Institutional Members: CEPR, NBER and Università Bocconi

WORKING PAPER SERIES

Economic Rationality: Investigating the Links between Uncertainty, Complexity, and Sophistication

Ilke Aydogan, Loïc Berger, and Valentina Bosetti

Working Paper n. 653

This Version: October, 2019

IGIER – Università Bocconi, Via Guglielmo Röntgen 1, 20136 Milano –Italy
http://www.igier.unibocconi.it

ECONOMIC RATIONALITY: INVESTIGATING THE LINKS BETWEEN UNCERTAINTY, COMPLEXITY, AND SOPHISTICATION*

Ilke Aydogan† Loïc Berger‡ Valentina Bosetti§

Abstract

We report on a laboratory experiment measuring the preferences of a unique pool of risk professionals over various sources of uncertainty that entail different degrees of complexity. We then compare these preferences with those of a control group composed of social science students to obtain a deeper understanding of the mechanisms driving behaviors under risk and ambiguity. We find that (1) ambiguity aversion is robust to subjects' degree of sophistication in probabilistic reasoning and background. (2) An association exists between attitudes toward ambiguity and compound risk for students/less sophisticated subjects, and is mainly explained by their attitudes toward complexity. Such an association does not exist for risk professionals/more sophisticated subjects. (3) The failure to reduce compound risk emerges as a *sufficient,* but not a *necessary,* condition for ambiguity non-neutrality. These findings suggest that decision making under ambiguity cannot be reduced to decision making under risk.

Keywords: Ambiguity aversion, reduction of compound lotteries, non-expected utility, model uncertainty, model misspecification

JEL Classification: D81

*We are grateful to Hans-Joachim Zwiesler for the opportunity to run the experiment at ICA 2018 and thank the organizing committee for their help with the experiment. We also thank Diana Valerio for her help with the laboratory experiment. The research leading to these results received funding from the French Agence Nationale de la Recherche (ANR), under grant ANR-17-CE03-0008-01 (project INDUCED) and from the European Research Council under the European Community's Programme "Ideas" - Call identifier: ERC-2013-StG/ERC grant agreement n° 336703 (project RISICO "RISk and uncertainty in developing and Implementing Climate change pOlicies"). Logistic support from the Bocconi Experimental Laboratory in the Social Sciences (BELSS) for hosting our experimental sessions is kindly acknowledged.

†Department of Economics, Bocconi University, Italy (ilke.aydogan@unibocconi.it).

‡IESEG School of Management, CNRS, Univ. Lille, UMR 9221 - LEM, F-59000 Lille, France; Bocconi University, RFF-CMCC European Institute on Economics and the Environment (EIEE), and Centro Euro-Mediterraneo sui Cambiamenti Climatici, Italy (l.berger@ieseg.fr).

§Department of Economics and IGIER, Bocconi University, and RFF-CMCC European Institute on Economics and the Environment (EIEE), Centro Euro-Mediterraneo sui Cambiamenti Climatici, Italy (valentina.bosetti@unibocconi.it).

1 Introduction

In 1961, Ellsberg proposed several experiments challenging the validity of Savage's (1954) expected utility model for making decisions in situations of uncertainty. These experiments have given rise to a vast literature studying the phenomenon of ambiguity aversion (i.e. preference for known probabilities, or risk, over unknown probabilities, or ambiguity) at both theoretical and empirical levels. However, the normative status of this preference, i.e. whether it constitutes a rational response to uncertainty or an irrational form of cautiousness, possibly related to computational difficulties, remains an open question. In this paper, we shed light on this question by empirically investigating the preferences of a unique pool of risk professionals, which we compare with those of social science students. Our results demonstrate that ambiguity preferences cannot be explained solely by computational inability or irrational aversion toward complexity, leaving room for an interpretation of ambiguity aversion as a rational response to the presence of ambiguity.

Ambiguity–also called Knightian uncertainty–is present in virtually all real-life situations and plays a major role in most economic problems. For example, Giannoni (2002) shows that a *robust* optimal monetary policy rule that takes ambiguity aversion into account (in the form advocated by Hansen and Sargent, 2001, 2008) may lead to amplification rather than attenuation in the response of the optimal policy to shocks. In the health domain, Berger et al. (2013) show that ambiguity aversion affects treatment decisions when there is ambiguity in either the diagnosis or the treatment itself. Finally, Millner et al. (2013) and Drouet et al. (2015) show that ambiguity aversion has a notable effect on climate change policies.

Although ambiguity aversion has been acknowledged as one of the most important behavioral deviations from the subjective expected utility theory of Savage (1954) (hereafter SEU; see Trautmann and van de Kuilen, 2015), the rationality of ambiguity aversion, and thus the normative status of ambiguity models and their use for prescriptive applications, has been the topic of heated debates among economists (e.g. Al-Najjar and Weinstein, 2009; Gilboa et al., 2009). Recent empirical studies (e.g. Halevy, 2007; Armantier and Treich, 2016; Chew et al., 2017) have challenged the normative status of ambiguity aversion by documenting a tight association between ambiguity non-neutrality and the failure to reduce compound or complex risks, which constitutes a clear violation of rationality. This association leaves little room for a normative interpretation of ambiguity models. As this question may have profound implications for policy making in several domains, we aim to clarify the extent to which ambiguity aversion can be a genuine preference that is not associated with mistakes in probabilistic reasoning. To this end, we focus on the roles of complexity and computational limitations in reasoning about uncertainty and investigate the preferences of a unique sample of risk professionals, who have been trained to deal with computationally complex problems that require probabilistic reasoning.

Through two laboratory experiments, we explore this problem along three dimensions simultaneously. The first dimension concerns different sources of uncertainty: we investigate attitudes toward simple risk, compound risk, and ambiguity. Under classical theories of decision making, such as the SEU of Savage (1954), (a) any source of ambiguity is reduced to a source of risk through the assignment of subjective probabilities and (b) compound risk is reduced to simple risk in accordance with the reduction of compound lotteries axiom. These classical principles are prescribed to any decision makers, regardless of their background, and for any decision problems under uncertainty, regardless of their level of complexity. Thus, the second and third dimensions we consider concern the background of the subjects and the task complexity. Specifically, instead of focusing exclusively on university students, as is typical in such experiments, we study an original sample of 84 risk professionals. To reach them, we set up a temporary experimental laboratory at the 31st International Congress of Actuaries, which was held in Berlin in 2018 (see Section 3 for details). Our subjects in this experiment thus consist of a unique pool of specialists, typically working in insurance related jobs and possessing a high level of training in the fields of mathematics, statistics, and actuarial science. Given their background, these subjects carry not only a high level of quantitative sophistication but also a high level of proficiency in probabilistic reasoning. Finally, by altering the spread of the potential probability distributions in a simple experimental design, we isolate the role played by complexity in behavior under risk and ambiguity.

Our main results can be summarized as follows. (1) Both actuaries and students persistently prefer sources of risk over sources of ambiguity. Nonetheless, as expected given their training, we find that actuaries are better able to reduce compound risk than students. (2) Consequently, the tight association between attitudes toward compound risk and ambiguity, which exists for students, does not hold for risk professionals. Furthermore, the association that exists for students is mainly explained by their attitudes toward the complexity of uncertain situations. (3) Overall, and contrary to what has been reported in previous studies, when one controls for task complexity and sophistication in probabilistic reasoning, non-reduction of compound risk is *a sufficient* but not *a necessary* condition for ambiguity non-neutrality. In other words, ambiguity aversion is a behavior not necessarily grounded in irrational reasoning, so that decision making under ambiguity cannot be reduced to decision making under risk.

2 Related literature

This paper relates to several strands of the literature. The first concerns the rationality of preferences in Ellsberg-type problems. From a philosophical point of view, the proponents of Savage's (1954) SEU provide logical arguments demonstrating violations of internal consistency, and hence emphasize the normative value of SEU for decision-making under uncertainty (Raiffa, 1961). Al-Najjar and Weinstein (2009) discuss other instances in which ambiguity aversion can lead to irrational behaviors such as sensitivity to sunk costs and aversion to information. In contrast, studies by Gilboa et al. (2008, 2009, 2012); Mukerji (2009) and Gilboa and Marinacci (2013) argue that satisfying internal consistency in line with Savage's axioms is neither necessary nor sufficient for rationality of choice under uncertainty. In particular, according to these authors, ambiguity aversion is not necessarily a failure of probabilistic reasoning but can as well be a rational response in the face of situations with insufficient information.

The question concerning the rationality of ambiguity aversion has also been an implicit topic of empirical research. For example, experimental studies by Halevy (2007) and Chew et al. (2017) document tight associations between attitudes toward compound risk and ambiguity, while Armantier and Treich (2016) also report tight associations between attitudes toward ambiguity and complex risk situations.[1] These results suggest that non-reduction of compound/complex risk, which constitutes a violation of a basic rationality tenet, is *necessary* for ambiguity non-neutrality. According to these studies, Ellsberg's paradox would thus be mainly explained by failures to handle objective probabilities correctly.[2] However, Bernasconi and Loomes (1992), Abdellaoui et al. (2015), and Aydogan et al. (2018) find weaker associations, questioning therefore the possibility of completely characterizing ambiguity by means of compound risk.

Recent empirical studies on this topic also suggest the importance of controlling for other factors, such as individual characteristics (e.g., subjects' level of comprehension or sophistication) and task complexity, when studying ambiguity attitudes (Chew et al., 2018). In the paper closest to ours, Abdellaoui et al. (2015) investigate the influence of quantitative sophistication on the relationship between compound risk and ambiguity by comparing a sample of advanced engineering students with a sample of students from non-engineering fields. They find a weaker association between compound risk and ambiguity preferences among engineering students than among non-engineering students. However, they also find the reduction of compound risk among engineering students to be surprisingly low, and not different from non-engineering students, suggesting that

[1] Under the *complex* risk of Armantier and Treich (2016), the risk is not presented in multiple stages as under compound risk, but the calculation of final objective probabilities is non-trivial.

[2] Halevy (2007, p. 532) notes: "The results suggest that failure to reduce compound (objective) lotteries is the underlying factor of the Ellsberg paradox."

their degree of quantitative sophistication may not translate into proficiency in probabilistic reasoning. Along the same lines, Berger and Bosetti (2019) investigate the preferences of climate policy makers while controlling for their performance in a belief updating task. They find that the subjects who performed better in this task were also more successful in reducing compound risk, although their ambiguity attitudes were not different from those who performed worse. Their analysis only focuses on simple tasks however and is therefore silent on the role played by complexity. Lastly, Chew et al. (2018) run an experiment in which they control for subjects' cognitive abilities. Their data do not reveal any strong effects of cognitive abilities on attitudes toward compound risk and ambiguity. In comparison, the current study controls for the role of sophistication in terms of probabilistic reasoning and for the role of task complexity. Hence, we contribute to the debate on the rationality of Ellsberg choices by shedding light on the extent to which ambiguity aversion is related to a genuine preference in the face of uncertainty or to a cognitive failure to deal with complex uncertain situations.

Our study also provides some insight for the theoretical literature that explores the implications of ambiguity aversion through analytical models. This literature, motivated by Ellsberg's experiments, has proposed various models for decision making under uncertainty. The theories of Segal (1987) and Seo (2009), for instance, model ambiguity as two-stage compound risk and connect ambiguity non-neutrality to failure to reduce compound risk. In contrast, the theories of Klibanoff et al. (2005), Nau (2006), and Ergin and Gul (2009) model ambiguity as two-stage uncertainty and account for ambiguity aversion by differentiating between objective and subjective uncertainties present at each stage, without relating it to compound risk reduction (see Marinacci, 2015). Finally, the source approach of Abdellaoui et al. (2011) accounts for ambiguity attitudes by different attitudes toward sources entailing objective and subjective uncertainties without relying on a two-stage structure. Our experiment contributes to this theoretical literature by shedding new light on the descriptive validity and normative value of theories that disentangle preferences under compound risk and ambiguity.

Lastly, we contribute to a strand of the experimental literature that has studied, from a descriptive point of view, the ambiguity preferences of individuals with different backgrounds. While most of the previous studies consider Western university students as subjects, several other studies use samples from the general population (Dimmock et al., 2015, 2016), children and adolescents (Sutter et al., 2013), business owners (Viscusi and Chesson, 1999), and policymakers (Berger and Bosetti, 2019). Ambiguity preferences of actuaries are investigated in studies by Hogarth and Kunreuther (1989), Cabantous (2007) and Cabantous et al. (2011). These studies use non-incentivized surveys with decision tasks in particular insurance contexts. They typically document ambiguity aversion among insurers, who ask higher insurance premia under ambiguity than under risk. In contrast to those studies, our experiment with actuaries is, to our knowledge, the first to adopt a simple context-free design, which enables us to use real monetary incentives and allows for comparisons with other samples of subjects (in our case university students).

3 Experiments

3.1 Samples

For this study, we run two laboratory experiments. The first experiment was run on a unique pool of risk professionals (actuaries). It took place during the 31st International Congress of Actuaries (ICA) in Germany.[3] A specific conference room with 20 computers was made available for five days for the purpose of the study (see Appendix A). The experiment was organized in 12 sessions, each of which lasted approximately 45 minutes, including instructions and payment. The second experiment was run on a standard pool of university students. It was conducted at the Bocconi Experimental Laboratory for Social Sciences (BELSS) in Bocconi University, Italy. The experiment was organized in five sessions. Each session lasted approximately one hour, including instructions and payment.

Experiment 1: Actuaries at ICA We collected data from 84 subjects from 33 different countries. The majority (77%) of the subjects were from the EU, among which most were from Germany (43 out of 84). The average age was slightly less than 40 and 44% of the subjects were female. Most of the subjects (57%) did not have children and were highly educated: 58 subjects (69%) held a master's degree and 18 subjects (21%) held a PhD. 46 subjects reported that their highest degree was obtained in a field related to mathematics and statistics, while 17 subjects reported it related to actuarial sciences. The remaining subjects reported diplomas in physics (2), engineering (1), finance (1), economics and management (3), or did not report anything (14). Finally, these subjects were highly experienced in the insurance and finance industries, with an average of 13 years of relevant work experience.

Experiment 2: University students We collected data from 125 students in economics, finance, law, management, political science, or data science at Bocconi University. At the time of the experiment, 80 of them (64%) were in a bachelor's program in one of the aforementioned fields while 34 (27%) were in a master's program, and the

[3] ICA is a conference organized by the International Actuarial Association every four years that gathers more than 2,500 actuaries, academics, and high-ranking representatives from the international insurance and financial industry. The 31st congress was held from June 4 to 8, 2018, in Berlin. More information is available at https://ica2018.com/

rest (9%) were in a PhD program. 42% of the subjects were female, and the average age was 20.5.

3.2 Design

We use a within subject design to study individual choices under simple risk, compound risk, and ambiguity. For ambiguity, we follow Marinacci (2015) and consider instances of *model uncertainty*, in which a set of possible probability distributions (or models) is explicitly given, and there is subjective uncertainty about the true distribution within this set.[4] The experiment entails betting on the color of a card drawn from a deck in different situations. We consider the following four distinct *sources* of uncertainty[5] that are constructed in a standard two-color Ellsberg setting.

1. (R) *Risk* entails a deck that contains an equal proportion of black and red cards.

2. (CR) *Compound Risk* entails, with equal probabilities, either a deck that contains p% red and $(1-p)$% black cards, or a deck that contains p% black and $(1-p)$% red cards.

3. (MU) *Model Uncertainty* entails, with unknown probabilities, either a deck that contains p% red and $(1-p)$% black cards, or a deck that contains p% black and $(1-p)$% red cards.

4. (E) *Ellsberg* ambiguity entails a deck of 100 cards that contains an unknown proportion of black and red cards.

The sources CR and MU explicitly entail two stages of uncertainty with two possible deck compositions in the second stage. They differ from each other in the type of uncertainty they entail in the first stage. Specifically, the two possible deck compositions are unambiguously assigned objective probabilities of 0.5 under CR, whereas these probabilities are unknown in the case of MU. On the basis of a symmetry argument, probabilities of 0.5 could be assigned to the two possible decks under MU, but these probabilities are then subjectively (rather than objectively) determined. Note that, strictly speaking, E is also an instance of model uncertainty, where there are 101 possible deck compositions (Marinacci, 2015).

[4] We also examined instances of model misspecification (Hansen, 2014; Hansen and Marinacci, 2016), in which there is also uncertainty about the set of possible distributions. However, those situations are the focus of another study and are not reported here (see Aydogan et al., 2018).

[5] Sources of uncertainty are defined as "groups of events that are generated by the same mechanism of uncertainty, which implies that they have similar characteristics" (Abdellaoui et al., 2011, p. 696).

3.3 Procedure

Both experiments were run on computers in English. Subjects were recruited on a voluntary basis and could sign up in advance for a particular time slot. Similar recruitment procedures were applied in both experiments. The experiment at ICA was advertised on the conference website and through notifications on the conference app. The participants could register online for an available time slot directly. For the experiment at the university, an internal recruitment system was used and participants could directly register online. Subjects gave their consent prior to the experiment by signing an informed consent document. The experiments were anonymized. In both cases, each subject was authorized to participate only once. The experiments were organized into different sessions taking place over several days. Each session started with the experimental instructions, examples of the stimuli, and comprehension questions. Subjects could not communicate with each other during the experiments. Complete instructions are presented in the Online Appendix.

Stimuli For both CR and MU, we consider two distinct cases that are characterized by different levels of complexity. The first case is with $p = 0\%$, which means that the deck features a degenerate distribution: it contains either only red cards or only black cards. We denote the corresponding situations $CR0$ and $MU0$. We consider this case because of its minimal computational complexity. In particular, although it is presented in two stages, all the uncertainty stems only from the first stage. This case also provides a test of a time neutrality condition (Segal, 1987), i.e. indifference between uncertainty resolving in the first or in the second stage. The second case considers $p = 25\%$, which means that the deck contains either 25% red (and 75% black) cards or 25% black (and 75% red) cards. We denote the corresponding situations $CR25$ and $MU25$. The degree of computational complexity in these situations is higher, as uncertainty is present in both stages. In addition, this case presents a mean-preserving contraction of second stage probabilities with respect to the case with $p = 0\%$. This enables us to observe the attitudes toward spreads in first-order probabilities as modeled by the smooth model of Klibanoff et al. (2005). Studying these two distinct cases also allows for comparisons with previous studies (see Section 5.3).

The implementation of CR and MU was as follows. Subjects were presented with a pile of decks and told that one deck would be picked randomly from that pile. In $CR0$ ($CR25$), exactly 50% of the decks in the pile contained 0% red–100% black (25% red–75% black) cards and the remaining 50% of the decks contained 0% black–100% red (25% black–75% red) cards. The situations under MU were similar, except that the proportions of the decks with different compositions in the pile were unknown. To implement R and E, the subjects were presented with a single deck of cards. In particular, the deck contained an equal proportion of red and black cards in R, and an unknown proportion of red and black cards in E. All the decks and piles were constructed before the experiment by one of the authors, who was not present in the room during the experimental sessions. The subjects were informed about this to avoid the effects of comparative ignorance (Fox and Tversky, 1995), i.e. an extra aversion to ambiguity induced by a comparison with a more knowledgeable someone (in this case, the experimenter).

For each of the six situations $(R, CR0, CR25, MU0, MU25, E)$, the subjects faced a bet on on the color of a card randomly drawn from the deck. For every bet, the winning color was determined by the subjects themselves. We elicited the certainty equivalents (CEs) of the bets using a choice-list design, in which subjects were asked to make 12 binary choices between the bet and a sure monetary amount (see Figure 1). We take the

Option 1	Option 2
Have €200 if the card drawn is black	Have €0 for sure
Have €200 if the card drawn is black	Have €10 for sure
Have €200 if the card drawn is black	Have €30 for sure
Have €200 if the card drawn is black	Have €50 for sure
Have €200 if the card drawn is black	Have €70 for sure
Have €200 if the card drawn is black	Have €90 for sure
Have €200 if the card drawn is black	Have €110 for sure
Have €200 if the card drawn is black	Have €130 for sure
Have €200 if the card drawn is black	Have €150 for sure
Have €200 if the card drawn is black	Have €170 for sure
Have €200 if the card drawn is black	Have €190 for sure
Have €200 if the card drawn is black	Have €200 for sure

Figure 1: Example of a typical choice list faced by subjects in the experiment

midpoint of an indifference interval implied by a switching point on the list as a proxy for the CE of a bet. The order of the bets was randomized, except for E, which was always presented at the end, as it was the only situation in which the number of cards in the deck was explicitly mentioned.[6] After completing the choice lists, the subjects answered a short survey with demographic questions.

[6]The objective here was to prevent a priming effect about the pre-specified number of cards in the decks. Although this is not directly relevant for the sources of uncertainty considered in the current study, it is crucial for constructing model misspecification (see Aydogan et al., 2018). As the same ordering was implemented for both groups of subjects, this design feature does not affect our conclusions.

Incentives The two experiments were incentivized, with different stakes offered for correct bets in view of the income gap between the two groups. In the experiment with actuaries, bets yielded either €200 or €0, while in the experiment with students, they yielded either €20 or €0. To equate the expected experimental costs in the two experiments and limit monetary transactions at ICA, only a fraction of actuaries (one out of 10) was paid based on one of their choices (i.e. between-subject random incentives),[7] whereas all student subjects in the lab were paid based on one of their choices in the experiment (i.e. within-subject random incentives).[8] At the end of the experiments, actuary subjects were offered goodies and drinks for their participation, and student subjects were offered a €5 participation fee.

In both experiments, the choice question implemented for determining the payment was selected randomly prior to the choices and resolutions of uncertainty (Johnson et al., 2015). Specifically, in the experiment with actuaries, each subject randomly drew, at the beginning of each experimental session, a sealed envelope that contained one of the uncertain situations printed on paper and another sealed envelope that contained one of the choice questions from the corresponding choice lists. The envelopes were kept by the subjects until the end of the experiment and opened only if the subject was selected to play his/her choice for real. A similar prior incentive system was employed in the experiment with students. In this case, the same pre-selected choice situation was implemented for determining the payment of all the subjects in the same session. In practice, at the beginning of the experiment, one volunteer in each session randomly drew an envelope containing one of the uncertain situations and another envelope containing one of the choice questions from the choice lists. The two sealed envelopes were attached to a board visible to all subjects. The contents of the envelopes were revealed only at the end of the experiment and every subject in the same session was paid based on his/her recorded decision in the choice situation corresponding to the contents of the envelopes. In both experiments, the draws from the piles and/or from the decks were made (physically) in front of the subjects according to the uncertain situation contained in the envelope. This prior incentive system ensured that subjects in both experiments made their choices knowing that every choice question could be implemented with equal chance.

[7] In practice, every actuary subject drew one sealed envelope from a box of envelopes, among which one out of 10 contained an image of a happy face, while the rest contained a sad face. Among the subjects whose envelope contained a happy face, one of their choices was implemented for real. In order to induce choices conditional on being selected to play for real, the draws were made before the experiment started, and the subjects kept their envelopes sealed until the end of the experiment.

[8] Previous studies in the literature on random incentive systems have not indicated behavioral differences between these two systems (Beaud and Willinger, 2015; Charness et al., 2016).

4 Indexes of compound risk and ambiguity attitudes

To examine attitudes toward compound risk and ambiguity, we introduce the following measure.

Definition 1. The relative *premium* Π_i for a bet on $i \in \{CR0, CR25, MU0, MU25, E\}$ is the difference between the CE of the bet on the simple risk (CE_R) and the CE of the bet on the uncertain situation i (CE_i), expressed in % of the CE of the bet on the preferred situation. Mathematically, it is defined as

$$\Pi_i \equiv \frac{CE_R - CE_i}{\max\{CE_R, CE_i\}} \quad \forall i \in \{CR0, CR25, MU0, MU25, E\}. \tag{1}$$

Intuitively, two cases can be distinguished. If the individual is averse to compound risk or to ambiguity, the preferred bet is the simple risk R, and the relative premium represents the percentage of extra money that an individual is ready to sacrifice to avoid betting on compound risk or on ambiguity, relative to the value of the bet on simple risk. Instead, if the individual is compound risk (ambiguity) seeking, the preferred situation is compound risk (ambiguity), and symmetrically, the relative premium Π_i represents the extra money that would be sacrificed to avoid betting on the simple risk, relative to the value of the bet on compound risk (ambiguity). This measure of the premium presents some desirable properties. First, the relative premium Π_i is symmetric around zero across averse and seeking preferences. Second, the relative premium Π_i belongs to the interval $[-1; 1]$, which also makes it easy to interpret in terms of percentages. Lastly, the normalization with respect to the maximum certainty equivalent allows more robust comparisons of actuaries and students by controlling for payoff differences in addition to the subjects' level of simple risk attitudes. In Appendix S3, we further provide a comparison of our definition with alternative definitions of premium that use normalizations with respect to expected value, CE_R (Trautmann and van de Kuilen, 2015), or $CE_R + CE_i$ (Sutter et al., 2013). We also show that our conclusions do not differ when using such alternative definitions.

5 General results

5.1 Quality of data and consistency

Our data consist, for every subject, of six choice lists, each of which corresponds to one uncertain situation described in Section 3.3. Four actuaries were discarded from the analysis as they show multiple-switching, reverse-switching, or no-switching patterns in all six choice lists, suggesting a lack of sufficient attention to the tasks in the experiment.

Without these subjects, the proportion of such inconsistencies amounts to 3.3% in both groups (16 out of 480 choice lists for actuaries and 25 out of 750 choice lists for students). These observations were not included in the following analysis as they do not imply clear measurement of CE and may be due to confusion on the subjects' part. We do not observe any order effects on treatments for the actuaries or students (see Online Appendix S2).

5.2 Attitudes toward ambiguity and compound risk

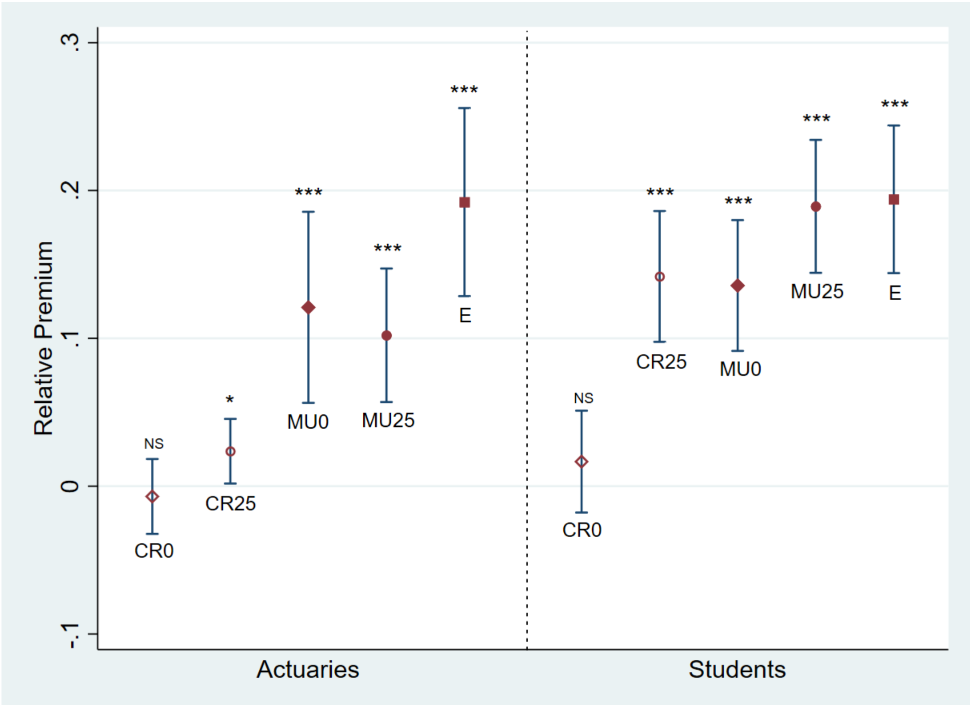

Figure 2: Mean relative premia and corresponding 95% confidence intervals for the actuaries and students. Asterisks indicate differences of relative premia from zero: *** significant at 0.001, ** significant at 0.01, * significant at 0.05, NS not significant.

Figure 2 summarizes the statistics on relative premia that are calculated based on Definition 1 (descriptive statistics are provided in Appendix B). Focusing on the left part of Figure 2, we see that actuaries exhibit aversion toward MU and E, paying positive premia for these sources (t-tests, p<0.001 for all)[9]. The average relative premium for $CR0$ does not differ from zero (t-test, $p = 0.59$) and is slightly positive for $CR25$ (t-test, $p = 0.14$ after bonferroni correction). Overall, actuaries exhibit different attitudes toward the sources CR and MU (MANOVA with repeated measures, p<0.001). Comparing with students on the right part of Figure 2, the differences between the two

[9]Testing multiple hypotheses may require Bonferroni corrections. Here, we report Bonferroni corrections only when they affect the test result.

groups are particularly marked for $CR25$ (t-tests, p<0.001). The distinction between CR and MU are also present in the student sample (MANOVA with repeated measures, p<0.001). In both samples, E has the highest relative premia among all uncertain situations.

5.3 Correlations of relative premia

Table 1 reports the Spearman's rank correlation coefficients of the relative premia across the cases of CR, MU, and E. As can be observed from the right panel, the correlations of CR with MU and E are all relatively high and significantly positive for the students, except the correlation between $CR0$ and E (p=0.26). These correlation coefficients are comparable to those observed in the literature (for a direct comparison with Chew et al., 2017, see Appendix C). Turning to the actuaries (shown on the left panel), we observe weaker correlations between CR and the sources of ambiguity. In fact, for these subjects, the only correlation that is significant between these sources is the one between $MU0$ and $CR25$ (p=0.03). By contrast, the correlations between the premia of MU and E are consistently significant and highly positive, with similar magnitudes across the two groups of subjects.

Table 1: Spearman correlations between premia

	Actuaries				**Students**		
	Π_{MU0}	Π_{MU25}	Π_E		Π_{MU0}	Π_{MU25}	Π_E
Π_{CR0}	0.023	-0.083	-0.065	Π_{CR0}	0.244**	0.291**	0.108
Π_{CR25}	0.253*	0.186	0.146	Π_{CR25}	0.400***	0.635***	0.446***
Π_E	0.597***	0.586***	1	Π_E	0.617***	0.626***	1

Notes: *** significant at 0.001, ** significant at 0.01, * significant at 0.05.

5.4 Ambiguity neutrality and reduction

Table 2 reports the proportions of subjects reducing compound risk or exhibiting ambiguity neutrality (in the sense of having relative premia equal to zero), along with those observed in some recent studies in the literature. As expected, actuaries exhibit higher abilities in probabilistic reasoning. In particular, these subjects reveal significantly higher proportions of reduction under CR than the student subjects (whether those in the current study or those in the previous studies of Halevy, 2007; Abdellaoui et al., 2015; and Chew et al., 2017). The proportions of ambiguity neutrality among actuaries are also higher than those among students. Nonetheless, confirming the distinction between CR and MU, the proportion of reduction under CR is higher than

Table 2: Reduction of Uncertainty and Ambiguity Neutrality

	Literature				This study	
	Halevy (2007)	Abdellaoui et al. (2015)	Chew et al. (2017)	Berger and Bosetti (2019)	Actuaries	Students
Reduction of Compound Risk						
$\Pi_{CR0} = 0$	33.1%	34.8%	38.3%	64.6%	82.4%	58.5%^{1}
$\Pi_{CR25} = 0$			30.3%		82.7%	43.6%^{1}
Ambiguity Neutrality						
$\Pi_{MU0} = 0$			42.6%	30.7%***	63.5%**	38.7%^{**1}
$\Pi_{MU25} = 0$			37.2%		65.8%**	39.3%^{1}
$\Pi_{E} = 0$	19.7%	26.1%	36.7%	23.9%	57.3%	37.8%^{2}

Notes: Frequencies are indicated in %. Asterisks indicate differences of model uncertainty from respective cases of compound risk: *** significant at 0.001, ** significant at 0.01, * significant at 0.05. The tests are based on McNemar's χ^2. Numbers indicate differences of students from actuaries: ^{1} significant at 0.001, ^{2} significant at 0.01, ^{3} significant at 0.05. The tests are based on two-sample tests of proportions.

that under the corresponding cases of MU for both group of subjects in the current study.

6 The relationship between ambiguity preferences, complexity, and sophistication

In this section, we examine the links between ambiguity preferences, complexity, and an index of the subjects' degree of sophistication in probabilistic reasoning. Rather than simply identifying sophistication with the subjects' background, we adopt a behavioral definition that is based on the subjects' ability to reduce compound probabilities under risk.

Definition 2. A subject is said to be *sophisticated* if she exhibits consistent choices[10] and behaves in accordance with the reduction of compound lotteries axiom. Mathematically, the condition is satisfied when

$$\Pi_{CR0} = \Pi_{CR25} = 0. \tag{2}$$

This behavioral definition accommodates the fact that some students might possess at least as much sophistication in probabilistic reasoning as the actuaries (although we

[10]The subjects who exhibit multiple- or no switching patterns in any of the choice lists are regarded as non-sophisticated, as these cases demonstrate clear violations of monotonicity.

expect the index to be correlated with the background of subjects). Table 3 reports the proportions of non-sophisticated and sophisticated subjects based on Definition 2. In line with our predictions, our index of sophistication is related to the subjects' background (Fisher's exact test, $p<0.001$). In particular, the proportion of sophisticated subjects is 70% among the actuaries, which is larger than the 30% among the students ($p<0.001$).

Table 3: Classification of subjects

	Actuaries	Students
Non-sophisticated	**30.3%** *(23/76)*	**70%**[1] *(84/120)*
Ambiguity neutral	8.7% *(2/23)*	6% *(5/84)*
Ambiguity non-neutral	91.3% *(21/23)*	94% *(79/84)*
Sophisticated	**69.7%** *(53/76)*	**30%**[1] *(36/120)*
Ambiguity neutral	71.7% *(38/53)*	52.8%[3] *(19/36)*
Ambiguity non-neutral	28.3% *(15/53)*	47.2%[3] *(17/36)*

Notes: Numbers indicate differences of students from actuaries: [1] significant at 0.001, [2] significant at 0.01, [3] significant at 0.05. The tests are based on proportion tests. Four actuaries and five students are not reported here because their ambiguity attitudes cannot be determined (see Section 6.1, footnote 11).

6.1 Ambiguity neutrality and sophistication

To examine the links between ambiguity preferences and sophistication, we first adopt a comprehensive definition of ambiguity neutrality that encompasses neutrality under both cases of MU and E. This leads us to a classification of individuals into two categories: ambiguity neutral individuals, for whom Π_i=0 $\forall i \in \{MU0, MU25, E\}$, and ambiguity non-neutral individuals, for whom this does not hold for at least one situation.[11]

Table 3 shows persistent proportions of ambiguity non-neutrality among sophisticated subjects, which amount to 28% among actuaries and 47% among students.[12] This difference in ambiguity non-neutrality between sophisticated actuaries and sophisticated students is mainly due to the higher proportion of perfectly Bayesian subjects (i.e. who reduce compound risk and, at the same time, are neutral toward ambiguity) among

[11] The ambiguity attitude of subjects who have missing observations due to multiple- or no switching patterns is determined based on the remaining non-missing observations of premia. Accordingly, a subject is said to be ambiguity neutral if all the non-missing premia for $MU0$, $MU25$, and E are zero, and non-neutral if any of the non-missing premia is non-zero. We cannot determine ambiguity attitudes for four actuaries and five students as the ambiguity premia are missing for all three situations. These subjects do not enter the analysis.

[12] The majority of these sophisticated and ambiguity non-neutral subjects (12 out of 15 actuaries and 12 out of 17 students) paid non-zero premium in at least two out of three ambiguous situations (i.e. $MU0$, $MU25$, and E).

the actuaries. Specifically, while the proportion of perfectly Bayesian subjects is 50% (38 out of 76) among actuaries, it is only 16% (19 out of 120) among students.[13] This finding might be explained by actuaries' familiarity with the Bayesian approach, coming from their occupational practice and training.[14]

The finding that ambiguity non-neutrality is still observed among sophisticated subjects who are perfectly capable of reducing compound risk implies that the failure to reduce compound risk may not be considered as a *necessary* condition for ambiguity non-neutrality. However, our data provides evidence that the failure to reduce compound risk is a *sufficient* condition for ambiguity non-neutrality. Indeed, we show that almost all non-sophisticated subjects exhibit ambiguity non-neutrality: the proportion of non-neutrality is 91.3% for actuaries (21 out of 23) and 94% for students (79 out of 84).

6.2 Complexity and sophistication

We now examine the role played by task complexity and the way our subjects respond to it in relation to their degree of sophistication in probabilistic reasoning. Recall that for each of the two sources CR and MU, we have two distinct cases: one in which there are either $p = 0\%$ red or $p = 0\%$ black cards in the deck and another in which there are either $p = 25\%$ red or $p = 25\%$ black cards in the deck. The former is designed to entail less *complexity* than the latter, in the sense that it is more easily reducible to a simple risk situation (see discussion above). We then define the notion of complexity premium in analogy with the definition of relative premium.

Definition 3. The *complexity premium* K_i for the source $i \in \{CR, MU\}$ is the difference between the CEs of the bets in the less and more complex cases of i, expressed in % of the CE of the bet on the preferred situation. Mathematically, it is defined as:

$$K_i \equiv \frac{(CE_{i0} - CE_{i25})}{\max\{CE_{i0}, CE_{i25}\}} \quad \forall i \in \{CR, MU\}. \tag{3}$$

Intuitively, the complexity premium measures the strength of preference for less complex situations over more complex ones. It belongs to the interval $[-1; 1]$. The interpretation is analogous to the one in Definition 1. If the individual is indifferent between the more and less complex situations, her complexity premium is equal to 0. If she prefers less complex situations, her premium is positive and represents the percentage of extra

[13] Among the 38 sophisticated actuaries exhibiting ambiguity neutrality, 18 (or 47%) are also risk neutral (i.e. they exhibit $CE_R =$ €100). These subjects (24% of all actuaries, or 18/76) are fully expected *value* maximizers. By contrast, only five students (out of 120) are fully expected value maximizers.

[14] Indeed, informal post-experiment interviews with some actuaries confirmed the use of Bayesian arguments to justify the equal treatment of subjective and objective probabilities in the choices they made.

money she is ready to sacrifice to avoid betting on the more complex case of source i, relative to the value of the bet on the less complex case. Alternatively, if the more complex situation is more desirable, the complexity premium K_i is negative and represents the extra money that an individual would be ready to sacrifice to avoid betting on the less complex case of source i, relative to the value of the bet on the more complex case.

It is important to note that different factors can be at play when evaluating the complexity premia under CR and MU. The first one, associated with the level of difficulty of the calculations, implies positive premia under aversion toward complexity. This factor may be at play under both sources, CR and MU. The second factor, associated with the degree of ambiguity, is only relevant under MU. Specifically, $MU0$ has a larger spread of first-order probabilities (0% and 100%) than $MU25$ (75% and 25%), which may be interpreted as a higher degree of ambiguity (see Jewitt and Mukerji, 2017; Berger, 2019). Thus, a preference for less ambiguous situations would imply a preference for $MU25$ over $MU0$ and therefore a negative complexity premium. As both factors are at play under MU, the sign of the premium will depend on which of the two factors prevails.

Table 4 reports the mean complexity premia paid under CR and MU. Part I shows that the average complexity premium among actuaries is positive under CR (t-test, $p = 0.04$) and negative under MU, although this latter effect is not significant (t-test, $p = 0.09$). In contrast, students pay on average positive complexity premia for both CR and MU (t-tests, $p < 0.001$ and $p = 0.01$ respectively). These premia are furthermore larger than those paid by the actuaries (t-tests, p=0.002 for CR and p=0.004 for MU), suggesting that students are affected by complexity more than actuaries. For both groups of subjects, the complexity premium is higher under CR than under MU (t-tests, p=0.02 for actuaries, and p=0.01 for students), which is possibly due to the increasing degree of ambiguity under MU.

Part II of Table 4 further examines complexity premia in relation to the subjects' degree of sophistication. Here, we pool the samples of actuaries and students and distinguish them solely based on their degree of sophistication in probabilistic reasoning.[15] For non-sophisticated subjects, we observe that the average complexity premia are positive under both CR and MU (t-test, $p < 0.001$ and $p = 0.03$, respectively), and higher under CR than under MU ($p = 0.005$). Turning to the sophisticated subjects, while they are not affected by complexity under CR ($K_{CR} = 0$ by Definition 2), their average complexity premium is negative under MU (t-test, $p = 0.04$), which suggests sensitivity toward different degrees of ambiguity among these subjects. This interpretation is further explored in the next subsection.

[15] Actuaries and students within the classes of sophisticated and non-sophisticated do not differ from each other (t-tests, p>0.05 for all).

Table 4: Mean Complexity Premia

Part I: By background

	Actuaries	Students
Compound Risk	0.031^{*}	0.123^{***2}
Model Uncertainty	-0.057^{+}	0.051^{*+2}

Part II: By sophistication level

	Sophisticated	Non-sophisticated
Compound Risk	$0\ ^{a}$	0.161^{***1}
Model Uncertainty	-0.051^{*+}	0.059^{*++2}

Notes: ^{a} For sophisticated subjects, the complexity premium under compound risk is zero by definition. Asterisks indicate whether the complexity premium significantly differs from zero. *** significant at 0.001, ** significant at 0.01, * significant at 0.05. Plus signs indicate differences of model uncertainty from compound risk: $^{+++}$ significant at 0.001, $^{++}$ significant at 0.01, $^{+}$ significant at 0.05. Numbers indicate differences between the two groups: students from actuaries in Part I, and non-sophisticated from sophisticated in Part II: ^{1} significant at 0.001, ^{2} significant at 0.01, ^{3} significant at 0.05. The tests are based on t-tests.

6.3 Ambiguity preferences and complexity

To complete the picture, we now study the direct link between attitudes toward complexity and attitudes toward Ellsberg ambiguity. Table 5 reports Spearman's correlation coefficients between relative premia for Ellsberg ambiguity (Π_E) and complexity premia under CR (K_{CR}) and MU (K_{MU}). As discussed in the previous section, the premium K_{CR} embeds individuals' reaction to complexity alone, while K_{MU} may also encompass a form of ambiguity attitude. According to this interpretation, if lower values of K_{MU} are induced by an aversion to more ambiguous situations, they should be associated with higher values of Π_E (i.e. higher aversion toward Ellsberg ambiguity), so that these two premia are negatively correlated. In particular, this should especially be the case for sophisticated subjects, for whom complexity has no role under CR, so that K_{MU} is expected to capture the effect due to the degree of ambiguity.

As can be observed in Part I, Ellsberg ambiguity is negatively correlated with complexity under MU (p=0.01) for actuaries, whereas its correlation with complexity under CR is positive, but not significant (p=0.13). Conversely, for students, we observe a positive and significant correlation between Ellsberg ambiguity and complexity under CR (p<0.001), but no significant correlation between Ellsberg ambiguity and complexity under MU (p=0.56). Looking at the correlations for sophisticated and non-sophisticated subjects in Part II, we observe a very similar pattern: Ellsberg ambiguity is negatively correlated with complexity under MU for sophisticated subjects, and positively correlated with complexity under CR for non-sophisticated subjects.

Table 5: Spearman correlation between ambiguity premium and complexity premium

Part I: By background

	Π_E	
	Actuaries	Students
K_{CR}	0.178	0.321***
K_{MU}	-0.296*	0.054

Part II: By sophistication level

	Π_E	
	Sophisticated	Non-sophisticated
K_{CR}	0 [a]	0.302**
K_{MU}	-0.347***	-0.030

Notes: [a] For sophisticated subjects, the complexity premium under compound risk is zero by definition. Asterisks indicate whether the complexity premium significantly differs from zero. *** significant at 0.001, ** significant at 0.01, * significant at 0.05. The tests are based on t-tests.

Overall, these results suggest that ambiguity attitudes in the sense of Ellsberg (1961) are driven by different factors for subjects with different backgrounds and different levels of sophistication in probabilistic reasoning. For students/non-sophisticated subjects, attitudes toward Ellsberg ambiguity are not only tightly linked to compound risk and model uncertainty attitudes (as shown in Table 1), but also to attitudes toward complexity under compound risk. For actuaries/sophisticated subjects, we have already seen that Ellsberg ambiguity attitudes are more closely linked to attitudes toward model uncertainty, while they now appear to be also affected by higher level properties, such as an aversion to spreads in first-order probabilities (i.e. a measure of the degree of ambiguity in several models of choice; see above and also Jewitt and Mukerji, 2017).

7 Conclusion

Although the shortcomings of Savage's (1954) expected utility theory for making decisions in the presence of ambiguity have been recognized at least since Ellsberg (1961), models of ambiguity aversion have nonetheless received relatively little attention in applications with prescriptive purposes. One reason for this can be found in the ongoing debate about the mechanisms behind ambiguity aversion and the related issue concerning its normative status. In this paper, we attempt to tackle this question from an empirical point of view. We examine the preferences over different sources of uncertainty–entailing objective and subjective probabilities, and different levels of complexity–of a unique pool of risk professionals, who are trained to deal with computationally complex situations. Comparing their preferences with those of a control group of social science students, we gain insights about the driving mechanisms behind risk and ambiguity attitudes.

There are several key findings to take away from our study. (1) Ambiguity aversion is robust to the degree of sophistication in probabilistic reasoning and to the background of the subjects. Different sources of uncertainty are treated differently, in line with what was previously observed in Abdellaoui et al. (2011). In particular, sources entailing objectively known probabilities are systematically preferred to sources in which these probabilities are unknown and therefore need to be subjectively determined. (2) The strong relationship between attitudes toward compound risk and ambiguity that is observed with students does not hold for risk professionals. Specifically, while our results suggest that there is an association between ambiguity and compound risk attitudes for the students or less sophisticated subjects, we find that this is mainly explained by their attitudes toward the complexity of the compound risk tasks. In contrast, the association between ambiguity and compound risk attitudes is not robust for risk professionals or more sophisticated subjects. For these groups of subjects, the effect of complexity is not found to play a significant role in preferences, and attitudes toward ambiguity are distinct from those toward compound risk. (3) We find that the failure to reduce compound risk is a *sufficient,* but not a *necessary,* condition for ambiguity non-neutrality. Indeed, a non-negligible proportion of sophisticated subjects who reduce compound risk are still ambiguity non-neutral. This result shows that the failure to reduce compound risk is not the only factor underlying ambiguity aversion.

These findings have several implications. First, by demonstrating that compound risk and ambiguity attitudes may not be tightly linked, depending on subjects' background/ sophistication, our findings leave room for normative interpretations of ambiguity aversion. Second, our findings also highlight the importance of testing alternative theories using different groups of subjects to clarify the mechanisms underlying revealed preferences. Third, similar to Armantier and Treich (2016), our findings shed light on the role of complexity while accounting for ambiguity attitudes. However, by taking the level of sophistication in probabilistic reasoning into account, our findings do not support the view that ambiguity aversion is a special case of complexity aversion. Empirically demonstrating that ambiguity non-neutrality is not only confined to under-trained and non-sophisticated decision makers, our study calls for a greater role of ambiguity attitudes in economic models.

Appendix

A Further experimental details

Figure A.1 illustrates the way the experimental lab was set up at the ICA 2018 conference.

Figure A.1: Left: The temporary laboratory room during ICA 2018. Right: example of a typical session with risk professionals (actuaries).

B Descriptive statistics

Tables B.1 and B.2 show the descriptive statistics of the data we collected in both experiments. Results are presented in terms of certainty equivalents and relative premia, respectively. Figures B.1 and B.2 present the distributions of the CEs for the six uncertain situations $(R, CR0, CR25, MU0, MU25, E)$ in the two experiments.

Table B.1: Descriptive Statitstics of the CEs

Actuaries

	Mean	SD	Min	Max	Obs.
R	89.08	34.07	5	195	76
$CR0$	91.38	37.00	5	195	76
$CR25$	87.60	34.20	5	195	77
$MU0$	78.31	40.79	5	195	77
$MU25$	80.57	37.64	5	195	79
E	71.20	39.70	5	195	79

Students

	Mean	SD	Min	Max	Obs.
R	9.55	3.52	0.5	19.5	120
$CR0$	9.46	3.59	0.5	19.5	122
$CR25$	8.12	3.56	0.5	19.5	119
$MU0$	8.31	3.80	0.5	19.5	121
$MU25$	7.75	3.71	0.5	19.5	120
E	7.75	3.83	0.5	19.5	123

Table B.2: Descriptive Statitstics of the relative Premia

Actuaries

	Mean	SD	Min	Max	Obs.
Π_{CR0}	-0.007	0.109	-0.6	0.286	74
Π_{CR25}	0.023	0.094	-0.2	0.429	75
Π_{MU0}	0.121	0.279	-0.5	0.95	74
Π_{MU25}	0.106	0.198	-0.286	0.8	76
Π_E	0.190	0.273	0	0.95	75

Students

	Mean	SD	Min	Max	Obs.
Π_{CR0}	0.010	0.186	-0.5	0.667	118
Π_{CR25}	0.136	0.234	-0.333	0.8	117
Π_{MU0}	0.130	0.238	-0.5	0.6	119
Π_{MU25}	0.181	0.246	-0.375	0.857	117
Π_E	0.191	0.272	-0.429	0.95	119

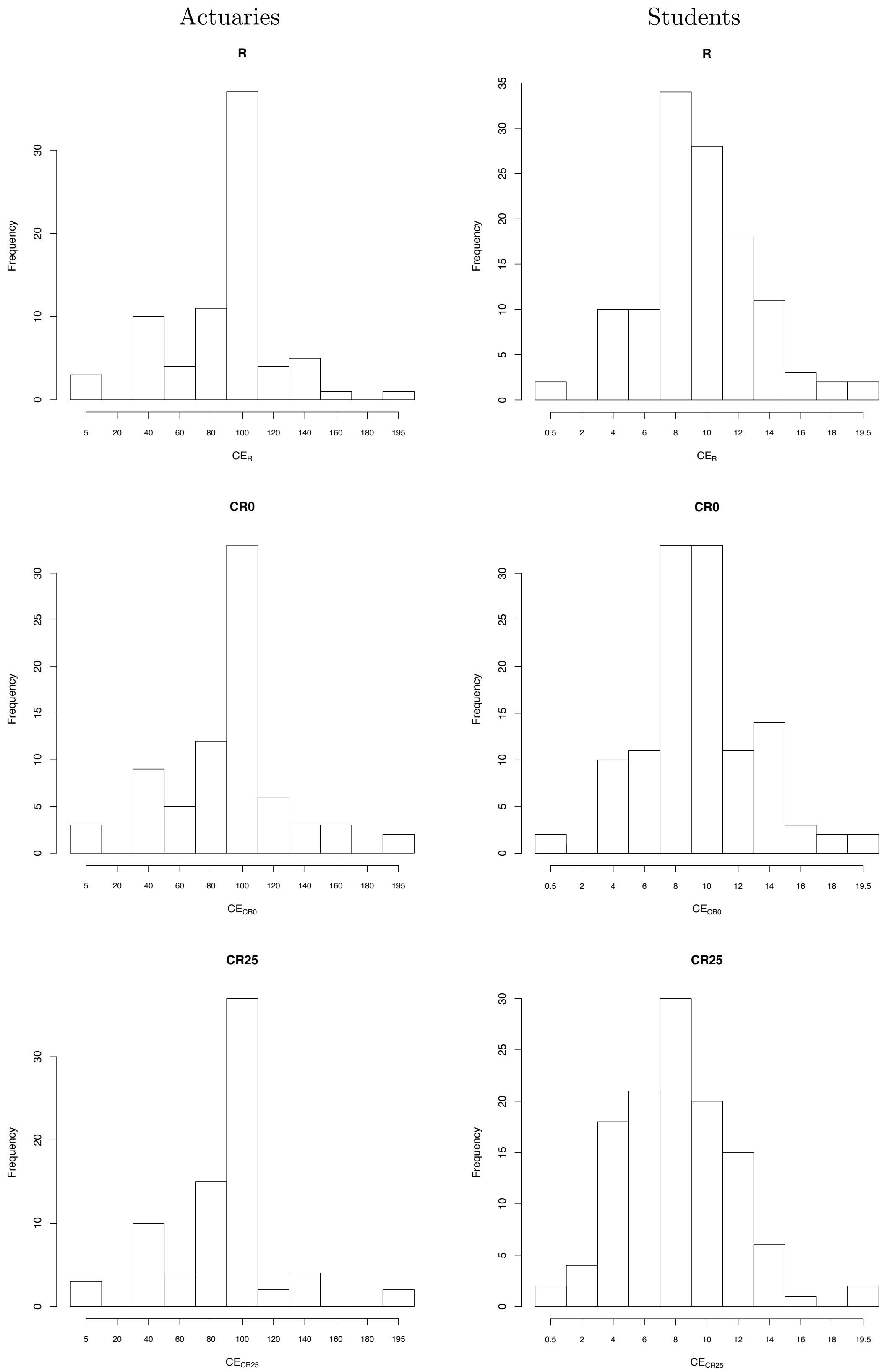

Figure B.1: Distributions of CEs for R, $CR0$, and $CR25$ for actuaries (on the left) and students (on the right)

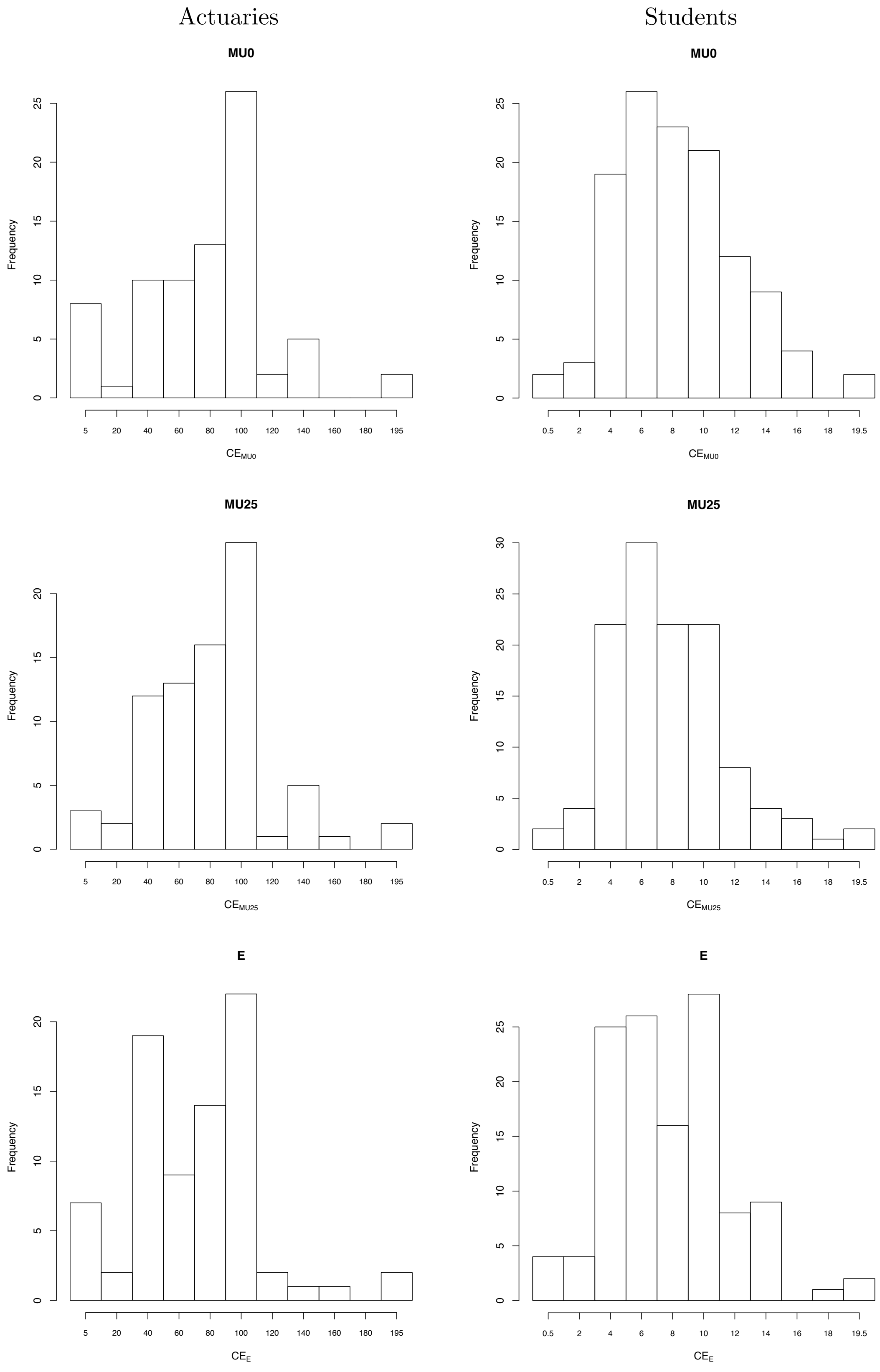

Figure B.2: Distributions of CEs for $MU0$, $MU25$, and E for actuaries (on the left) and students (on the right)

C Ambiguity and compound risk: comparison with the literature

Table C.1: Spearman correlations between premia: Comparison with the literature

	This study (students)				Chew et al. (2017)		
	Π_{MU0}	Π_{MU25}	Π_E		Π_{MU0}	Π_{MU25}	Π_E
Π_{CR0}	0.244**	0.291**	0.108	Π_{CR0}	0.467***	0.282***	0.222**
Π_{CR25}	0.400***	0.635***	0.446***	Π_{CR25}	0.302***	0.587***	0.411***
Π_E	0.617***	0.626***	1	Π_E	0.196**	0.504***	1

Notes: *** significant at 0.001, ** significant at 0.01, * significant at 0.05.

Table C.1 compares the correlations between the premia under compound risk and ambiguity obtained from the student subjects in Chew et al. (2017) and in the current study. The premia are defined as in Definition 1. Both studies find positive correlations of similar magnitudes between the cases of compound risk and ambiguity, where the correlation of $CR0$ with Ellsberg ambiguity is weaker than that of $CR25$. Compared to the current study, $MU0$ is found to be more similar to $CR0$ in Chew et al. (2017), and its correlation with Ellsberg ambiguity is weaker in that study.

References

Abdellaoui, M., A. Baillon, L. Placido, and P. P. Wakker (2011). The rich domain of uncertainty: Source functions and their experimental implementation. *The American Economic Review 101*(2), 695–723.

Abdellaoui, M., P. Klibanoff, and L. Placido (2015). Experiments on compound risk in relation to simple risk and to ambiguity. *Management Science 61*(6), 1306–1322.

Al-Najjar, N. and J. Weinstein (2009). The ambiguity aversion literature: A critical assessment. *Economics and Philosophy 25*(3), 249–284.

Armantier, O. and N. Treich (2016). The rich domain of risk. *Management Science 62*, 1954–1969.

Aydogan, I., L. Berger, V. Bosetti, and N. Liu (2018). Three layers of uncertainty and the role of model misspecification: An experiment. IGIER Working Paper 623, Bocconi University.

Beaud, M. and M. Willinger (2015). Are people risk vulnerable? *Management Science 61*(3), 624–636.

Berger, L. (2019). What is partial ambiguity? mimeo.

Berger, L., H. Bleichrodt, and L. Eeckhoudt (2013). Treatment decisions under ambiguity. *Journal of Health Economics 32*(3), 559–569.

Berger, L. and V. Bosetti (2019). Are policymakers ambiguity averse? *Economic Journal* (2).

Bernasconi, M. and G. Loomes (1992). Failures of the reduction principle in an Ellsberg-type problem. *Theory and Decision 32*(1), 77–100.

Cabantous, L. (2007). Ambiguity aversion in the field of insurance: Insurers' attitude to imprecise and conflicting probability estimates. *Theory and Decision 62*(3), 219–240.

Cabantous, L., D. Hilton, H. Kunreuther, and E. Michel-Kerjan (2011). Is imprecise knowledge better than conflicting expertise? Evidence from insurers' decisions in the United States. *Journal of Risk and Uncertainty 42*(3), 211–232.

Charness, G., U. Gneezy, and B. Halladay (2016). Experimental methods: Pay one or pay all. *Journal of Economic Behavior & Organization 131*, 141–150.

Chew, S. H., B. Miao, and S. Zhong (2017). Partial ambiguity. *Econometrica 85*(4), 1239–1260.

Chew, S. H., B. Miao, and S. Zhong (2018). Ellsberg meets Keynes at an urn. Technical report.

Chew, S. H., M. Ratchford, and J. S. Sagi (2018). You need to recognise ambiguity to avoid it. *The Economic Journal 128*(614), 2480–2506.

Dimmock, S. G., R. Kouwenberg, O. S. Mitchell, and K. Peijnenburg (2016). Ambiguity aversion and household portfolio choice puzzles: Empirical evidence. *Journal of Financial Economics 119*(3), 559–577.

Dimmock, S. G., R. Kouwenberg, and P. P. Wakker (2015). Ambiguity attitudes in a large representative sample. *Management Science 62*(5), 1363–1380.

Drouet, L., V. Bosetti, and M. Tavoni (2015). Selection of climate policies under the uncertainties in the fifth assessment report of the IPCC. *Nature Climate Change 5*, 937–940.

Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. *The Quarterly Journal of Economics 75*, 643–669.

Ergin, H. and F. Gul (2009). A theory of subjective compound lotteries. *Journal of*

Economic Theory 144(3), 899–929.

Fox, C. R. and A. Tversky (1995). Ambiguity aversion and comparative ignorance. *The Quarterly Journal of Economics 110*(3), 585–603.

Giannoni, M. P. (2002). Does model uncertainty justify caution? Robust optimal monetary policy in a forward-looking model. *Macroeconomic Dynamics 6*(1), 111–144.

Gilboa, I. and M. Marinacci (2013). Ambiguity and the bayesian paradigm. In *Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Econometric Society.* D. Acemoglu, M. Arellano, and E. Dekel (Eds.). New York: Cambridge University Press.

Gilboa, I., A. Postlewaite, and D. Schmeidler (2009). Is it always rational to satisfy Savage's axioms? *Economics and Philosophy 25*(3), 285–296.

Gilboa, I., A. Postlewaite, and D. Schmeidler (2012). Rationality of belief or: why Savage's axioms are neither necessary nor sufficient for rationality. *Synthese 187*(1), 11–31.

Gilboa, I., A. W. Postlewaite, and D. Schmeidler (2008). Probability and uncertainty in economic modeling. *The Journal of Economic Perspectives 22*(3), 173–188.

Halevy, Y. (2007). Ellsberg revisited: An experimental study. *Econometrica 75*(2), 503–536.

Hansen, L. and T. Sargent (2008). *Robustness.* Princeton University Press.

Hansen, L. P. (2014). Nobel lecture: Uncertainty outside and inside economic models. *Journal of Political Economy 122*(5), 945–987.

Hansen, L. P. and M. Marinacci (2016). Ambiguity aversion and model misspecification: An economic perspective. *Statistical Science 31*, 511–515.

Hansen, L. P. and T. J. Sargent (2001). Robust control and model uncertainty. *American Economic Review 91*(2), 60–66.

Hogarth, R. M. and H. Kunreuther (1989). Risk, ambiguity, and insurance. *Journal of Risk and Uncertainty 2*(1), 5–35.

Jewitt, I. and S. Mukerji (2017). Ordering ambiguous acts. *Journal of Economic Theory 171*, 213–267.

Johnson, C., A. Baillon, H. Bleichrodt, Z. Li, D. Van Dolder, and P. Wakker (2015). Prince: An improved method for measuring incentivized preferences.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005). A smooth model of decision making under ambiguity. *Econometrica 73*, 1849–1892.

Marinacci, M. (2015). Model uncertainty. *Journal of the European Economic Association 13*(6), 1022–1100.

Millner, A., S. Dietz, and G. Heal (2013). Scientific ambiguity and climate policy. *Environmental and Resource Economics 55*(1), 21–46.

Mukerji, S. (2009). Foundations of ambiguity and economic modelling. *Economics and Philosophy 25*(03), 297–302.

Nau, R. F. (2006). Uncertainty aversion with second-order utilities and probabilities. *Management Science 52*(1), 136–145.

Raiffa, H. (1961). Risk, ambiguity, and the Savage axioms: Comment. *The Quarterly Journal of Economics 75*(4), 690–694.

Savage, L. (1954). *The Foundations of Statistics.* New York: J. Wiley. Second revised edition, 1972.

Segal, U. (1987). The Ellsberg paradox and risk aversion: An anticipated utility approach. *International Economic Review*, 175–202.

Seo, K. (2009). Ambiguity and second-order belief. *Econometrica 77*(5), 1575–1605.

Sutter, M., M. G. Kocher, D. Glätzle-Rützler, and S. T. Trautmann (2013). Impatience and uncertainty: Experimental decisions predict adolescents' field behavior. *American Economic Review 103*(1), 510–31.

Sutter, M., M. G. Kocher, D. Glï¿œtzle-Rï¿œetzler, and S. T. Trautmann (2013, February). Impatience and uncertainty: Experimental decisions predict adolescents' field behavior. *American Economic Review 103*(1), 510–31.

Trautmann, S. T. and G. van de Kuilen (2015). *Ambiguity Attitudes*, Chapter 3, pp. 89–116. The Wiley Blackwell Handbook of Judgment and Decision Making, John Wiley & Sons, Ltd.

Viscusi, W. K. and H. Chesson (1999). Hopes and fears: The conflicting effects of risk ambiguity. *Theory and Decision 47*(2), 157–184.