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WORKING PAPER SERIES

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Working Paper n. 623

This Version: June, 2018

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http://www.igier.unibocconi.it

THREE LAYERS OF UNCERTAINTY: AN EXPERIMENT*

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Abstract

We experimentally explore decision-making under uncertainty using a framework that decomposes uncertainty into three distinct layers: (1) physical uncertainty, entailing inherent randomness within a given probability model, (2) model uncertainty, entailing subjective uncertainty about the probability model to be used and (3) model misspecification, entailing uncertainty about the presence of the true probability model among the set of models considered. Using a new experimental design, we measure individual attitudes towards these different layers of uncertainty and study the distinct role of each of them in characterizing ambiguity attitudes. In addition to providing new insights into the underlying processes behind ambiguity aversion –failure to reduce compound probabilities or distinct attitudes towards unknown probabilities– our study provides the first empirical evidence for the intermediate role of model misspecification between model uncertainty and Ellsberg in decision-making under uncertainty.

Keywords: Ambiguity aversion, reduction of compound lotteries, non-expected utility, model uncertainty, model misspecification

JEL Classification: D81

*Berger acknowledges the support of the French Agence Nationale de la Recherche (ANR), under grant ANR-17-CE03-0008-01 (project INDUCED). Bosetti acknowledges the financial support provided by the ERC-2013-StG 336703-RISICO. Logistic support from the Bocconi Experimental Laboratory in the Social Sciences (BELSS) for hosting our experimental sessions is kindly acknowledged.

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1 Introduction

Uncertainty is pervasive and plays a major role in economics. Whether economic agents in the market pursuing individual goals or policy makers pursuing social objectives, decision makers rarely have complete information about objective probability distributions over relevant states of the world. Descriptively valid understanding of individual behavior in the face of uncertainty is of great importance for constructing realistic economic models capable of making accurate predictions, as well as improving decision making processes in prescriptive applications.

Following the early insights from Arrow (1951), and the recent discussions in Hansen (2014), Marinacci (2015) and Hansen and Marinacci (2016), the framework guiding our investigations decomposes uncertainty into three distinct layers of analysis. Specifically, we consider a decision maker (DM), who possesses ex-ante information about a set of possible *probability models* characterizing inherent randomness within a phenomenon of interest, but is uncertain about the true probability model among the set of possible models. Thus, a distinction is made between (i) *risk*, where the uncertainty –of aleatory or physical type– is about the possible outcomes *within* a given probability model, and (ii) *model uncertainty* where there exists a second layer of uncertainty –of epistemic nature– concerning which alternative model should be used to assign probabilities. More specifically, *risk* represents situations where the consequences of the actions taken by a DM depend on the states of the world over which there is an *objectively* known probability distribution. *Model uncertainty* characterizes situations with limited information where the DM cannot identify a single probability distribution corresponding to the phenomenon of interest. As such, this second layer of uncertainty may be quantified by means of *subjective* probabilities across the models under consideration. In addition, the DM may face situations where the set of probability models under consideration may not even include the true model. This third layer of uncertainty is known as (iii) *model misspecification.* It represents the approximate nature of probability models, which are often simplified representations of more complex phenomena.

These three distinct layers of uncertainty are inherent to any decision problem under uncertainty where the DM adopts probabilistic theories about the outcomes of a phenomenon and forms beliefs over their relevance. In practice, this decomposition of uncertainty into layers provides a useful framework to analyze the vast majority of decision problems under ambiguity.[1] As an example, in Ellsberg's

[1]Following the seminal work of Ellsberg (1961), *ambiguity* is the term that has emerged in the literature to characterize the situations in which "the DM does not have sufficient information to quantify through a single probability distribution the stochastic nature of the problem she is facing" (Cerreia-

(1961) classical experiment, which has been the standard tool to study ambiguity in economics, the two-color ambiguous urn with a total number of N balls displays both the layers of risk and model uncertainty. Specifically, an ambiguous urn with N balls provides $N + 1$ possible physical compositions of the urn, each of which constitutes a *risk*, whereas the distribution over the compositions, unknown to the DM, constitutes the epistemic uncertainty in the second layer (note that, by construction, there is no third layer in this case). In real-life problems, such as the choice of an optimal environmental policy in the face of climate change, the first layer may represent the probability distribution of the long term temperature response to greenhouse gas emissions. As multiple instances of this distribution exist –depending on the climate models employed or the type of data used to estimate the probabilistic relationship– a second layer of uncertainty emerges as the uncertainty surrounding these different models. Lastly, the potential misspecification of the existing climate models gives rise to the third layer. Thus, the climate policy maker has to deal with uncertainty consisting of the three distinct layers.

Until now, the research in economics has generally focused on the layers of risk and model uncertainty when considering decisions under ambiguity. Our study is the first which goes beyond these two layers and examines the role played by model misspecification. When modeling uncertain situations, DMs use their best available information to specify uncertainties, correcting and removing any model misspecification that they are aware of. From this perspective, it may seem impossible to implement the third layer of model misspecification in an experiment, at least without using deception. In our experiment, we overcome the difficulties by using a modified Ellsberg setting encompassing all the three layers. We are thus able to pin down the relative importance of each distinct layer to the total effect of uncertainty.

Traditionally, the way economists have dealt with uncertainty is by following the Subjective Expected Utility approach (Savage 1954, henceforth, SEU). In line with the Bayesian tradition, this approach holds that any source of uncertainty can be quantified in probabilistic terms and in this sense can be treated similarly as *risk*, reducing uncertainty *de facto* to its first layer. In this approach, there is no role for ambiguity attitudes. Whether for purely descriptive purposes, or with a clear normative appeal, several lines of research have then been developed to accommodate the results of the research initiated by Ellsberg (1961). In particular, two types of non-SEU theories that relate to the multi-layer representation of uncertainty entail relaxing some of the critical assumptions of SEU: reduction of compound probabilities and source independence (i.e. no distinction between

Vioglio et al., 2013a).

physical and epistemic uncertainty).[2]

The first approach models ambiguity attitudes by relaxing the reduction principle between uncertainty presented in different stages, while still holding the source independence assumption. Segal's (1987) Anticipated Utility approach (See Quiggin, 1982) and Seo's (2009) model posit considering any source of ambiguity as compound risk.[3] While these theories adopt a representation of ambiguity with multiple *stages* of risk, they therefore do not distinguish between *layers* entailing distinct types of uncertainty present at different stages. Accordingly, no distinction exists between compound risk (where there are two *stages* of physical uncertainty) and model uncertainty (where there are distinct *layers* of physical and epistemic uncertainty). As non-neutral ambiguity attitudes result from the violation of an elementary rationality condition, these theories assign to ambiguity attitudes a purely descriptive status.

The second approach models ambiguity attitudes by source dependence, assuming different attitudes towards different layers of uncertainty (e.g. a preference for physical uncertainty over epistemic uncertainty). The smooth ambiguity model of Klibanoff et al. (2005) (see also Nau, 2006 and Ergin and Gul, 2009) distinguishes the layers of risk and model uncertainty by assuming different utility functions for attitudes towards aleatory and epistemic uncertainty. Hence, the reduction principle still holds for compound risk (physical uncertainty present in different stages) but not for model uncertainty. Other studies by Chew and Sagi (2008), and Abdellaoui et al. (2011) also explicitly model ambiguity attitudes by source dependence without any implications about compound risk (as they do not adopt a multi-stage approach).

The main objective of this paper is to elicit individuals' attitudes towards different sources of uncertainty consisting of multiple layers and to understand to what extent these attitudes are associated with attitudes towards ambiguity. Our investigation makes two contributions to the ambiguity literature in economics. First, we shed new light on the explanatory power of different theoretical approaches proposed in the literature to accommodate non-neutral ambiguity attitudes. Specifically, exploring model uncertainty along with the corresponding instances of compound risk, our laboratory experiment reveals the interaction of

[2]Note that we here mainly focus on theories consistent with the Bayesian tradition which uses a single probability measure to quantify probabilistic judgments within each layer of uncertainty. Multiple prior models such as the one proposed by Gilboa and Schmeidler (1989) are discussed later in the paper.

[3]Segal (1987) writes "Indeed, most writers in this area, including Ellsberg himself, suggested a distinction between ambiguity (or uncertainty) and risk. One of the aims of this paper is to show that (at least within the anticipated utility framework) there is no real distinction between these two concepts." (p 178-179). Thus, he proposes "risk aversion and ambiguity aversion are two sides of the same coin, and the rejection of the Ellsberg urn does not require a new concept of ambiguity aversion, or a new concept of risk aversion." (p 179).

source dependence and reduction of compound probabilities, and their relative importance in explaining ambiguity attitudes. Second, we provide the first experimental evidence on the role of model misspecification in decision making under uncertainty. While the importance of model misspecification has been conjectured by several studies (Hansen and Marinacci, 2016; Berger and Marinacci, 2017) no theory has formally incorporated it yet.[4] By gauging how much can be gained by incorporating model misspecification, our study informs future research in this direction.

There are four main findings emerging from our analysis. First, attitudes towards ambiguity and uncertainty explicitly presented in different stages are closely related. Second, the association with ambiguity attitudes is however stronger for model uncertainty than for compound risk. Thus, we find strong evidence for the role of source dependence. Third, we find that model misspecification is an intermediate case between model uncertainty and ambiguity, although it is not the main driver of attitudes towards ambiguity. Ambiguity attitudes are mostly captured by attitudes towards model uncertainty. Lastly, our results indicate that the degree of complexity of the decision problem plays an important role. In particular, when the level of complexity of the task is reduced, or when only more sophisticated subjects are considered, the association between attitudes towards ambiguity and compound risk tends to disappear, suggesting separate normative and descriptive considerations for each of them. Overall, these findings also contribute to the debate on whether ambiguity preferences found in experiments should be considered as a deviation from rationality, or instead, could be seen as a rational way to cope with uncertainty.

2 Experimental design

This section presents our experimental design. We use a within-subject design to examine choices under different sources[5] of uncertainty generated in an Ellsberg setting. The experiment is run with student subjects, with real monetary incentives.

[4]Modeling it is challenging as it requires a trade-off between "tractability and conceptual appeal" (Hansen and Marinacci, 2016, pg. 511).

[5]We here adopt the definition of *sources of uncertainty,* proposed by Abdellaoui et al. (2011), as "groups of events that are generated by the same mechanism of uncertainty, which implies that they have similar characteristics" (p. 696).

2.1 The sources of uncertainty

We consider the following six sources of uncertainty in an Ellsberg two-color setting with decks containing black and red cards.

1. *Simple Risk*, denoted by SR, entails a deck containing an equal proportion of black and red cards;

2. *Compound Risk*, denoted by CR, entails a deck that contains either p% red ($(100-p)$% black) or p% black ($(100-p)$% red) cards with equal probability;

3. *Model Uncertainty*, denoted by MU, entails a deck that contains either p% red ($(100-p)$% black) or p% black ($(100-p)$% red) cards with unknown probability;

4. *Model Misspecification*, denoted by MM, entails a deck that is likely to contain either p% red ($(100-p)$% black) or p% black ($(100-p)$% red) cards, and may or may not contain any other proportion of red and black cards.

5. *Extended Ellsberg*, denoted by EE, entails a deck that contains an unknown proportion of black and red cards;

6. *Standard Ellsberg*, denoted by SE, entails a deck of 100 cards that contains an unknown proportion of black and red cards.

In sources 2-4, we consider two situations: one with $p = 0$ and the other with $p = 25$. For example, CR with $p = 0$ entails a deck that contains either 0% red (100% black) or 0% black (100% red) with equal probability, whereas CR with $p = 25$ entails a deck that contains either 25% red (75% black) or 25% black (75% red) with equal probability. The cases of MU and MM are constructed similarly for the two proportions. We denote these respective cases as $CR0$, $CR25$, $MU0$, $MU25$, $MM0$, and $MM25$.

The sources CR, MU and MM differ in the layers of uncertainty they encompass (and hence in the type of second order probabilities considered). Specifically, CR entails only the layer of risk (even if it is presented in a compound way, using different stages). Under CR, the two possible deck compositions, p% and $(100-p)$% red (or black) cards, are unambiguously assigned *objective* probabilities 50%. Conversely, the source MU entails both a layer of model uncertainty and one of risk. Under MU, the two possible deck compositions can only be assigned *subjective* probabilities. On the basis of symmetry, defined in Section 4.1, these subjective probabilities will be assumed to be 50%. Finally, the source MM entails the three layers of uncertainty together.

Our treatment MM can be interpreted as a form of MU, where a larger number of models is considered. For many subjects, the treatment MM might be psychologically similar to model misspecification, and our framing serves to induce this perception. Indeed, in many applications, the term model misspecification has been used in our sense (Hansen and Sargent, 2001b,a; Hansen et al., 2006; Hansen and Marinacci, 2016), for what formally can be taken as an extra layer of model uncertainty. Hence, the treatment MM serves as a good proxy for the third layer, from which useful insights can be obtained.

While EE corresponds, in spirit, to Ellsberg's (1961) ambiguous situation where "numerical probabilities are inapplicable", it has to be noted that it slightly differs from the situation SE originally used by Ellsberg, where the total number of cards in the deck is known. Here, we consider SE for the sake of comprehensiveness and for allowing comparisons with previous literature. Yet, remark that formally speaking, SE can be interpreted as an instance of MU with two layers of uncertainty only, since 101 physical compositions of the deck are possible. Therefore, EE and SE differ in that the former (where the number of cards composing the deck is unknown) may be seen as entailing the three layers of uncertainty together (as a set of probability models can be postulated, but this set may not contain the true model).

2.2 Procedure

The experiment was run on computers. Subjects were seated in cubicles, and could not communicate with each other during the experiment. Each session started with the experimental instructions, examples of the stimuli, and comprehension questions. Complete instructions and comprehension questions are presented in the Online Appendix.

Subjects Five experimental sessions were conducted at Bocconi Experimental Laboratory for Social Sciences (BELSS) in Bocconi University, Italy. The subjects were 125 Bocconi University students having various academic degrees, mostly from economics, management and marketing departments (average age 20.5 years, 52 female). Each session lasted approximately one hour including instructions and payment.

Stimuli During the experiment, subjects faced nine monetary prospects under the different uncertain situations introduced earlier: SR, $CR0$, $CR25$, $MU0$, $MU25$, $MM0$, $MM25$, EE, and SE. Each prospect gave the subjects either €20 or €0 depending on the color of a card randomly drawn from a deck. In every prospect, the color giving €20 was picked by the subjects themselves. The prospects under SR, $CR0$, $CR25$, $MU0$, $MU25$, $MM0$, $MM25$, and EE were constructed with decks containing an unspecified number of cards. In SR, the subjects were instructed that the deck contained an equal proportion of red and black cards. In the cases of CR, MU, MM, and EE the subjects were instructed that the deck was going to be picked randomly from a pile of decks. In $CR0$ ($CR25$), the pile was composed of decks containing 0% black (25% black) cards and decks containing 0% red (25% red) cards, with an equal proportion of each. In $MU0$ ($MU25$), the pile was also composed of decks containing 0% black (25% black) and decks containing 0% red (25% red) cards, but with an unknown proportion of each. In $MM0$ ($MM25$), the majority (at least half) of the pile consisted of decks containing 0% black (25% black) and decks containing 0% red (25% red) cards with an unknown proportion of each. Notably, the subjects were instructed that the pile may or may not contain decks with compositions other than the two described. In EE, the pile was composed of decks containing red and black cards each with an unknown composition. Lastly, SE involved a single deck containing 100 cards with an unknown proportion of black and red cards.

All the decks and piles were constructed in advance by one of the authors, who was not present in the room during the experimental sessions. Thus, no one in the room, including the experimenters, had any additional information about the content of the decks and piles, other than what was described in the experimental instructions. The subjects were informed accordingly to prevent the effects of comparative ignorance (Fox and Tversky, 1995). The subjects were also reminded that they could check the piles and the decks at the end of the experiment to verify the truthfulness of the descriptions of prospects.

We elicited the certainty equivalents (CE) of the nine prospects using a choice-list design. Specifically, in each prospect, the subjects were asked to make twelve binary choices between the prospect of receiving €20 and receiving a sure monetary amount ranging between €0 and €20. The sure amounts were incremented by €2 between €1 and €19. In what follows, we take the midpoint of an indifference interval implied by a switching point as a proxy for the CE of the prospect. Switching in the middle of the list implies a CE equal to the expected payoff.

The order of SR, $CR0$, $CR25$, $MU0$, $MU25$, $MM0$, $MM25$, and EE were randomized, whereas SE was always presented at the end to prevent a priming effect about the number of cards in the decks. After completing the nine choice lists, the subjects answered six multiple-choice questions that intended to measure their numeracy skills. These questions entailed calculation of probabilities in a chance game involving random draws from two decks with specific proportions of red and black cards. Lastly, the subjects were presented with several items intended to elicit self-reported risk attitudes in real life situations. The questionnaire ended with demographics questions.

Incentives The subjects received a €5 show-up fee. In addition, they received a variable amount depending on one of the choices that they made during the experiment. The choice situation on which the payment was based was the same for every subject in a given session. In practice, twelve binary choice questions on the choice lists (each containing a decision problem between the prospect of receiving €20 based on the color of the card to be drawn and different monetary amounts) and the descriptions of nine uncertain situations (under which the card was going to be drawn) were printed on paper and physically enclosed in sealed envelopes before every experimental session. In each experimental session, a volunteer from the subjects randomly picked two envelopes before the experiment started: one from the nine envelopes each containing an uncertain situation and another from the twelve envelopes each containing a question from the choice lists. The two envelopes picked, still sealed, were then attached to a white board visible to all participants. The subjects were informed that the choice situation that would matter for their payment was contained in the envelopes, which would remain visible and closed until the end of the experiment. When all the subjects completed the questionnaire, the envelopes were opened, and the contents were revealed to the subjects. The draws from the piles and/or from the decks were made as described under the uncertain situation contained in the first envelope, and the subjects were paid according to their recorded decision in the choice question contained in the second envelope.[6]

3 Related experimental literature

Previous experimental studies have investigated the link between different sources of uncertainty and ambiguity. Most of these studies questioned the possibility to completely characterize ambiguity by means of compound risks. Using urns presenting simple risk, compound risk and ambiguity, Yates and Zukowski (1976) and Chow and Sarin (2002) found that simple risk is most preferred, ambiguity is least preferred, and compound risk is intermediate between the two. In the same vein, Bernasconi and Loomes (1992) found less aversion towards compound

[6] Note that this prior incentive system (Johnson et al., 2015) slightly differs from the standard random incentive system in that it performs the randomization before, rather than after, the choices and the resolutions of uncertainty. Its theoretical incentive compatibility in Ellsberg experiments was proved in Baillon et al. (2014).

risk than what is typically found under ambiguity, questioning therefore the possibility to characterize completely ambiguity by means of compound risk. More recently, Halevy (2007) reported the results of an experiment confirming that, on average, subjects prefer compound risk situations to ambiguous ones. However, his experiment also suggested a tight association between ambiguity neutrality and reduction of compound risk (ROCR). Qualitatively similar results concerning the association between attitudes towards ambiguity and compound risk were reported by Dean and Ortoleva (2015). Armantier and Treich (2016) also suggested a tight association between attitudes towards ambiguity and *complex* risks, where probabilities are objective but non-trivial to compute.[7] On the contrary, using a setup close to Halevy's, Abdellaoui et al. (2015) found significantly less association between compound risk reduction and ambiguity neutrality. In particular, they showed that, for more sophisticated subjects, compound risk reduction is compatible with ambiguity non-neutrality, suggesting that failure to reduce compound risk and ambiguity non-neutrality do not necessarily share the same behavioral grounds. Relatedly, in an experiment with children, Prokosheva (2016) obtained a significant relationship between arithmetic test scores and compound risk reduction, while no such relationship was found between ambiguity neutrality and these scores. Finally, using designs closer to ours, Chew et al. (2017) and Berger and Bosetti (2017) extended the investigations to the role of model uncertainty. Whereas Chew et al. (2017) observed very similar attitudes towards compound risk and model uncertainty,[8] Berger and Bosetti (2017) only reported a significant association between attitudes towards ambiguity and model uncertainty, but not between attitudes towards ambiguity and compound risk.

4 Theoretical predictions

Following the exposition of our experimental design, we now describe the preferences predicted by different theories of choice under uncertainty for the prospects considered in the experiment. Given that uncertainty is explicitly represented through different stages in the prospects we present, we focus on theoretical models that accommodate such representation of uncertainty. Since –to our knowledge– no theoretical setup has so far explicitly accommodated all three layers of risk, model uncertainty and model misspecification together, we concentrate on

[7] Kovářík et al. (2016) also experimentally studied attitudes towards both complexity and ambiguity, but without considering the association between them.

[8] Note that Chew et al. (2017) did not refer to "model uncertainty" to characterize two-layer uncertainty, but rather talked about "partial ambiguity". Their partial ambiguous prospects are then used to investigate the association with compound risk.

theories with a clear two-layer perspective. These include subjective expected utility (SEU), maxmin preferences, families of smooth preferences and families of recursive non-expected utility preferences.

4.1 The setting

We denote the set of states of the world as S and the set of consequences as C. Formally, a prospect is a function $\mathcal{P} : S \to C$, mapping states into consequences. That is, $\mathcal{P}(s)$ is the consequence of prospect $\mathcal{P}$ when $s \in S$ obtains. In our setting, each of the nine prospects involves a bet on the color of a card drawn being either red or black. While the state space consists of 2^9 states, we restrict our attention to 9 payoff-relevant events each describing whether the bet is correct or not in a given situation. Hence, a prospect $\mathcal{P}_i$ results in a consequence $c \in \{€0, €20\}$ depending on which state of the world $s_i \in \{\text{red}, \text{black}\}$ realizes in situation $i \in \mathbb{P} = \{SR, CR0, CR25, MU0, MU25, MM0, MM25, EE, SE\}$. States are thus seen as realizations of underlying random variables that are part of a data generating mechanism. We assume that the DM has a complete and transitive preference relation $\succsim$ over prospects.

Abstracting from the issue of model misspecification, we assume that the DM knows that states are generated by a *probability model* m which belongs to a collection M.[9] Each model m therefore describes a possible data generating mechanism (i.e. a possible composition of the deck) and as such represents the inherent randomness that states feature. In our experiment, M is either singleton, as in the case of the risky prospect SR, or contains two elements (except for SE, in which $|M| = 101$). To ease the derivation and presentation of our theoretical predictions, we now impose a symmetry assumption and define then notion of relative premium.

Symmetry condition: For each uncertain prospect, the DM is indifferent to the color on which to bet (red or black).[10]

Definition: The (relative) *premium* Π_i is defined as the difference

$$\Pi_i \equiv CE_{SR} - CE_i \quad \forall i \in \mathbb{P} \setminus \{SR\}.$$

[9]Formally, we assume the existence of a measurable space (S, Σ), where Σ is an algebra of events of S. A model $m : \Sigma \to [0, 1]$ is thus a probability measure, and the collection M is a finite subset of $\Delta(S)$, the collection of all probability measures.

[10]The symmetry condition has been supported empirically in preceding studies by Abdellaoui et al. (2011), Chew et al. (2017) and Epstein and Halevy (2018). In our experiment, given that our subjects pick their own color to bet on in the ambiguous prospects, asymmetric beliefs implies only an underestimation of ambiguity aversion.

In words, this premium represents the difference between the certainty equivalent for the simple risk and the certainty equivalent for the uncertain prospect. The premium is positive (resp. zero, or negative) when a subject is more (resp. as much, or less) averse to the uncertain prospect than to the simple risk. This premium represents in turn the *compound risk premium* (Abdellaoui et al., 2015), or the *ambiguity premium* (Berger, 2011; Maccheroni et al., 2013), depending on the prospect considered.

4.2 Subjective expected utility

The benchmark model we consider is the subjective expected utility (SEU) model originally due to Savage (1954). In its two-layer version axiomatized by Cerreia-Vioglio et al. (2013b), it is assumed that the DM has a subjective prior probability $\mu : 2^M \to [0,1]$ quantifying the epistemic uncertainty about models. This subjective prior reflects the structural information received and some personal information the DM may have on models. The subjective expected utility of a bet on prospect $\mathcal{P}_i$ is

$$V_{\text{SEU}}(\mathcal{P}_i) = \sum_m \mu\left(m\right) \left(\sum_s p(s|m) u\left(\mathcal{P}_i(s)\right) \right). \tag{1}$$

In this expression, $u : C \to \mathbb{R}$ is the von Neumann-Morgenstern utility function capturing risk attitude, and $p(s|m)$ is the objective probability of state s conditional on model m. Criterion (1) is a Bayesian two-stage criterion that describes both layers of uncertainty via standard probability measures. The same attitude is considered towards both risk and model uncertainty. In its reduced form due Savage (1954), it might be rewritten

$$V_{\text{SEU}}(\mathcal{P}_i) = \sum_s \bar{\mu}(s) u\left(\mathcal{P}_i(s)\right), \tag{2}$$

where $\bar{\mu}(s) = \sum_m \mu(m)p(s|m)$ is the predictive subjective probability induced by prior μ through reduction. Unsurprisingly, when we normalize $u(0) = 0$, we obtain:

$$V_{\text{SEU}}\left(\mathcal{P}_i\right) = 0.5u(20) \qquad \forall i \in \mathbb{P}. \tag{3}$$

In other words, SEU predicts that all uncertain prospects lead to the same expected utility level. To see this, remark that in the case of SR, M is a singleton, so that criterion (1) reduces to the standard von Neumann-Morgenstern, where epistemic uncertainty does not play any role. In the cases of $CR0$ and

$CR25$, it is assumed that the subjective prior beliefs over models coincide with the objective probabilities, and the two stages of risk are reduced into a single one (i.e. ROCR). In the cases of $MU0$ and $MU25$, the symmetry condition imposes $\mu(m) = 0.5$ for each model, while in the standard Ellsberg case (SE), the result follows from the symmetry condition imposing $\mu(m) = \mu(m')$ for all m, m' such that $p(s|m) = 1 - p(s|m')$. Finally, note that while the prospects $MM0, MM25$ and EE do not have a formal existence within such a two-layer setup (where the true model is assumed to belong to M, which is itself finite), we can infer from the preceding analysis that they lead to exactly the same level of expected utility under the symmetry condition. In terms of premia, the predictions in the SEU case are then summarized as:

$$\Pi_i = 0 \qquad \forall i \in \mathbb{P}. \tag{4}$$

4.3 Maxmin models

The family of theories we now examine relax the assumption of equal treatment between the layers of risk and model uncertainty. These theories thus depart from the Bayesian framework presented above. The first decision criterion, which is due to Wald (1950), is the most extreme in that it considers only the worst among the possible models affected by epistemic uncertainty. The second criterion is less extreme and originates in the work of Gilboa and Schmeidler (1989) and Schmeidler (1989).

Wald The decision criterion due to Wald (1950) fosters an extreme form of ambiguity aversion in that it makes the DM consider only the model giving her the lowest expected utility level:

$$V_{\text{Wald}}(\mathcal{P}_i) = \min_m \sum_s p(s|m) u\left(\mathcal{P}_i(s)\right). \tag{5}$$

It should be noted that the layer of risk is not affected by the extreme cautiousness entailed by such criterion. When the maxmin criterion is derived in order to address the epistemic uncertainty of the second layer (as in Marinacci, 2015), it makes no prediction regarding the way the DM evaluates compound risks. It is therefore perfectly conceivable to assume that the three prospects entailing the layer of risk only ($SR, CR0, CR25$) are treated the same way, as in the SEU case. The predictions under Wald's criterion may then be written as[11]

[11] In the presentation of the predictions that follows, we consider deviations from ambiguity neutrality as strict.

$$0 = \Pi_{CR0} = \Pi_{CR25} < \Pi_{MU25} < \Pi_i \qquad \forall i \in \mathbb{P} \setminus \{SR, CR0, CR25, MU25\}. \quad (6)$$

Multiple priors In the multiple priors (MP) model axiomatized by Gilboa and Schmeidler (1989), the incompleteness of information regarding the correct model may result in the prior probability measure to be non-singleton. Instead, the DM has a set C of priors μ, and makes her decision based on the prior giving rise to the least favorable SEU. The two-layer version of this criterion is written:

$$V_{\text{MP}}(\mathcal{P}_i) = \min_{\mu \in \text{C}} \sum_m \mu(m) \left(\sum_s p(s|m) u\left(\mathcal{P}_i(s)\right) \right). \quad (7)$$

This formulation encompasses the original version of the MP model proposed by Gilboa and Schmeidler (1989), which is recovered when considering predictive subjective probabilities $\bar{\mu}(s)$. In our case, the symmetry conditions translates to a symmetric set of priors C, giving therefore rise to ambiguity aversion. For the risk situations $(SR, CR0, CR25)$, the MP model –which is built within the Anscombe and Aumann (1963) framework– predicts the same CEs for all (compound) risks with the same expected utility. Together, these predictions are summarized as:

$$0 = \Pi_{CR0} = \Pi_{CR25} < \Pi_i \qquad \forall i \in \mathbb{P} \setminus \{SR, CR0, CR25\}. \quad (8)$$

It should be noted that this criterion is not as extreme as it appears at first sight. Under the MP model, the minimization is realized over the set C which incorporates both a taste component (the attitude towards ambiguity) and an information component (the way ambiguity is perceived). These two components are inherently indistinguishable so that a smaller set C may reflect better information and/or less ambiguity aversion. This flexibility allowed in the construction of C moreover leads to a wide range of possible choice behavior in the ambiguous prospects we present, preventing therefore any finer ranking order.[12] Finally, when the set of priors C is singleton, we are back to the SEU criterion (1), which predicts equality among all the relative premia.

4.4 Smooth models

Contrary to the maxmin theories, the next family of decision criteria model the different layers of uncertainty via standard probability measures (i.e. unique μ). In

[12] Note that the same predictions under ambiguity aversion would hold for the more general α-version of the MP model that has been axiomatized by Ghirardato et al. (2004) and in which both the "max" and the "min" appear with weights α and $1 - \alpha$.

this case however, the independence assumption between the stages of uncertainty is dropped to allow for distinct treatment of simple risk, and either compound risk or model uncertainty situations. The utility of betting on prospect i under the smooth criterion is

$$V_{\mathrm{smt}}(\mathcal{P}_i) = \sum_{m} \mu\left(m\right) \phi \left(\sum_{s} p(s|m) u\left(\mathcal{P}_i(s)\right) \right), \tag{9}$$

where $\phi : \mathrm{Im}\ u \subseteq \mathbb{R} \to \mathbb{R}$ is a strictly increasing and continuous function representing in turn preferences towards ambiguity (KMM's version) and compound risk (Seo's version).[13]

KMM By dropping the independence assumption between the layers of uncertainty, the version of the smooth model due to Klibanoff, Marinacci, and Mukerji (2005, hereafter KMM) and Marinacci (2015) allows for a distinct treatment of risk and model uncertainty. While it has been implicitly assumed that identical attitudes were considered towards uncertainty quantified via objective and subjective probabilities in (1), the more general version (9) distinguishes these attitudes. In particular, this is made explicit once we write $v = \phi \circ u$, where $v : C \to \mathbb{R}$ captures the attitude towards model uncertainty (i.e. towards epistemic uncertainty, Marinacci, 2015). In this sense, the ambiguous prospect may be regarded as being evaluated in two steps: for each model m, the DM first computes a certainty equivalent $c(m)$ using her risk attitude modeled by u, while in a second step she evaluates the overall prospect by taking the expected utility over these certainty equivalents using her subjective prior μ and her attitude towards model uncertainty, modeled by v. In this respect, aversion to model uncertainty is represented by a concave function v, which is interpreted as aversion to mean preserving spreads in the certainty equivalents induced by each model. Ambiguity aversion in this model (concave ϕ) therefore results from a higher degree of aversion to model uncertainty than to risk, while the SEU case is recovered when both degrees are identical. Since compound risk prospects feature two stages of the same layer of risk, each stage is evaluated using risk aversion u only. This theory therefore incorporates ROCR. Finally, also remark that $MU0$ presents the maximum spread in the space of certainty equivalents $c(m)$ and should therefore be considered as the least favorable prospect by each subject exhibiting ambiguity aversion. In terms of premia, the predictions for an ambiguity averse subject are

[13] Note that Nau (2006) and Ergin and Gul (2009) characterized representations that, at least in special cases, can take the same representation as (9) and share the same interpretation as KMM's version.

$$
\begin{aligned}
0 = \quad & \Pi_{CR0} && = \Pi_{CR25} && (10)\\
0 < \quad & \Pi_{MU25} && < \Pi_{MU0} && (11)\\
0 < \quad & \Pi_{SE} && \leq \Pi_{MU0}. && (12)
\end{aligned}
$$

As before, extending loosely the criterion to account for the third layer of misspecification by considering, instead, the set of models $M = [0, 1]$, enables us to draw the following additional predictions:

$$0 < \Pi_i \quad \leq \quad \Pi_{MU0} \qquad \forall i \in \{MM0, MM25, EE\}. \tag{13}$$

In words, expression (13) says that, if anything, misspecification is perceived at least as good as $MU0$, which is characterized by the extreme spread of models (since in this case, non-degenerate probability distributions may not be excluded).

Seo In the approach proposed by Seo (2009), the distinction is not only made between the layers of risk and model uncertainty, but also between the first and the second stages of risk. In that sense, ambiguity aversion may as well result from non-reduction of objective compound risk. Attitudes towards objective probabilities presented in two stages or towards model uncertainty and ambiguity are thus closely related (in the words of Seo (2009), ROCR implies neutrality to ambiguity). Formulation (9) implies distinct expected utilities in the different stages. When ϕ is linear, the DM reduces the two stages of uncertainty into a single one and V_{smt} collapses to the SEU formulation (2). Consistent with what precedes, Seo's (2009) predictions under ambiguity aversion may be summarized as follows

$$
\begin{aligned}
0 < \Pi_{CR25} = \Pi_{MU25} \quad & < \quad \Pi_{CR0} = \Pi_{MU0} && (14)\\
0 < \Pi_{SE} \quad & \leq \quad \Pi_{CR0} = \Pi_{MU0}. && (15)
\end{aligned}
$$

Once the criterion is extended to allow for considering misspecification, we furthermore have

$$0 < \Pi_i \quad \leq \quad \Pi_{CR0} = \Pi_{MU0} \qquad \forall i \in \{MM0, MM25, EE\}. \tag{16}$$

4.5 Recursive non-expected utility models

Other approaches which violate the ROCR axiom and expected utility theory have been proposed. An example is the theory proposed by Segal (1987; 1990) which uses, to evaluate the first and second stage of uncertainty, either Quiggin's (1982) rank dependent utility or Gul's (1991) disappointment aversion. According to these approaches, ambiguous prospects are seen as two-stage risks which are evaluated by the DM using the previously mentioned two-step procedure: each second-stage lottery is first replaced by its certainty equivalent before the overall value of the prospect is computed at the first-stage. The difference with previous theories, however, lays in the way the certainty equivalents and the global prospect are evaluated. In what follows, we outline two distinct approaches.

Recursive rank dependent utility In the rank dependent utility (RDU) model of Quiggin (1982), the lottery $x = (x(1), p(1); ...; x(n), p(n))$ with $x(1) \geq ... \geq x(n)$ is evaluated by

$$V_{\mathrm{RDU}}(x) = u(x(n)) + \sum_{s=2}^{n} \left[u(x(s-1)) - u(x(s))\right] f\left(\sum_{t=1}^{s-1} p(t)\right). \qquad (17)$$

In this expression, $f : [0,1] \to [0,1]$, with $f(0) = 0$ and $f(1) = 1$, is an increasing transformation function, which is furthermore convex under uncertainty aversion. A certainty equivalent $c(m) = u^{-1}\left(V_{\mathrm{RDU}}(x|m)\right)$ may then be computed for each model m separately, and the overall prospect is then evaluated recursively using (17) and the priors μ on these CE's. The recursive rank dependent utility (RRDU) of prospects SR, $CR0$ and $CR25$, giving 20 if the bet is correct and 0 otherwise, are then for example computed as:

$$\begin{aligned}
V_{\mathrm{RRDU}}(\mathcal{P}_{SR}) &= V_{\mathrm{RRDU}}(\mathcal{P}_{CR0}) \\
&= u(20)f(0.5) \\
&> V_{\mathrm{RRDU}}(\mathcal{P}_{CR25}) \\
&= u(20)f(0.25) + \left[u^{-1}\left(u(20)f(0.75) - u^{-1}\left(u(20)f(0.25)\right)\right] f(0.5).
\end{aligned}$$

Under the symmetry assumption, compound risk and model uncertainty prospects are evaluated the same way, so that $V_{\mathrm{RRDU}}(\mathcal{P}_{CR0}) = V_{\mathrm{RRDU}}(\mathcal{P}_{MUO})$ and $V_{\mathrm{RRDU}}(\mathcal{P}_{CR25}) = V_{\mathrm{RRDU}}(\mathcal{P}_{MU25})$. When f is convex, $CR0$ is preferred to SE.[14] Together, these pre-

[14]Note that the common empirical finding in the literature is uncertainty seeking for low likelihood events and uncertainty aversion for moderate and high likelihood events, which implies inverse S-shaped –first concave and then convex– f (Wakker, 2010). Here, we focus on moderate probabilities where

dictions are written

$$
\begin{aligned}
0 = \Pi_{CR0} = \Pi_{MU0} &< \Pi_{CR25} = \Pi_{MU25} && (18)\\
0 &\leq \Pi_{SE}. && (19)
\end{aligned}
$$

Recursive disappointment aversion In the disappointment aversion (DA) model of Gul (1991), the value $V_{\mathrm{DA}}(x)$ of the lottery $x = (x(1), p(1); ...; x(n), p(n))$ is given by the unique solution of the equation:

$$
v = \frac{\sum_{\{s:u(x(s))\geq v\}} p(s)u(x(s)) + (1+\beta)\sum_{\{s:u(x(s))<v\}} p(s)u(x(s))}{1+\beta\sum_{\{s:u(x(s))<v\}} p(s)}. \qquad (20)
$$

In this expression, $\beta \in (-1, \infty)$ is the coefficient of disappointment aversion (if $\beta > 0$) or elation seeking (if $\beta < 0$). The outcomes are separated into two groups: the elating outcomes (which are preferred to the lottery x) and the disappointing outcomes (which are worse than the lottery x). The DM then evaluates x in an expected utility way, except that disappointing outcomes are given a uniformly extra weight under disappointment aversion. As before, a certainty equivalent is then computed for each model m separately, and the overall value of the two-stage prospect is evaluated recursively using the same preferences on these CEs and the prior measure μ. Unsurprisingly, when $\beta = 0$, this criterion collapses to the SEU criterion (1). Using the recursive disappoint aversion (RDA) model, it is then easy to see that a disappointment averse DM always prefers any two-stage prospect to be resolved in a single stage (or to be degenerate in the second stage). In particular, if in our case a bet gives 20 if correct and 0 otherwise, we have:

$$
\begin{aligned}
V_{\mathrm{RDA}}(\mathcal{P}_{SR}) &= V_{\mathrm{RDA}}(\mathcal{P}_{CR0}) = V_{\mathrm{RDA}}(\mathcal{P}_{MUO})\\
&= u(20)\\
&> V_{\mathrm{RDA}}(\mathcal{P}_{CR25}) = V_{\mathrm{RDA}}(\mathcal{P}_{MU25})\\
&= \frac{0.5}{1+0.5\beta}\left(\frac{0.75u(20)}{1+0.25\beta}\right) + \frac{0.5(1+\beta)}{1+0.5\beta}\left(\frac{0.25u(20)}{1+0.75\beta}\right).
\end{aligned}
$$

Under the interpretation that ambiguity aversion amounts to preferring objective simple risks to compound (non-degenerate) ones, Artstein and Dillenberger (2015) show that a disappointment averse DM exhibits ambiguity aversion for any pos-

uncertainty aversion is prevalent.

sible beliefs about the model. The predictions under the RDA approach are then the same as under RRDU, summarized in (14) and (15).

5 Results

5.1 Quality of data and consistency

The data we collected consist of 124 observations for $MU25$, and 125 observations for the rest of the prospects.[15] A total of 39 (3.5% of all) choice lists from 14 different subjects exhibited multiple-switching, no-switching or reverse-switching patterns. These observations were not included in the following analysis as the CEs for these patterns do not imply a clear measure and may be due to confusion. We do not observe any order treatment effect on the CEs (details are reported in Appendix D).

5.2 General results

One of our main objectives is to observe the attitudes towards different sources of uncertainty, possibly encompassing distinct layers of uncertainty. Figure 1 summarizes statistics on relative premia for CR, MU, MM, EE, and SE (the complete descriptive statistics are presented in Appendix A). First, we can observe that, in line with the standard findings in the literature, the average relative premia are positive, indicating an aversion to compound risk and ambiguity. The premia differ from zero in all cases (t-test, p-value<0.001),[16] except for $CR0$ (t-test, p-value $= 0.580$) indicating indifference between simple and compound risk in this case, consistent with the ROCR. Interestingly, reduction is rejected in the case of $MU0$, where the complexity of the problem is the same as in $CR0$, but where there is a second layer involving subjective probabilities.

Second, a multivariate analysis of variance (MANOVA) with repeated measures, indicates that the relative premia for the sources CR, MU and MM are different from each other (p-value<0.001). Looking at the pairwise comparisons, CR differs from both MU and MM (MANOVA with repeated measures, p-value<0.001 for both). MU and MM are marginally different (MANOVA with repeated measures, p-value=0.054). Overall, our data suggest a strong increasing trend in relative premia moving from compound risk, to model uncertainty and model misspecification, within both $p = 0$ and $p = 25$ (Page's L-test for increasing

[15] One subject omitted answering to choice situation $MU25$ by mistake.

[16] Throughout, non-parametric Wilcoxon tests give the same conclusions on rejecting or not rejecting the null hypothesis.

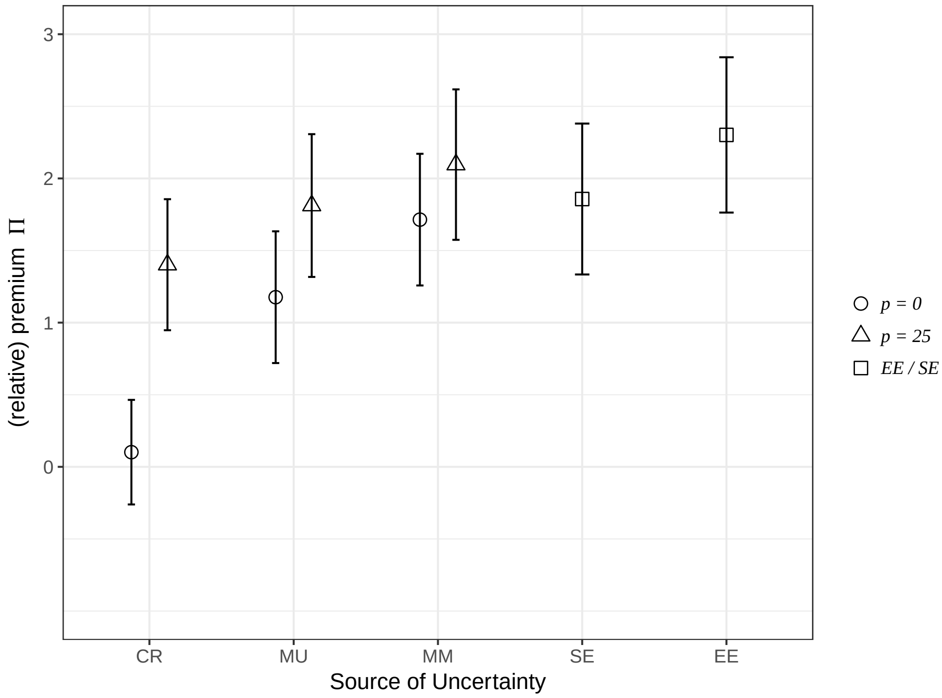

Figure 1: Mean (relative) premia and corresponding 95% confidence intervals for the 9 prospects

trend, p-value<0.001 for $p = 0$ and p-value=0.003 for $p = 25$). The relative premia are also on average higher for $p = 25$ than for $p = 0$ across the three sources of uncertainty (repeated measures MANOVA, p-value<0.001). The premia difference between the treatments with $p = 25$ and $p = 0$ is significant for CR (t-test, p-value<0.001) and for MU (t-test, p-value=0.006), and marginally significant for MM (t-test, p-value=0.053) .

Finally, our data do not reveal a significant difference between relative premia for EE and SE (t-test, p-value=0.110). The slight preference for SE can be ascribed to the distinction between inherent model uncertainty and model misspecification in SE and EE respectively, which is consistent with our observations on MU and MM. This is an interesting feature of SE: being technically composed of two layers of uncertainty only (no misspecification by construction), it is still virtually able to replicate behaviors under EE, encompassing the three layers.

5.3 Associations

The relationship between ambiguity and compound risk has been extensively discussed in the literature (Halevy, 2007; Abdellaoui et al., 2015; Chew et al., 2017). Here, while re-examining the strength of this relationship, we are able to further extend the analysis by looking at the association of ambiguity with uncertainty presented in two- (MU) and three layers (MM). The conjecture we want to test is that stronger associations exist between ambiguity and the latter two sources of uncertainty, than between ambiguity and CR. In what follows, we focus on EE as representing ambiguity in the spirit of Ellsberg, since it better reflects a situation of unmeasurable uncertainty encompassing the three layers of uncertainty altogether. Our findings are also robust to the use of SE as representing ambiguity.

5.3.1 Ambiguity neutrality and reduction

Here, we distinguish between types of reduction under different sources of uncertainty. We first replicate the analysis of preceding studies with contingency tables relating ambiguity neutrality (AN) and ROCR using our data. Then, we extend the analysis by considering reduction of MU (ROMU) and reduction of MM (ROMM). In what follows, a subject is classified as reducing compound risk if she assigns zero relative premia to both $CR0$ and $CR25$. ROMU and ROMM are defined similarly (i.e. $\Pi_{MU0} = \Pi_{MU25} = 0$ and $\Pi_{MM0} = \Pi_{MM25} = 0$, respectively). A subject is classified as AN if she assigns zero relative premia to EE. Table 1 reports the contingency tables relating ROCR, ROMU and ROMM with AN.

Table 1: Association between ambiguity neutrality and ROCR, ROMU, ROMM

	ROCR ($\Pi_{CR0} = \Pi_{CR25} = 0$)		**ROMU** ($\Pi_{MU0} = \Pi_{MU25} = 0$)		**ROMM** ($\Pi_{MM0} = \Pi_{MM25} = 0$)		
Ambiguity neutrality ($\Pi_{EE} = 0$)	No	Yes	No	Yes	No	Yes	**Total**
No	61 *(51.8)* 52.6%	15 *(24.2)* 12.9%	69 *(55)* 59.5%	7 *(21)* 6%	73 *(59.6)* 62.9%	3 *(16.4)* 2.6%	**76** 65.5%
Yes	18 *(27.2)* 15.5%	22 *(12.8)* 19%	15 *(29)* 12.9%	25 *(11)* 21.6%	18 *(31.4)* 15.5%	22 *(8.6)* 19%	**40** 34.5%
Total	**79** 68.1%	**37** 31.9%	**84** 72.4%	**32** 27.6%	**91** 78.4%	**25** 21.6%	**116** 100%
Fisher's exact tests (2-sided):	p<0.001		p<0.001		p<0.001		

Notes: Expected frequency under a null hypothesis of independence in parentheses. Relative frequencies indicated in %.

As can be observed, our data confirm the previous findings in the literature by rejecting the independence hypothesis between AN and ROCR, although the association found in our data is relatively weak. Specifically, we observe that among the 37 subjects who reduce compound risk, 22 (59.5%) are also ambiguity neutral, and among the 40 subjects who are ambiguity neutral, 22 (55%) reduce compound risk. Compared to the preceding studies, the proportion of AN conditional on ROCR is significantly lower in our data than the 96% (22 out of 23 subject) found in the data of Halevy (2007) (p-value=0.002), or the 95% (39 out of 41 subjects) found in the data of Chew et al. (2017) (p-value<0.001).[17] In Appendix B, we provide a more comprehensive comparison of our results with the ones previously obtained in the literature.

Turning to ROMU, our data suggest a larger overlap between ROMU and AN than between ROCR and AN. In the direction of ROMU implying AN, out of the 32 subjects reducing MU, 25 (78%) exhibited AN. This proportion is higher than the proportion of AN conditional on ROCR (59.5%), although the difference is marginal (p-value=0.097). Looking at the converse implication, out of the 40 subjects exhibiting AN, 25 (62.5%) reduced MU. This proportion is also slightly higher than the proportion of ROCR conditional on AN in our data (55%), however the difference is not significant (p-value=0.496).

Lastly, our data indicate an even larger overlap between AN and ROMM. Out of the 25 subjects reducing MM, 22 (88%) exhibited AN, and out of the 40 subjects exhibiting AN, 22 (55%) reduced MM. The proportion of AN conditional on ROMM is higher than the proportion conditional on ROCR (59.5%) (p-value=0.015). The proportion of AN conditional on ROMM is also slightly higher than the proportion conditional on MU (78%) but this difference is not significant (p-value=0.331) .

5.3.2 Associations of attitudes

We now extend the previous analysis concerning ambiguity neutrality and reduction by examining the associations of attitudes towards the different sources of uncertainty. Accordingly, the contingency tables, reported in Table 2, relate aversion, seeking and neutrality attitudes towards EE and the other sources of uncertainty. Here, a subject is classified as CR averse (seeking) if she assigns a positive (negative) relative premium for both $CR0$ and $CR25$. As in the previous section, CR neutrality is defined as zero relative premia for both $CR0$ and $CR25$. We define attitudes towards MU and MM analogously.

[17] Similarly, the proportion of ROCR conditional on AN in our data is significantly lower than the 79% (22 out of 28 subjects) found in the data of Halevy (p-value=0.045). The strength of the association suggested in the data of Abdellaoui et al. (2015) is comparable to our study (p-value=0.22 for AN conditional on ROCR, and p-value=0.334 for the converse implication).

Table 2: Association between ambiguity attitude and attitudes towards CR, MU, and MM

Ambiguity	Compound Risk (CR)				Model Uncertainty (MU)				Model Misspecification (MM)			
	Averse ($\Pi_{CR0} > 0$ & $\Pi_{CR25} > 0$)	Neutral ($\Pi_{CR0} =$ $\Pi_{CR25} = 0$)	Seeking ($\Pi_{CR0} < 0$ & $\Pi_{CR25} < 0$)	**Total**	Averse ($\Pi_{MU0} > 0$ & $\Pi_{MU25} > 0$)	Neutral ($\Pi_{MU0} =$ $\Pi_{MU25} = 0$)	Seeking ($\Pi_{MU0} < 0$ & $\Pi_{MU25} < 0$)	**Total**	Averse ($\Pi_{MM0} > 0$ & $\Pi_{MM25} > 0$)	Neutral ($\Pi_{MM0} =$ $\Pi_{MM25} = 0$)	Seeking ($\Pi_{MM0} < 0$ & $\Pi_{MM25} < 0$)	**Total**
Averse ($\Pi_{EE} > 0$)	16 *(9.8)* 25%	13 *(17.3)* 20.3%	1 *(2.8)* 1.6%	**30** 46.9%	44 *(29.1)* 52.4%	7 *(19.4)* 8.3%	0 *(2.4)* 0%	**51** 60.7%	51 *(34.7)* 58.6%	2 *(15.2)* 2.3%	0 *(3)* 0%	**53** 60.9%
Neutral ($\Pi_{EE} = 0$)	3 *(9.2)* 4.7%	22 *(16.2)* 34.4%	3 *(2.6)* 4.7%	**28** 43.8%	3 *(17.1)* 3.6%	25 *(11.4)* 29.8%	2 *(1.4)* 2.4%	**30** 35.7%	4 *(19)* 4.6%	22 *(8.3)* 25.3%	3 *(1.7)* 3.5%	**29** 33.3%
Seeking ($\Pi_{EE} < 0$)	2 *(2)* 3.1%	2 *(3.5)* 3.1%	2 *(0.6)* 3.1%	**6** 9.4%	1 *(1.7)* 1.2%	0 *(1.1)* 0%	2 *(0.1)* 2.4%	**3** 3.6%	2 *(3.3)* 2.3%	1 *(1.4)* 1.2%	2 *(0.3)* 2.3%	**5** 5.8%
Total	**21** 32.8%	**37** 57.8%	**6** 9.4%	**64** 100%	**48** 57.1%	**32** 38.1%	**4** 4.8%	**84** 100%	**57** 65.5%	**25** 28.7%	**5** 5.8%	**87** 100%
Independence test:	Fischer's exact test (2-sided): $p = 0.001$				Fischer's exact test (2-sided): $p < 0.001$				Fischer's exact test (2-sided): $p < 0.001$			

Notes: Expected frequency under a null hypothesis of independence in parentheses. Relative frequencies indicated in %.

The results indicate that, similar to the previous contingency tables with neutrality and reduction, there is a significant relation between attitudes towards EE and CR. Furthermore, we replicate the stronger associations between EE and MU, and between EE and MM. In particular, the proportion of observations on the diagonals is significantly higher in the tables for MU and MM compared to the table for CR (p-value=0.002 for MU and p-value<0.001 for MM).

The differences in associations are also revealed in the pairwise correlations of the relative premia, reported in Table 3. The multivariate tests of correlations indicate that the relative premium for EE is more strongly correlated with the premium for MU than it is with the premium for CR. This result is valid for both $p = 0$ and $p = 25$ (p-value<0.05). There is also a stronger correlation between EE and MM than between EE and CR, although this difference is significant for $p = 0$ (p-value<0.001) but not for $p = 25$ (p-value=0.142).

Table 3: Correlation Matrices of Attitudes towards Different Sources

Ambiguity & Compound Risk				**Ambiguity & Model Uncertainty**				**Ambiguity & Model Misspecification**			
	Π_{EE}	Π_{CR0}	Π_{CR25}		Π_{EE}	Π_{MU0}	Π_{MU25}		Π_{EE}	Π_{MM0}	Π_{MM25}
Π_{EE}	1			Π_{EE}	1			Π_{EE}	1		
Π_{CR0}	0.3099	1		Π_{MU0}	0.6134^{**}	1		Π_{MM0}	0.6609^{***}	1	
Π_{CR25}	0.5593^{+}	0.4318	1	Π_{MU25}	0.7456^{*}	0.6118	1	Π_{MM25}	0.6786	0.7231	1

Notes: Star signs indicate differences across correlation matrices where the first matrix (ambiguity & compound risk) is the base. ***significantly different from the corresponding correlation under CR at 0.1% level; **significantly different from the corresponding correlation under CR at 1% level; *significantly different from the corresponding correlation under CR at 5% level. Plus signs indicate differences between the correlations of Π_{EE} with the other premia within the given correlation matrix. $^{++}$significantly different from the correlation between Π_{EE} and Π_i (where $i \in \{CR0, MU0, MM0\}$ represents the source within the given matrix at 1% level; $^{+}$significantly different from the correlation between Π_{EE} and Π_i within the given matrix at 5% level.

5.3.3 Further results

We now explore the role of complexity (i.e. making a distinction between relatively easy vs. more difficult tasks) and the role of numerical ability (i.e. making a distinction between relatively more quantitatively sophisticated vs. less sophisticated subjects) in the association between AN and the other sources of uncertainty.

The role of complexity As already noticed in Section 5.2, our results reveal a significant difference between the relative premia for the two cases of compound risk, $CR0$ and $CR25$. The distinct treatments between these two prospects is not really surprising as $CR0$ may be claimed to be more easily reducible than $CR25$ (for someone who wants to reduce the CR, degenerate probabilities in the second stage are indeed easier to manipulate).[18] In what follows, we focus on the relatively simpler prospects with degenerate risk in the second stage. Specifically, we report, in Table 4, the results of the association between AN and ROCR or ROMU respectively. The definition of ambiguity neutrality remains as in the analysis above (i.e. $\Pi_{EE} = 0$), while to define ROCR (ROMU) we now require only one equality to hold, namely $\Pi_{CR0} = 0$ ($\Pi_{MU0} = 0$). With such a definition

Table 4: Association between ambiguity neutrality and ROCR, ROMU in the case $p = 0$

Ambiguity neutrality ($\Pi_{EE} = 0$)	**ROCR** ($\Pi_{CR0} = 0$) No	**ROCR** ($\Pi_{CR0} = 0$) Yes	**ROMU** ($\Pi_{MU0} = 0$) No	**ROMU** ($\Pi_{MU0} = 0$) Yes	**Total**
No	35 *(30)* 30.4%	40 *(45)* 34.8%	59 *(45.7)* 51.3%	16 *(29.3)* 13.9%	**75** 65.2%
Yes	11 *(16)* 9.6%	29 *(24)* 25.2%	11 *(24.3)* 9.6%	29 *(15.7)* 25.2%	**40** 34.8%
Total	**46** 40%	**69** 60%	**70** 60.9%	**45** 39.1%	**115** 100%
Fisher's exact tests (2-sided):	p =0.049		p <0.001		

Notes: Expected frequency under a null hypothesis of independence in parentheses. Relative frequencies indicated in %.

of ROCR, focused on the simple task, 60% of our sample is indifferent between the simple and the compound risk (consistent with what is found in Berger and Bosetti, 2017). This is in contrast with the 39% of subjects who reduce the model uncertainty under the analogous simple task. Notably, the only difference between the two prospects, $CR0$ and $MU0$, is the nature of the probabilities in the first stage. Considering this simple task, the association between AN and ROCR is weaker and only significant at the 5% level. In particular, while only 35 subjects among the 75 ambiguity non-neutral subjects (46.7%) do not reduce CR, 59 subjects out of the same 75 subjects (78.7%) do not reduce model uncertainty. These proportions significantly differ (p-value<0.001). Similarly, while the proportion of ambiguity neutrality among subjects reducing CR was 42% (29 out of 69), the proportion among subjects reducing MU was 64.4% (29 out of 45). This proportions also differ significantly (p-value=0.0193).

The role of cognitive skills At the end of the experiment, six multiple-choice questions were used to test subjects' quantitative skills. The answers to these questions enable us to investigate the impact of numeracy on AN, ROCR and

[18] Note that given the low degree of complexity they carry, compound risks with degenerate second stage have also been used in the literature to test the "time neutrality" hypothesis (i.e. risk resolving entirely in the first stage of two, rather than in one stage).

ROMU. A latent-mixture model is estimated to classify subjects into two groups: a low-skilled group whose likelihood of getting the right answer in any multiple-choice question is assumed to be 1/6 (as there were 6 choice options with 1 correct answer), and a high-skilled group whose likelihood of getting the right answer is assumed to be higher. The model is estimated by Bayesian inference methods using Markov chain Monte Carlo (MCMC) algorithm run through WinBUGS software.[19] A subject is classified as a member of a group if her posterior distribution indicated that she belongs to the group with at least 95% probability. The estimations result in 95 classified subjects (55 high-skilled, 40 low-skilled). Table 5 reports the contingency tables relating AN and ROCR for these high- and low-numeracy groups. The contingency tables in Part 1 suggest that the relationship between AN and ROCR is weaker among the high numeracy group. In particular, both the proportion of AN conditional on ROCR and the proportion of ROCR conditional on AN are lower in the high numeracy group than in the low numeracy group. Hence, the hypothesis of independence between AN and ROCR is not rejected in the high numeracy group, whereas it is rejected in the low numeracy group despite smaller number of observations there. The same impact of numeracy skills is not observed on the relationship between AN and ROMU. The contingency tables in Part 2 indicate that the association between AN and ROMU was strong for both low and high numeracy groups. The association between AN and ROMM (whose contingency table is not reported here) is also robust to the differences in numeracy skills (p-value<0.001, Pearson χ^2, for both low and high numeracy subjects).[20]

5.4 Individual level analysis

In this section, we report the results of the individual level analysis. We classify subjects on the basis of the proximity of their preference patterns over different sources of uncertainty to the predictions of the theoretical models of choice presented in Section 4. Our classification is based on the five prospects (SR, $CR0$, $CR25$, $MU0$, and $MU25$), which gives us ten binary comparisons with $\Delta_{ij} = CE_i - CE_j$ where $i, j \in \{SR, CR0, CR25, MU0, MU25\}$ and $i \neq j$. We

[19]Two chains, each with 100.000 MCMC samples, are run, after a burn-in of 1000 iterations. Only every tenth observation is recorded to reduce the autocorrelation. The WinBUGs code is available upon request. For further details on the Bayesian modeling through WinBUGS, see Lee and Wagenmakers (2014) providing a practical introduction to the subject.

[20]The same patterns are also observed when the associations are between attitudes, i.e. aversion, seeking and neutrality, as in Table 2. The independence of ambiguity and CR attitudes is rejected among low numeracy group (p-value=0.003, Pearson χ^2) but it is not rejected among the high numeracy group (p-value=0.115, Pearson χ^2). The associations between ambiguity and MU/MM attitudes are robust in both groups (p-value<0.001, Pearson χ^2 for all the associations).

Table 5: ASSOCIATION BETWEEN AMBIGUITY NEUTRALITY AND ROCR/ROMU BY NUMERACY GROUPS

Part I

Ambiguity neutrality ($\Pi_{EE} = 0$)	Low Numeracy Subjects: **ROCR** ($\Pi_{CR0} = \Pi_{CR25} = 0$) No	Yes	**Total**	High Numeracy Subjects: **ROCR** ($\Pi_{CR0} = \Pi_{CR25} = 0$) No	Yes	**Total**
No	21 *(16.9)* 52.5%	4 *(8.1)* 10%	**25** 62.5%	22 *(19.2)* 40%	9 *(11.8)* 16.4%	**31** 56.4%
Yes	6 *(10.1)* 15%	9 *(4.9)* 22.5%	**15** 37.5%	12 *(14.8)* 21.8%	12 *(9.2)* 21.8%	**24** 43.6%
Total	**27** 67.5%	**13** 32.5%	**40** 100%	**34** 61.8%	**21** 38.2%	**55** 100%
Fisher's exact test (2-sided):		p =0.006			p = 0.163	

Part II

Ambiguity neutrality ($\Pi_{EE} = 0$)	Low Numeracy Subjects: **ROMU** ($\Pi_{MU0} = \Pi_{MU25} = 0$) No	Yes	**Total**	High Numeracy Subjects: **ROMU** ($\Pi_{MU0} = \Pi_{MU25} = 0$) No	Yes	**Total**
No	24 *(18.1)* 60%	1 *(6.9)* 2.5%	**25** 62.5%	28 *(21.4)* 50.9%	3 *(9.6)* 5.5%	**31** 56.4%
Yes	5 *(10.9)* 12.5%	10 *(4.1)* 25%	**15** 37.5%	10 *(16.6)* 18.2%	14 *(7.4)* 25.4%	**24** 43.6%
Total	**29** 72.5%	**11** 27.5%	**40** 100%	**38** 69.1%	**17** 30.9%	**55** 100%
Fisher's exact test (2-sided):		p <0.001			p < 0.001	

Notes: Expected frequency under a null hypothesis of independence in parentheses. Relative frequencies indicated in %.

consider the predictions of the following theories: (Classical) SEU, Smooth model of KMM (2005), RRDU of Segal (1987) and the theory of Seo (2009). SEU predicts reduction in both CR and MU. The theories of Segal and Seo both predict violations of ROCR and ROMU, and they do not distinguish between CR and MU. These two theories differ in their predictions of preferences over mean preserving spreads as discussed in Section 4. KMM model makes the same predictions on the mean preserving spreads as the theory of Seo, but it differs in predicting no violations of ROCR. Within each non-SEU theory, the preference patterns compatible with ambiguity aversion and ambiguity seeking are distinguished. Following this classification based on CR and MU, the compatibility with the observed ambiguity attitudes based on EE is examined. As before, subjects are classified as ambiguity averse (AA) if they exhibit $\Pi_{EE} > 0$, ambiguity neutral (AN) if $\Pi_{EE} = 0$, and ambiguity seeking (AS) if $\Pi_{EE} < 0$. The analysis is done using data collected from subjects who did not exhibit any multiple switching patterns, and thus had no missing CE data.

The classification into the different theoretical models is done using a latent mixture model estimation, which takes the stochastic component of responses into account. Each Δ_{ij} is assumed to follow a normal distribution with a mean being zero, non-positive, or non-negative. The estimation of the latent model is performed using Bayesian methods,[21] assuming uninformative uniform priors for mean (with a range between 0 and 10 for non-negative Δ_{ij} and between -10 and 0 for non-positive Δ_{ij}) and for standard deviation (ranging between 0 and 10) of Δ_{ij}. The analysis results in an estimated likelihood of each theoretical model for every subject. The subjects are classified with the theoretical model that is estimated as the most likely for them. Overall, our latent mixture model performs well in detecting distinct preference patterns predicted by the theoretical models. In particular, for 83% of the subjects (92 out of 111), the theoretical model that is estimated as the most likely is at least twice as likely as the model estimated as the second most likely.[22] The descriptive validity of the model is supported by the posterior prediction tests, where the estimated posterior distributions are able to predict the observed patterns in the data accurately (see Appendix C.1 for further details).

Table 6 reports the classification results based on our latent mixture analysis and their distribution across the observed ambiguity attitudes. Based on SR, CR, and MU, we observe that 61% of the subjects (68 out of 111) are classified as non-SEU. 30% of all observed preferences are consistent with the KMM model, whereas 22% are consistent with RRDU, and 9% with Seo's theory. Focusing on the compatibility with ambiguity attitudes, among subjects classified as SEU, 67% (29 out of 43) are consistent in satisfying ambiguity neutrality defined as $\Pi_{EE} = 0$. We also observe that for the majority of the non-SEU subjects, the preference patterns within CR and MU are compatible with the observed ambiguity attitudes

[21]Three chains, each with 10.000 MCMC samples, are run, after a burn-in of 1000 iterations. Only every tenth observation is recorded to reduce the autocorrelation.

[22]Among seven preference patterns under consideration (SEU, and ambiguity averse and ambiguity seeking classes in three non-SEU theories), the most likely theoretical model received at least 50% likelihood for 93% of the subjects (103 out of 111).

based on preferences towards EE. The proportion of compatibility is as high as 85% (28 out of 33) for KMM model, 80% (8 out of 10) for the theory of Seo, and 60% (15 out of 25) for RRDU.

Table 6: Individual types with two-stage perspective

Compound risk and model uncertainty attitudes		Ambiguity attitude: AA ($\Pi_{EE} > 0$)	AN ($\Pi_{EE} = 0$)	AS ($\Pi_{EE} < 0$)	**Total**
SEU	ambiguity neutral	13 *(24.8)* 11.7%	**29** *(15.1)* 26.1%	1 *(3.1)* 0.9%	**43** 38.7%
KMM	ambiguity averse	**25** *(16.1)* 22.5%	2 *(9.8)* 1.8%	1 *(2)* 0.9%	**28** 25.2%
	ambiguity seeking	1 *(2.9)* 0.9%	1 *(1.8)* 0.9%	**3** *(0.4)* 2.7%	**5** 4.5%
RRDU	ambiguity averse	**15** *(9.2)* 13.5%	1 *(5.6)* 0.9%	0 *(1.2)* 0%	**16** 14.4%
	ambiguity seeking	3 *(5.2)* 2.7%	6 *(3.2)* 5.4%	**0** *(0.6)* 0%	**9** 8.1%
Seo	ambiguity averse	**5** *(2.9)* 4.5%	0 *(1.8)* 0%	0 *(0.4)* 0%	**5** 4.5%
	ambiguity seeking	2 *(2.9)* 1.8%	0 *(1.8)* 0%	**3** *(0.4)* 2.7%	**5** 4.5%
Total		**64** 57.7%	**39** 35.1%	**8** 7.2%	**111** 100%

Notes: Expected frequency under a null hypothesis of independence in parentheses.
Relative frequencies indicated in %. Fisher's exact test (2-sided): $p < 0.001$.

In Appendix C.2, we provide the results of an individual level analysis based on a different, less sophisticated, method (i.e. counting the number of choice patterns consistent with each set of theoretical predictions), and show that similar conclusions may be drawn.

6 Conclusion

We have investigated the relationship between attitudes towards different sources of uncertainty, possibly encompassing different layers, namely risk, model uncertainty and model misspecification. By doing so, our study gives a new interpretation of the mechanisms behind ambiguity preferences. Specifically, our design enables us to shed new light on the relationship between ambiguity neutrality and different types of reduction, among which reduction of compound risk, reduction of model uncertainty and reduction of model misspecification. While we find some evidence on the relationship between ROCR and ambiguity neutrality, we also find that this association is far from the almost perfect one found in other studies (Halevy, 2007; Chew et al., 2017). Rather, we find that ambiguity preferences are more tightly associated with preferences towards other layers of uncertainty. Moreover, the relationship between CR and ambiguity attitudes seems to disappear when the level of complexity of the CR is reduced, or when the subjects are quantitatively more sophisticated. On the contrary, the association between MU (or MM) and ambiguity attitudes is robust to these factors. Overall, these findings could be seen as leaning in favor of the source dependence perspective in explaining ambiguity attitudes, while also supporting the idea that complexity is another important characteristic of a source of uncertainty (Armantier and Treich, 2016).

These experimental results bear important implications for the normative interpretation of ambiguity attitudes. In particular, if one sees the violation of independence in risky choices as a departure from rationality, and if subjects who are ambiguity non-neutral are also less likely to reduce CR, then this weakens the potential for ambiguity aversion models to claim a normative status. In this case indeed, evidence would support the idea of Ellsberg's type of behaviors as violating "Bayesian rationality", which assumes probabilistic sophistication (Machina and Schmeidler, 1992, 1995) –i.e. the existence of unique subjective probabilities, together with their updating and manipulation using classic probability formulas– usually coupled with SEU. On the contrary, the stronger association between attitudes towards ambiguity and towards sources of uncertainty encompassing different layers that we document in this paper leaves room for an interpretation of Ellsberg behaviors as rational responses to situations of uncertainty where probabilities are unknown. Thus, our results question the Bayesian notion of rationality –which has traditionally been the standard approach in economics– while giving support to the work of Gilboa et al. (2008, 2009, 2010, 2012); Gilboa and Marinacci (2013); Mukerji (2009), who have recently challenged it on the grounds of the inability of Bayesian priors to reflect the DM's lack of information. Accordingly, behaving differently in the absence of objective information compared to the situations where objective information is available could also be regarded as rational. In that sense, our results leave open the possibility to use ambiguity models with normative purposes in real world problems where the information is limited.

The present investigation also expands the existing literature by empirically studying the role played by the third layer of model misspecification on preferences towards ambiguity. Even if our treatment MM can be interpreted as a form of MU –simply with the number of probability models increased– we have argued that it can serve as a good proxy for model misspecification. In many applications, the term model misspecification has been used in the sense implied by our design, i.e. for what formally can be taken as an extra layer of uncertainty (Hansen, 2014; Hansen and Marinacci, 2016). Although, strictly speaking, it may seem impossible to generate genuine unawareness in an experiment, at least without using deception, by using our proxy, insights can be obtained into the importance of model misspecification. Specifically, we have shown that our MM source is an intermediate case lying between model uncertainty and ambiguity à la Ellsberg (SE). It is also the source of uncertainty most strongly associated with ambiguity. Although the difference between MU and MM is not always substantial in our experiment (since attitudes towards MU capture a good part of the uncertainty attitude), one can use our results to infer that misspecification issues are potentially important in real-life problems, where the symmetry condition does not play any role. As such, our study highlights the importance of future theoretical developments to capture decisions under MM.

Appendix

A Descriptive statistics

In Tables A.1 and A.2, we provide the descriptive statistics of the data we collected in terms of certainty equivalents and relative premia, respectively.

Table A.1: Descriptive Statitstics of the Certainty Equivalents

	Mean	Median	Mode	SD	Minimum	Maximum	Obs
SR	9.55	10	8	3.52	0.5	19.5	120
$CR0$	9.46	10	8^*	3.59	0.5	19.5	122
$CR25$	8.12	8	8	3.56	0.5	19.5	119
$MU0$	8.31	8	6	3.80	0.5	19.5	121
$MU25$	7.75	8	6	3.71	0.5	19.5	120
$MM0$	7.87	8	6	3.48	0.5	19.5	122
$MM25$	7.43	6	6	3.58	0.5	19.5	121
SE	7.75	8	10	3.83	0.5	19.5	123
EE	7.30	6	4	3.76	0.5	19.5	119

Notes: *Multiple modes exist. The lowest value is shown.

Table A.2: Descriptive Statitstics of the (relative) Premia

	Mean	Median	Mode	SD	Minimum	Maximum	Obs
Π_{CR0}	0.102	0	0	2.48	-6	8	118
Π_{CR25}	1.402	0	0	1.99	-4	10	117
Π_{MU0}	1.176	0	0	2.52	-4	8	119
Π_{MU25}	1.812	2	0	2.70	-6	12	117
Π_{MM0}	1.714	2	0	2.51	-4	10	119
Π_{MM25}	2.096	2	0	2.89	-4	10	120
Π_{SE}	1.857	2	0	2.88	-6	12	119
Π_{EE}	2.302	2	0	2.93	-4	12	116

B Association between ambiguity neutrality and reduction of two-stage uncertainty in the literature

In this appendix, we present the results of the association between ambiguity neutrality and ROCR or ROMU previously obtained in the literature. Table B.1 reports the contingency tables relating ROCR with AN as originally presented in different studies. While in general the results seem consistent in rejecting the independence hypothesis between AN and ROCR, it has to be noted that the definitions used for AN and ROCR vary from one study to another. For example, in Halevy (2007) the condition for AN is met if the subject is indifferent between a simple risk and Ellsberg's ambiguous urn with 10 balls, while the condition for ROCR is met if the subject reduces two instances of compound risk: one with a degenerate second stage (comparable to our $CR0$) and another with a uniform distribution over 11 possible second-stage probabilities. In Chew et al. (2017), AN corresponds to indifference between a simple risk and five different instances of model uncertainty, while ROCR corresponds to reduction of five different instances of compound risk.

This heterogeneity in the definitions used for ambiguity neutrality and reduction of compound lottery makes the comparison between different studies non-trivial. Yet, overall we can observe a pattern already discussed in Section 5.3.3: ROCR is more prevalent when the complexity of the task is reduced. This is for example the case in Prokosheva (2016), where 42% of the subjects meet the condition of ROCR, defined as a single indifference between a simple risk and a $CR25$ prospect generated with an urn containing 4 balls. In the same vein, reduction is also more often observed in our study (32% of our subjects, see Table 1) using only binary risk in the first stage than in the studies using more complex cases of compound risk as in Halevy (2007); Abdellaoui et al. (2015); Chew et al. (2017); Dean and Ortoleva (2015).

To overcome the problem of comparing results using various definitions of AN and ROCR, we present in Table B.2 a comparison between our results and the ones recently obtained by Chew et al. (2017) using common definitions. In this case, AN corresponds to $\Pi_{SE} = 0$, ROCR to $\Pi_{CR0} = \Pi_{CR25} = 0$, and ROMU to $\Pi_{MU0} = \Pi_{MU25} = 0$. As can be observed in the first part of the table, the results concerning AN are very close to each other, with 62% and 63% of subjects being ambiguity non-neutral (p-value=0.844, two-sided test of equality of proportions). Yet, in our experiment 32% of subjects reduce compound risk, while only 22% did in Chew et al.'s (2017) study (p-value=0.048, two-sided test of equality of proportions).

Table B.1: Association between ambiguity neutrality and ROCR in the literature

Reduction of Compound Risk (ROCR)

	Halevy (2007)[a]			**Abdellaoui et al. (2015)**[b]			**Chew et al. (2017)**[c]		
Ambiguity neutrality	No	Yes	**Total**	No	Yes	**Total**	No	Yes	**Total**
No	113 *(95.5)* 79.6%	1 *(18.5)* 0.7%	**114** 80.3%	81 *(72.4)* 70.4%	4 *(12.6)* 3.5%	**85** 73.9%	147 *(123.6)* 78.2%	1 *(24.4)* 0.5%	**148** 78.7%
Yes	6 *(23.5)* 4.2%	22 *(4.5)* 15.5%	**28** 19.7%	17 *(25.6)* 14.8%	13 *(4.4)* 11.3%	**30** 26.1%	10 *(33.4)* 5.3%	30 *(6.6)* 16%	**40** 21.3%
Total	**119** 83.8%	**23** 16.2%	**142** 100%	**98** 85.2%	**17** 14.8%	**115** 100%	**157** 83.5%	**31** 16.5%	**188** 100%
Independence test:	Fisher's exact test (2-sided): $p < 0.001$			Fisher's exact test (2-sided): $p < 0.001$			Pearson's chi-square test: $p < 0.001$		

	Dean and Ortoleva (2015)[d]			**Prokosheva (2016)**[e]			**This paper**[f]		
Ambiguity neutrality	No	Yes	**Total**	No	Yes	**Total**	No	Yes	**Total**
No	117 *(95.7)* 79.1%	3 *(24.3)* 2%	**120** 81.1%	66 *(44.5)* 48.9%	10 *(31.5)* 7.4%	**76** 56.3%	61 *(51.8)* 52.6%	15 *(24.2)* 12.9%	**76** 65.5%
Yes	1 *(22.3)* 0.7%	27 *(5.7)* 18.2%	**28** 18.9%	13 *(34.5)* 9.6%	46 *(24.5)* 34.1%	**59** 43.7%	18 *(27.2)* 15.5%	22 *(12.8)* 19%	**40** 34.5%
Total	**118** 79.7%	**30** 20.3%	**148** 100%	**79** 58.5%	**56** 41.5%	**135** 100%	**79** 68.1%	**37** 31.9%	**116** 100%
Independence test:	Fisher's exact test (2-sided): $p < 0.001$			Fisher's exact test (2-sided): $p < 0.001$			Fisher's exact test (2-sided): $p < 0.001$		

Notes: Expected frequency under a null hypothesis of independence in parentheses. Relative frequencies indicated in %. [a]AN corresponds to $\Pi_{SE} = 0$ with 10 balls in the urn, ROCR corresponds to $\Pi_{CR0} = \Pi_{CRunif} = 0$ with 10 balls, where $CRunif$ is the CR $(L_0, 1/11; ...; L_{10}, 1/11;)$, yielding the binary lottery L_i with probability 1/11 (the binary lottery $L_i \equiv x_{i/10}0$ gives x with probability $i/10$ and 0 otherwise). [b]AN corresponds to $\Pi_{SE} = 0$ with 2 balls in the urn, ROCR corresponds indifference between the simple risk $x_{0.5}0$ and the following CR: $CR0 \equiv (x0, 1/2; x1 0, 1/2)$, $(x0, 1/3; x_{0.5}0, 1/3; x_1 0, 1/3)$, $(x0, 1/4; x_{0.5}0, 1/2; x_1 0, 1/4)$. [c]AN corresponds to indifference between the binary risk $x_{0.5}0$ and the following MU situations: $(x_{0.25}0, ?; x_{0.26}0, ?; ...; x_{0.74}0, ?; x_{0.75}0, ?)$, $(x_0 0, ?; x_{0.01}0, ?; x_{0.02}0, ?; ....; x_{0.99}0, ?; x_1 0, ?)$, $(x_0 0, ?; x_{0.01}0, ?; ....; x_{0.25}0, ?; x_{0.75}0, ?; ; x_{0.76}0, ?; ...; x_1 0, ?)$, $MU0 \equiv (x_0 0, ?; x_1 0, ?)$, $MU25 \equiv (x_{0.25}0, ?; x_{0.75}0, ?)$, where the probabilities each binary lottery is the true one are unknown (as marked by ?), ROCR corresponds to indifference between the binary risk $x_{0.5}0$ and the following CR: $(x_{0.25}0, 1/51; x_{0.26}0, 1/51; ...; x_{0.74}0, 1/51; x_{0.75}0, 1/51)$, $(x_0 0, 1/101; x_{0.01}0, 1/101; x_{0.02}0, 1/101; ....; x_{0.99}0, 1/101; x_1 0, 1/101)$, $(x_0 0, 1/52; x_{0.01}0, 1/52; ....; x_{0.25}0, 1/52; x_{0.75}0, 1/52; ; x_{0.76}0, 1/52; ...; x_1 0, 1/52)$, $CR0 \equiv (x_0 0, 1/2; x_1 0, 1/2)$, $CR25 \equiv (x_{0.25}0, 1/2; x_{0.75}0, 1/2)$. [d]AN corresponds to $\Pi_{SE} = 0$ with 40 balls in the urn, ROCR corresponds to $\Pi_{CRunif} = 0$ with 40 balls, where $CRunif$ is the CR $(L_0, 1/41; ...; L_{40}, 1/41;)$, yielding the binary lottery L_i with probability 1/41 (the binary lottery $L_i \equiv x_{i/40}0$ gives x with probability $i/40$ and 0 otherwise). [e]AN corresponds to $\Pi_{SE} = 0$ with 4 balls in the urn, ROCR corresponds to $\Pi_{CR25} = 0$ with 4 balls. [f]AN corresponds to $\Pi_{EE} = 0$, ROCR corresponds to $\Pi_{CR0} = \Pi_{CR25} = 0$.

Table B.2: COMPARISON ASSOCIATION BETWEEN AMBIGUITY NEUTRALITY AND ROCR (PART I) OR ROMU (PART II) USING COMMON DEFINITIONS

Part I

	Reduction of Compound Risk (ROCR): $\Pi_{CR0} = \Pi_{CR25} = 0$					
	Chew et al. (2017)[a]			**This paper**[b]		
Ambiguity neutrality $\Pi_{SE} = 0$	No	Yes	**Total**	No	Yes	**Total**
No	117 *(93)* 62.2%	2 *(26)* 1.1%	**119** 63.3%	60 *(50.4)* 50.4%	14 *(23.6)* 11.8%	**74** 62.2%
Yes	30 *(54)* 16%	39 *(15)* 20.7%	**69** 36.7%	21 *(30.6)* 17.6%	24 *(14.4)* 20.2%	**45** 37.8%
Total	**147** 78.2%	**41** 21.8%	**188** 100%	**81** 68.1%	**38** 31.9%	**119** 100%
Independence test:	Fisher's exact test (2-sided): $p < 0.001$			Fisher's exact test (2-sided): $p < 0.001$		

Part II

	Reduction of Model Uncertainty (ROMU): $\Pi_{MU0} = \Pi_{MU25} = 0$					
	Chew et al. (2017)[a]			**This paper**[b]		
Ambiguity neutrality $\Pi_{SE} = 0$	No	Yes	**Total**	No	Yes	**Total**
No	112 *(84.8)* 59.6%	7 *(34.2)* 3.7%	**119** 63.3%	65 *(54.1)* 54.6%	9 *(19.9)* 7.6%	**74** 62.2%
Yes	22 *(49.2)* 11.7%	47 *(19.8)* 25%	**69** 36.7%	22 *(32.9)* 18.5%	23 *(12.1)* 19.3%	**45** 37.8%
Total	**134** 71.3%	**54** 28.7%	**188** 100%	**87** 73.1%	**32** 26.9%	**119** 100%
Independence test:	Fisher's exact test (2-sided): $p < 0.001$			Fisher's exact test (2-sided): $p < 0.001$		

Notes: Expected frequency under a null hypothesis of independence in parentheses. Relative frequencies indicated in %. [a]The $MU0$ situation is represented by an deck with 100 cards, that can be either all red or all black. [b]The $MU0$ situation is represented by an Urn with only 1 ball, that can be either red or black. [c]The $MU0$ situation is represented by an Urn with 100 balls, that can be either all red or all black.

Turning to ROMU, which is presented in the second part of the table, we observe that unlike ROCR, the proportions of subjects who reduce MU are similar in the two experiments (p-value=0.728, two-sided test of equality of proportions). In terms of association, our findings are comparable to Chew et al. (2017) for the association between AN and ROMU (two-sided tests of equality of proportions in two data sets: p-value=0.081 for proportions of AN conditional on ROCR, and p=0.068 for the converse implication). Concerning the association between AN and ROCR, we found a lower association in our data than in Chew et al. (2017) (two-sided tests of equality of proportions in two data sets: p-value=0.0004 for proportions of AN conditional on ROCR, and p=0.738 for the converse implication).

C Extended individual level analysis

In this appendix, we provide further results on the individual analysis described in Section 5.4.

C.1 Descriptive validity of latent mixture model

Figure C.1 below shows the mean and 95% confidence intervals of the predicted data for the CE differences based on the posterior distributions of the latent mixture model. The predictive distributions are obtained by taking the mean of the posterior predictive samples as the predictor of each observation in the data. The resulting predictive distributions are compared with the actual data observed. The predictions are close to the actual data observed, with some over-prediction of the difference between SR and $MU25$, and the difference between $CR0$ and $MU25$. The sign of the differences are always predicted correctly.

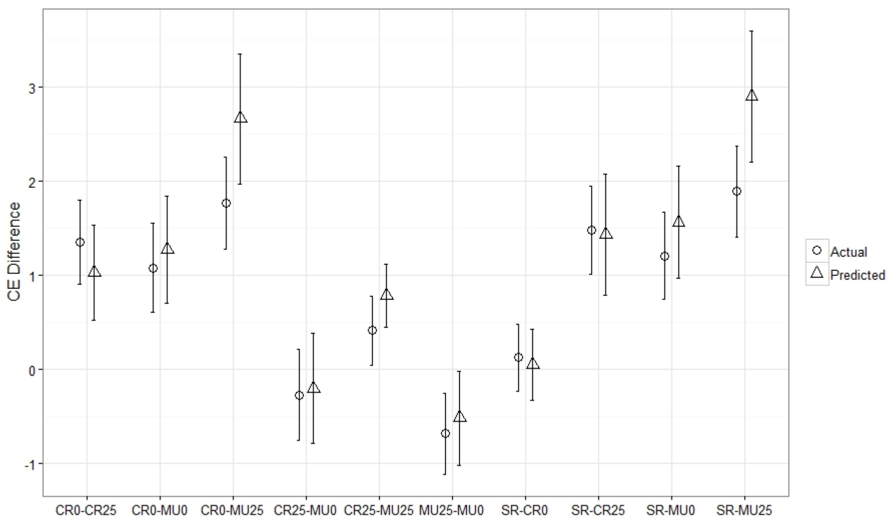

Figure C.1: Posterior Predictive Distributions of CE differences

C.2 Simple individual analysis

We here report the results of a simple individual type analysis completing the results of Section 5.4. As before, we classify subjects in terms of the proximity of the observed choices to the predictions of the theory of choice under uncertainty that specifically consider uncertainty in two stages (since we focus on SR, CR, and MU). We classify the subjects either as SEU, KMM, RRDU or Seo as before, using the 10 possible binary comparisons between SR, $CR0$, $CR25$, $MU0$ and $MU25$. For each of the 10 binary comparisons, we can count the number of consistent choice patterns of each subject for each set of theoretical predictions, and associate the subject with the type delivering the highest number of consistent choices. When this number is identical for several alternative theories, the weight received by each of them is uniformly distributed. For example, if a subject reveals five choices consistent with SEU and five choices consistent with ambiguity aversion under KMM model, a weight 1/2 is associated with each of these two theories for this subject. This procedure prevents any double counting of subjects and ensures that the sum of individual types is the same as the sample size.

Table C.1: Individual types with two-stage perspective II

Compound risk and model uncertainty attitudes		AA ($\Pi_{EE} > 0$)	AN ($\Pi_{EE} = 0$)	AS ($\Pi_{EE} < 0$)	**Total**
		Ambiguity attitude			
SEU	ambiguity neutral	7.5 *(18.5)* 6.8%	**24.5** *(11.2)* 22.1%	0 *(2.3)* 0%	**32** 28.8%
KMM	ambiguity averse	**21.83** *(16.3)* 19.7%	5.5 *(10)* 5%	1 *(2)* 0.9%	**28.33** 25.5%
	ambiguity seeking	1 *(2.9)* 0.9%	1 *(1.8)* 0.9%	**3** *(0.4)* 2.7%	**5** 4.5%
RRDU	ambiguity averse	**25.83** *(16.9)* 23.3%	3 *(10.3)* 2.7%	0.5 *(2.1)* 0.5%	**29.33** 26.4%
	ambiguity seeking	0 *(1.7)* 0%	3 *(1.1)* 2.7%	**0** *(0.2)* 0%	**3** 2.7%
Seo	ambiguity averse	**5.83** *(4.5)* 5.3%	2 *(2.8)* 1.8%	0 *(0.6)* 0%	**7.83** 7.1%
	ambiguity seeking	2 *(3.2)* 1.8%	0 *(1.9)* 0%	**3.5** *(0.4)* 3.2%	**5.5** 5%
Total		**64** 57.7%	**39** 35.1%	**8** 7.2%	**111** 100%

Notes: Expected frequency under a null hypothesis of independence in parentheses.
Relative frequencies indicated in %.

Table C.1 reports the results of the classification based on the simple individual analysis and shows their relation to the observed ambiguity attitudes. As can be observed, the results are consistent with the ones obtained using the latent mixture analysis in Section 5.4. In this simple type analysis however, we observe that 71% of the subjects (79 out of 111) are classified as non-SEU. KMM model still constitutes 30% of all observed preferences, whereas RRDU has now 29%, and the theory of Seo has 12%. Turning to the compatibility with ambiguity attitudes defined in terms of Π_{EE}, among subjects classified as SEU, 77% (24.5 out of 32) are consistent in satisfying ambiguity neutrality. As with the latent mixture model, we also observe that for the majority of non-SEU subjects, the preference patterns within CR and MU are compatible with the observed ambiguity attitudes based on preferences towards EE. The proportion of compatibility is in this case 74%

(24.83 out of 33.33) for KMM model, 80% (25.83 out of 32.33) for RRDU, and 70% (9.33 out of 13.33) for the theory of Seo.

D Order treatment

The eight situations $\{SR, CR0, CR25, MU0, MU25, MM0, MM25, EE\}$ appeared to the subjects in a random order. We test the order effects by correlating the CEs with the order of the situations. No order effect is detected. Table D.1 presents the average CEs of each situation by the order of appearance in the experiment. No correlation between certainty equivalents of the prospect in any situation and the order of the situation is significantly different from zero, neither when each situation is examined independently nor pooled together.

Table D.1: Variation in the Certainty Equivalents as a function of the order of scenarios

Order	1st	2nd	3rd	4th	5th	6th	7th	8th	**Pearson correlation**	**p-value**
SR	8.91	10.00	9.00	8.93	10.31	10.00	11.14	8.81	**0.05**	**0.60**
$CR0$	9.50	9.22	8.81	10.50	9.00	9.65	10.42	7.88	**-0.03**	**0.77**
$CR25$	7.73	9.81	8.43	8.00	8.55	7.43	7.81	7.73	**-0.08**	**0.39**
$MU0$	7.38	9.83	8.33	9.88	8.88	6.75	8.43	6.50	**-0.12**	**0.19**
$MU25$	8.31	6.55	8.00	6.80	6.90	10.27	6.92	8.90	**0.12**	**0.20**
$MM0$	8.59	8.27	7.16	7.29	7.97	7.60	7.65	8.50	**-0.03**	**0.78**
$MM25$	7.89	8.39	8.25	5.68	7.47	6.40	7.23	7.71	**-0.07**	**0.45**
EE	6.00	6.44	8.14	7.67	7.12	8.44	8.23	6.69	**0.09**	**0.30**
Pooled	***8.00***	***8.51***	***8.27***	***8.38***	***8.21***	***8.40***	***8.25***	***7.78***	**-0.02**	**0.54**

Notes: each number in columns 2 to 9 is the average of CEs of the scenario shown in the corresponding order. Pearson correlation is calculated between the scenario order and the CEs.

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