 M_T}{2\beta^2}$$

Thus

$$\begin{aligned} & A'_\beta - \frac{1}{2\beta}A \\ = & -\frac{3\left(\pi_2-\pi_1\right)^2(n-1)}{\beta^2} - \frac{1}{2\beta}n\pi_1 + \frac{3\left(\pi_2-\pi_1\right)^2 d_T}{2\beta^2} + \frac{3\left(\pi_2-2\pi_1\right)\pi_2 M_T}{2\beta^2} \end{aligned}$$

which is clearly ambiguous.

Proof of Proposition 7 and Corollary 3: The total welfare $\mathcal{W}^{\pi_1}$ when $p^*_{T,i} = \pi_1$ is given by:

$$\mathcal{W}^{\pi_1} = U_T^{\pi_1} + \sum_{i \in N_T} U_i(e_i^*) + \sum_{j \in \mathbf{z}_{T,i}, j \neq i} U_j$$

Let us start with firm T. Her expected utility (see (10)) is given by:

$$\begin{aligned} U_T^{\pi_1} &= \frac{\beta}{2}\left(e_T^{\pi_1}\right)^2 \\ &= \frac{\pi_1^2}{2\beta}\left[n - \frac{\left(\pi_2-\pi_1\right)\left(n-1-d_T\right)}{\beta} - \frac{\pi_2}{\beta}M_T\right]^2 \end{aligned}$$

The expected utility of a firm $i \in N_T$ is:

$$U_i^* = e_T^{\pi_1}\left[\frac{\left(\pi_2-\pi_1+\pi_2 c_i\right)^2}{2\beta}\right]$$

where (see (9))

$$e_T^{\pi_1} = \frac{\pi_1}{\beta}\left[n - \frac{\left(\pi_2-\pi_1\right)\left(n-1-d_T\right)}{\beta} - \frac{\pi_2}{\beta}M_T\right]$$

To calculate U_i^*, we have used (5) where we have set $p^*_{T,i} = \pi_1$ and multiplied the utility by $e_T^{\pi_1}$ since this utility is obtained conditional on the fact that firm T has innovated. As a result,

$$\begin{aligned}
\sum_{i\in N_T} U_i(e_i^*) &= \sum_{i\in N_T} \frac{e_T^{\pi_1}}{2\beta}(\pi_2-\pi_1+\pi_2 c_i)^2 \\
&= \sum_{i\in N_T} \frac{e_T^{\pi_1}}{2\beta}\left[(\pi_2-\pi_1)^2+\pi_2^2 c_i^2+2(\pi_2-\pi_1)\pi_2 c_i\right] \\
&= \frac{e_T^{\pi_1}}{2\beta}\sum_{i\in N_T}(\pi_2-\pi_1)^2+\frac{\pi_2^2 e_T^{\pi_1}}{2\beta}\sum_{i\in N_T} c_i^2+\frac{(\pi_2-\pi_1)\pi_2 e_T^{\pi_1}}{\beta}\sum_{i\in N_T} c_i \\
&= \frac{e_T^{\pi_1}(\pi_2-\pi_1)^2}{2\beta}d_T+\frac{\pi_2^2 e_T^{\pi_1}}{2\beta}M_T+\frac{(\pi_2-\pi_1)\pi_2 e_T^{\pi_1}}{\beta}(n-1-d_T) \\
&= \frac{e_T^{\pi_1}(\pi_2-\pi_1)\pi_2(n-1)}{\beta}-\frac{e_T^{\pi_1}(\pi_2^2-\pi_1^2)d_T}{2\beta}+\frac{e_T^{\pi_1}\pi_2^2}{2\beta}M_T \\
&= \frac{e_T^{\pi_1}}{2\beta}\left[2(\pi_2-\pi_1)\pi_2(n-1)-\left(\pi_2^2-\pi_1^2\right)d_T+\pi_2^2 M_T\right]
\end{aligned}$$

Since $\sum_{i\in N_T} U_i^* > 0$, it has to be that

$$2(\pi_2-\pi_1)\pi_2(n-1)-\left(\pi_2^2-\pi_1^2\right)d_T+\pi_2^2 M_T > 0 \qquad (21)$$

Finally, the expected utility of each firm j belonging to the $i-$induced subnetwork is given by:

$$U_{j\in \mathbf{z}_{T,i}} = e_T^{\pi_1}\left[e_i^*(\pi_2-\pi_2)+(1-e_i^*)(\pi_1-\pi_1)\right]=0$$

Indeed, after firm T has innovated (probability $e_T^{\pi_1}$), either firm i has innovated with probability e_i^* and firm j gets $\pi_2-\pi_2$ (profit from using the new technology minus its price) or firm i has not innovated (probability $1-e_i^*$) and firm j obtains a profit of $\pi_1-\pi_1$ (profit from using the old technology minus its price). As a result, the expected utility of any firm j belonging to the $i-$induced subnetwork is equal to zero.

We can now write the total welfare $\mathcal{W}^{\pi_1}$. It is given by:

$$\mathcal{W}^{\pi_1} = U_T^{\pi_1}+\sum_{i\in N_T} U_i^*$$

Using the expressions above, we obtain:

$$\mathcal{W}^{\pi_1} = \frac{\beta}{2}\left(e_T^{\pi_1}\right)^2+\frac{e_T^{\pi_1}}{2\beta}\left[2(\pi_2-\pi_1)\pi_2(n-1)-\left(\pi_2^2-\pi_1^2\right)d_T+\pi_2^2 M_T\right] \qquad (22)$$

From (9), we have:

$$e_T^{\pi_1} = \frac{\pi_1}{\beta}\left[n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T\right]$$

with

$$\frac{\partial e_T^{\pi_1}}{\partial d_T} = \frac{\pi_1(\pi_2-\pi_1)}{\beta^2} > 0$$

$$\frac{\partial e_T^{\pi_1}}{\partial M_T} = -\frac{\pi_1\pi_2}{\beta^2} < 0$$

Assume condition (8), i.e.

$$n > \frac{\pi_2 M_T - (\pi_2-\pi_1)(1+d_T)}{\beta-(\pi_2-\pi_1)}$$

so that $e_T^{\pi_1} > 0$. Then,

$$\begin{aligned}
\frac{\partial \mathcal{W}^{\pi_1}}{\partial d_T} &= \beta e_T^{\pi_1}\frac{\partial e_T^{\pi_1}}{\partial d_T} + \frac{\partial e_T^{\pi_1}}{\partial d_T}\frac{1}{2\beta}\left[2(\pi_2-\pi_1)\pi_2(n-1) - \left(\pi_2^2-\pi_1^2\right)d_T + \pi_2^2 M_T\right] - \frac{e_T^{\pi_1}}{2\beta}\left(\pi_2^2-\pi_1^2\right) \\
&= e_T^{\pi_1}\left[\beta\frac{\partial e_T^{\pi_1}}{\partial d_T} - \frac{(\pi_2^2-\pi_1^2)}{2\beta}\right] + \frac{\partial e_T^{\pi_1}}{\partial d_T}\frac{1}{2\beta}\left[2(\pi_2-\pi_1)\pi_2(n-1) - \left(\pi_2^2-\pi_1^2\right)d_T + \pi_2^2 M_T\right] \\
&= -e_T^{\pi_1}\frac{(\pi_2-\pi_1)^2}{2\beta} + \frac{\partial e_T^{\pi_1}}{\partial d_T}\frac{1}{2\beta}\left[2(\pi_2-\pi_1)\pi_2(n-1) - \left(\pi_2^2-\pi_1^2\right)d_T + \pi_2^2 M_T\right] \\
&= -\frac{\pi_1(\pi_2-\pi_1)^2}{2\beta^2}\left[n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T\right] \\
&\quad + \frac{\pi_1(\pi_2-\pi_1)}{2\beta^3}\left[2(\pi_2-\pi_1)\pi_2(n-1) - \left(\pi_2^2-\pi_1^2\right)d_T + \pi_2^2 M_T\right]
\end{aligned}$$

Thus

$$\frac{\partial \mathcal{W}^{\pi_1}}{\partial d_T} > 0$$

$$\Leftrightarrow 2(\pi_2-\pi_1)\pi_2(n-1) - \left(\pi_2^2-\pi_1^2\right)d_T + \pi_2^2 M_T > \beta(\pi_2-\pi_1)\left[n - \frac{(\pi_2-\pi_1)(n-1-d_T)}{\beta} - \frac{\pi_2}{\beta}M_T\right]$$

$$\Leftrightarrow 2(\pi_2-\pi_1)\pi_2(n-1) + \pi_2 M_T(2\pi_2-\pi_1) + 2\pi_1(\pi_2-\pi_1)d_T > (\pi_2-\pi_1)[n\beta - (\pi_2-\pi_1)(n-1)]$$

$$\Leftrightarrow 2(\pi_2-\pi_1)\pi_2(n-1) - (\pi_2-\pi_1)[n\beta - (\pi_2-\pi_1)(n-1)] + \pi_2 M_T(2\pi_2-\pi_1) + 2\pi_1(\pi_2-\pi_1)d_T > 0$$

$$\Leftrightarrow 2(\pi_2-\pi_1)\pi_2(n-1) - (\pi_2-\pi_1)n\beta + (\pi_2-\pi_1)^2(n-1) + \pi_2 M_T(2\pi_2-\pi_1) + 2\pi_1(\pi_2-\pi_1)d_T > 0$$

$$\Leftrightarrow n(\pi_2-\pi_1)(3\pi_2-\pi_1-\beta) + (\pi_2-\pi_1)\pi_1(2d_T+1) + \pi_2 M_T(\pi_2-\pi_1) + 2\pi_1\pi_2 + \pi_2^2(2M_T-3) > 0$$

It should be clear that the left-hand side of this inequality is positive as long as β is not too large and thus increasing d_T increases total welfare.

Let us determine the impact of M_T on total welfare $\mathcal{W}^{\pi_1}$. We have:

$$\begin{aligned}\frac{\partial \mathcal{W}^{\pi_1}}{\partial M_T} &= \beta e_T^{\pi_1}\frac{\partial e_T^{\pi_1}}{\partial M_T}+\frac{\partial e_T^{\pi_1}}{\partial M_T}\frac{1}{2\beta}\left[2\left(\pi_2-\pi_1\right)\pi_2\left(n-1\right)-\left(\pi_2^2-\pi_1^2\right)d_T+\pi_2^2M_T\right]+\frac{e_T^{\pi_1}}{2\beta}\pi_2^2\\ &= \frac{\pi_2 e_T^{\pi_1}}{\beta}\left(\frac{\pi_2-2\pi_1}{2}\right)+\frac{\partial e_T^{\pi_1}}{\partial M_T}\frac{1}{2\beta}\left[2\left(\pi_2-\pi_1\right)\pi_2\left(n-1\right)-\left(\pi_2^2-\pi_1^2\right)d_T+\pi_2^2M_T\right]\end{aligned}$$

If $\frac{\pi_2}{\pi_1}<2$, then $\frac{\partial \mathcal{W}^{\pi_1}}{\partial M_T}<0$.

If $\frac{\pi_2}{\pi_1}>2$, then

$$\begin{aligned}\frac{\partial \mathcal{W}^{\pi_1}}{\partial M_T} &= \frac{\pi_2 e_T^{\pi_1}}{\beta}\left(\frac{\pi_2-2\pi_1}{2}\right)+\frac{\partial e_T^{\pi_1}}{\partial M_T}\frac{1}{2\beta}\left[2\left(\pi_2-\pi_1\right)\pi_2\left(n-1\right)-\left(\pi_2^2-\pi_1^2\right)d_T+\pi_2^2M_T\right]\\ &= \frac{\pi_2 e_T^{\pi_1}}{\beta}\left(\frac{\pi_2-2\pi_1}{2}\right)-\frac{\pi_1\pi_2}{2\beta^3}\left[2\left(\pi_2-\pi_1\right)\pi_2\left(n-1\right)-\left(\pi_2^2-\pi_1^2\right)d_T+\pi_2^2M_T\right]\end{aligned}$$

$$\frac{\partial \mathcal{W}^{\pi_1}}{\partial M_T}\gtreqless 0$$

$$\Leftrightarrow \beta^2 e_T^{\pi_1}\left(\pi_2-2\pi_1\right)\gtreqless \pi_1\left[2\left(\pi_2-\pi_1\right)\pi_2\left(n-1\right)-\left(\pi_2^2-\pi_1^2\right)d_T+\pi_2^2M_T\right]$$

$$\Leftrightarrow \beta^2 e_T^{\pi_1}\left(\pi_2-2\pi_1\right)\gtreqless 2\left(\pi_2-\pi_1\right)\pi_1\pi_2\left(n-1\right)-\pi_1\left(\pi_2^2-\pi_1^2\right)d_T+\pi_1\pi_2^2M_T$$

$$\begin{aligned}&\Leftrightarrow \quad \beta^2\left(\pi_2-2\pi_1\right)\pi_1\left[n\beta-\left(\pi_2-\pi_1\right)\left(n-1\right)\right]+\beta^2\left(\pi_2-2\pi_1\right)\pi_1\left(\pi_2-\pi_1\right)d_T-\beta^2\left(\pi_2-2\pi_1\right)\pi_1\pi_2M_T\\ &\gtreqless \quad 2\left(\pi_2-\pi_1\right)\pi_1\pi_2\beta^2\left(n-1\right)-\pi_1\left(\pi_2^2-\pi_1^2\right)\beta^2 d_T+\pi_1\pi_2^2\beta^2M_T\end{aligned}$$

$$\begin{aligned}&\Leftrightarrow \quad n\beta\left(\pi_2-2\pi_1\right)\pi_1+\left(\pi_2-\pi_1\right)d_T\left(2\pi_1\pi_2-\pi_1^2\right)\\ &\gtreqless \quad \left(\pi_2-\pi_1\right)\left(n-1\right)\pi_1\left(3\pi_2-2\pi_1\right)+2\pi_1\pi_2M_T\left(\pi_2-\pi_1\right)\end{aligned}$$

Thus, when $\frac{\pi_2}{\pi_1}>2$, then the higher (lower) is d_T and the lower (higher) is M_T, the more likely that M_T has a positive (negative) impact on total welfare. If n is large enough, then $\frac{\partial \mathcal{W}^{\pi_1}}{\partial M_T}<0$.

Proof of Proposition 8: When firm T will charge different prices to firms $i\in N_T$ and firms $j\in \mathbf{z}_{T,i}$, we have seen in Lemma 3 that the optimal price policy for firm T is to set a price of $p_{T,i}^*$ to all firms $i\in N_T$ and, then, if firms i do not innovate, to set a price equal to $p_{T,j}^*=\pi_1$ to all the other firms in the network, i.e. all firms $j\neq i$ belonging to the i-induced subnetwork $\boldsymbol{z}_{T,i}$. This implies that all firms but firm T will have an (expected) utility of zero and, as a result, the total welfare will be reduced to the (expected) utility of firm T. Using Proposition 5, we thus have:

$$\mathcal{W} = U_T(e) = \frac{\beta}{2}(e_T^{NC})^2$$

where e_T^{NC} is defined by (17). As a result, we can use the comparative statics results of Proposition 5, which shows that

$$\frac{\partial U_T^{NC}}{\partial M_T} \gtreqqless 0 \Leftrightarrow \frac{\pi_2}{\pi_1} \lesseqqgtr 2$$

and

$$\frac{\partial U_T^{NC}}{\partial d_T} < 0$$

which implies that

$$\frac{\partial \mathcal{W}}{\partial M_T} \gtreqqless 0 \Leftrightarrow \frac{\pi_2}{\pi_1} \lesseqqgtr 2$$

and

$$\frac{\partial \mathcal{W}}{\partial d_T} < 0$$

As a result,

(i) If $\frac{\pi_2}{\pi_1} < 2$, then $\frac{\partial \mathcal{W}}{\partial M_T} > 0$ and thus the planner should target firms with the highest M_T, i.e. peripheral firms. On the contrary, if $\frac{\pi_2}{\pi_1} > 2$, then $\frac{\partial \mathcal{W}}{\partial M_T} < 0$ and thus the planner should target firms with the lowest M_T, i.e. central firms.

(ii) The effect of d_T on welfare is always negative, i.e. $\frac{\partial \mathcal{W}}{\partial d_T} < 0$, and thus the planner should target the firm with the lowest degree d_T.

Proof of Observation 2: The cases of $0 < \beta < 10.167$ and $\beta > 23.1$ are obvious since they are just an application of Proposition 9 and Observation 1.

Let us thus focus on case (ii) when $10.167 < \beta < 23.1$. We have seen in Observation 1 that, in that case, if the target firm is d, i.e., $T = d$, then she sets a high price of $p_{d,i}^* = \overline{U}_i$. On the contrary, if the target firm is c, i.e., $T = c$, then she sets a low price of $p_{c,i}^* = \pi_1$.

If the target firm is d ($T = d$), using (19), the total welfare can be written as:

$$\mathcal{W}^{\overline{U}_i} = U_d^{\overline{U}_i} = \frac{1}{2\beta}\left(2 + \frac{121}{2\beta}\right)^2 = \frac{(4\beta + 121)^2}{4\beta^2} = \frac{16\beta^2 + 968\beta + 14\,641}{4\beta^2}$$

If the target firm is c ($T = c$), then using (22), the total welfare can be written as:

$$\begin{aligned}
\mathcal{W}^{\pi_1} &= U_c^{\pi_1} + \sum_{i \in N_T} U_i(e_i^*) \\
&= \frac{\beta}{2}\left(e_c^{\pi_1}\right)^2 + \frac{e_c^{\pi_1}}{2\beta}\left[2\left(\pi_2 - \pi_1\right)\pi_2\left(n-1\right) - \left(\pi_2^2 - \pi_1^2\right)d_c + \pi_2^2 M_c\right] \\
&= \frac{\beta}{2}\left(e_c^{\pi_1}\right)^2 + \frac{35 e_c^{\pi_1}}{2\beta}
\end{aligned}$$

where $e_c^{\pi_1}$ is defined by (9), that is:

$$\begin{aligned}
e_c^{\pi_1} &= \frac{\pi_1}{\beta}\left[n - \frac{\left(\pi_2 - \pi_1\right)\left(n - 1 - d_c\right)}{\beta} - \frac{\pi_2}{\beta} M_c\right] \\
&= \frac{(7\beta - 13)}{\beta^2}
\end{aligned}$$

Thus,

$$\mathcal{W}^{\pi_1} = \frac{\beta}{2}\left(e_c^{\pi_1}\right)^2 + \frac{35 e_c^{\pi_1}}{2\beta} = \frac{\left(7\beta - 13\right)^2 + 35\left(7\beta - 13\right)}{2\beta^3} = \frac{49\beta^2 + 63\beta - 286}{2\beta^3}$$

We now need to check that

$$\begin{aligned}
\mathcal{W}^{\overline{U}_i} &\gtreqless \mathcal{W}^{\pi_1} \\
&\Leftrightarrow \frac{16\beta^2 + 968\beta + 14641}{4\beta^2} \gtreqless \frac{49\beta^2 + 63\beta - 286}{2\beta^3} \\
&\Leftrightarrow 16\beta^3 + 968\beta^2 + 14641\beta \gtreqless 98\beta^2 + 126\beta - 572 \\
&\Leftrightarrow 16\beta^3 + 870\beta^2 + 14515\beta + 572 \gtreqless 0
\end{aligned}$$

Since the expression on the left-hand side of the inequality is always positive, $\mathcal{W}^{\overline{U}_i} > \mathcal{W}^{\pi_1}$, and thus the planner targets firm d.

Appendix 2: Unknown network

We now consider the case where the network is not observed by the firms. This is relevant, for example, in the case in which each firm's contact are considered just as private information.[15] or when the network is very big. To be more precise, we assume that firms know the number of firms n in the network, that each firm i observes her own degree d_i (i.e. number of links) in the directed network, and all firms share a common belief about others' degree, which is given by d^e.[16] For our analysis, what is relevant is $E_i[c_i]$, the expected value of the diffusion centrality of firm i for firm i and $E_T[c_i]$, the expected value of the diffusion centrality of firm i for firm T, since these two terms determine the incentives faced by firms i and T. We have:

$$E_i[c_i] = \frac{(n-1-d^e)}{d^e} \tag{23}$$

Indeed, each firm i thinks that she has a market (diffusion centrality), which is composed of all the firms that do not belong to the (expected) neighborhood of T (i.e. $n-1-d^e$ firms), divided by the number of (expected) branches departing from T (i.e. d^e). We assume n to be large enough so that $E_i[c_i] > d_i$, for all i. Moreover,

$$E_T[c_i] = \frac{(n-1-d_T)}{d_T} \tag{24}$$

$E_T[c_i]$ has a similar interpretation as $E_i[c_i]$. Observe, however, that $E_T[c_i]$ differs from $E_i[c_i]$ since firm T knows her exact number of neighbors d_T while firm i does not and can therefore compute the exact number of expected firms composing each neighbor's market. If we first consider first i's choices, we have

$$\begin{aligned} E_i[U_i] &= e_i\pi_2\left(1+E_i[c_i]\right) + (1-e_i)\pi_1 - \frac{\beta}{2}e_i^2 - p_{T,i} \\ &= e_i\left[\frac{\pi_2\,(n-1)}{d^e}\right] + (1-e_i)\pi_1 - \frac{\beta}{2}e_i^2 - p_{T,i} \end{aligned} \tag{25}$$

Each firm i chooses e_i that maximizes $E_i[U_i]$. We obtain:

$$e_i^* = \frac{1}{\beta}\left[\frac{\pi_2\,(n-1)}{d^e} - \pi_1\right] \tag{26}$$

[15]We keep assuming, however, that the network is cycle-free.

[16]Galeotti and Goyal (2009), Galeotti et al. (2010), Fainmesser and Galeotti (2016) and Belhaj and Derioan (2016) make a similar assumption.

Notice that $E_T[e_i] = e_i^*$, since firm T, knowing $d^e = d_T$, perfectly predicts i's optimal effort. It is then straightforward to notice that e_i^* is decreasing in d^e. This is because a higher d^e induce lower $E_i[c_i]$ and thus reduces its incentives to produce effort. The equilibrium utility is equal to:

$$E_i[U_i^*] = \pi_1 + \frac{1}{2\beta}\left[\frac{\pi_2 (n-1)}{d^e} - \pi_1\right]^2 - p_{T,i} \tag{27}$$

Consider now the choice of the target firm T. As in the previous section, consider first committed prices (fixed prices). In that case, firm T can choose either a low price equal to $p_{T,i}^* = \pi_1$ or a high price equal to $E_i[U_i^*]$, i.e.

$$p_{T,i}^* = \overline{U}_i = \pi_1 + \frac{1}{2\beta}\left[\frac{\pi_2 (n-1)}{d^e} - \pi_1\right]^2 \tag{28}$$

We have the following two cases. If $p_{T,i}^* = \pi_1$, using (7), (24) and (26), we have:

$$\begin{aligned} E_T\left[U_T^{\pi_1}\right] &= e_T\left\{\pi_1 + \sum_{i\in N_T}\left[e_i^*\pi_1 + (1-e_i^*)\pi_1\left(1+E_T[c_i]\right)\right]\right\} - \frac{\beta}{2}e_T^2 \\ &= e_T\left\{\pi_1 - \pi_1\left(\frac{n-1-d_T}{d_T}\right)\sum_{i\in N_T} e_i^* + \pi_1(n-1)\right\} - \frac{\beta}{2}e_T^2 \\ &= e_T\left\{\pi_1 n - \frac{\pi_1(n-1-d_T)}{\beta}\left[\frac{\pi_2(n-1)}{d^e} - \pi_1\right]\right\} - \frac{\beta}{2}e_T^2 \end{aligned}$$

If $p_{T,i}^* = \overline{U}_i$, using (11) and (27), we have:

$$\begin{aligned} E_T\left[U_T^{\overline{U}_i}\right] &= e_T\left(\pi_1 + \sum_{i\in N_T} E_i[U_i^*]\right) - \frac{\beta}{2}e_T^2 \\ &= e_T\left(\pi_1 + \pi_1 d_T + \frac{d_T}{2\beta}\left[\frac{\pi_2(n-1)}{d^e} - \pi_1\right]^2\right) - \frac{\beta}{2}e_T^2 \end{aligned}$$

We have the following result.

Proposition 10. *If firms do not know the network but share a common belief about other firms' degree, then*

$$e_T^{\pi_1} = \frac{\pi_1}{\beta}\left\{n - \frac{(n-1-d_T)}{\beta}\left[\frac{\pi_2(n-1)}{d^e} - \pi_1\right]\right\}$$

$$e_T^{\overline{U}_i} = \frac{1}{\beta}\left\{\pi_1(1+d_T) + \frac{d_T}{2\beta}\left[\frac{\pi_2\,(n-1)}{d^e} - \pi_1\right]^2\right\}$$

Furthermore,

$$\frac{\partial e_T^{\pi_1}}{\partial d_T} > 0, \quad \frac{\partial e_T^{\pi_1}}{\partial d^e} > 0, \quad \frac{\partial e_T^{\overline{U}_i}}{\partial d_T} > 0, \quad \frac{\partial e_T^{\overline{U}_i}}{\partial d^e} < 0$$

Consider first the case in which firm T chooses $p^*_{T,i} = \pi_1$. The optimal effort is increasing both in d_T and d^e. In particular, the higher d_T, the higher is the number of direct neighbors that pay the price π_1. Then, the higher d^e, the lower is $E_i[c_i]$. This means that each firm i decreases her own effort inducing firm T to increase her own effort since the probability of having her own innovation spread increases. Consider now the case in which $p^*_{T,i} = \overline{U}_i$. If d_T increases, firm T's effort increases since T has a higher number of neighbors thinking they have a higher level of rents than they will actually experience, and then T can extract this higher expected rent level. On the other side if d^e increases, $E_i[c_i]$ decreases and effort is reduced. Then firms with a larger degree d_T will always exert a higher level of innovation effort, while the effect of the expected degree of others changes with the pricing policy.

We now focus on the choice of the pricing policy by the T firm.

Proposition 11. *Assume that firm T can charge a price of either $p^*_{T,i} = \pi_1$ for all $i \in N_T$ or $p^*_{T,i} = \overline{U}_i$ for all $i \in N_T$. Then, there exist a $\overline{\beta}$, a $\overline{d}^e$ and a $\overline{d}_T$ such that $p^*_{T,i} = \overline{U}_i$ (resp. $p^*_{T,i} = \pi_1$) for all $i \in N_T$ if and only if $\beta < \overline{\beta}$, $d^e > \overline{d}^e$, and/or $d_T > \overline{d}_T$ (resp. $\beta > \overline{\beta}$, $d^e < \overline{d}^e$, and/or $d_T < \overline{d}_T$).*

Indeed, if the expected degree is very high, then firm T chooses to implement a high price policy $p^*_{T,i} = \overline{U}_i$. This is because high d^e would increase e^*_i, thus increasing the probability of innovation for i, which would also increase $E_i[c_i]$, thus increasing the expected rents of i and the price i is willing to pay to T. Similarly, firms T with a high number of neighbors also charge a high price. In this case, the expected utility for both pricing policies increase, but it increase more by imposing $p^*_{T,i} = \overline{U}_i$. As noticed earlier, this kind of policy may harm the diffusion process since, if any i does not innovate, then the innovation cannot flow to the rest of the network given the high price set by firm T.

Consider now the effect of property rights protection β. As for the case of known network, the incentives are such that property rights protection has almost opposite effects in the two cases. If $p^*_{T,i} = \overline{U}_i$, then firm T can extract all the expected rents form i, so that a high β lowers these rents and decreases firm T's effort. Noticing that a high β harms also second step innovation, if firms use this high pricing mechanism, then a very low level of property rights protection may seem appropriate. If $p^*_{T,i} = \pi_1$ then the incentives are different and the optimal effort is concave and non monotone in β so that there exists a level

$$\overline{\beta} = \frac{1}{n} 2 \left(n - 1 - d_T\right) \left[\frac{\pi_2 \left(n - 1\right)}{d^e} - \pi_1\right]$$

such that T's effort is increasing in β if and only if $\beta \leq \overline{\beta}$. In this way, innovation is maximal for middle level property rights protection.

Proposition 12. *Any targeted firm would choose full price discrimination (flexible prices). In that case, the planner, whose aim is to maximize total welfare, will target the firm with the highest degree* d_T.

This last result states two important results. First, as in the perfect information case, the targeted firm with always choose not to commit to a single price. Second, and more importantly, when firms do not know the network, a policy maker, who does not know the network either, will always target the firm with the highest degree. This is a similar result to the one obtain in the perfect information case.